Renormalization of the Yukawa and Quartic Couplings in $\mathcal{N} = 1$ Supersymmetric QCD

M. Costa [1], H. Herodotou [1*], H. Panagopoulos [1]

[1]Department of Physics, University of Cyprus,
POB 20537, 1678, Nicosia, Cyprus

August 2023

University of Cyprus

2023 LATTICE

*Speaker

Contents

Supersymmetric QCD on the Lattice (1)

- To study the strong interactions between the particles and their superpartners → study the theory of SQCD → serves as a prototype for SUSY models which necessitate a non-perturbative study, and for which SUSY is necessarily broken by the regularization [1] [2]
- Extend Wilson's formulation of the QCD action → superpartner fields [3] [4]
- Standard discretization → quarks (ψ), squarks ($A_\pm$) and gluinos (λ) → on the lattice points whereas gluons (u_μ) → on the links between adjacent points:

$$U_\mu(x) = \exp[igaT^\alpha u_\mu^\alpha(x + a\hat{\mu}/2)] \qquad (1)$$

[1] J. Giedt, Int. J. Mod. Phys. A24 (2009) 4045-4095

[2] D. Schaich, PoS (LATTICE2018) 005

[3] D. Schaich, Eur. Phys. J. ST 232 (2023) no.3, 305-320

[4] G. Bergner and S. Catterall, Int. J. Mod. Phys. A 31 (2016) no.22, 1643005

Supersymmetric QCD on the Lattice (2)

- For Wilson-type quarks and gluinos, the Euclidean action $\mathcal{S}^L_{\rm SQCD}$ on the lattice becomes [5]:

$$\mathcal{S}^L_{\rm SQCD} = a^4 \sum_x \Big[\frac{N_c}{g^2} \sum_{\mu,\,\nu} \Big(1 - \frac{1}{N_c} \,{\rm Tr}\, U_{\mu\,\nu} \Big) + \sum_\mu {\rm Tr}\,\left(\bar\lambda \gamma_\mu \mathcal{D}_\mu \lambda\right) - a\frac{r}{2} {\rm Tr}\,\left(\bar\lambda \mathcal{D}^2 \lambda\right)$$
$$+ \sum_\mu \Big(\mathcal{D}_\mu A_+^\dagger \mathcal{D}_\mu A_+ + \mathcal{D}_\mu A_- \mathcal{D}_\mu A_-^\dagger + \bar\psi \gamma_\mu \mathcal{D}_\mu \psi \Big) - a\frac{r}{2} \bar\psi \mathcal{D}^2 \psi$$
$$+ i\sqrt{2} g \big(A_+^\dagger \bar\lambda^\alpha T^\alpha P_+ \psi - \bar\psi P_- \lambda^\alpha T^\alpha A_+ + A_- \bar\lambda^\alpha T^\alpha P_- \psi - \bar\psi P_+ \lambda^\alpha T^\alpha A_-^\dagger \big)$$
$$+ \frac{1}{2} g^2 (A_+^\dagger T^\alpha A_+ - A_- T^\alpha A_-^\dagger)^2 - m(\bar\psi\psi - m A_+^\dagger A_+ - m A_- A_-^\dagger) \Big], \tag{2}$$

- $P_\pm = (1 \pm \gamma_5)/2$ and $U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a\hat\mu) U_\mu^\dagger(x + a\hat\nu) U_\nu^\dagger(x)$
- m $\rightarrow$ the mass of the matter fields (which may be flavor-dependent)
- $\mathcal{D}$ $\rightarrow$ the standard covariant derivative in the fundamental/adjoint representation [5]
- a $\rightarrow$ lattice spacing, r $\rightarrow$ Wilson parameter, N_c $\rightarrow$ number of colors
- T^α $\rightarrow$ generators of SU(N_c), g $\rightarrow$ coupling constant

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Symmetries of the Supersymmetric QCD Action (1)

