The conformal fixed point of the SU(3) with 10 fundamental flavors

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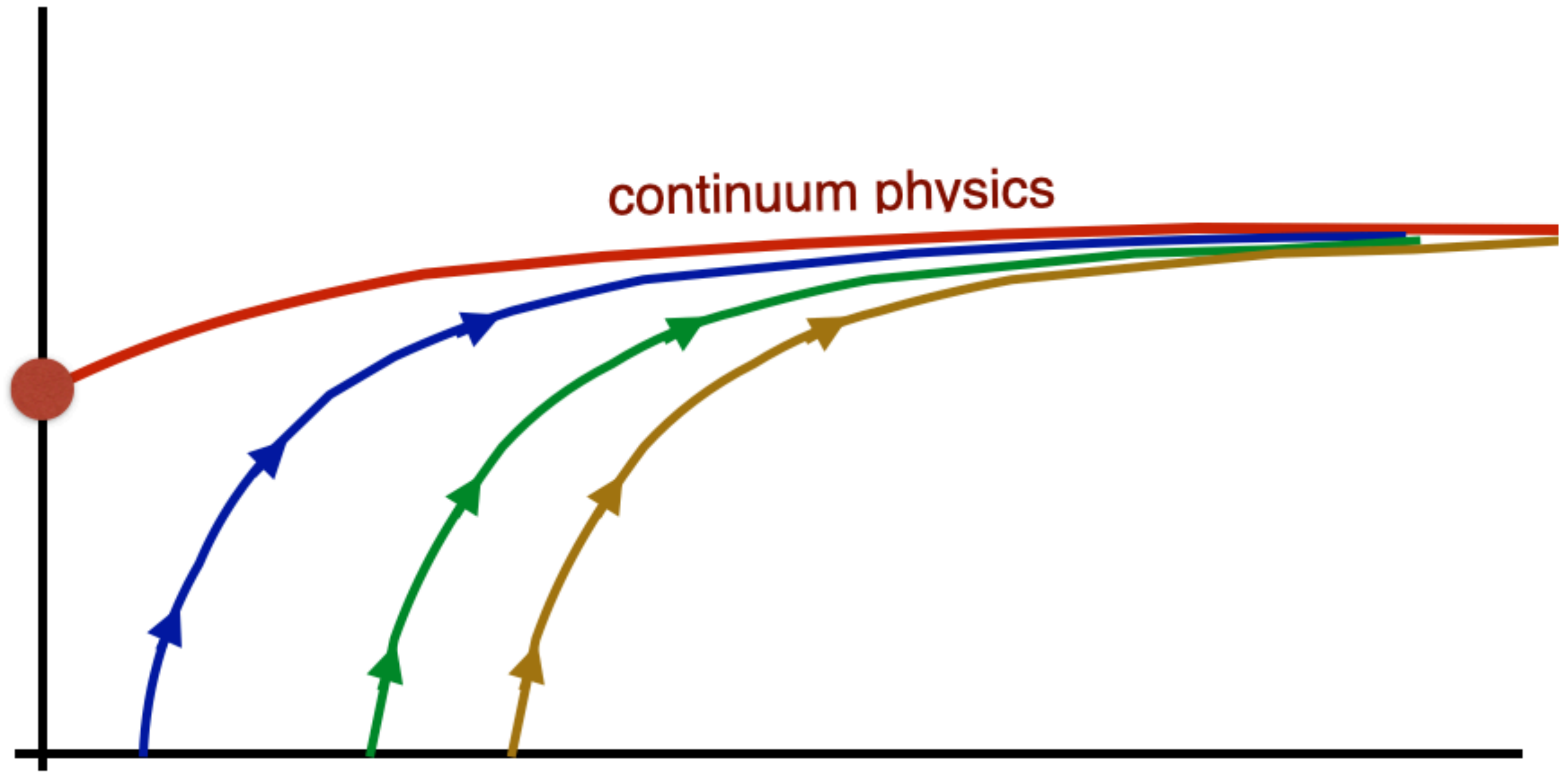


Based on recent work by A.H.,E.Neil, Y.Shamir, B.Svetitsky, O.Witzel, arXiv:2306.07236

Gradient flow vs continuous RG transformations

A. Carosso, AH, E. Neil,
PRL 121,201601 (2018)

GF can be *interpreted* as continuous real space RG with $\mu \propto 1/\sqrt{8t}$

- for *local* operators
- in the infinite volume limit

Ex-1: $g^2_{GF} = \mathcal{N} t^2 < E(t) > \quad \Longrightarrow \beta(g^2)$

Ex-2: $\mathcal{O} = \bar{\psi}(x)\Gamma\psi(x) \quad \Longrightarrow \gamma_{\mathcal{O}}(g^2)$

Continuous β function

$$g^2_{GF} = \mathcal{N} t^2 < E(t) > \quad \Longrightarrow \quad \beta_{GF}(a; g^2_{GF}) = -t \frac{dg^2_{GF}(a;t)}{dt}$$

- Fodor et al,EPJWeb Conf.175, 08027 (2018)
- AH, O.Witzel *Phys.Rev.D* 101 (2020) 3, 034514

- estimate flowed energy density $\langle E(t) \rangle$: W, C or S
- $\beta(g^2)$ defined in $am_f = 0$ chiral limit : simulations are at κ_{cr}

Analysis:

(1) infinite volume limit : extrapolate in $(a/L)^4 \to 0$ at fixed bare coupling and t/a^2
(2) interpolate $\beta(g^2; t)$ vs $g^2(t)$, all t/a^2
(3) continuum limit : $t/a^2 \to \infty$ while keeping g^2_{GF} fixed

Same approach as $N_f = 0$

- AH, C.T.Peterson, J.VanSickle,O.Witzel, *Phys.Rev.D* 108 (2023) 1, 014502

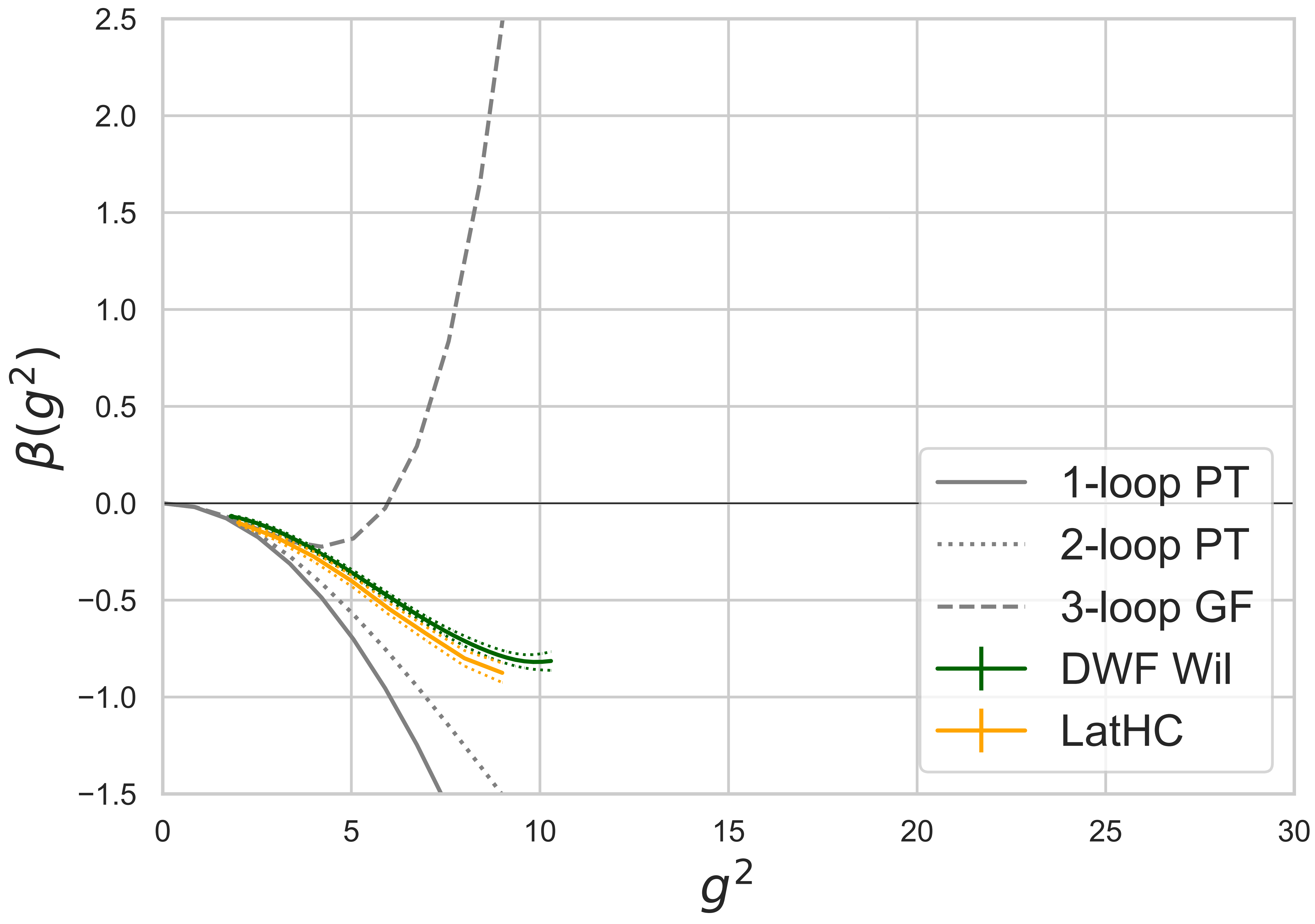
SU(3) gauge, $N_f = 10$ flavors

This system is close to the conformal sill
Prior simulations were limited by strong lattice artifacts

➡ restricted to $g^2 \lesssim 10$

➡ no prediction of IR dynamics

- *AH,C.Rebbi,O.Witzel, Phys.Rev.D* 101 (2020) 11, 114508
- O. Witzel, this conference
- J.Kuti,Z.Fodor,K.Hollad,C-H. Wong arXiv: 2203.15847



