The double-pole nature of the Λ(1405) from lattice QCD

Fernando Romero-López

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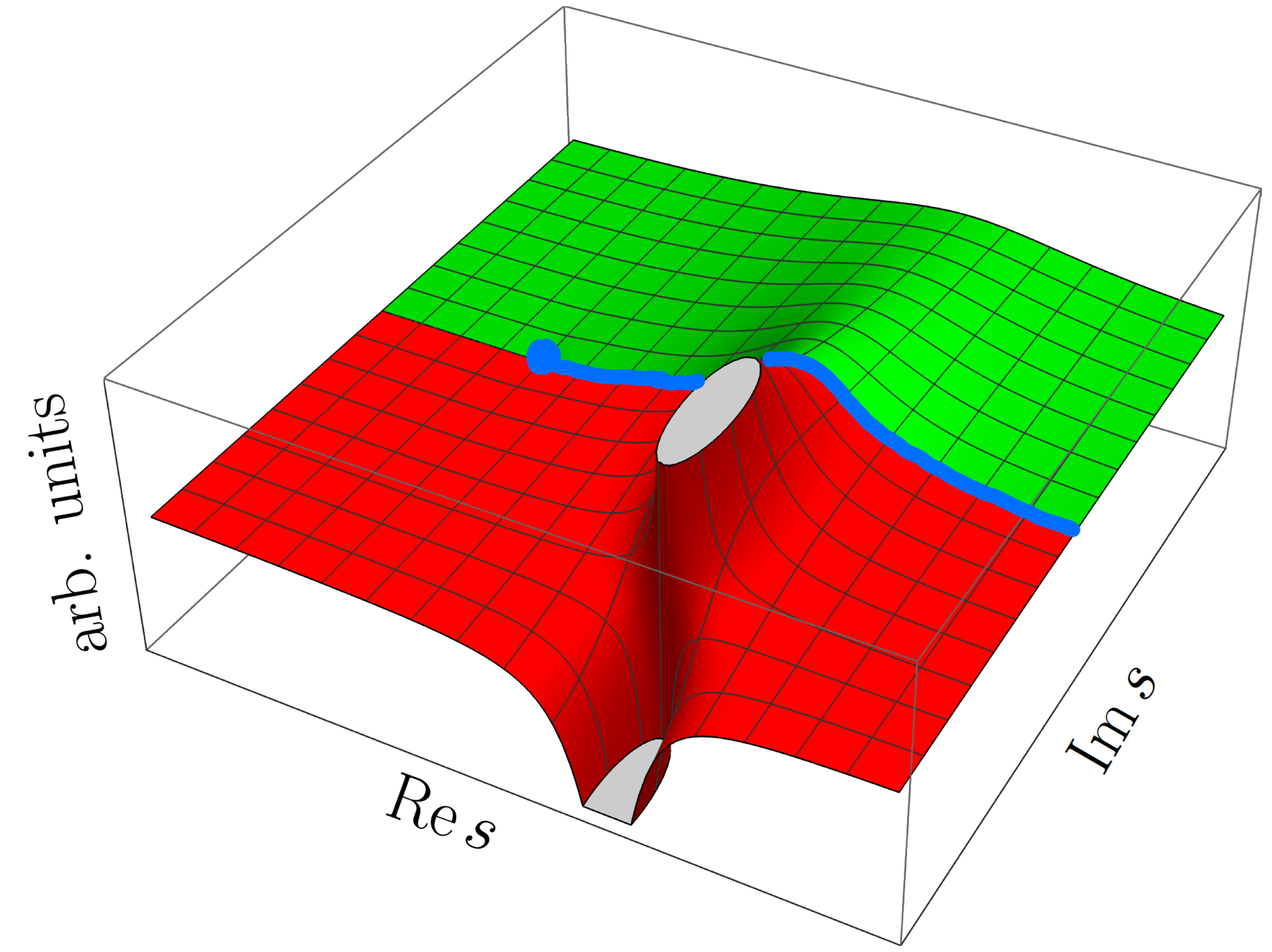
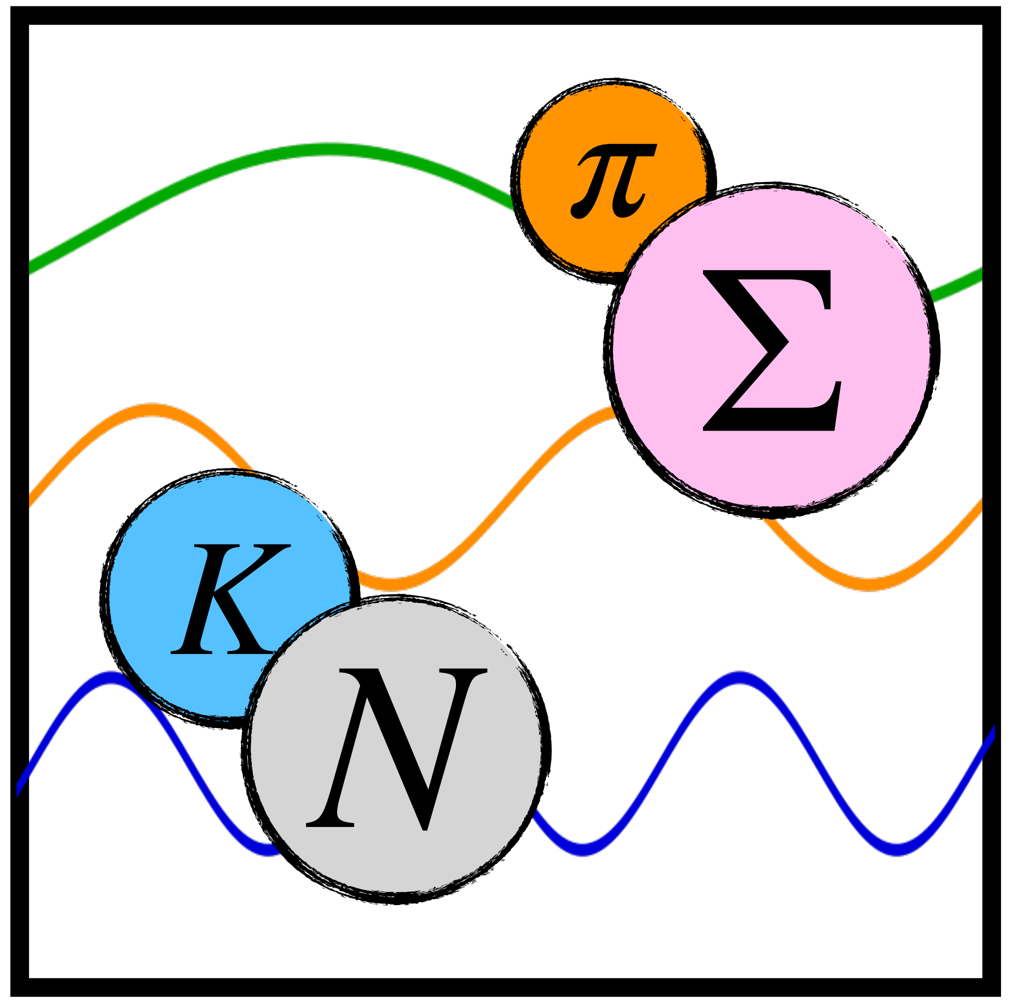
fernando@mit.edu





