Meson-meson scattering at large N_c

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The large N_c limit of QCD

QCD simplifies at **large N_c**

$N_c \to \infty \quad N_f = \text{const}$

- Keeps **non-perturbative features**
- Has **predictive power** in the non-perturbative regime
- Lattice allows to study **subleading N_c effects**

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Long-term goal: Understand QCD at large N_c

- Meson masses and decay constants [Hernández et al. 2019]
- $K \to (\pi\pi)_{I=0,2}$ [Donini et al. 2019]
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$N_f = 4$ QCD

$m_u = m_d = m_s = m_c$

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This work: Energy-dependent meson-meson scattering

$\pi\pi$ scattering at large N_c

$N_f = 4 \rightarrow$ **7 scattering channels**

$$15 \otimes 15 = \mathbf{84}\,(\boldsymbol{SS}) \oplus 45 \oplus 45 \oplus \mathbf{20}\,(\boldsymbol{AA}) \oplus 15 \oplus 15 \oplus 1$$

[JBB et al. 2022] $\pi^+\pi^+$ $D_s^+\pi^+ - D^+K^+$

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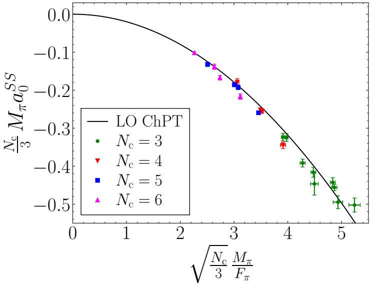
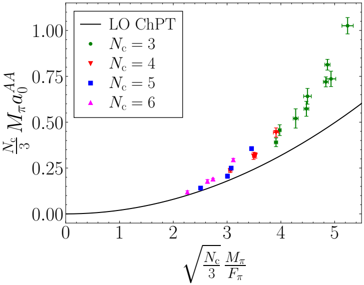
***SS* channel** (repulsive)



***AA* channel** (attractive)



$\pi\pi$ scattering at large N_c

AA channel is **attractive** → **Possible tetraquark**

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Recently found **exotic states at LHCb** [LHCb 2020, 2022]:

$$J=0\text{: }\begin{array}{l} T^0_{cs0}(2900)\text{ in }D^+K^- \\ T^{++}_{c\bar{s}0}(2900)\text{ and }T^0_{c\bar{s}0}(2900)\text{ in }D_s^\pm\pi^+ \end{array} \longrightarrow \textbf{\textit{AA} channel}$$

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$$J=1:\ T^0_{cs1}(2900) \text{ in } D^+K^- \longrightarrow 84 \oplus \mathbf{45} \oplus \mathbf{45} \oplus 20 \oplus 15 \oplus 15 \oplus 1$$

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$T^{++}_{c\bar{s}0}(2900)$ and $T^0_{c\bar{s}0}(2900)$ in $D_s^{\pm}\pi^+$ $\longrightarrow$ ***AA* channel**

$J = 1$: $T^0_{cs1}(2900)$ in D^+K^- $\longrightarrow$ $84 \oplus \mathbf{45} \oplus \mathbf{45} \oplus 20 \oplus 15 \oplus 15 \oplus 1$

Below $D_s^*\rho$ threshold → Described as **meson-meson bound states**

Meson-meson scattering at large N_c

Goal: study meson-meson scattering at large N_c

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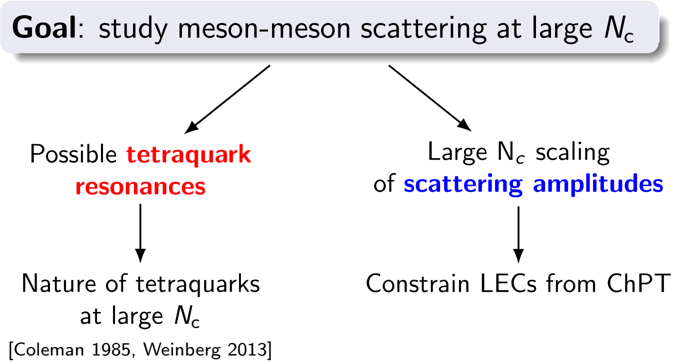
Possible **tetraquark resonances**

Nature of tetraquarks at large N_c

[Coleman 1985, Weinberg 2013]

Meson-meson scattering at large N_c



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Large N_c scaling of **scattering amplitudes**

Constrain LECs from ChPT

Meson-meson scattering at large N_c

Goal: study meson-meson scattering at large N_c

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$$C_{SS} = D - C + (p_1 \leftrightarrow p_2)$$
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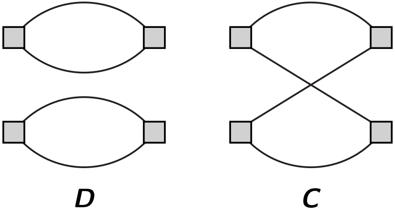