Continuous β function
- staggered fermions (LatHC)
- Mobius DW

give consistent results

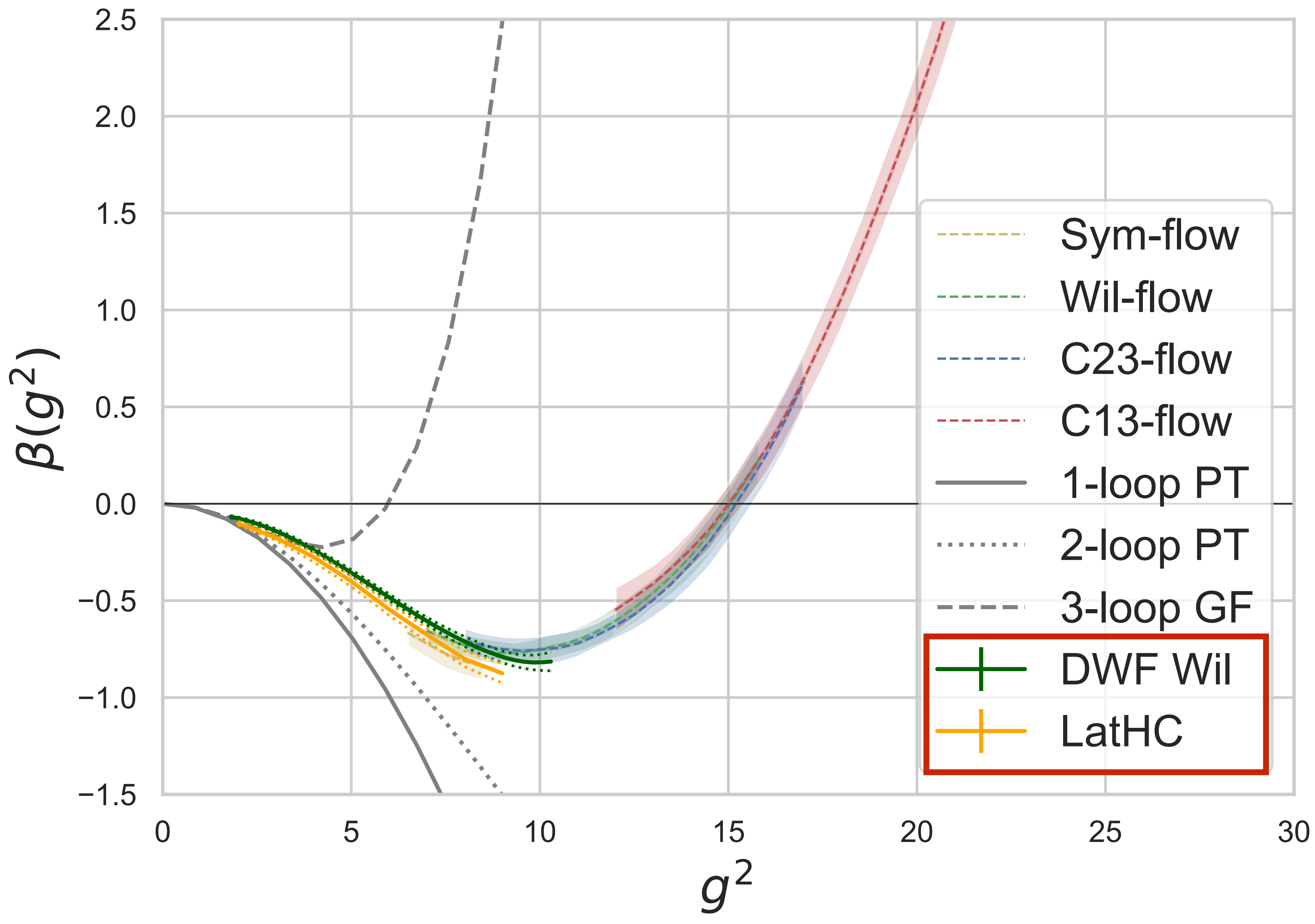
SU(3) gauge, $N_f = 10$ flavors

Our work with Wilson fermions reaches $g^2 \gtrsim 25$

➡ reveals IRFP at $g_*^2 = 15.0(5)$

➡ consistent with prior results

- A.H., E.Neil, Y.Shamir, B.Svetitsky, O.Witzel, arXiv:2306.07236



We utilize several improvements#

- PV improved action
- different flows
- bootstrap error analysis

#See applications in:
- A.H., E.Neil, Y.Shamir, B.Svetitsky, O.Witzel, *Phys.Rev.D* 107 (2023) 11, 114504
- Y. Shamir, this conference

Improved action: heavy PV bosons

AH, Y. Shamir, B. Svetitsky,
PRD104, 074509 (2021)

Many flavor systems suffer from *bulk phase transitions* in strong coupling

- Fermions generate a *positive* effective gauge action (hopping expansion)

$$S_{eff}^{(f)} = \frac{N_s}{(2am_f)^4}\sum_p ReTrV_\square + c\frac{N_s}{(2am_f)^6}\sum_{6link} ReTrV_{\text{6-link}}\cdots$$

Bare gauge coupling g_0^2 *increases* to compensate:

➡ rough gauge configurations, large cutoff effects

Improved action: heavy PV bosons

AH, Y. Shamir, B. Svetitsky,
PRD104, 074509 (2021)

Compensate with **heavy Pauli-Villars bosons**

- same interaction as fermions but with *bosonic statistics :* $S_{eff}^{(PV)} = -S_{eff}^{(f)}$
- g_0^2 decreases, cutoff effects decrease
- keep $am_{PV} \sim \mathcal{O}(1)$ fixed: in the IR $(a \to 0)$ the PV bosons decouple
  - e.g.: $N_f = 10,\ \beta = 6.3,\ \ \kappa_f = 0.12677\ :\ am_{AWI} \lesssim 10^{-4}\ ,\ am_{PS} \propto 1/L$

    $\kappa_{PV} = 0.1\ (\ am_{PV} = 1)\ :\ am_{AWI} = 0.73,\ am_{PS} = 1.80$

Improved action: heavy PV bosons

AH, Y. Shamir, B. Svetitsky,
PRD104, 074509 (2021)

Add many PV bosons to reduce the lattice fluctuations:

➡ extends accessible renormalized coupling range

➡ only a local effective gauge action - IR properties are unchanged

Different flows / RG transformation

Vary the action for gradient flow (improved RG):

$$S_{flow} = c_p S_{\rm p} + c_r S_{\rm rec} \;, \quad c_p + 8c_r = 1$$

S : $c_p = 5/3$

W: $c_p = 1$

C23: $c_p = 2/3$

C13: $c_p = 1/3$

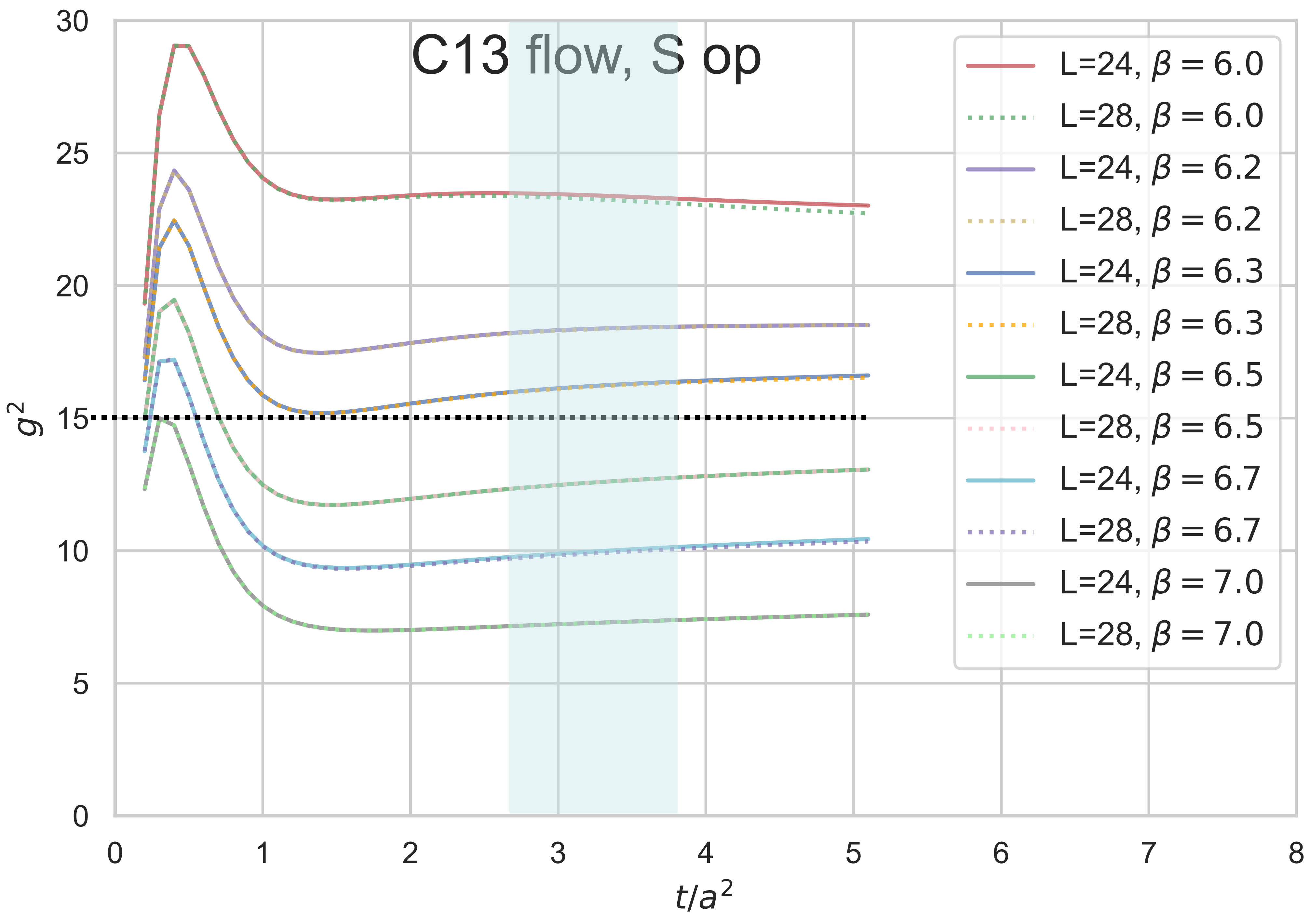
All flows should give the same continuum limit

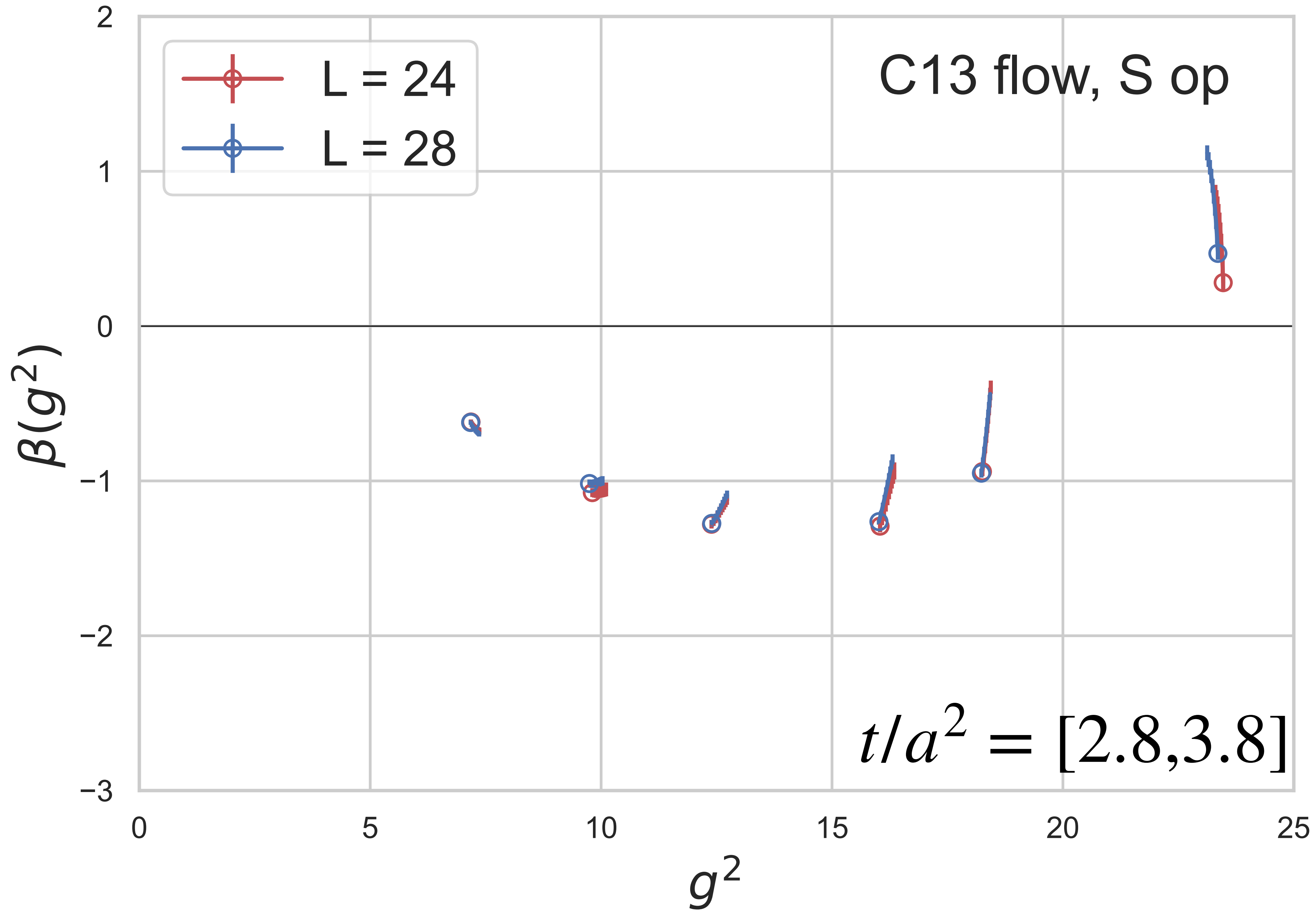
g^2 range is increasing as c_p decreases