Acknowledgements

John Bulava,[1] Bárbara Cid-Mora,[2] Andrew D. Hanlon,[3] Ben Hörz,[4] Daniel Mohler,[5,2] Colin Morningstar,[6]
Joseph Moscoso,[7] Amy Nicholson,[7] Fernando Romero-López,[8] Sarah Skinner,[6] and André Walker-Loud[9]
(for the Baryon Scattering (BaSc) Collaboration)

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Based on:
[arXiv:2307.10413] (letter)
[arXiv:2307.13471] (long paper)

Andrew Hanlon (BNL)
[Plenary, Thursday 9.00 am]

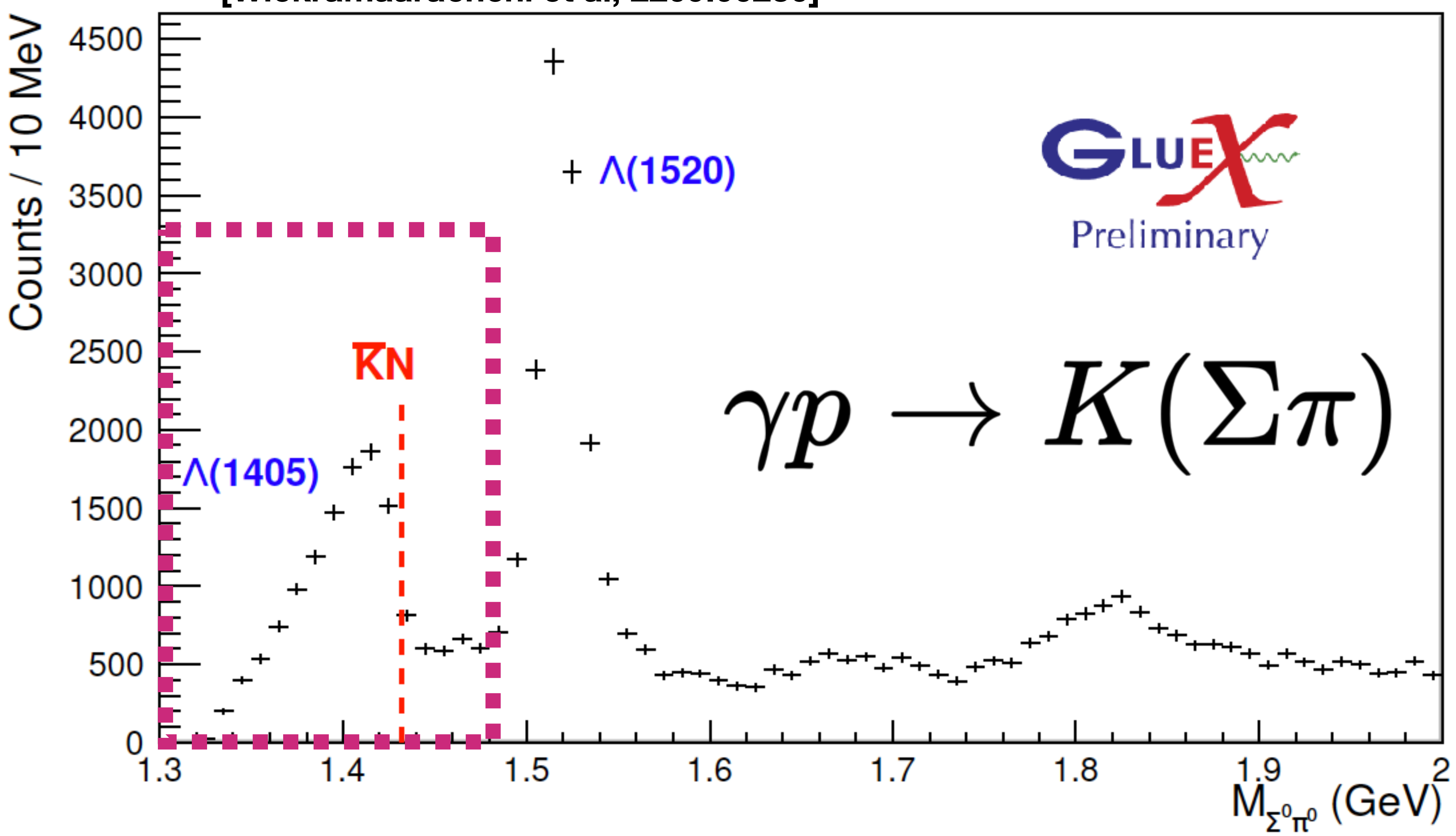
Sarah Skinner (CMU)
[Wednesday 10:20 am]

Bárbara Cid-Mora (GSI)
[Poster on Tuesday]

The Λ(1405)

- **Known since 1950s, still under investigation**
  [Dalitz, Tuan PRL 1959]

[Wickramaarachchi et al, 2209.06230]



- **Appears in coupled-channel scattering**

$$\begin{pmatrix} \pi\Sigma \to \pi\Sigma & \pi\Sigma \to Kp \\ Kp \to \pi\Sigma & Kp \to Kp \end{pmatrix}$$