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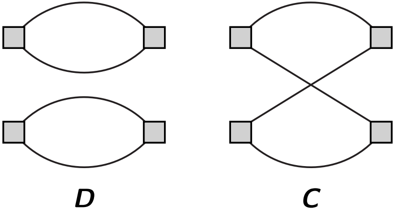

This talk: Preliminary results for AA with $N_c = 3$ and SS with $N_c = 3, 4$

Lattice computations

Lattice computations performed with **HiRep** [Del Debbio et al. 2010]

- Iwasaki gauge action with $N_f = 4$ clover-improved Wilson fermions
- $N_c = 3, 4, 5, 6$ ensembles with $a \sim 0.075$ fm and $M_\pi \sim 590$ MeV

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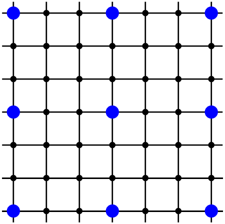
Operator basis: $\pi\pi + \rho\rho +$ local tetraquark

- Two-particle operators → $\mathbb{Z}_2 \times \mathbb{Z}_2$ noise
- Local tetraquark operators → **Point sources in a regular subgrid $\tilde{\Lambda}$**



$$T(\boldsymbol{P}) \propto \sum_{\boldsymbol{x}\in\tilde{\Lambda}} e^{-i\boldsymbol{P}\boldsymbol{x}} T(\boldsymbol{x})$$

Finite-volume energy spectra

Project to cubic-group irreps and solve GEVP

$$C(t)v_n(t) = \lambda_n(t)C(t_0)v_n(t)$$

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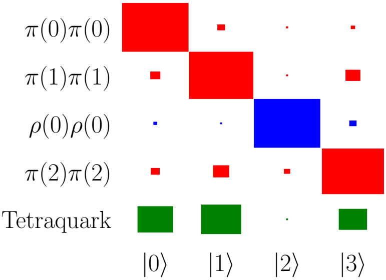
Eigenvectors provide intuition on the effect of each operator

AA channel, $N_c = 3$, $A_1^+(0)$, $M_\rho \sim 1.7M_\pi$

Area ∝ Relative overlap



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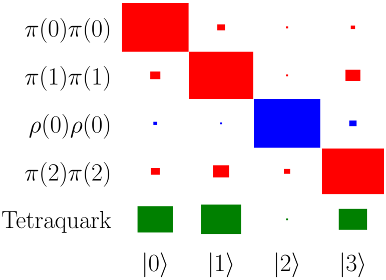

Negligible effect on $\pi\pi$

Significant overlap with $\pi\pi$

Finite-volume energy spectra

Non-negligible **thermal effects**

$\tilde{t} = t - \frac{T}{2}$

$\Delta E = E_{\mathbf{k}_1} - E_{\mathbf{k}_2}$

$$C_{\mathbf{k}_1,\mathbf{k}_2}(t) = A\cosh(E_{\mathbf{k}_1,\mathbf{k}_2}\tilde{t}\,) + \tilde{A}\cosh(\Delta E\tilde{t}\,)$$

$$C_{\mathbf{k}_1}(t)C_{\mathbf{k}_2}(t) = B\left[\cosh(E^{\text{free}}_{\mathbf{k}_1,\mathbf{k}_2}\tilde{t}\,) + \cosh(\Delta E\tilde{t}\,)\right]$$

Time-dependent thermal effect

Finite-volume energy spectra

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Time-dependent thermal effect

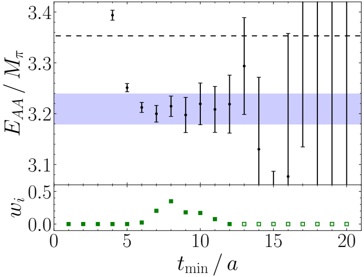
Fit ratio to 3 ($\boldsymbol{k}_1 \neq \boldsymbol{k}_2$) or 2 ($\boldsymbol{k}_1 = \boldsymbol{k}_2$) parameters

$$R(t) = \frac{\partial_0 C_{\boldsymbol{k}_1,\boldsymbol{k}_2}(t)}{\partial_0[C_{\boldsymbol{k}_1}(t)C_{\boldsymbol{k}_2}(t)]}$$

Average plateaux using **Akaike Information Criterion**

[Jay and Neil 2020]