Parity ($\mathcal{P}$) is a symmetry of the continuum theory that is preserved exactly in the lattice formulation:

$$\mathcal{P}: \left\{ \begin{array}{lll} U_0(x) \to U_0(x_P)\,, & U_k(x) \to U_k^\dagger(x_P - a\hat{k})\,, & k = 1,2,3 \\ \psi_f(x) \to \gamma_0 \psi_f(x_P) & & \\ \bar{\psi}_f(x) \to \bar{\psi}_f(x_P)\gamma_0 & & \\ \lambda_f^\alpha(x) \to \gamma_0 \lambda_f^\alpha(x_\mathcal{P}) & & \\ \bar{\lambda}_f^\alpha(x) \to \bar{\lambda}_f^\alpha(x_\mathcal{P})\gamma_0 & & \\ A_\pm(x) \to A_\mp^\dagger(x_P) & & \\ A_\pm^\dagger(x) \to A_\mp(x_P) & & \end{array} \right\}$$

where $x_P = (-\mathrm{x}, x_0)$

Symmetries of the Supersymmetric QCD Action (2)

- Charge conjugation ($\mathcal{C}$) is also a symmetry of the continuum theory that is preserved exactly in the lattice formulation:

$$\mathcal{C}: \left\{ \begin{array}{l} U_\mu(x) \to U_\mu^\star(x)\,, \quad \mu = 0,1,2,3 \\ \psi(x) \to -C\bar{\psi}(x)^T \\ \bar{\psi}(x) \to \psi(x)^T C^\dagger \\ \lambda(x) \to C\bar{\lambda}(x)^T \\ \bar{\lambda}(x) \to -\lambda(x)^T C^\dagger \\ A_\pm(x) \to A_\mp(x) \\ A_\pm^\dagger(x) \to A_\mp^\dagger(x) \end{array} \right\}$$

- The matrix C satisfies: $(C\gamma_\mu)^T = C\gamma_\mu$, $C^T = -C$ and $C^\dagger C = 1$

Symmetries of the Supersymmetric QCD Action (3)

- $U(1)_R$ which rotates the quark and gluino fields in opposite direction:

$$\mathcal{R}: \left\{ \begin{array}{l} \psi_f(x) \rightarrow e^{i\theta\gamma_5}\psi_f(x) \\ \bar{\psi}_f(x) \rightarrow \bar{\psi}_f(x)e^{i\theta\gamma_5} \\ \lambda(x) \rightarrow e^{-i\theta\gamma_5}\lambda(x) \\ \bar{\lambda}(x) \rightarrow \bar{\lambda}(x)e^{-i\theta\gamma_5} \end{array} \right\}$$

- $U(1)_A$ which rotates the squark and the quark fields in the same direction as follows:

$$\chi: \left\{ \begin{array}{l} \psi_f(x) \rightarrow e^{i\theta\gamma_5}\psi_f(x) \\ \bar{\psi}_f(x) \rightarrow \bar{\psi}_f(x)e^{i\theta\gamma_5} \\ A_\pm(x) \rightarrow e^{i\theta}A_\pm(x) \\ A_\pm^\dagger(x) \rightarrow e^{-i\theta}A_\pm^\dagger(x) \end{array} \right\}$$

- Two terms with the Wilson parameter $\rightarrow$ break these symmetries $\rightarrow$ remedy the fermion doubling problem

What do we calculate and why? (1)

- We calculate the renormalization factors of the Yukawa and quartic couplings of the $\mathcal{N}=1$ Supersymmetric QCD, discretized on a Euclidean lattice
- Introduce the appropriate counterterms to the regularised Lagrangian so as to fine-tune the bare parameters [6]
- Calculate perturbatively the relevant three-point and four-point Green's functions using both dimensional and lattice regularizations
- Exploit some symmetries of the action → reduce the number of counterterms significantly → have a lattice discretization which preserves as many as possible of the continuum symmetries

[6] P. Athron and D. J. Miller, Phys. Rev. D 76 (2007), 075010

What do we calculate and why? (2)

- Lagrangian parameters of the classical action do not include quantum fluctuations $\rightarrow$ not the physically measured parameters $\rightarrow$ bare parameters
- Restore Supersymmetry in the continuum limit [7] [8] [9]
- Important ingredients in extracting nonperturbative information for supersymmetric theories through lattice simulations
- This work is a sequel to earlier investigations on SCQD and completes the one-loop fine-tuning of the SQCD action on the lattice $\rightarrow$ paving the way for numerical simulations of SQCD [5] [10]