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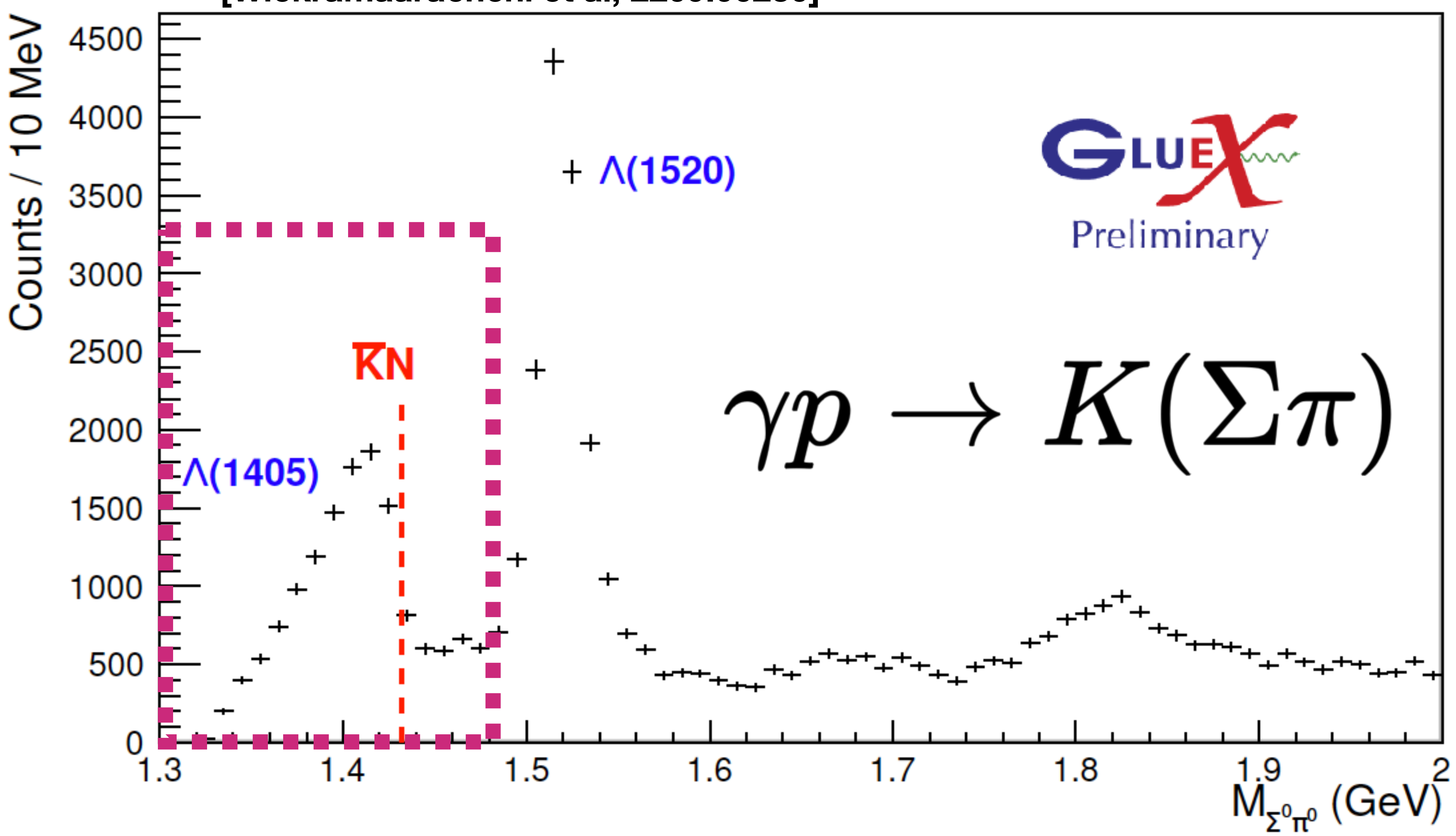

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- Latest PDG lists two resonances in the energy region

$\Lambda(1405)\ 1/2^-$ $I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗ 😀

$\Lambda(1380)\ 1/2^-$ $J^P = \frac{1}{2}^-$ Status: ∗∗ 😐

∗∗∗∗	Existence is certain.
∗∗∗	Existence is very likely.
∗∗	Evidence of existence is fair.
∗	Evidence of existence is poor.

One or two resonances?

The nature of the $\Lambda(1405)$ is a theoretical and experimental challenge

One or two resonances?

The nature of the $\Lambda(1405)$ is a theoretical and experimental challenge

Experiment

- Quantum numbers $J^P = 1/2^-$ @ CLAS
  [CLAS Collaboration, arXiv:1402.22967]

- Different CLAS analysis favor two poles:
  [Mai, Meißner, EPJA 2014] [Roca, Oset, PRC 2013]

- BGOOD & ALICE consistent with two poles
  [BGOOD, arXiv:2108.12235] [ALICE, arXiv:2205.15176]

- Preliminary GlueX analysis supports two poles
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- J-PARC consistent with one pole
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Theory

- Simple quark models predict one state
  [Isgur, Karl PRD 1987]
- Chiral Unitarity approach predicts two poles
  [Oller, Meißner, hep-ph/0402261] See review: [Mai, arXiv:2010.00056]
- Lattice QCD studies, but no coupled scattering
  [Gubler et al PRD 2016], [Menadue et al PRL 2012], [Engel et al PRD 2013], [Hall et al, PRL 2015] , [Takahashi, Oka, PRD 2010], [Talk by Kotaro Murakami]

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This talk!