(1) Infinite volume extrapolation

Volume dependence at leading order $\propto (t^2/L^4)$ (Wilsonian RG in finite volume)

- we use $t/a^2 = [2.8,3.8]$, $L = 24, 28$: $t^2/L^4 < 6 \times 10^{-5}$
- take $(a/L)^4 \to 0$ for g^2, $\beta(g^2)$ for *each* bare coupling and flow time

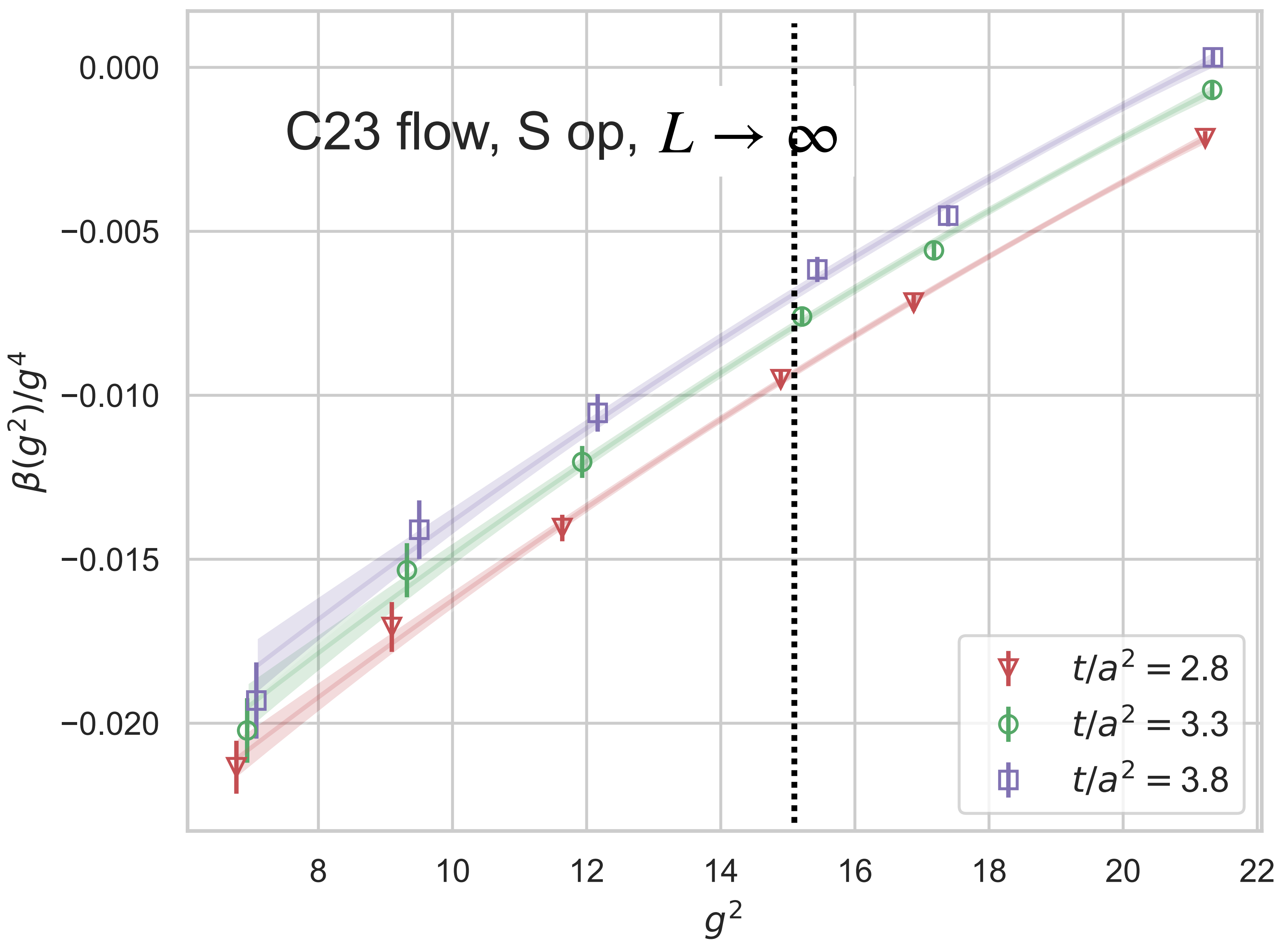




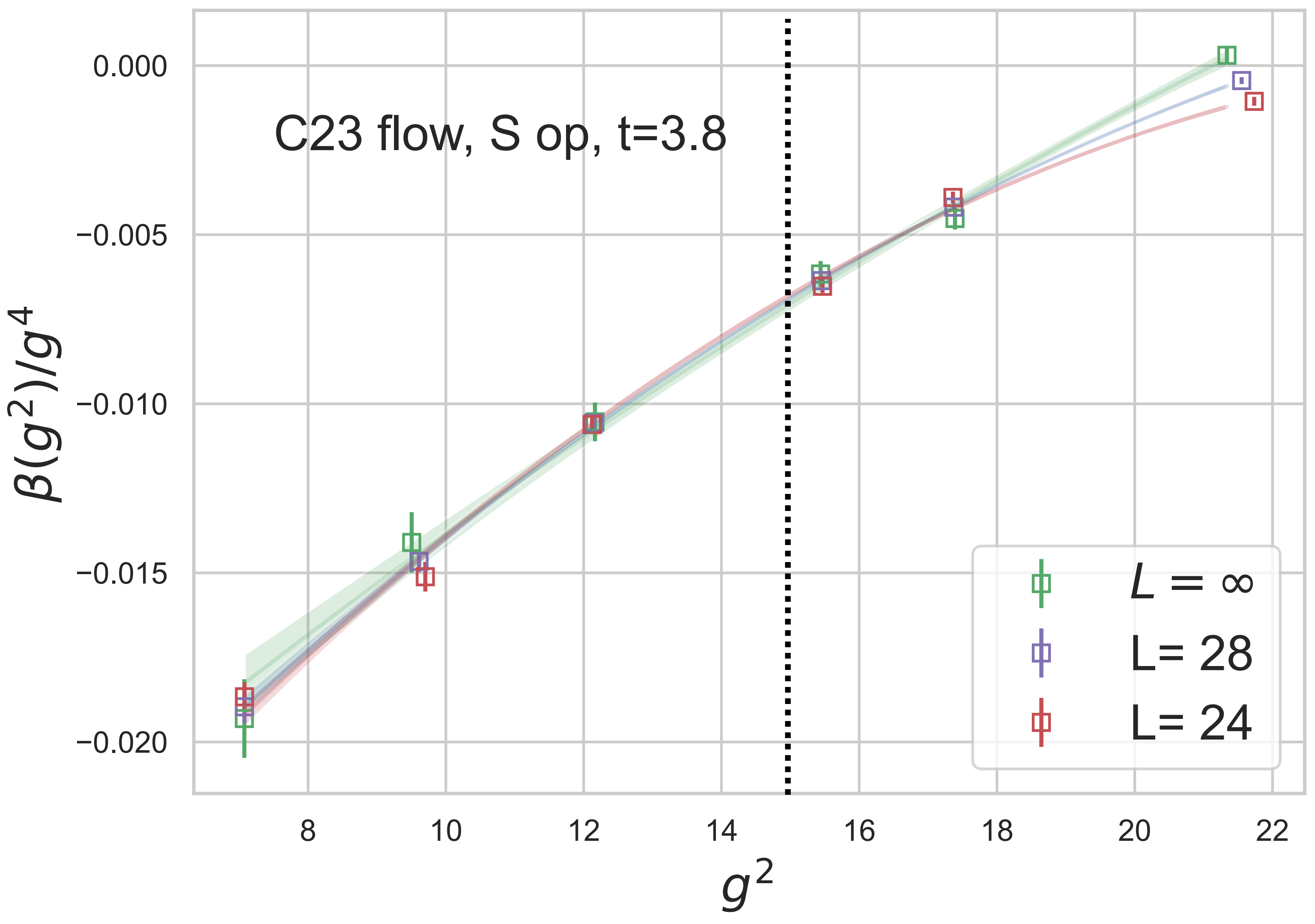
.... IRFP at $g^2 \approx 15$

(2) Interpolation

We need the value of $\beta(g^2)$ for every g^2 and t/a^2 : polynomial interpolation



