Composite Higgs model from the lattice:  
Infrared fixed point and anomalous dimensions

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Composite Higgs

- Hierarchy problem: [un]naturalness of the Higgs mass
- Composite Higgs: make the Higgs a [pseudo] Nambu-Goldstone boson
- **Hypercolor**: new strong sector with scale \( \Lambda_{HC} \gtrsim 1 \text{ TeV} \)
- Ferretti & Karateev list of HC models:
  a) asymptotically free
  b) vector-like
  c) two fermion reps: \( Q, q \)
  d) SM symmetries embedded in unbroken \( \mathcal{H} \) of global \( \mathcal{G} \).
  e) Partially composite top quark
      top is the only quark with mass \( m_t \sim m_{\text{higgs}} \)
Partial compositeness

- $t$ couples linearly to “chimera” HC baryon $B = Qqq$
- Requires 4-fermion interactions $G(\bar{t}B + \bar{B}t)$ where $G \sim \Lambda_{EHC}^{-2}$
  \[ t \xrightarrow{G} B \xrightarrow{G} t \]
- Must be induced by *extended hypercolor* with some yet higher scale $\Lambda_{EHC}$
- Problem (same as Technicolor): flavor violations coming from similar 4-fermion interactions among SM fermions $\Rightarrow \Lambda_{EHC} \gg \Lambda_{HC}$
- Problem (cont.): To generate realistic top mass, need to overcome naive suppression factor $\sim (\Lambda_{HC}/\Lambda_{EHC})^4$
- Solution: a) *Near-conformal* HC theory
  b) *Large anomalous dimension* for $B$ such that suppression factor reduces to $(\Lambda_{HC}/\Lambda_{EHC})^{2(2-\gamma_B)}$
  $\Rightarrow$ Ideally $\gamma_B \sim 2$ all the way from EHC to HC scale!
Prototype: $2 + 2$ model

- Ferretti & Karateev “M6”: SU(4) gauge theory with $3 \times \text{fund.} + 5 \times \text{sextet}$
- $2+2$ model: $2 \times \text{fund.} + 4 \times \text{sextet}$ (Note: $4 \times \text{Majorana} = 2 \times \text{Dirac}$)

But:

$2+2$ model is QCD like

matrix element $\langle 0 | B_i | B \rangle$
of partial compositeness is very small

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Looking for (near) conformality: 4 + 4 model

- Analytic estimates for the conformal window (dashed lines)
  SU(4) with $N_f \times \text{Dirac fund.} + n_f \times \text{Maj. sextets}$
Improvement!

Plaquette action and Wilson fermions with:

- Clover term
- nHYP smeared links
- nHYP dislocation suppressing gauge term
- PV bosons \textit{PRD 104 (2021) 7, 074509}

decouple in continuum limit, while generating
well behaved effective gauge action

$\Rightarrow$ Payoff: ever stronger renormalized coupling with each improvement
Continuous RG

- Gradient flow

\[
\frac{\partial B_\mu}{\partial t} = -\frac{\partial S_g}{\partial B_\mu}, \quad \left. B_\mu \right|_{t=0} = A_\mu
\]

\[
g^2(t) = C t^2 \langle E(t) \rangle, \quad E(t) = G^a_{\mu\nu} G^a_{\mu\nu}(t)
\]

- Flow as RG: \( g^2(t) \) is the running coupling at scale \( \mu^2 = 1/t \), and

\[
\beta(g^2) = -t \frac{\partial g^2}{\partial t}
\]

- Discretizations:

\[
S^\text{latt}_g = c_p S^{\text{plaq}} + c_r S^{\text{rec}} \quad \text{with} \quad c_p + 8c_r = 1.
\]

  - Symanzik flow: \( c_p = 5/3 \)
  - C43 flow: \( c_p = 4/3 \)
  - Wilson flow: \( c_p = 1 \)
  - C23 flow: \( c_p = 2/3 \)
  - C13 flow: \( c_p = 1/3 \)

- \( E(t) \) operators: Symanzik, Wilson, Clover
Raw flow, finite volume

- We use $2.4 \leq t/a^2 \leq 3.2$
- We only estimated FV effects
- Infinite volume limit $\Rightarrow$ small shift of $g^2$
Interpolation

\[ g^2(g_0, t), \quad \beta(g_0, t) \]

Interpolate \[ \Rightarrow \beta(g^2, t) \]
Continuum limit: \( a^2/t \to 0 \) at fixed \( g^2 \)

Precise condition:

Require consistency of \( S \) and \( W \):

\[
\langle \beta_S - \beta_W \rangle \leq \sqrt{2} \sigma_S
\]
Beta function

Use 5 different flows

Cover $4 \lesssim g^2 \lesssim 17$

Infrared fixed point

$g^2 \sim 15.5$
Anomalous dimensions

- Fermion flow
  \[ \frac{\partial \chi}{\partial t} = \Delta \chi, \quad \chi\big|_{t=0} = \psi \]

- Flowed meson correlators \[ \langle X(0) X'(x_4, t) \rangle \]
  source \( X = \bar{\psi} \Gamma \psi \) (not flowed; Gaussian point-split)
  sink \( X' = \bar{\chi} \Gamma \chi \) (flowed; point operator, \( \vec{p} = 0 \))

- Correct for anomalous dim. of \( \psi \) field:
  divide by conserved current \( \Rightarrow \) anomalous dim. of \( X \) composite

\[ \gamma_X = -2 \frac{t}{R} \frac{\partial R}{\partial t} \quad R = \frac{\langle X(0) X'(x_4, t) \rangle}{\langle V(0) V'(x_4, t) \rangle} \]
\( x_4 \) plateau

- \( x_4 \)-dependence: look for plateau \( x_4 \gg \sqrt{8t} \)
  where separation \( \gg \) smearing by flow

- For the rest, same as for beta function

- Generalizes to chimera operators
Anomalous dimensions

mass anomalous dim.  chimera anom. dim. (wanted $\gamma \sim 2$ !!)

$\gamma \sim 2$ !!
In summary

• $4 + 4$ model is IR conformal: $g^2_* \sim 15.5$

• By giving some fermions a (large) mass
  can obtain M6 or M11 models of the Ferretti-Karateev list

• Large mass anomalous dimensions at the IRFP; but:

• If we hoped for large $\gamma_{\text{chimera}} \sim 2$
  what we actually find is small $\gamma_{\text{chimera}} \lesssim 0.5$