---

[7] G. Curci and G. Veneziano, Nucl. Phys. B292 (1987) 555

[8] F. Farchioni et al., Eur. Phys. J. D 76 (2002), 719

[9] S. Ali et al., Eur. Phys. J. C 78 (2018) 404

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

[10] M. Costa and H. Panagopoulos, Phys. Rev. D 99 (2019) no.7, 074512

Coupling Constants of the Action of SQCD

- Bare coupling constants appearing in the lattice action are not typically all identical
- Gauge coupling g $\rightarrow$ gluons couple with quarks, squarks, gluinos and other gluons with the same gauge coupling constant
- Yukawa interactions (between quarks, squarks and gluinos) and four-squark interactions contain a potentially different coupling constant $\rightarrow$ must be fine-tuned on the lattice
- A similar situation holds for quark and squark masses [5]

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Computational Setup (1)

- For the purpose of studying Yukawa couplings, we examine the behavior under $\mathcal{P}$ and $\mathcal{C}$ of all gauge invariant and flavor singlet dimension-4 operators having one gluino, one quark and one squark field

Operators	$\mathcal{C}$	$\mathcal{P}$
$A_+^\dagger \bar\lambda P_+ \psi$	$-\bar\psi P_+ \lambda A_-^\dagger$	$A_- \bar\lambda P_- \psi$
$\bar\psi P_- \lambda A_+$	$-A_- \bar\lambda P_- \psi$	$\bar\psi P_+ \lambda A_-^\dagger$
$A_- \bar\lambda P_- \psi$	$-\bar\psi P_- \lambda A_+$	$A_+^\dagger \bar\lambda P_+ \psi$
$\bar\psi P_+ \lambda A_-^\dagger$	$-A_+^\dagger \bar\lambda P_+ \psi$	$\bar\psi P_- \lambda A_+$
$A_+^\dagger \bar\lambda P_- \psi$	$-\bar\psi P_- \lambda A_-^\dagger$	$A_- \bar\lambda P_+ \psi$
$\bar\psi P_+ \lambda A_+$	$-A_- \bar\lambda P_+ \psi$	$\bar\psi P_- \lambda A_-^\dagger$
$A_- \bar\lambda P_+ \psi$	$-\bar\psi P_+ \lambda A_+$	$A_+^\dagger \bar\lambda P_- \psi$
$\bar\psi P_- \lambda A_-^\dagger$	$-A_+^\dagger \bar\lambda P_- \psi$	$\bar\psi P_+ \lambda A_+$

Computational Setup (2)

- Two linear combinations of Yukawa-type operators which are invariant under $\mathcal{P}$ and $\mathcal{C}$:

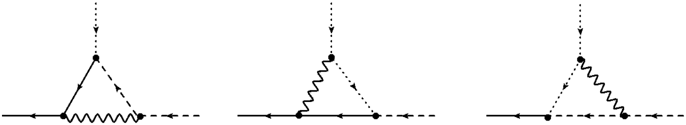
$$A_+^\dagger \bar{\lambda} P_+ \psi - \bar{\psi} P_- \lambda A_+ + A_- \bar{\lambda} P_- \psi - \bar{\psi} P_+ \lambda A_-^\dagger \qquad (3)$$

$$A_+^\dagger \bar{\lambda} P_- \psi - \bar{\psi} P_+ \lambda A_+ + A_- \bar{\lambda} P_+ \psi - \bar{\psi} P_- \lambda A_-^\dagger \qquad (4)$$

- All terms within each of the combinations in Eqs. (3) and (4) are multiplied by the same Yukawa coupling, g_{Y_1} and g_{Y_2}, respectively
- In the absence of anomalies, $\chi \times \mathcal{R}$ leaves invariant each of the four constituents of the Yukawa term (Eq. (3)), but it changes the constituents of the "mirror" Yukawa term (Eq. (4)) $\rightarrow$ guarantees the absence of a "mirror" Yukawa term

Computational Setup (3)

- Compute, perturbatively, the relevant three-point Green's functions using both dimensional regularization (DR) in $D = 4 - 2\epsilon$ dimensions and lattice regularization (LR) with external gluino-quark-squark fields
- Three one-loop Feynman diagrams that enter the computation of the three-point amputated Green's functions for the Yukawa couplings (a wavy (solid) line $\rightarrow$ gluons (quarks) and a dotted (dashed) line $\rightarrow$ squarks (gluinos)):