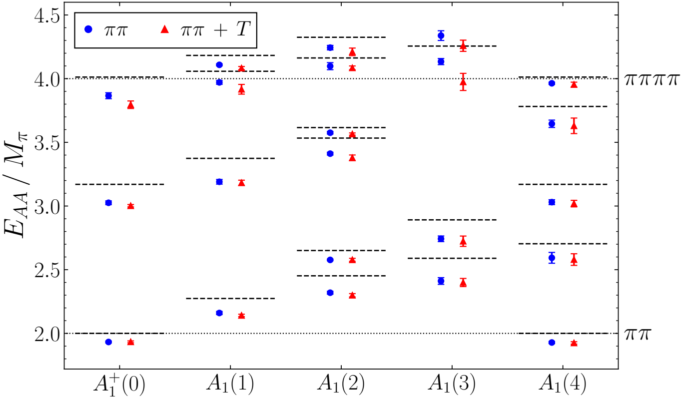
AA channel, $N_c = 3$, $A_1(3)$



Finite-volume energies: *AA* channel

We study the effect of different operators for $N_c = 3$:

$\pi\pi$ vs $\pi\pi$ + Local tetraquarks



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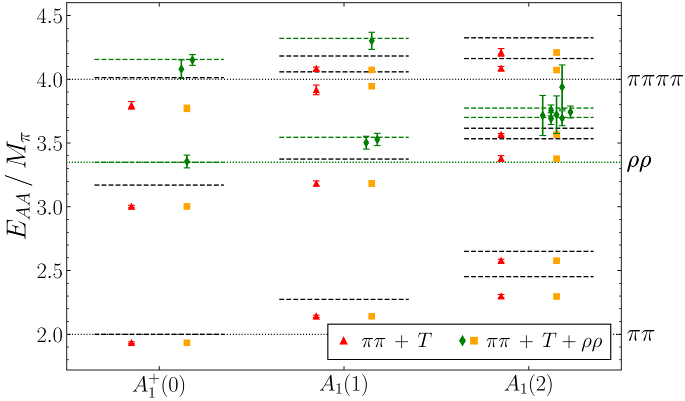
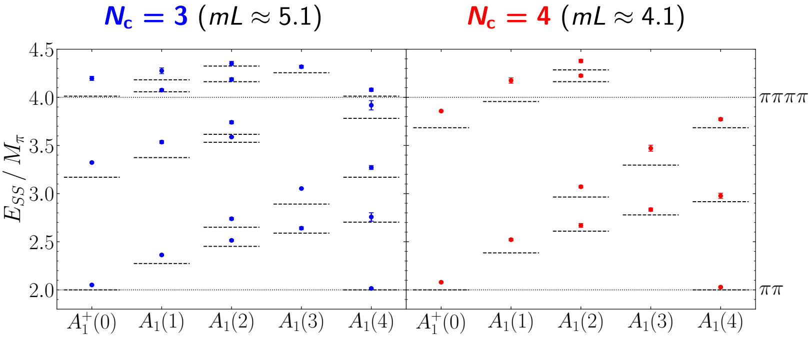
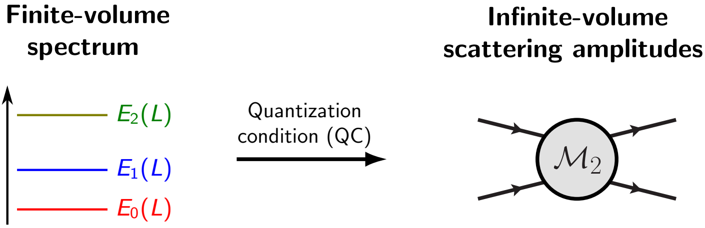


Finite-volume energies: *SS* channel ($\pi\pi$ only)



Finite-volume scattering formalism



Finite-volume spectrum

Quantization condition (QC)

Infinite-volume scattering amplitudes

Two-particle QC [Lüscher 1986, Rummukainen and Gotlieb 1995, He et al. 2005]:

$$\det[\underbrace{\tilde{\mathcal{K}}^{-1}}_{\pi\pi\text{-}\rho\rho\text{ mixing}}(E)+\underbrace{B}_{J\text{ mixing}}(\boldsymbol{P},L;E)]=0$$

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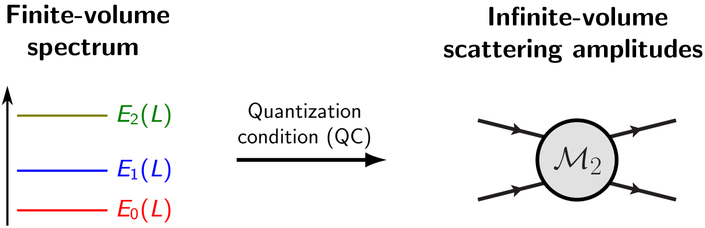