Strategy

- Use D200 CLS ensemble, Stochastic LapH method, single and two-hadron operators

[Peardon et al, 0905.2160]

a[fm]	$(L/a)^3 \times T/a$
0.0633(4)(6)	$64^3 \times 128$

(Open BCs)

[Straßberger et al, 2112.06696]

$$m_\pi \simeq 200 \text{ MeV}$$

$$m_K \simeq 487 \text{ MeV}$$

Chiral trajectory: $2m_{ud} + m_s \simeq \text{const}$

$\Lambda(\boldsymbol{d}^2)$	Operators
$G_{1u}(0)$	$\Lambda[G_{1u}(0)]_0$
	$\Lambda[G_{1u}(0)]_1$
	$\Lambda[G_{1u}(0)]_2$
	$\Lambda[G_{1u}(0)]_3$
	$\bar{K}[A_{1u}(0)]_0 \; N[G_{1g}(0)]_0$
	$\pi[A_{1u}^-(0)]_0 \; \Sigma[G_{1g}(0)]_0$
	$\bar{K}[A_2(1)]_1 \; N[G_1(1)]_0$
	$\pi[A_2^-(1)]_1 \; \Sigma[G_1(1)]_0$

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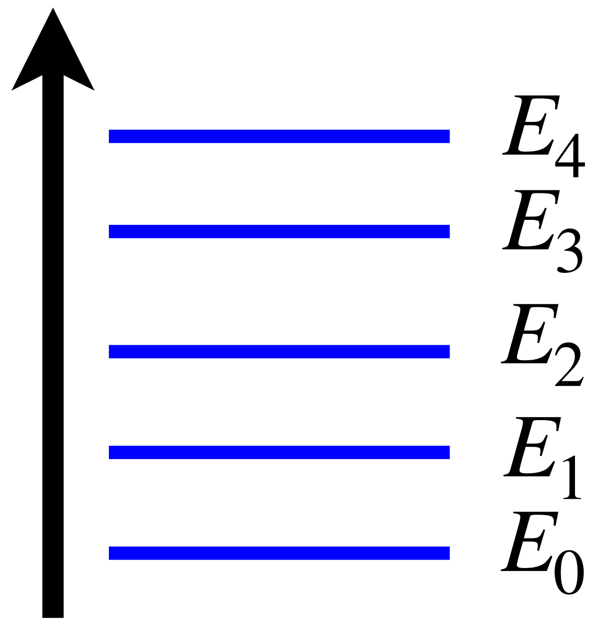
Spectrum



$\longrightarrow$ Quantization condition $\det_{\ell m}\left[\widetilde{K} + F^{-1}\right] = 0$ $\longrightarrow$ K-matrix $\widetilde{K}$ $\longrightarrow$ Resonance poles $\mathcal{T}^{-1}(E_{\text{pole}}) = 0$

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Energy determinations

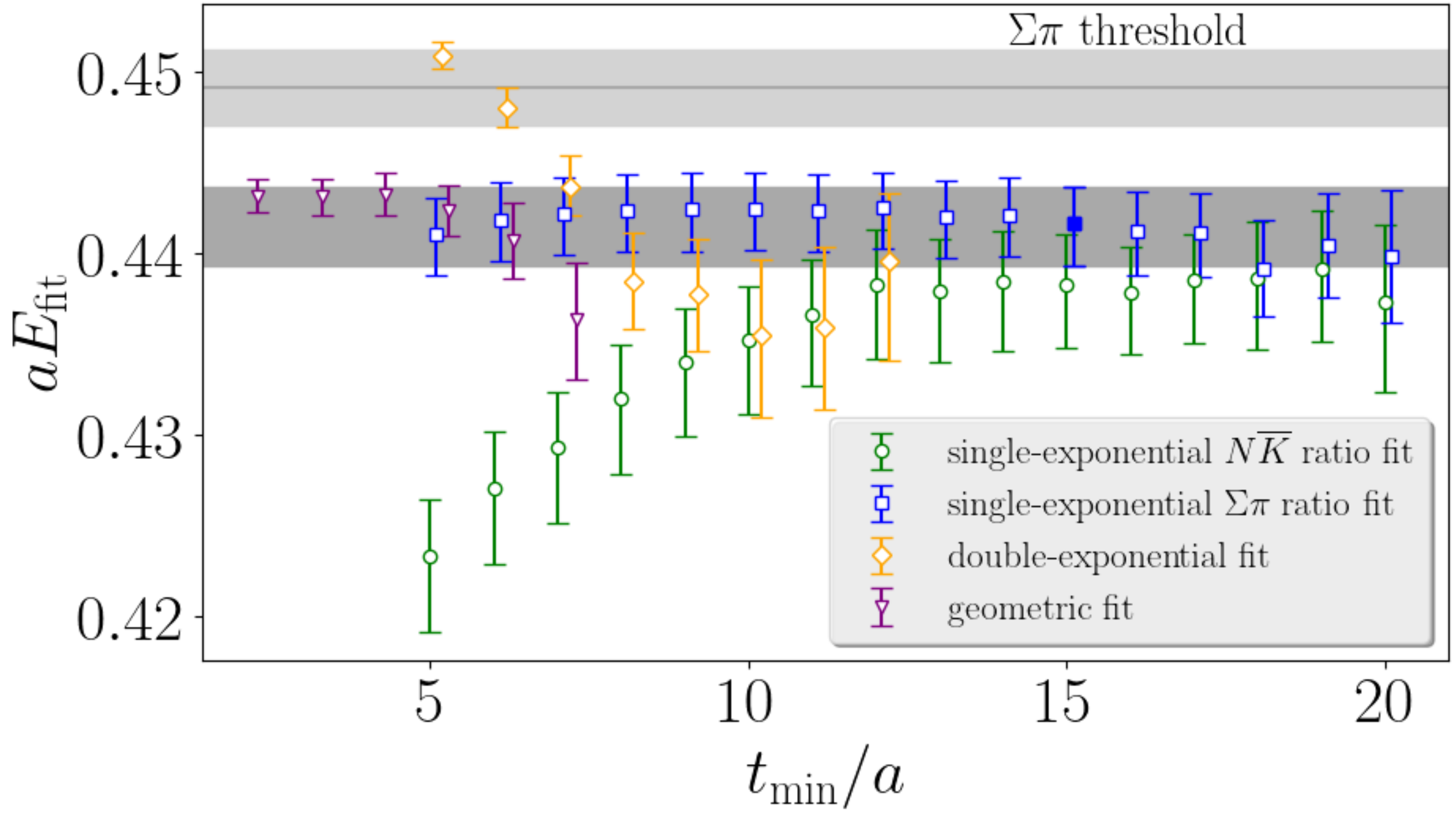
- **GEVP + Energy shift from ratio fits**