- An arrow entering (exiting) a vertex $\rightarrow$ a $\lambda, \psi, A_+, A_-^\dagger$ ($\bar\lambda, \bar\psi, A_+^\dagger, A_-$) field

Computational Setup (4)

- We impose renormalization conditions which result in the cancellation of divergences in the corresponding bare three-point amputated Green's functions with external gluino-quark-squark fields
- The renormalization factors are defined in such a way as to remove all divergences
- The application of the renormalization factors on the bare Green's functions leads to the renormalized Green's functions, which are independent of the regulator (ϵ in DR, a in LR) $\rightarrow$ renormalized Green's functions at a given scheme, but derived via different regularizations, should coincide

Renormalization Factors of the Fields and the Coupling Constants

- The definition of the renormalization factors of the fields and the gauge coupling constant are the following:

$$\psi \equiv \psi^B = Z_\psi^{-1/2}\, \psi^R, \tag{5}$$

$$u_\mu \equiv u_\mu^B = Z_u^{-1/2}\, u_\mu^R, \tag{6}$$

$$\lambda \equiv \lambda^B = Z_\lambda^{-1/2}\, \lambda^R, \tag{7}$$

$$c \equiv c^B = Z_c^{-1/2}\, c^R, \tag{8}$$

$$g \equiv g^B = Z_g^{-1}\, \mu^\epsilon\, g^R \tag{9}$$

- The Yukawa coupling is renormalized as follows:

$$g_Y \equiv g_Y^B = Z_Y^{-1} Z_g^{-1} \mu^\epsilon g^R \tag{10}$$

- The components of the squark fields may mix at the quantum level, via a 2×2 mixing matrix (Z_A). We define the renormalization mixing matrix for the squark fields as follows:

$$\begin{pmatrix} A_+^R \\ A_-^{R\,\dagger} \end{pmatrix} = \left(Z_A^{1/2}\right) \begin{pmatrix} A_+^B \\ A_-^{B\,\dagger} \end{pmatrix} \tag{11}$$

Renormalization Condition in DR

- In the *DR* and $\overline{\mathrm{MS}}$ scheme this 2×2 mixing matrix is diagonal[5]
- Green's function in *DR* with external squark field $A_+ \rightarrow$ the renormalization condition up to g^2 will be given by:

$$\langle \lambda(q_1) A_+(q_3) \bar{\psi}(q_2) \rangle \Big|^{\overline{\mathrm{MS}}} = Z_\psi^{-1/2} Z_\lambda^{-1/2} (Z_A^{-1/2})_{++} \langle \lambda(q_1) A_+(q_3) \bar{\psi}(q_2) \rangle \Big|^{\mathrm{bare}} \quad (12)$$

- In the right-hand side all coupling constants must be expressed in terms of their renormalized values
- The left-hand side $\rightarrow$ the $\overline{\mathrm{MS}}$ (free of pole parts) renormalized Green's function
- The other renormalization conditions which involve the external squark fields $A_+^\dagger, A_-, A_-^\dagger$ are similar

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Renormalization Factor of the Yukawa Coupling in DR (1)

- Free to make appropriate choices of the external momenta → have checked that no superficial IR divergences will be generated given that the quark and squark fields are massive → calculation of the corresponding diagrams by setting to zero only one of the external momenta
- We present the one-loop Green's function for the Yukawa coupling for zero gluino momentum in *DR* with external squark field A_+:

$$\langle \lambda^{\alpha_1}(0)\bar{\psi}(q_2)A_+(q_3)\rangle^{DR,\text{1loop}} = -i\,(2\pi)^4\delta(q_2-q_3)\frac{g_Y g^2}{16\pi^2}\frac{1}{4\sqrt{2}N_c}T^{\alpha_1}\times$$
$$\Bigg[-3(1+\gamma_5)+((1+\alpha)(1+\gamma_5)+8\gamma_5 c_{\rm hv})N_c^2$$
$$+(1+\gamma_5)(-\alpha+(3+2\alpha)N_c^2)\left(\frac{1}{\epsilon}+\log\left(\frac{\bar{\mu}^2}{q_2^2}\right)\right)\Bigg] \qquad (13)$$

- $c_{\rm hv}=0,1$ for the naïve and 't Hooft-Veltman (HV) prescription of γ_5, respectively [11] [12] and α → gauge parameter

