Baryonic screening masses in high temperature QCD

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In collaboration with:
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- The baryonic screening spectrum is analysed at 12 temperatures between ~ 1 GeV and ~ 160 GeV
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- Study properties of **Quark-Gluon Plasma** from first principles
- Probe nature of QCD in the high temperature regime

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- **Screening mass** characterizes exponential fall-off (inverse correlation length)

$$C_{N^\pm}(x_3) \underset{x_3\to\infty}{\sim} \exp\{-m_{N^\pm} x_3\}\left(1 + O\left(e^{-\Delta m_{N^\pm} x_3}\right)\right)$$

$$m_{N^\pm} = \lim_{x_3\to\infty} \underbrace{-\frac{d}{dx_3}\ln\left[C_{N^\pm}(x_3)\right]}_{m^{\mathrm{eff}}_{N^\pm}(x_3)} \qquad \text{Free theory: } m_{N^\pm} = 3\pi T$$

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- Opposite parity channels related by chiral symmetry: $C_{N^+}(x_3) \xrightarrow[\text{Continuum}]{SU(2)_A\ \text{W.I.}} -C_{N^-}(x_3)$

Degeneracy of screening masses when chiral symmetry is restored

T	T[GeV]	L_0/a	$n_{\rm mdu}$	$n_{\rm skip}$
T_0	164.6(5.6)	4	300	10
		6	390	10
T_1	82.3(2.8)	4	300	10
		6	310	10
		8	500	10
		10	500	10
T_2	51.4(1.7)	4	300	10
		6	320	10
		8	490	10
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T_3	32.8(1.0)	4	300	10
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T_8	3.040(78)	4	320	10
		6	300	10
		8	500	10
		10	500	10
T_9	2.833(68)	4	400	10
		6	390	10
		8	390	10
T_{10}	1.821(39)	4	410	10
		6	400	10
		8	400	10
T_{11}	1.167(23)	4	400	10
		6	390	10
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- 12 temperature values $T_{0,\dots,11}$ between **1.167** GeV and **164.6** GeV
- 4 lattice spacings $L_0/a = 4, 6, 8, 10$, $L_i/a = 288 \Rightarrow TL \sim 20 - 50$
  Finite volume effects suppressed exponentially in $g^2 TL$
- $N_f = 3$ flavors of $O(a)$ improved massless Wilson fermions

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- Shifted boundary conditions [Giusti, Meyer 2011-13]

$$U_\mu(x_0 + L_0, \mathbf{x}) = U_\mu(x_0, \mathbf{x} - L_0\boldsymbol{\xi})$$
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$$L_0\sqrt{1+\boldsymbol{\xi}^2} \implies T = 1/\left(L_0\sqrt{1+\boldsymbol{\xi}^2}\right)$$

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Phase factor: $\omega_0 x_0 \to \omega_0\left(x_0 + x_1\right)/2$

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- Mesonic case [Dalla Brida et al. 2021]

Renormalization & scale setting

- Hadronic scheme: $M_{\text{Had}} = M_{\text{Had}}^{\text{Phys}}$ would require $a \ll \dfrac{1}{T} \ll \dfrac{1}{M_{\text{Had}}} \ll L$

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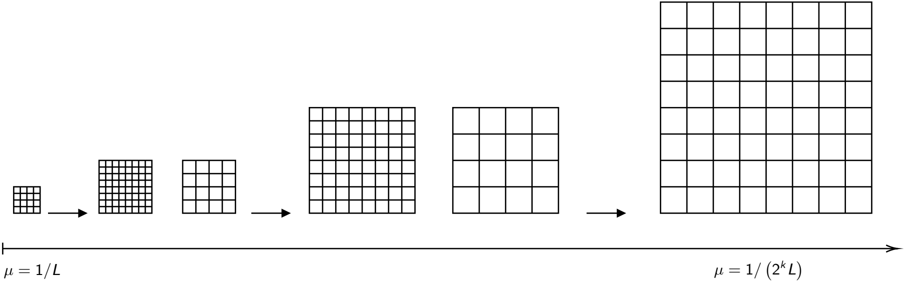
- **Non-perturbative** $\bar{g}^2(\mu)$

$$\bar{g}^2(\mu)\begin{cases} T_0 - T_8:\ \text{Schrödinger functional},\ T = \frac{1}{\sqrt{2}L_0} = \frac{1}{\sqrt{2}L_{\text{SF}}} = \frac{\mu_{\text{SF}}}{\sqrt{2}} \\ T_9 - T_{11}:\ \text{Gradient flow},\quad T = \frac{1}{\sqrt{2}L_0} = \frac{2}{\sqrt{2}L_{\text{GF}}} = \sqrt{2}\mu_{\text{GF}} \end{cases}$$