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$\pi\pi$-$\rho\rho$ mixing ← $\tilde{\mathcal{K}}^{-1}(E)$

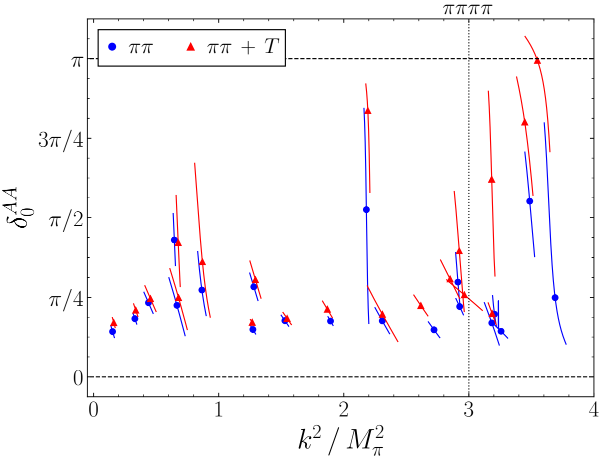
$B(\boldsymbol{P}, L; E)$ → J mixing

Single-channel, *s*-wave ⟶

$$k \cot\delta = \frac{2}{\gamma L \pi^{1/2}} \mathcal{Z}_{00}^{\boldsymbol{P}}\left(\frac{kL}{2\pi}\right)$$

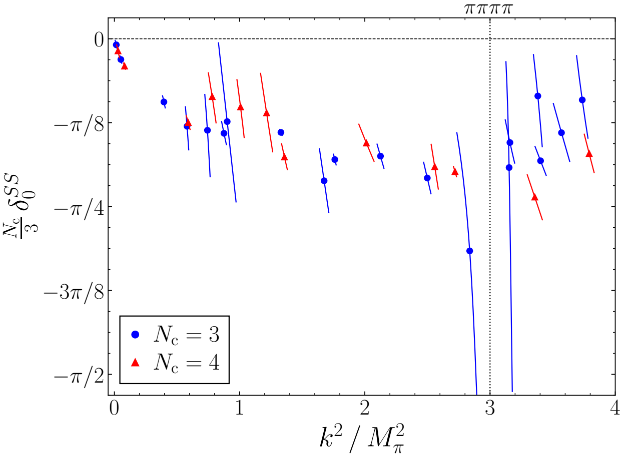
Scattering phase shift: *AA* channel

Compare amplitude for $\pi\pi$ vs $\pi\pi$ + local tetraquarks



Scattering phase shift: SS channel ($\pi\pi$ only)

Expected large N_c scaling: $\boldsymbol{\mathcal{M} \sim N_c^{-1}}$



Summary and outlook

Goal: study meson-meson scattering at large N_c

✓ We have determined the finite-volume energy spectra in the *AA* and *SS* channels including two-particle and local tetraquark operators

✓ In the AA channel, we have found a significant effect from tetraquark operators in the finite-volume energies

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Thank you for your attention!

Two-particle $\pi\pi$ isospin states

$$\mathbf{84}\,(\boldsymbol{SS}) \oplus \mathbf{45}\,(\boldsymbol{SA}) \oplus \mathbf{45}\,(\boldsymbol{AS}) \oplus \mathbf{20}\,(\boldsymbol{AA}) \oplus 15 \oplus 15 \oplus 1$$

$$O_{SS}(p_1,p_2) = \frac{1}{2}\left[\pi^+(p_1)D_s^+(p_2) + \pi^+(p_2)D_s^+(p_1) + K^+(p_1)D^+(p_2) + K^+(p_2)D^+(p_1)\right]$$

$$O_{SA}(p_1,p_2) = \frac{1}{2}\left[\pi^+(p_1)D_s^+(p_2) - \pi^+(p_2)D_s^+(p_1) - K^+(p_1)D^+(p_2) + K^+(p_2)D^+(p_1)\right]$$

$$O_{AS}(p_1,p_2) = \frac{1}{2}\left[\pi^+(p_1)D_s^+(p_2) - \pi^+(p_2)D_s^+(p_1) + K^+(p_1)D^+(p_2) - K^+(p_2)D^+(p_1)\right]$$

$$O_{AA}(p_1,p_2) = \frac{1}{2}\left[\pi^+(p_1)D_s^+(p_2) + \pi^+(p_2)D_s^+(p_1) - K^+(p_1)D^+(p_2) - K^+(p_2)D^+(p_1)\right]$$