[11] M. S. Chanowitz et al., Nucl. Phys. B 159 (1979), 225-243
[12] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B 44 (1972), 189-213

Renormalization Factor of the Yukawa Coupling in DR (2)

- Recall the following fundamental results on the renormalization factor in *DR* which appear in the right-hand side of Eq. (12) [5]:

$$Z_\psi^{DR,\overline{\rm MS}} = 1 + \frac{g^2\, C_F}{16\,\pi^2}\frac{1}{\epsilon}\,(1+\alpha)\,, \qquad Z_A^{DR,\overline{\rm MS}} = \left(1 + \frac{g^2\, C_F}{16\,\pi^2}\frac{1}{\epsilon}\,(-1+\alpha)\right)\mathbb{1}$$

$$Z_\lambda^{DR,\overline{\rm MS}} = 1 + \frac{g^2}{16\,\pi^2}\frac{1}{\epsilon}\,(\alpha\, N_c + N_f)\,, \quad Z_g^{DR,\overline{\rm MS}} = 1 + \frac{g^2}{16\,\pi^2}\frac{1}{\epsilon}\left(\frac{3}{2}N_c - \frac{1}{2}N_f\right)$$

- $C_F = (N_c^2-1)/(2\,N_c)$ $\rightarrow$ the quadratic Casimir operator in the fundamental representation and N_f $\rightarrow$ number of flavors
- By using Eq. (12) and for all Green's functions and all choices of the external momenta which we consider, we obtain the same value of $Z_Y^{DR,\overline{\rm MS}}$:

$$\boldsymbol{Z_Y^{DR,\overline{\rm MS}} = 1 + \mathcal{O}(g^4)} \tag{14}$$

- $Z_Y^{DR,\overline{\rm MS}}$ $\rightarrow$ **at the quantum-level, the renormalization process involving the Yukawa interaction is not affected by one-loop corrections** $\rightarrow$ expect that the corresponding renormalization on the lattice will be finite

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Renormalization Condition on the Lattice

- On the lattice the renormalization mixing matrix for the squark fields is non diagonal and the component $A_+(A_-)$ mixes with $A_-^\dagger(A_+^\dagger)$ [5]
- The $\chi \times \mathcal{R}$ symmetry is broken $\rightarrow$ in the calculation of the bare Green's functions on the lattice, we expect that mirror Yukawa term will arise at one-loop
- Introduction of the renormalization factor Z_{Y_1} and the mixing coefficient with mirror Yukawa term z_{Y_2}, where $Z = \mathbb{1} + \mathcal{O}(g^2)$ and $z = \mathcal{O}(g^2)$
- The renormalization condition is the following:

$$\langle \lambda(q_1) A_+(q_3) \bar{\psi}(q_2) \rangle \Big|^{\overline{\mathrm{MS}}} = Z_\psi^{-1/2} Z_\lambda^{-1/2} \langle \lambda(q_1) \Big((Z_A^{-1/2})_{++} A_+(q_3) + (Z_A^{-1/2})_{+-} A_-^\dagger(q_3) \Big) \bar{\psi}(q_2) \rangle \Big|^{\mathrm{bare}} \quad (15)$$

- The bare coupling g_{Y_2} arises in the right hand side of the renormalization condition

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Renormalization Factor of the Yukawa Coupling on the Lattice (1)

- Recall renormalization factors of fields and gauge coupling on the lattice [5]
- Having checked that alternative choices of the external momenta give the same results for these differences, we present it only for zero gluino momentum and with external squark field A_+:

$$\langle\lambda^{\alpha_1}(0)A_+(q_3)\bar\psi(q_2)\rangle^{\overline{\rm MS},1{\rm loop}} - \langle\lambda^{\alpha_1}(0)A_+(q_3)\bar\psi(q_2)\rangle^{LR,1{\rm loop}}$$
$$= -i\,(2\pi)^4\delta(q_2-q_3)\frac{g_Y g^2}{16\pi^2}\,\frac{1}{4\sqrt{2}N_c}\,T^{\alpha_1}\times$$
$$\Big[-3.7920\alpha(1+\gamma_5)+(-3.6920+5.9510\gamma_5+7.5840\alpha(1+\gamma_5)-8\gamma_5 c_{\rm hv})N_c^2$$
$$+(1+\gamma_5)(\alpha-(3+2\alpha)N_c^2)\log\left(a^2\bar\mu^2\right)\Big] \qquad (16)$$