Compare L=24, 28 and ∞

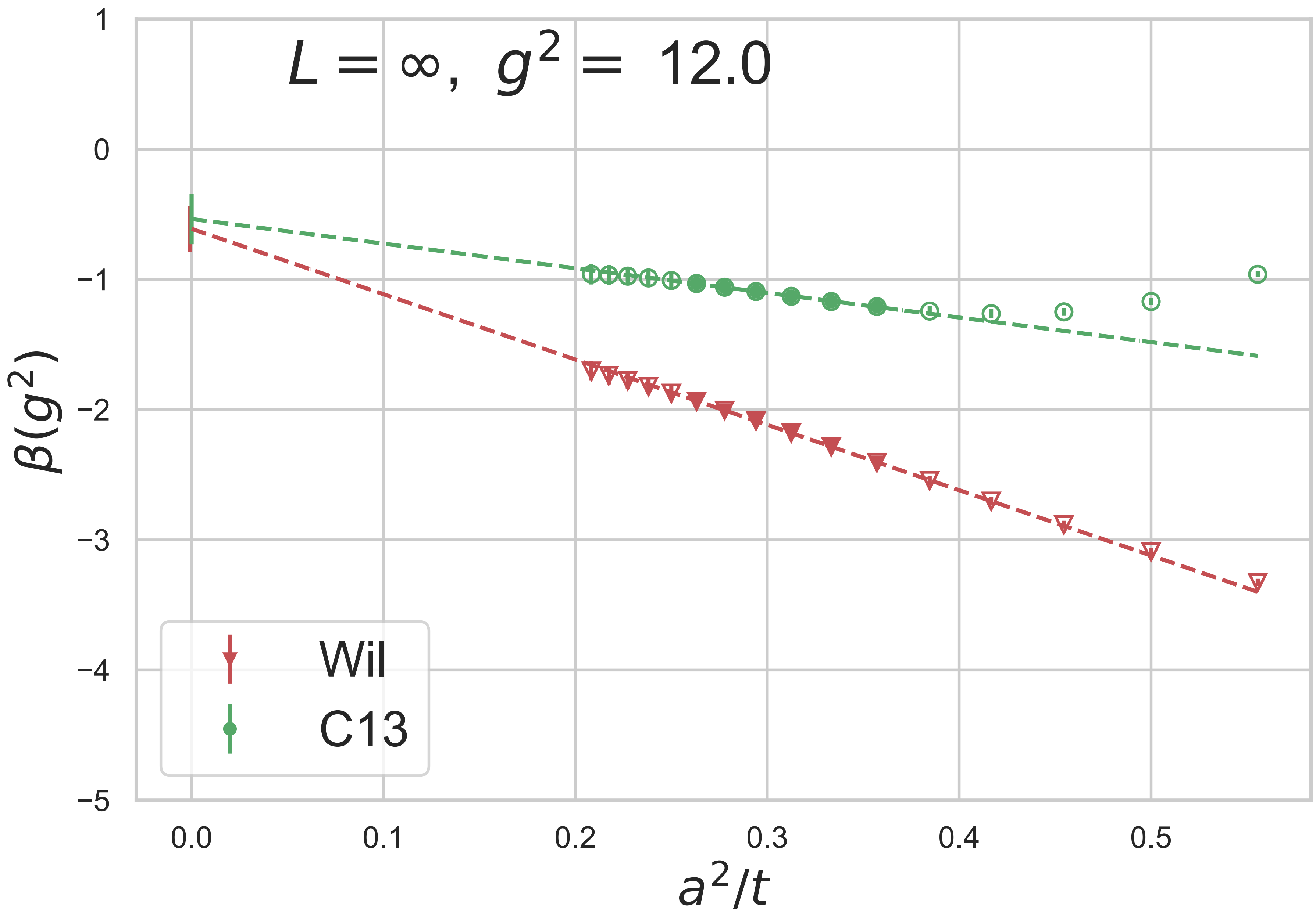


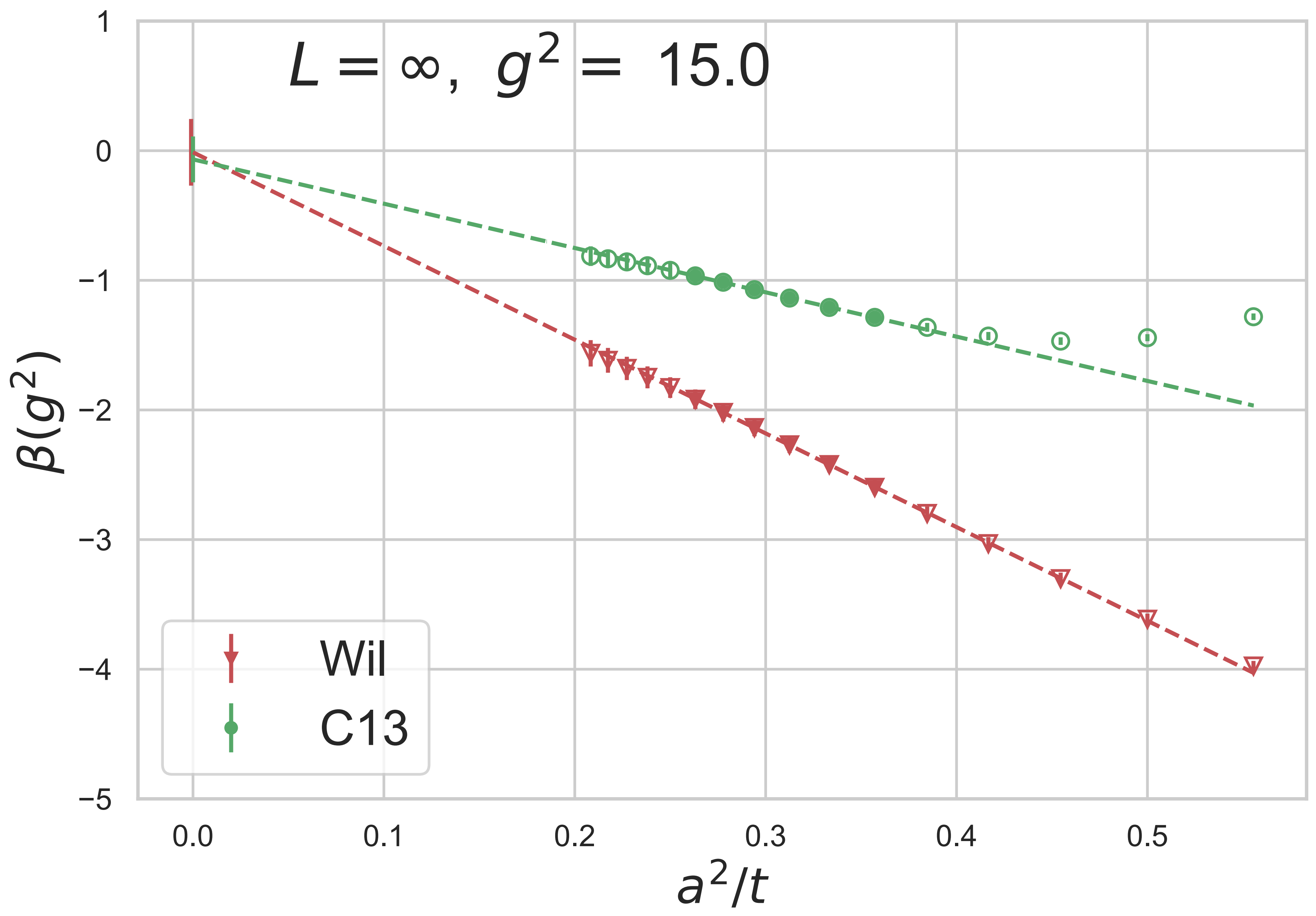
……… IRFP at $g^2 \approx 15$

(3) Continuum limit

Extrapolate linearly in $a^2/t \to 0$

$L = \infty$ continuum limit, S op



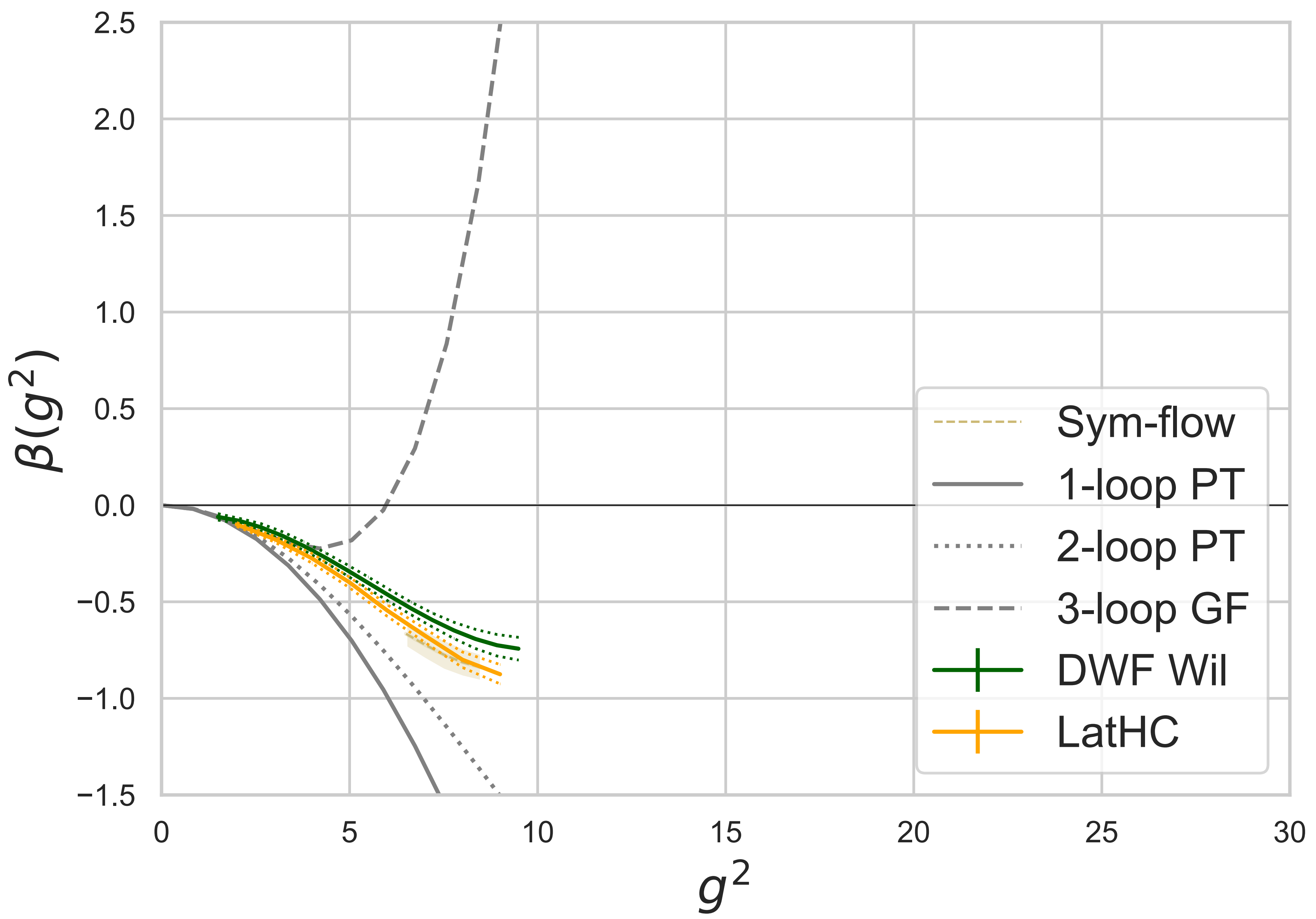


.... IRFP at $g^2 \approx 15$

Continuum limit

Require consistency between different operators :
- W agrees with S within 1 1σ
- C and W agree with S within 1σ

} Constrains g^2 range (details in extra slides)



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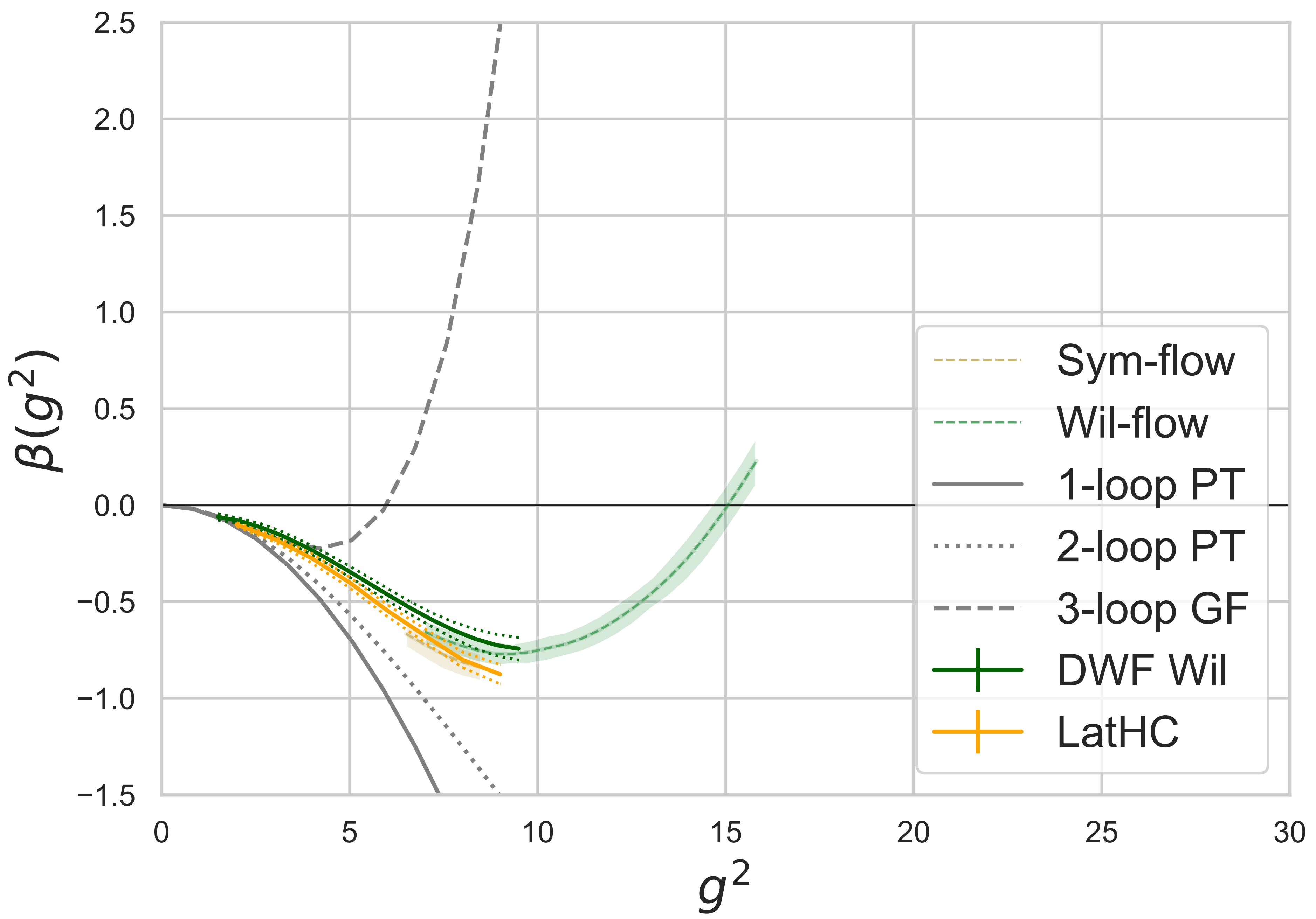