[ALPHA collaboration, '93,2016-18]

Bridge high energies (small L) to low energies (large L) via step scaling techniques



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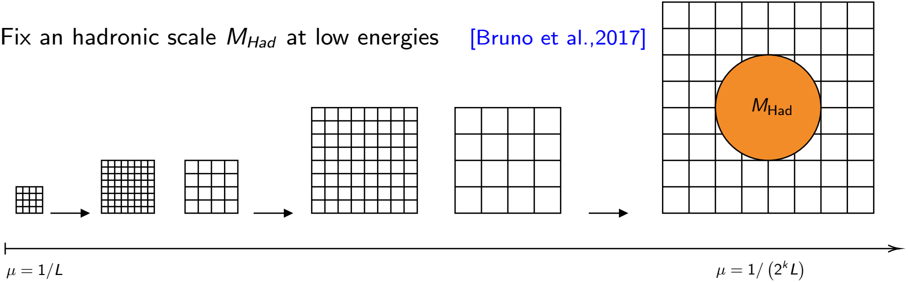
- Fix an hadronic scale M_{Had} at low energies [Bruno et al.,2017]



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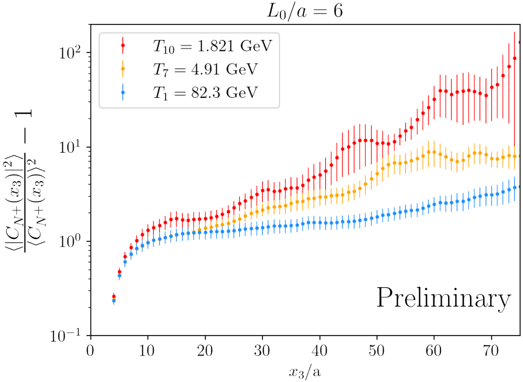

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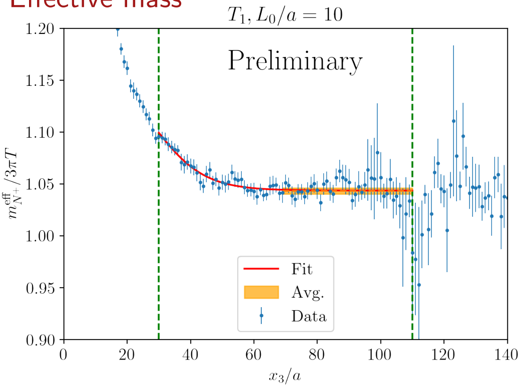
$T \to \infty$: $2m_{N\pm} \to 3\pi T$, $\quad 3m_{S/P} \to 2\pi T$ (arrows: $m_{N\pm} \downarrow 3\pi T$, $m_{S/P} \downarrow 2\pi T$)

Effective mass



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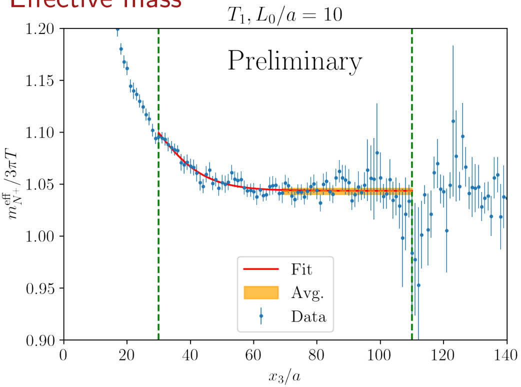

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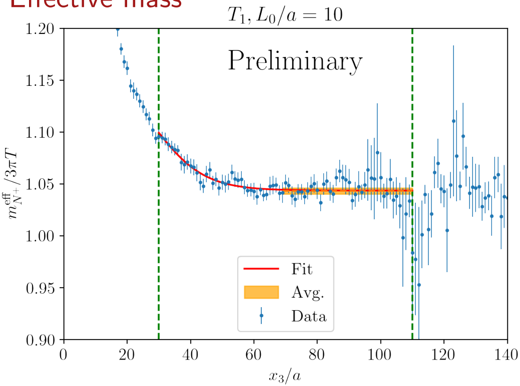