[5] M. Costa and H. Panagopoulos, Phys. Rev. D 96 (2017) no.3, 034507

Renormalization Factor of the Yukawa Coupling on the Lattice (2)

- By combining the lattice expressions with the $\overline{\rm MS}$-renormalized Green's functions calculated in the continuum and by recalling renormalization factors of fields and gauge coupling on the lattice we find for the renormalization factor and the mixing coefficient:

$$Z_{Y_1}{}^{LR,\overline{\rm MS}} = 1 + \frac{g^2}{16\,\pi^2}\left(\frac{1.45833}{N_c} + (4.40768 - 2c_{\rm hv})N_c + 0.520616 N_f\right) \tag{17}$$

$$z_{Y_2}{}^{LR,\overline{\rm MS}} = \frac{g^2}{16\,\pi^2}\left(\frac{-0.040580}{N_c} + (2.45134 - 2c_{\rm hv})N_c\right) \tag{18}$$

- The above factors are **gauge independent** $\rightarrow$ the $\overline{\rm MS}$ renormalization factors for gauge invariant objects are gauge-independent
- The multiplicative renormalization Z_{Y_1} and the mixing coefficient z_{Y_2} are **finite**

Computational Setup (1)

- Two squarks to lie in the fundamental representation and the other two in the antifundamental
- Ten cases for choosing the 4 external squarks:

$$(A_+^\dagger A_+)(A_+^\dagger A_+), \quad (A_- A_-^\dagger)(A_- A_-^\dagger),$$
$$(A_+^\dagger A_+)(A_- A_-^\dagger), \quad (A_+^\dagger A_-^\dagger)(A_+^\dagger A_-^\dagger), \quad (A_- A_+)(A_- A_+), \quad (A_- A_+)(A_+^\dagger A_-^\dagger),$$
$$(A_+^\dagger A_+)(A_+^\dagger A_-^\dagger), \quad (A_+^\dagger A_+)(A_- A_+), \quad (A_- A_-^\dagger)(A_+^\dagger A_-^\dagger), \quad (A_- A_-^\dagger)(A_- A_+)$$

- Pairs of squark fields in parenthesis $\rightarrow$ color-singlet combinations
- Take into account $\mathcal{C}$ and $\mathcal{P}$ to construct combinations which are invariant under these symmetries

Computational Setup (2)

- There are five combinations [13]:

Operators	$\mathcal{C}$	$\mathcal{P}$
$\lambda_1[(A_+^\dagger A_+)^2 + (A_- A_-^\dagger)^2]$	+	+
$\lambda_2[(A_+^\dagger A_-^\dagger)^2 + (A_- A_+)^2]$	+	+
$\lambda_3(A_+^\dagger A_+)(A_- A_-^\dagger)$	+	+
$\lambda_4(A_+^\dagger A_-^\dagger)(A_- A_+)$	+	+
$\lambda_5(A_+^\dagger A_-^\dagger + A_- A_+)(A_+^\dagger A_+ + A_- A_-^\dagger)$	+	+

- Operators which are gauge invariant, flavor singlets and with dimensionality 4
- The tree-level values of λ_i which satisfy Supersymmetry are:

$$\lambda_1 = \frac{1}{2}\, g^2\, \frac{N_c - 1}{2N_c}, \;\; \lambda_3 = \frac{1}{2}\, g^2\, \frac{1}{N_c}, \;\; \lambda_4 = -\frac{1}{2}\, g^2, \;\; \lambda_2 = \lambda_5 = 0 \qquad (19)$$

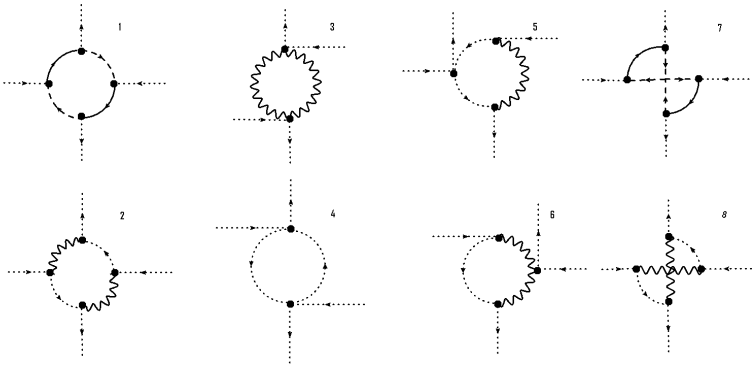
- Compute these tree-level values by using the following relation for the generators:

$$T^a_{ij} T^a_{kl} = \frac{1}{2}(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}) \qquad (20)$$