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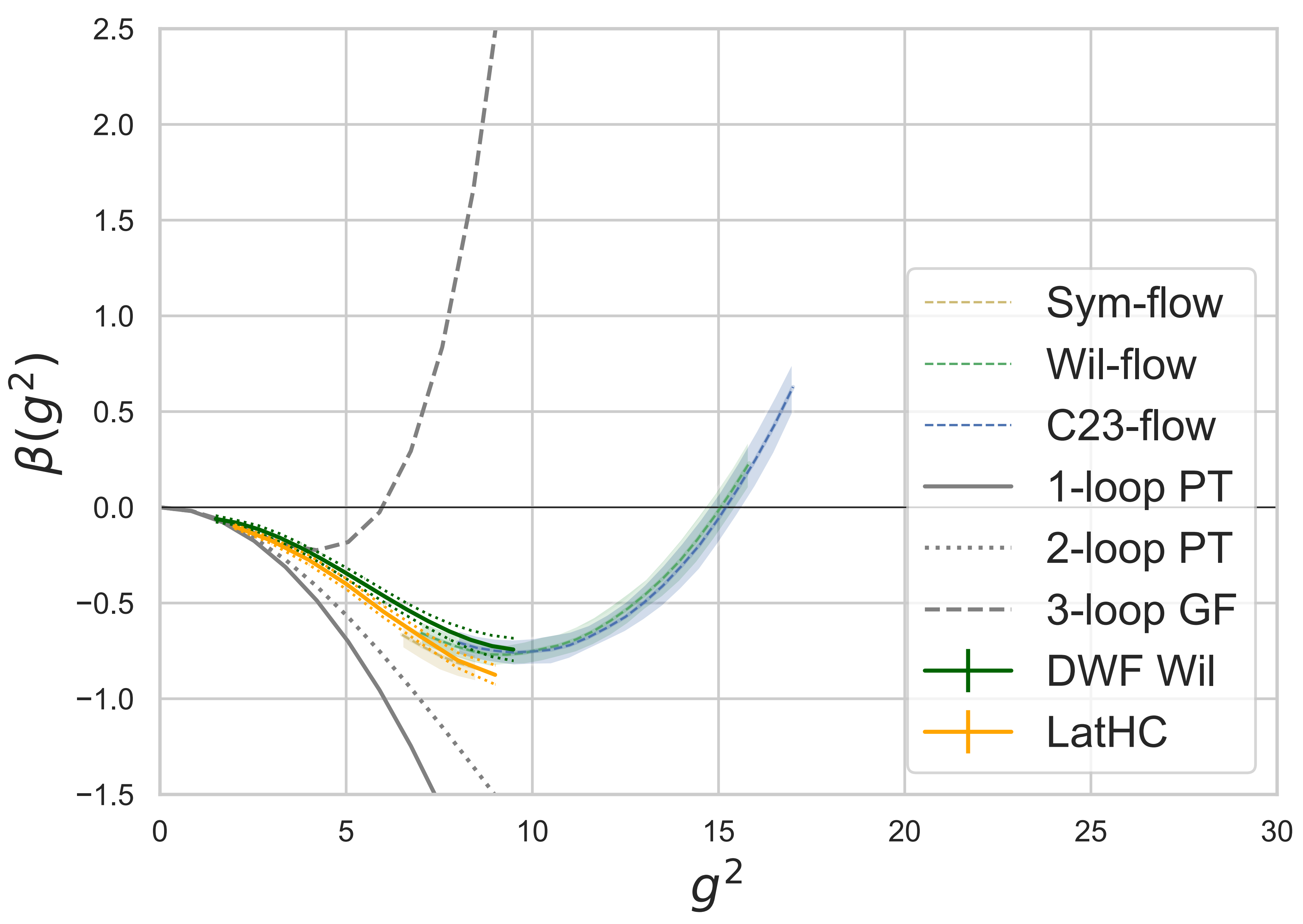
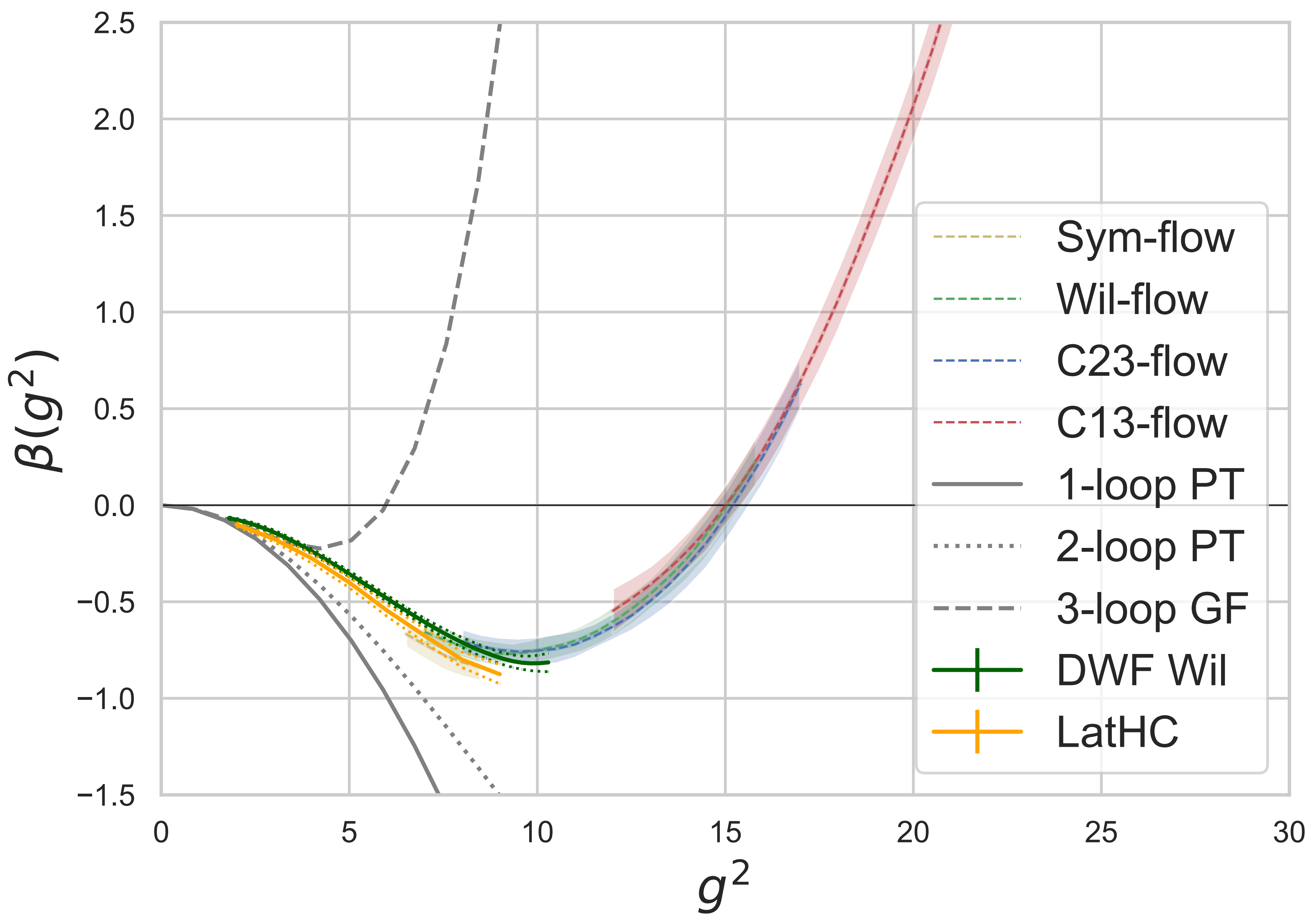


Continuum limit

Require consistency between different operators :
- W agrees with S within 1 1σ (bootstrap)
- Or: C and W agree with S within 1σ (bootstrap)