$$R_n(t) = \frac{C_{\text{meson}-\text{baryon}}(t)}{C_{\text{meson}}(t)C_{\text{baryon}}(t)}$$

  - Reduced uncertainties
  - Partial cancellation of (inelastic) excited states
  - But not positive definite

- **Look for consistency between methods**
  - Use different denominators in ratio
  - Check against multi-exp fits (non-ratio)

Energy as a function of fit range $t \in [t_{\text{min}}, t_{\text{max}} = 25]$



[More details in Poster by Bárbara Cid-Mora]

Amplitude analysis

- Use multi-channel two-particle scattering amplitude [Lüscher 89', ..., Briceño arXiv:1401.3312]
  - Keep only s waves, but checked the impact of higher partial waves

$$\det_{\ell m}\left[\begin{pmatrix} \widetilde{K}_{\pi\Sigma\to\pi\Sigma} & \widetilde{K}_{\pi\Sigma\to KN} \\ \widetilde{K}_{KN\to\pi\Sigma} & \widetilde{K}_{KN\to KN} \end{pmatrix} + \begin{pmatrix} F^{-1}_{\pi\Sigma}(E_n, \vec{P}, L) & 0 \\ 0 & F^{-1}_{KN}(E_n, \vec{P}, L) \end{pmatrix}\right] = 0$$

Multi-channel K-Matrix   Zeta function

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Multi-channel K-Matrix — Zeta function

- Test several parametrization for the K matrix and its inverse

distance to $\pi\Sigma$ threshold $\Delta_{\pi\Sigma} = \dfrac{E_{\rm cm}^2 - (m_\pi + m_\Sigma)^2}{(m_\pi + m_\Sigma)^2}$

Example 1: $\dfrac{m_\pi}{E_{\rm cm}}\widetilde{K}^{-1}_{ij} = \widetilde{A}_{ij} + \widetilde{B}_{ij}\Delta_{\pi\Sigma}$

Example 2: $\dfrac{E_{\rm cm}}{m_\pi}\widetilde{K}_{ij} = A_{ij} + B_{ij}\Delta_{\pi\Sigma}$

  - Flexible enough to accommodate none, one or two poles.