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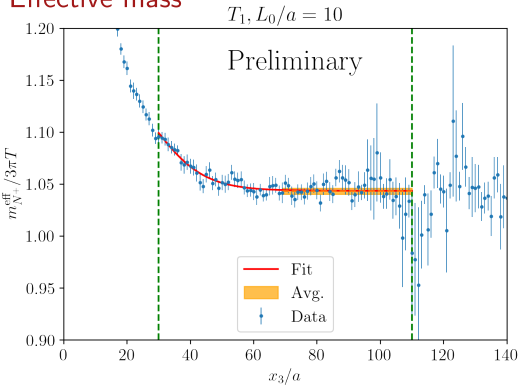

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Tree Level Improvement

Tree level screening mass at finite lattice spacing computed analytically : $m^{\rm Free}_{N^\pm}(L_0/a)$

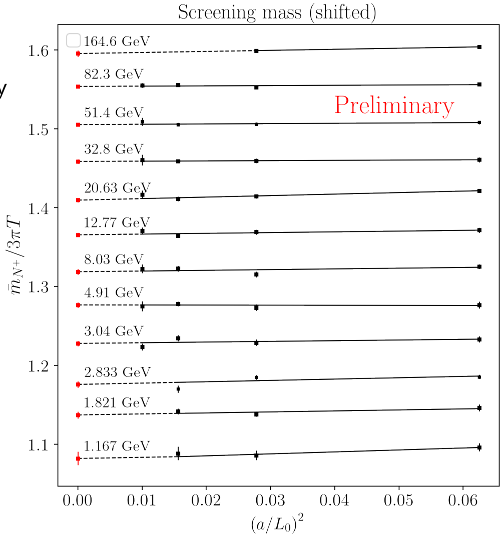
Accelerate continuum limit extrapolation by defining tree level improved screening mass:

$$\overline{m}_{N^\pm}(L_0/a) \equiv m_{N^\pm}(L_0/a) - (m^{\rm Free}_{N^\pm}(L_0/a) - 3\pi T)$$

Continuum extrapolation

Nucleon screening mass

- $O(a)$ improvement: excellent fit quality for constant + quadratic polynomial



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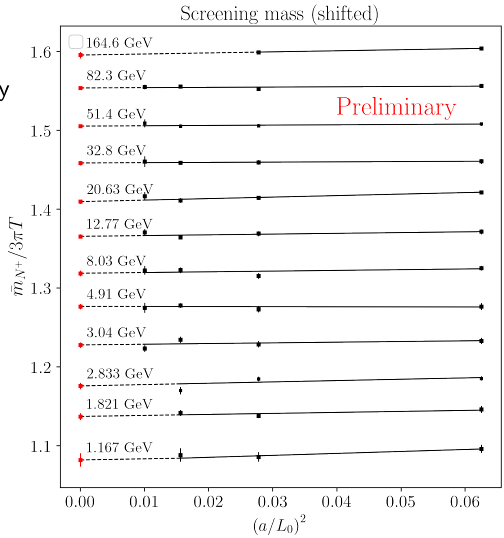

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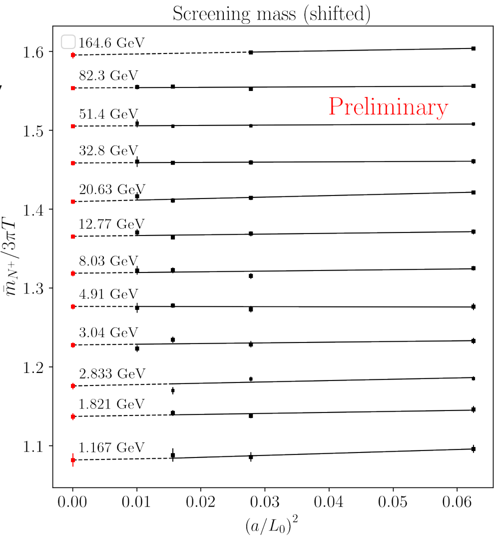

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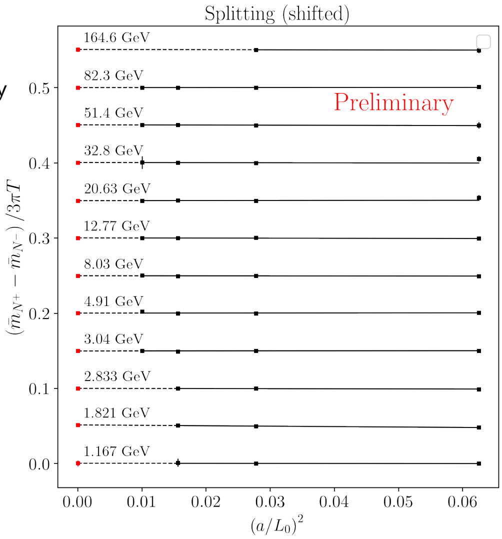
Parity splitting