Constrains g^2 range



All flows are consistent

IRFP at $g^2 \simeq 15$

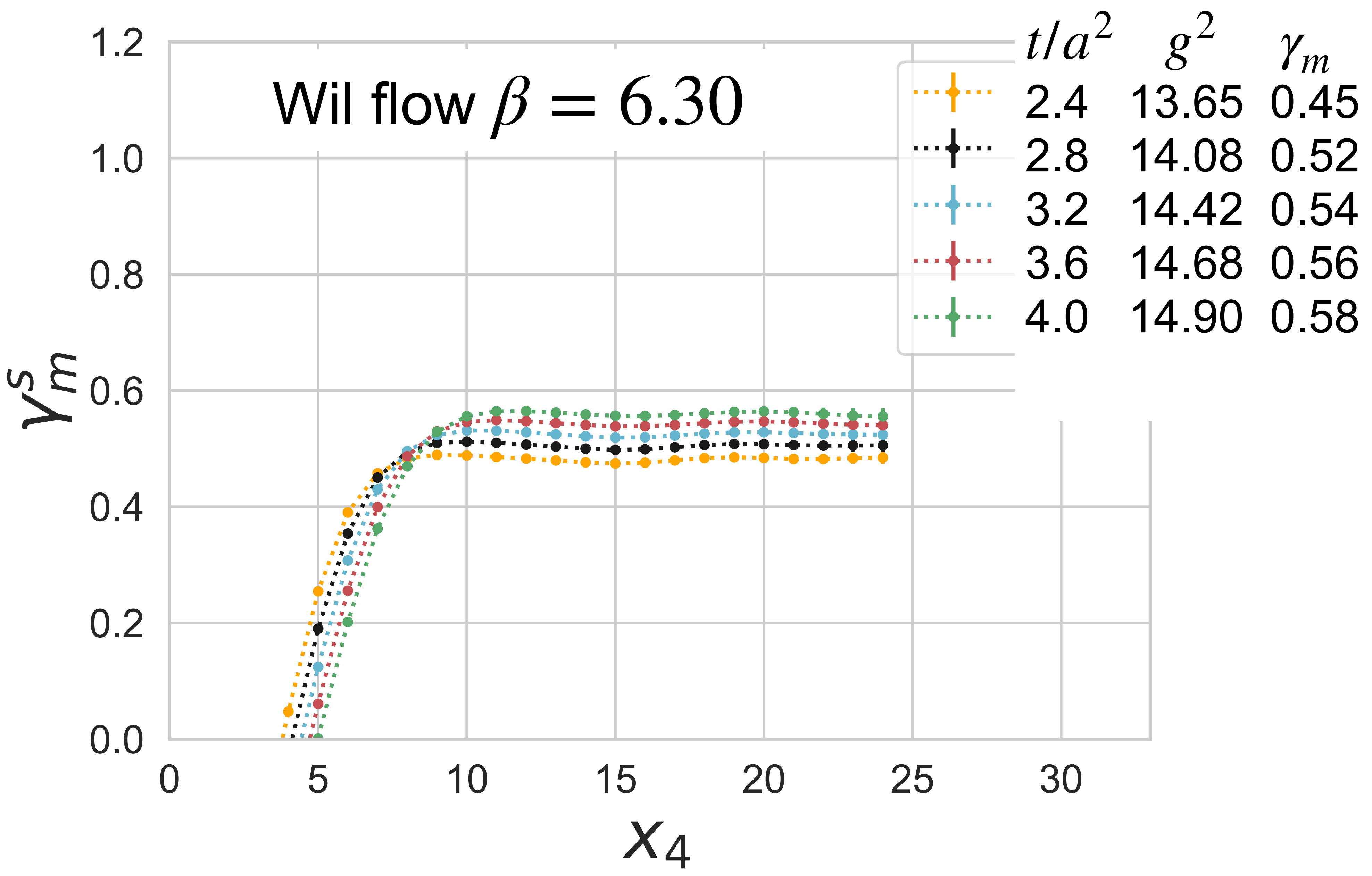
Anomalous dimension

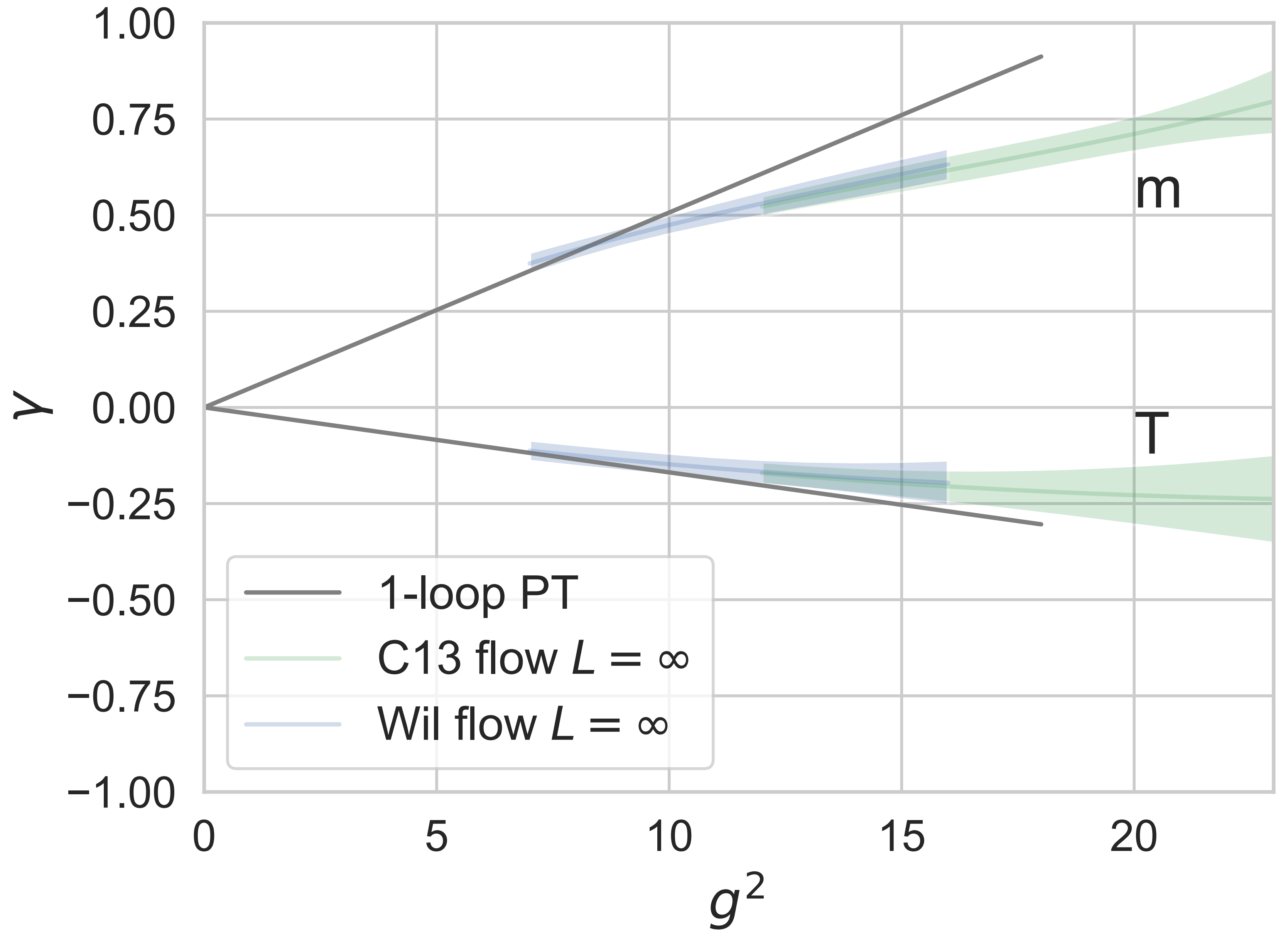
$\mathcal{O} = \bar{\psi}(x)\Gamma\psi(x)$ or $G_{\mathcal{O}}(x_4, t) = \langle\, \mathcal{O}(\bar{p} = 0, x_4; \mathbf{t})\; \mathcal{O}(\bar{p} = 0, 0; \mathbf{t} = \mathbf{0})\,\rangle$

- remove η_ψ (Z_ψ) by dividing with the vector operator :

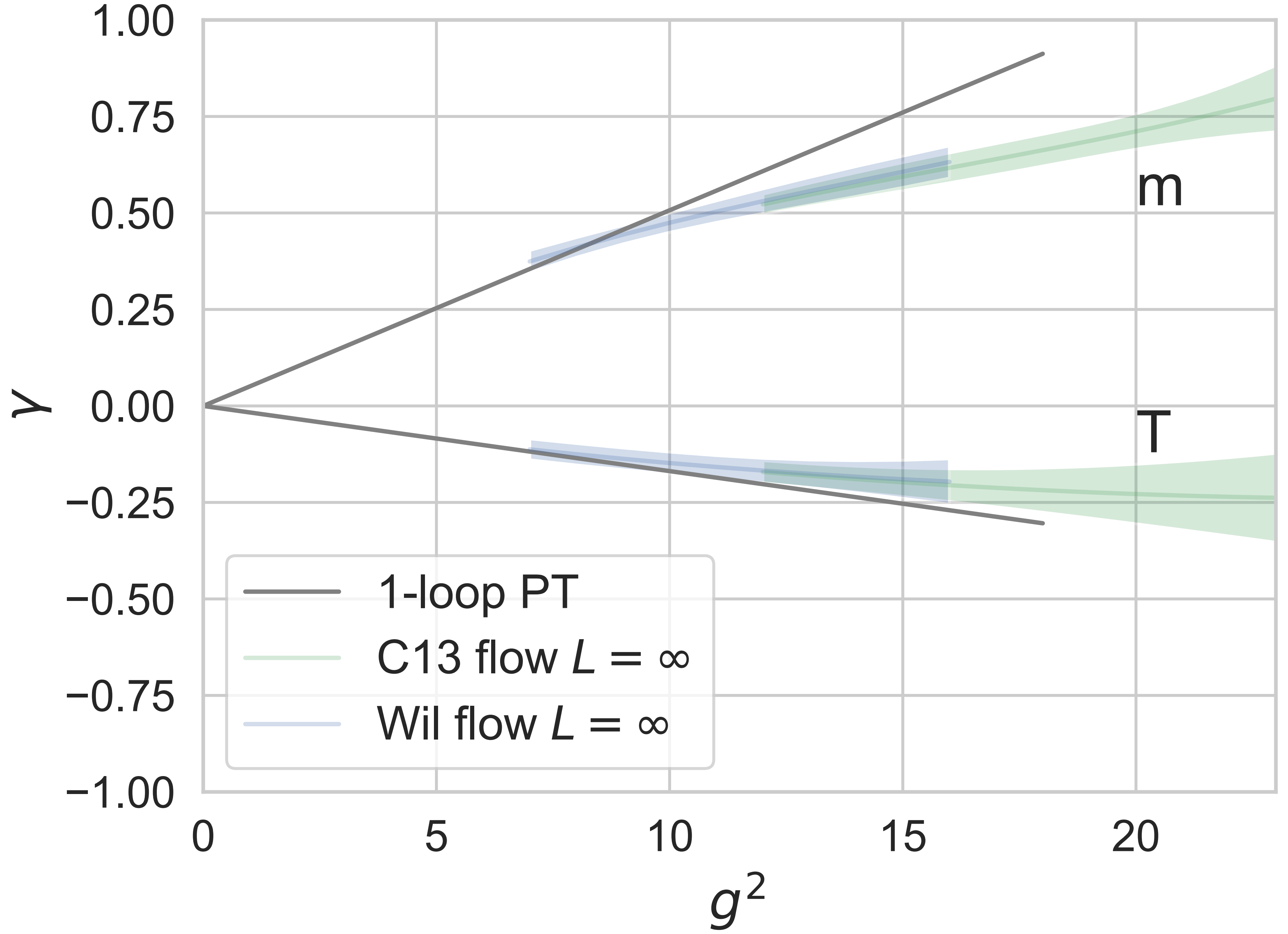
$$\mathcal{R}_{\mathcal{O}}(x_4, t) = \frac{G_{\mathcal{O}}(x_4, t)}{G_{\mathcal{V}}(x_4, t)} \implies \gamma_{\mathcal{O}}(a; t) = t\frac{d\log\mathcal{R}_{\mathcal{O}}(\mathrm{a}; \mathrm{t})}{dt}$$, no dependence on $x_4 \gg \sqrt{8t}$

- combine $\gamma_{\mathcal{O}}(t)$ with $g^2_{GF}(t)$ to obtain $\gamma_{\mathcal{O}}(g^2)$
- continuum limit : $a^2/t \to 0$ at fixed g^2





$\gamma(g^2)$



Anomalous dimension $\gamma_m^* \simeq 0.60$ (at $g^2_{IRFP} = 15.0(5)$)
(not even close to the conformal sill)

γ_m is consistent with prior hyperscaling determination
at $g^2 \simeq 10$, $\gamma_m = 0.46$

LSD coll., *Phys.Rev.D* 103 (2021) 1, 014504

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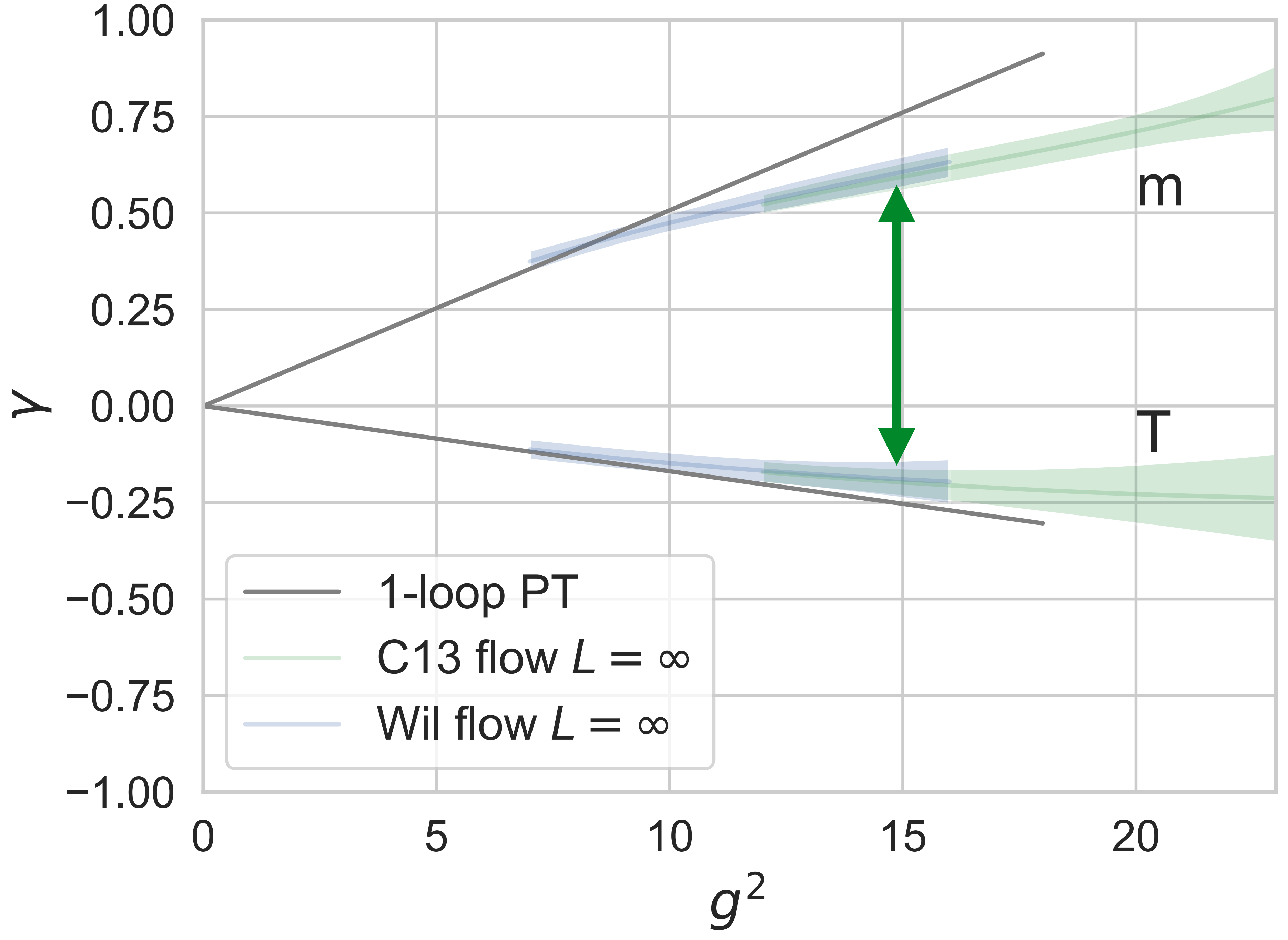