[13] B. Wellegehausen and A. Wipf, PoS LATTICE2018 (2018), 210

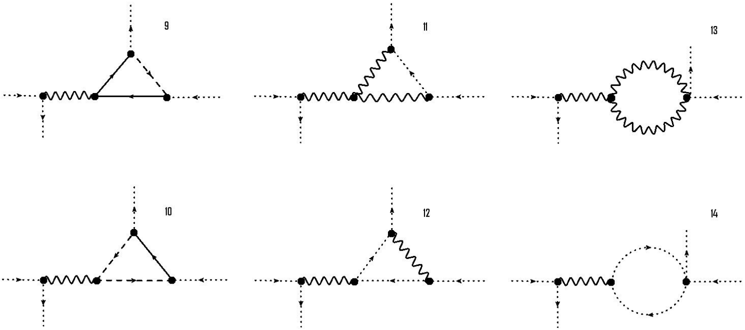
Computational Setup (3)

- These couplings receive quantum corrections, coming from the Feynman diagrams (a wavy (solid) line $\rightarrow$ gluons (quarks) and a dotted (dashed) line $\rightarrow$ squarks (gluinos)):



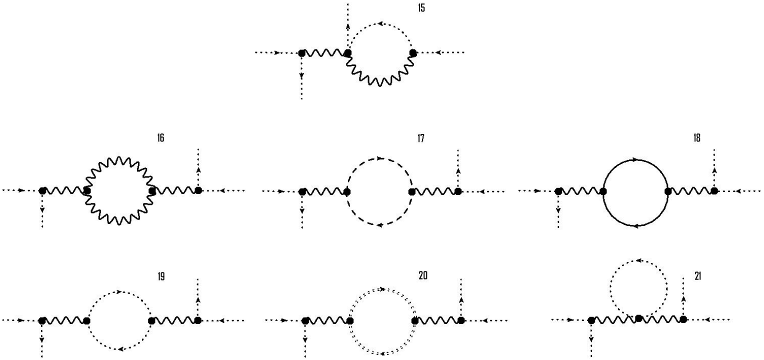
- Diagrams 7 and 8 are variants of diagrams 1 and 2, respectively
- Majorana nature of gluinos $\rightarrow$ in diagram 7, in which $\lambda - \lambda$ as well as $\bar{\lambda} - \bar{\lambda}$ propagators appear

Computational Setup (4)



- Majorana nature of gluinos $\rightarrow$ in diagram 10, in which $\lambda - \lambda$ as well as $\bar{\lambda} - \bar{\lambda}$ propagators appear

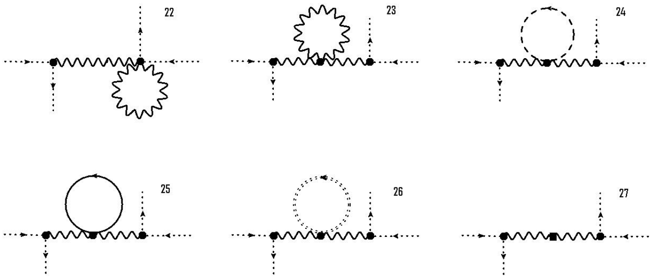
Computational Setup (5)



- Unlike gluon tadpoles which vanish in dimensional regularization, the massive squark tadpole gives a nonzero contribution (diagram 21)

Computational Setup (6)

- Additional one-loop Feynman diagrams leading to the fine tuning of the quartic couplings on the lattice (a wavy (solid) line → gluons (quarks) and a dotted (dashed) line → squarks (gluinos)):



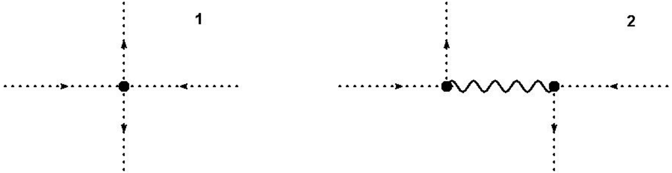
- The "double dashed" line is the ghost field and the solid box in diagram 27 comes from the measure part of the lattice action