- $(m_{N^+} - m_{N^-})$ compatible with 0 for all temperatures and lattice spacings



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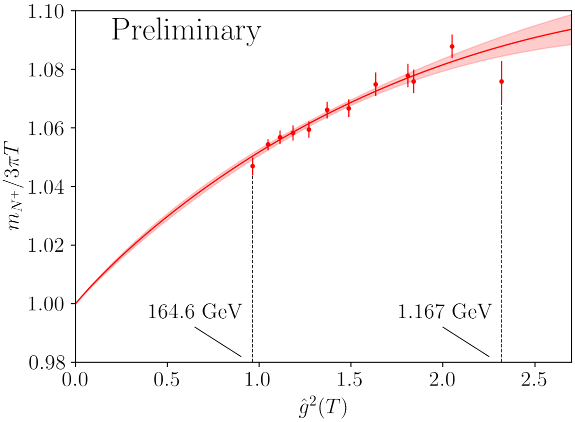

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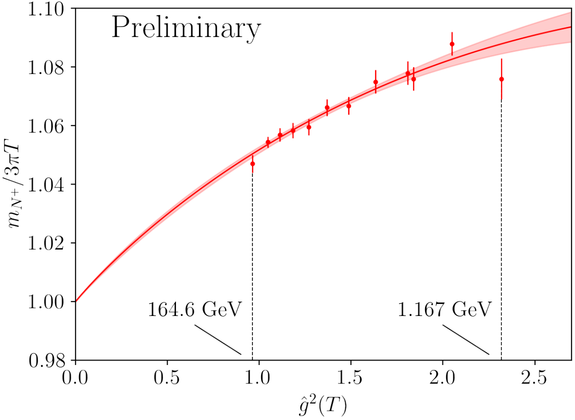

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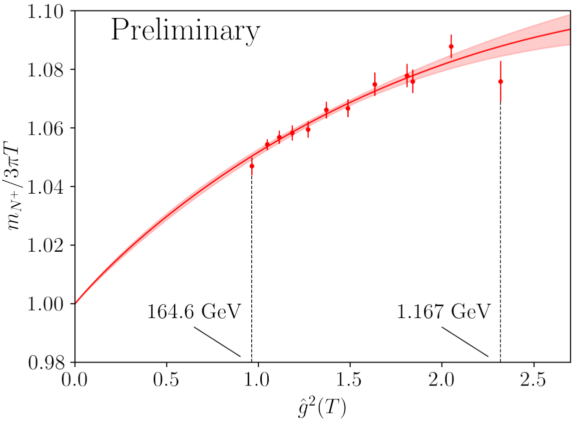
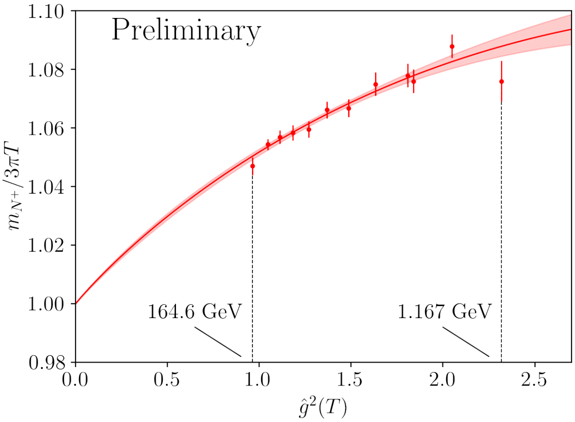
- Free theory result: constant term set to 1
  $\Rightarrow$ Single $\hat{g}^2$ term not enough to reproduce behaviour



- $\frac{m_{N^+}}{3\pi T}$ displayed as a function of $\hat{g}^2(T) \equiv \left(\frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}} + \frac{4}{9\pi^2} \ln \left(2 \ln \frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}} \right) \right)^{-1}$

- Data is fitted to a polynomial in $\hat{g}$
- Free theory result: constant term set to 1 $\Rightarrow$ Single $\hat{g}^2$ term not enough to reproduce behaviour
- Leaving constant term free (with curvature) yields fit results compatible with 1



Conclusions

- Bulk of the nucleon screening mass is given by free theory contribution $3\pi T$
- Interactions induce $\sim$ 4% to $\sim$ 8% deviations, clearly visible with high statistical precision of few per-mil
- Chiral symmetry restoration: opposite parity channels degenerate within accuracy
- Single $\hat{g}^2$ term unable to explain results in this temperature range

Outlook

- Conclude analysis and publish results
- Investigate screening masses of non-static mesonic fields in the same temperature range

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Thank you for your attention!