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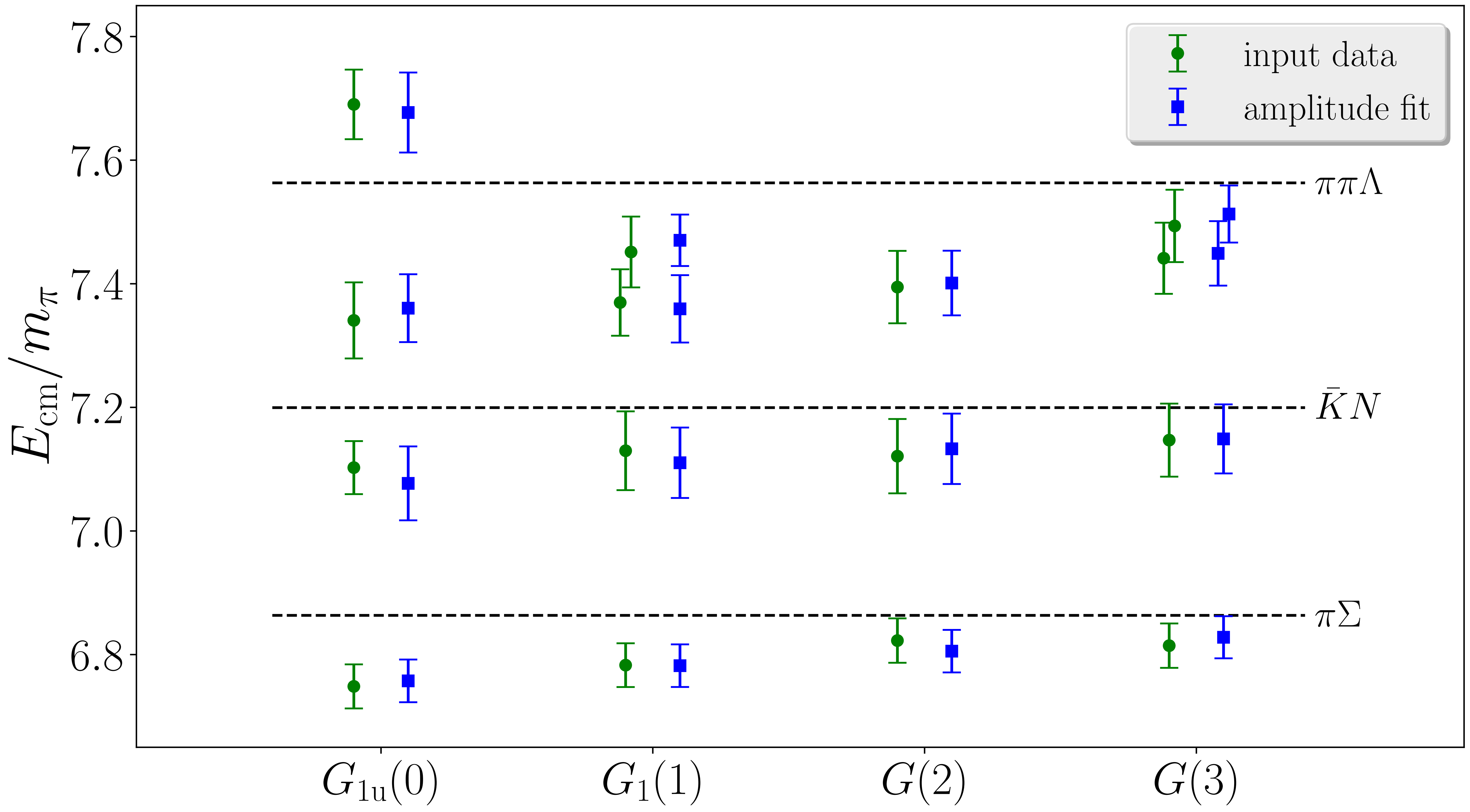
  - Flexible enough to accommodate none, one or two poles.

- Pole positions in the complex plane as vanishing eigenvalues in the inverse amplitude