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Summary:

With new methods we cover strongly coupled regime that *was not accessible before*

- Heavy PV bosons remove cutoff effects → stronger renormalized gauge couplings
- Use of different flows further extends the reach in g^2
- Different flows at same g^2 rely on different bare couplings $\Longrightarrow$ consistency check!
- Bootstrap analysis give reliable errors and consistency checks

Our results show that SU(3) with 10 flavors is IR conformal

- $g_*^2 = 15.0(5)$
- $\gamma_m^* \simeq 0.6$

EXTRA SLIDES

Simulation details

Plaquette action and Wilson fermions with:

- Clover term
- nHYP smeared links
- nHYP dislocation suppressing gauge term
- PV bosons : 3 per fermions, $\kappa_{PV} = 0.1$ $(am_{PV} \gtrsim 1.0)$
- well behaved effective gauge action

Different flows / RG transformation

Vary the action for gradient flow (improved RG):

$$S_{flow} = c_p S_{\rm p} + c_r S_{\rm rec} \ , \quad c_p + 8c_r = 1$$

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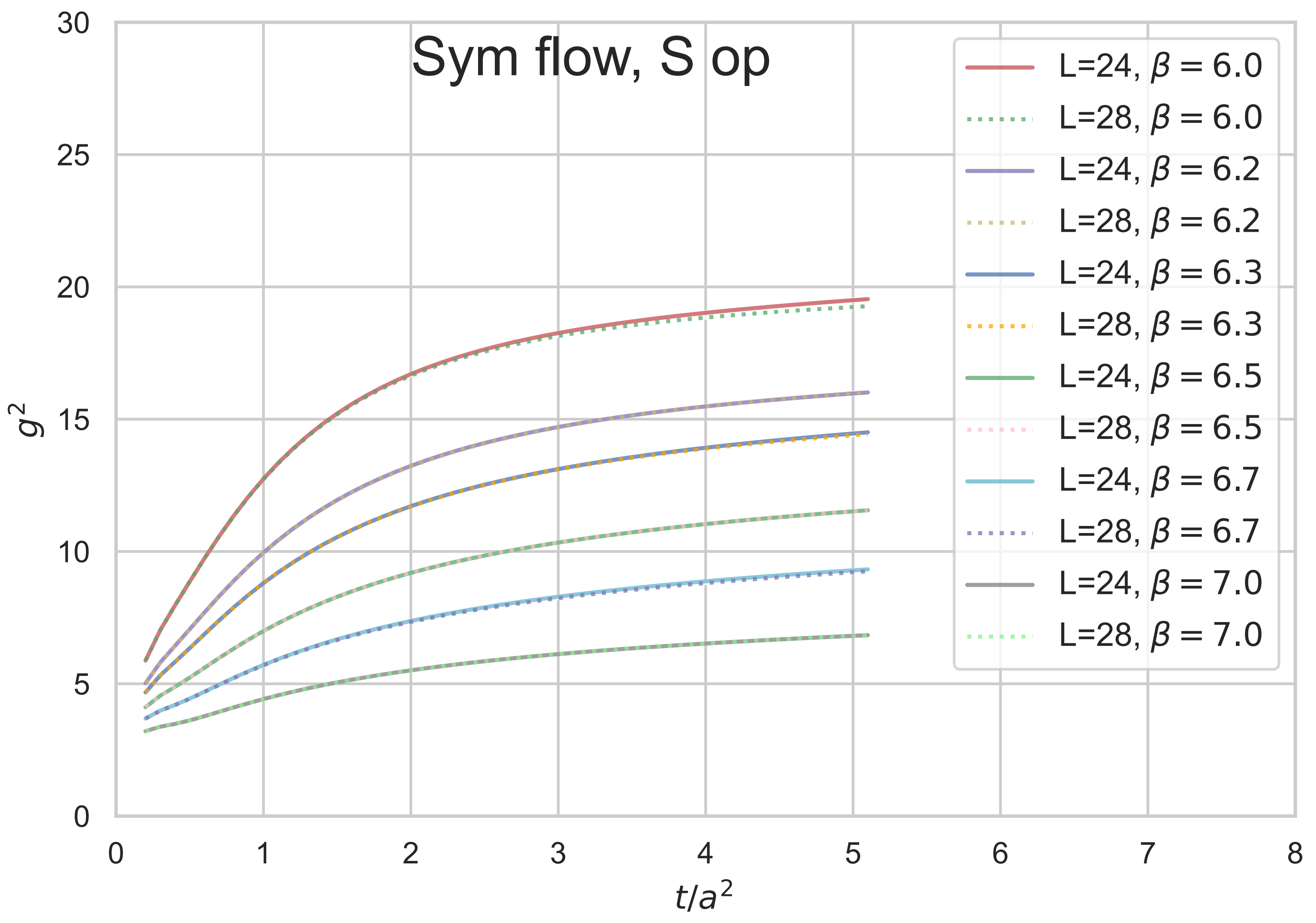
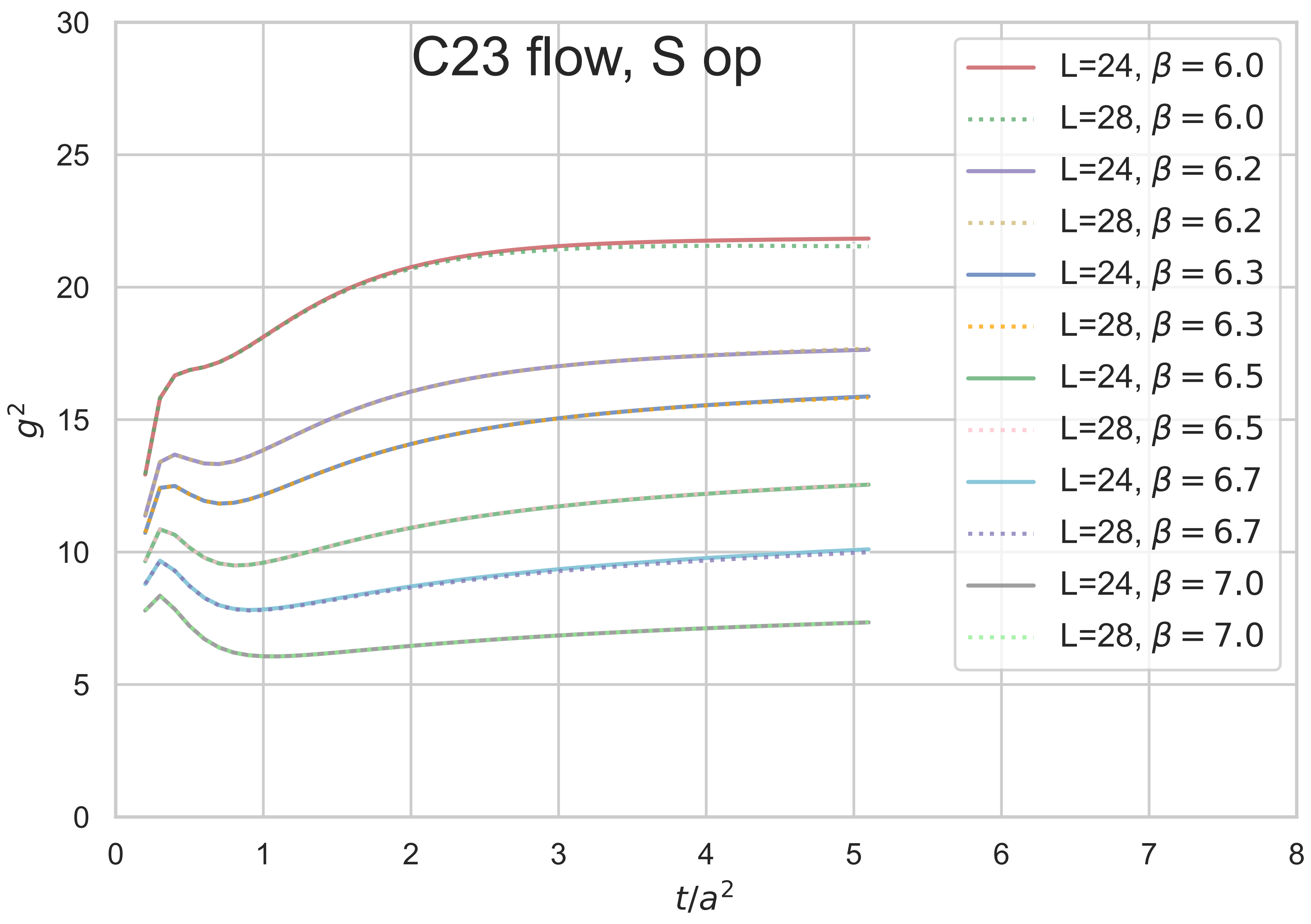
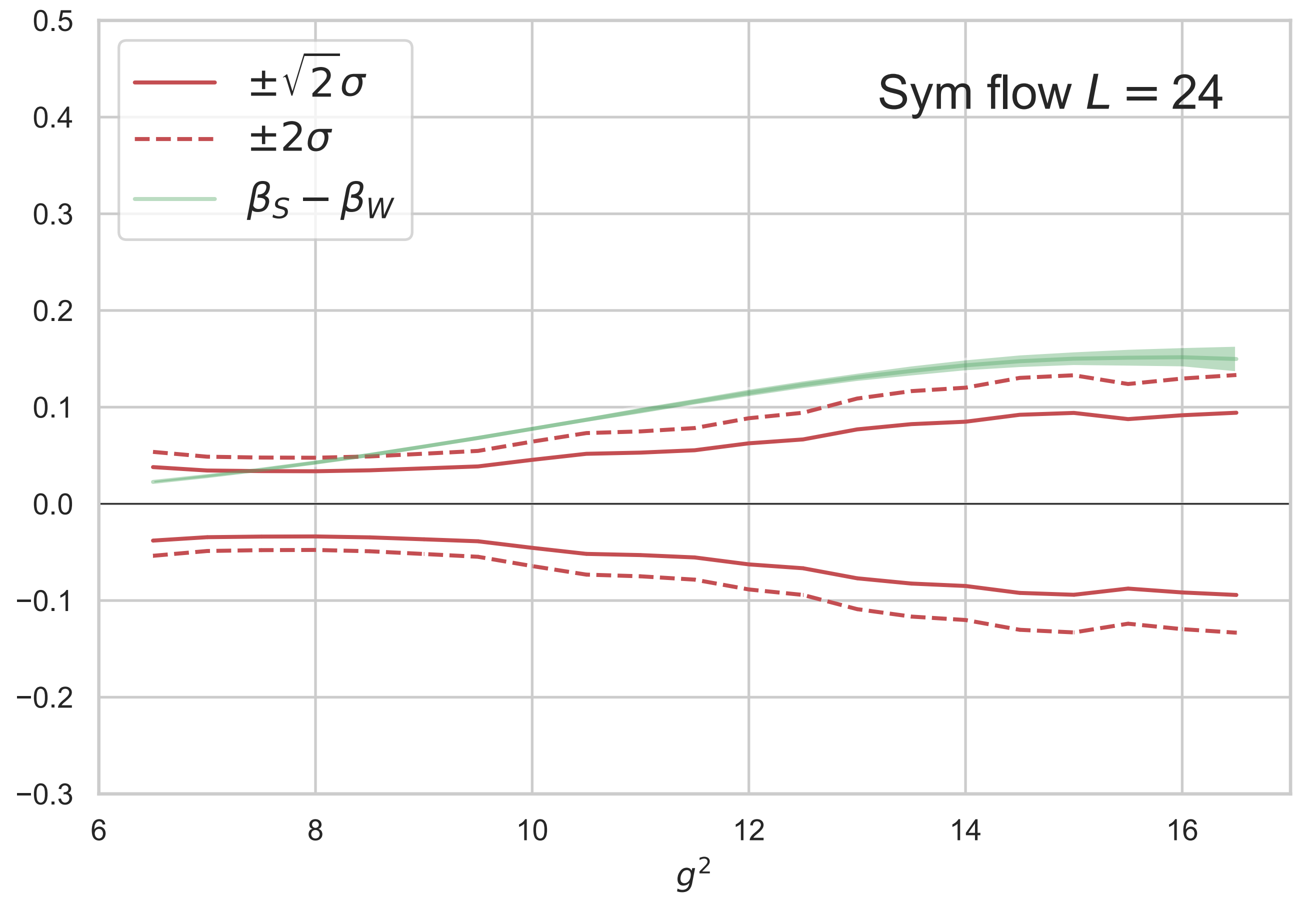
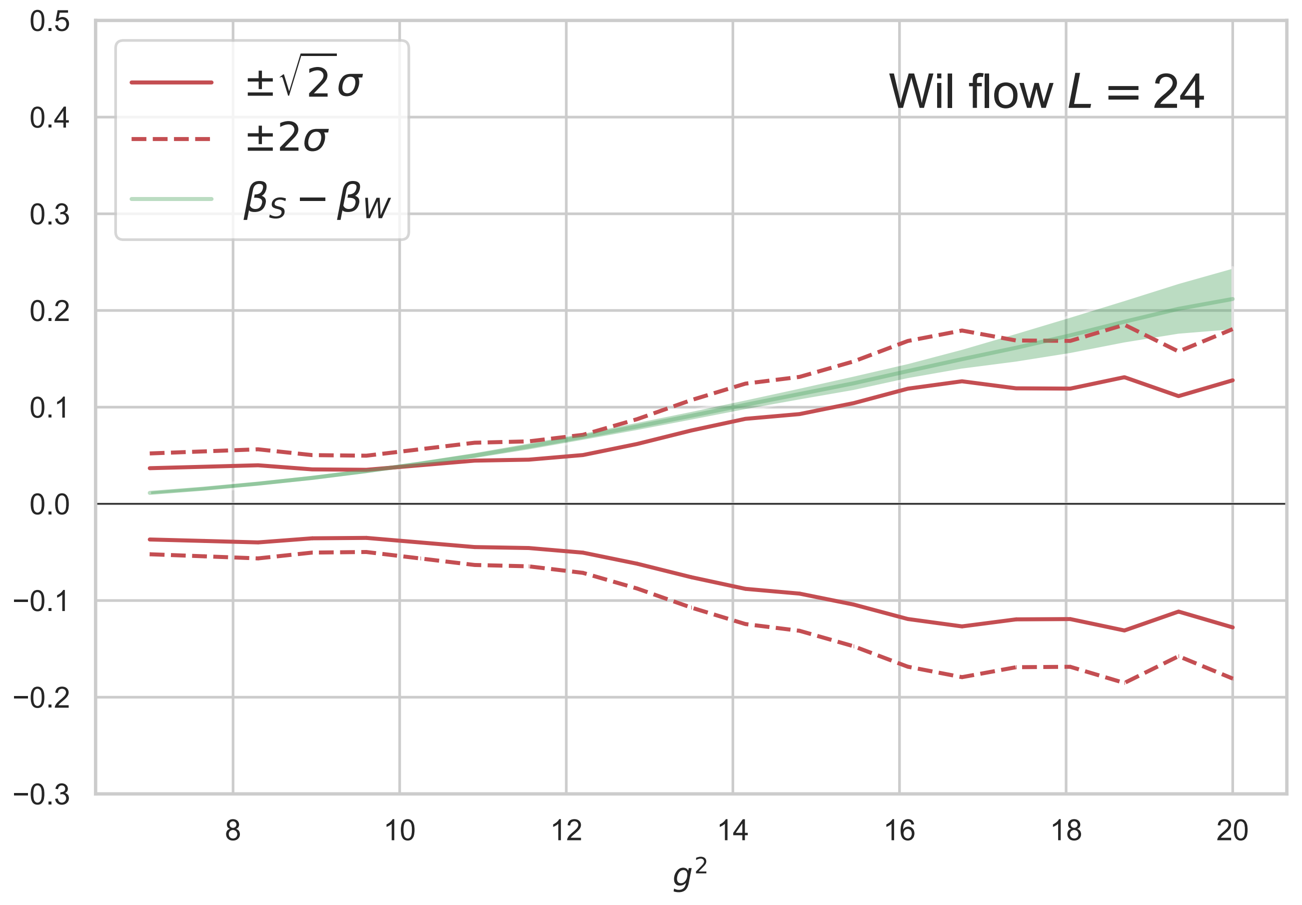
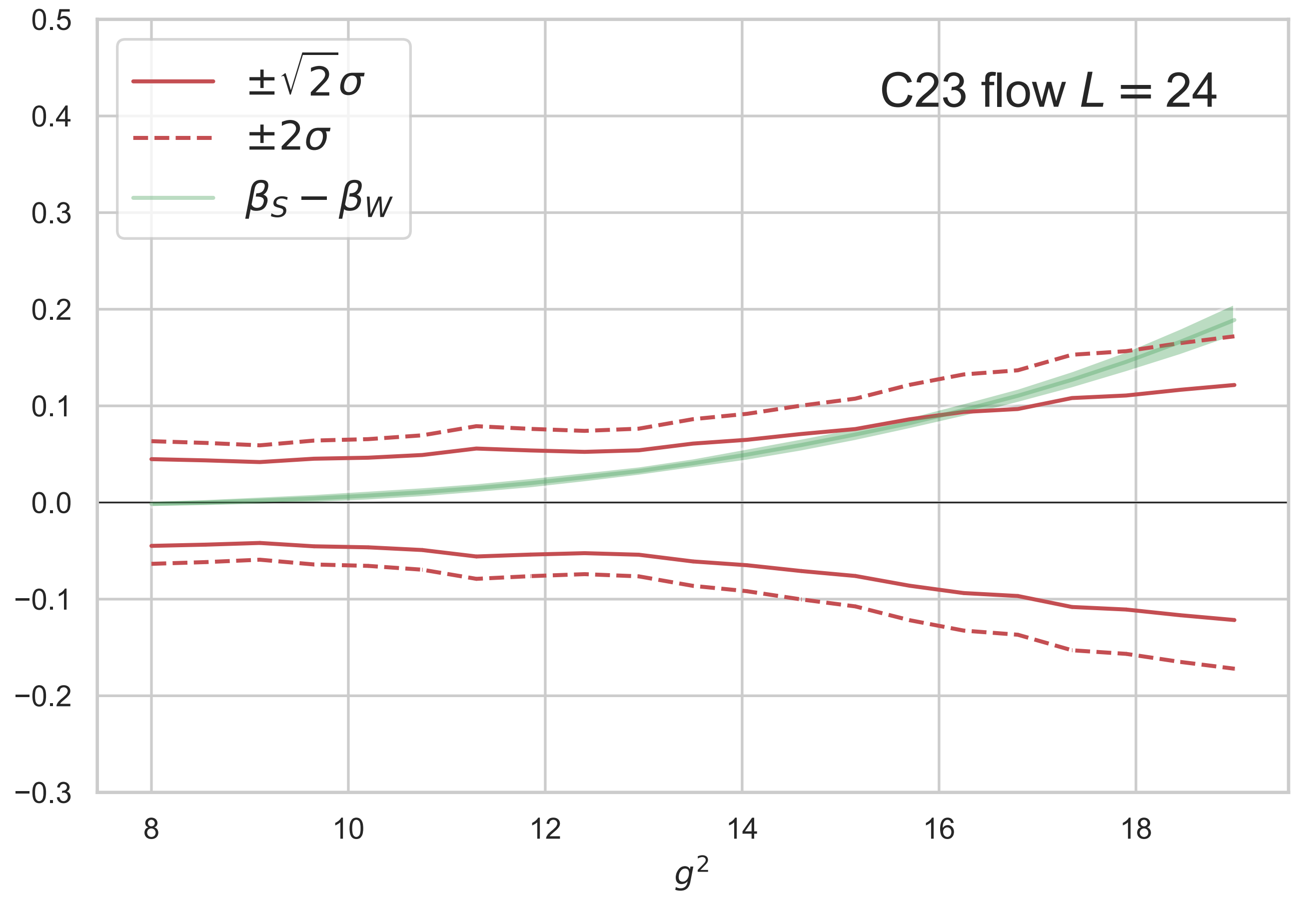