Computational Setup (7)

- For each diagram $\rightarrow$ nine Green's functions
- Compute the diagrams by setting 2 external squark momenta of fields in the fundamental representation (or antifundamental representation) to zero $\rightarrow$ this choice guarantees absence of IR singularities

Tree-level Green's Functions with four External Squarks

- Compute the tree-level Green's functions with 4 external squarks



- We present the tree-level Green's functions with four external squarks:

$$\langle A_+^{\dagger\alpha_1}(q_1)A_+^{\dagger\alpha_2}(q_2)\,A_+^{\alpha_3}(q_3)A_+^{\alpha_4}(q_4)\rangle^{\rm tree} = \frac{1}{2N_c}\,g^2\,(-2+\beta)\,(-1+N_c)\,(\delta^{\alpha_1\alpha_3}\delta^{\alpha_2\alpha_4}+\delta^{\alpha_1\alpha_4}\delta^{\alpha_2\alpha_3}) \quad (21)$$

$$\langle A_-^{\dagger\alpha_1}(q_1)A_-^{\dagger\alpha_2}(q_2)\,A_-^{\alpha_3}(q_3)A_-^{\alpha_4}(q_4)\rangle^{\rm tree} = \frac{1}{2N_c}\,g^2\,(-2+\beta)\,(-1+N_c)\,(\delta^{\alpha_3\alpha_1}\delta^{\alpha_4\alpha_2}+\delta^{\alpha_4\alpha_1}\delta^{\alpha_3\alpha_2}) \quad (22)$$

$$\langle A_+^{\dagger\alpha_1}(q_1)A_+^{\alpha_2}(q_2)\,A_-^{\dagger\alpha_3}(q_3)A_-^{\alpha_4}(q_4)\rangle^{\rm tree} = \frac{1}{2N_c}\,g^2\,\beta\,(N_c\delta^{\alpha_1\alpha_3}\delta^{\alpha_4\alpha_2}-\delta^{\alpha_1\alpha_2}\delta^{\alpha_4\alpha_3}) \quad (23)$$

- The rest of the tree-level Green's functions with four external squarks are zero

Renormalization Factor of the Quartic Couplings in DR

- The quartic couplings are renormalized as follows:

$$\lambda_1 = Z_{\lambda_1}^{-1} Z_g^{-2} \mu^{2\epsilon} \left[\frac{1}{2} \left(g^R\right)^2 \frac{N_c - 1}{2N_c} \right] \tag{24}$$

$$\lambda_3 = Z_{\lambda_3}^{-1} Z_g^{-2} \mu^{2\epsilon} \left[\frac{1}{2} \left(g^R\right)^2 \frac{1}{N_c} \right] \tag{25}$$

$$\lambda_4 = Z_{\lambda_4}^{-1} Z_g^{-2} \mu^{2\epsilon} \left[-\frac{1}{2} \left(g^R\right)^2 \right] \tag{26}$$

- The renormalization condition up to g^2 will be given by:

$$\langle A_+(q_1) A_+^\dagger(q_2) A_+(q_3) A_+^\dagger(q_4)\rangle \Big|^{\overline{\rm MS}} = \tag{27}$$

$$(Z_A^{-2})_{++} \langle A_+(q_1) A_+^\dagger(q_2) A_+(q_3) A_+^\dagger(q_4)\rangle \Big|^{\rm bare}$$

- Use the renormalization condition, the renormalization factors of the squark fields and the gauge coupling and the bare Green's functions → determine the appropriate renormalization factor of each quartic coupling in order to cancel the divergences
- In our ongoing investigation → calculating perturbatively the relevant four-point Green's functions on the lattice so as to deduce the renormalization facotrs of the quartic couplings on the lattice

Summary-Conclusions

- The renormalization of the Yukawa coupling is **finite** and there is **a finite mixing** with the mirror Yukawa term on the lattice
- The renormalization of the quartic couplings is **underway**

Thank you for your attention!

European Union
European Regional Development Fund
Κυπριακή Δημοκρατία
RESEARCH & INNOVATION FOUNDATION
2016-2020 RESTART RESEARCH