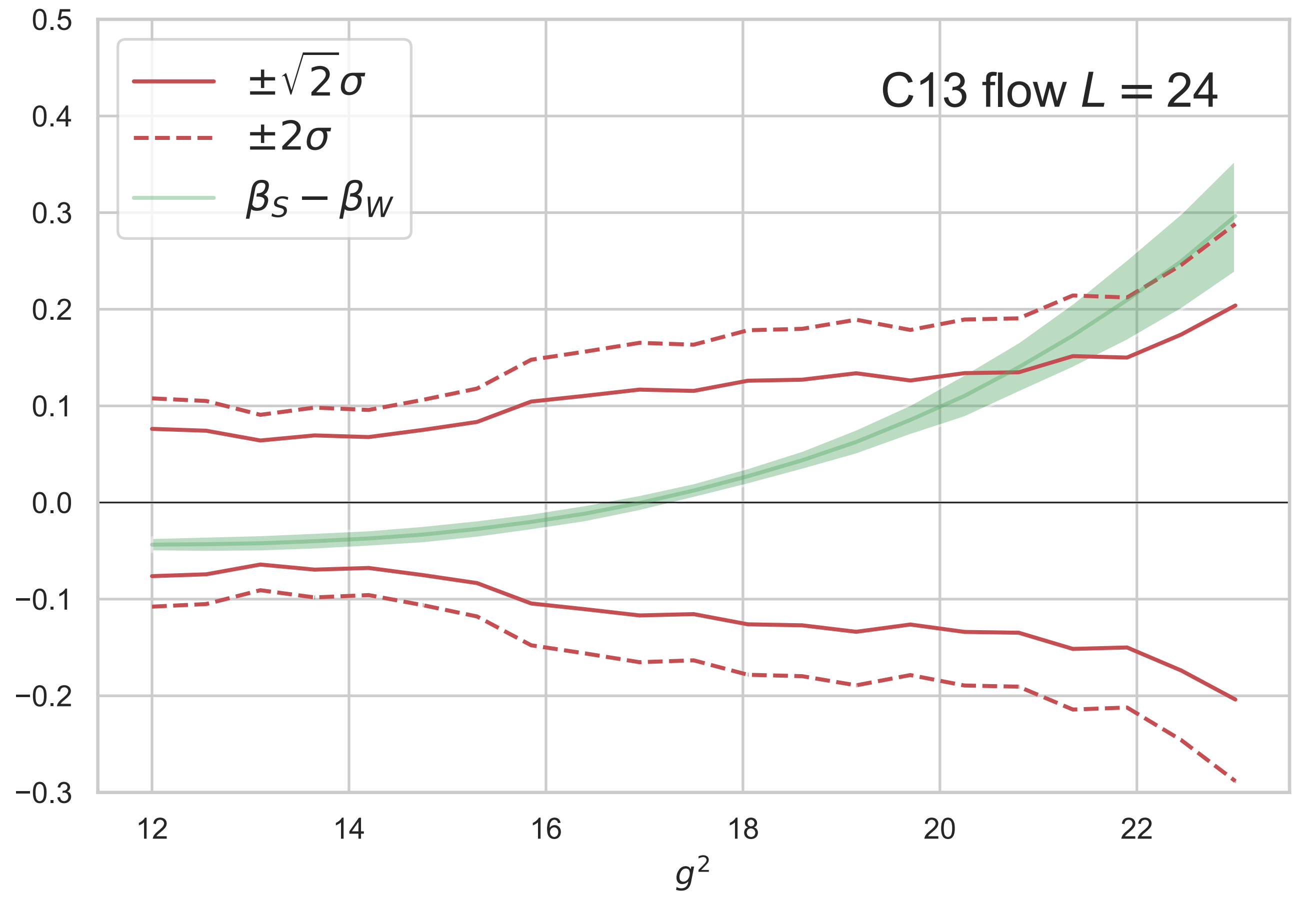


Consistency of flows









Continuum limit, $L \to \infty$