$$\mathcal{T}^{-1}(E_{\rm pole}) = 0$$

Fitting the spectrum



finite-volume irreps

- **Fit shifts w.r.t. non-interacting energies**

$$\Delta E_i = E_{\rm cm}^{\rm latt} - E_{\rm cm}^{\rm free}$$

- **Minimize correlated χ^2 with residues**

$$\delta_i = \Delta E_{{\rm cm},i} - \Delta E_{{\rm cm},i}^{\rm QC}$$

- **Preferred fit based on lowest AIC**

Akaike Information Criterion (AIC) $= \chi^2 - 2\,{\rm dof}$

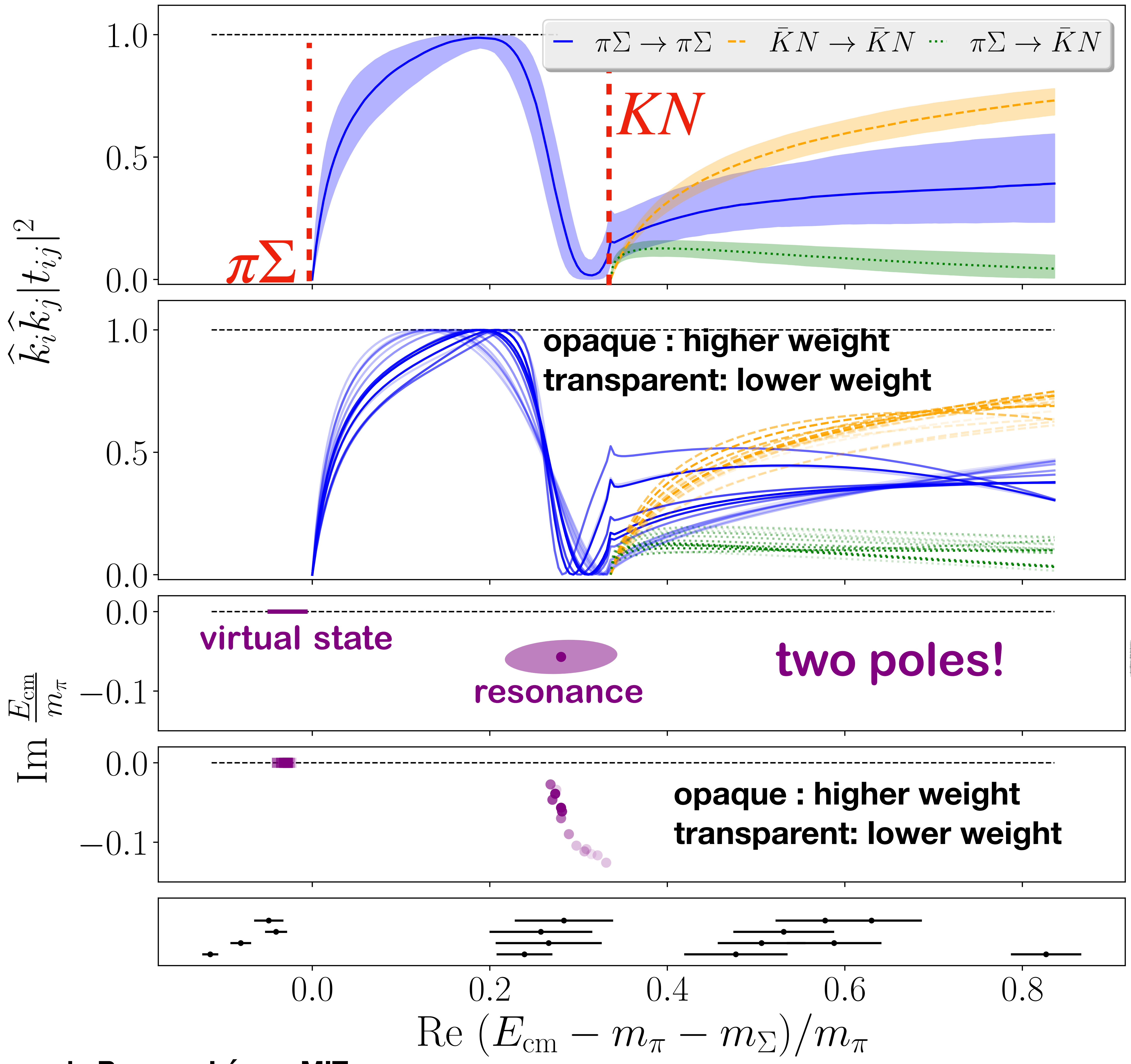
$$\frac{E_{\rm cm}}{m_\pi}\widetilde{K}_{ij} = A_{ij} + B_{ij}\Delta_{\pi\Sigma}$$

distance to $\pi\Sigma$ threshold

4 parameters $B_{11} = B_{00} = 0$ (fixed)

15 energies $\chi^2/{\rm dof} = 0.96$

Amplitudes and poles



- Scattering amplitudes for "preferred" fit
  i.e. with lowest $\mathrm{AIC} = \chi^2 - 2\,\mathrm{dof}$
- Scattering amplitudes for different parametrizations
- Pole positions for "preferred" fit
- Pole positions for for different parametrization
  All find two poles!
- Lattice QCD energies used in fits

Double-pole picture

Two poles with $(\text{sign Im}\ k_{\pi\Sigma}, \text{sign Im}\ k_{KN}) = (-,+)$

Virtual bound state

$$E_1 = 1392(9)_{\text{stat}}(2)_{\text{model}}(16)_a\ \text{MeV}$$

$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\text{stat}}(6)_{\text{model}}$$

Stronger coupling to $\pi\Sigma$

ratio of residues of the pole

Resonance pole

$$E_2 = [1455(13)_{\text{stat}}(2)_{\text{model}}(17)_a - i \times 11.5(4.4)_{\text{stat}}(4.0)_{\text{model}}(0.1)_a]\ \text{MeV}$$

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 0.53(9)_{\text{stat}}(10)_{\text{model}}$$

Stronger coupling to KN

Fernando Romero-López, MIT

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Stronger coupling to KN

- ☑ Qualitative agreement with chiral approaches [See PDG, section 83]

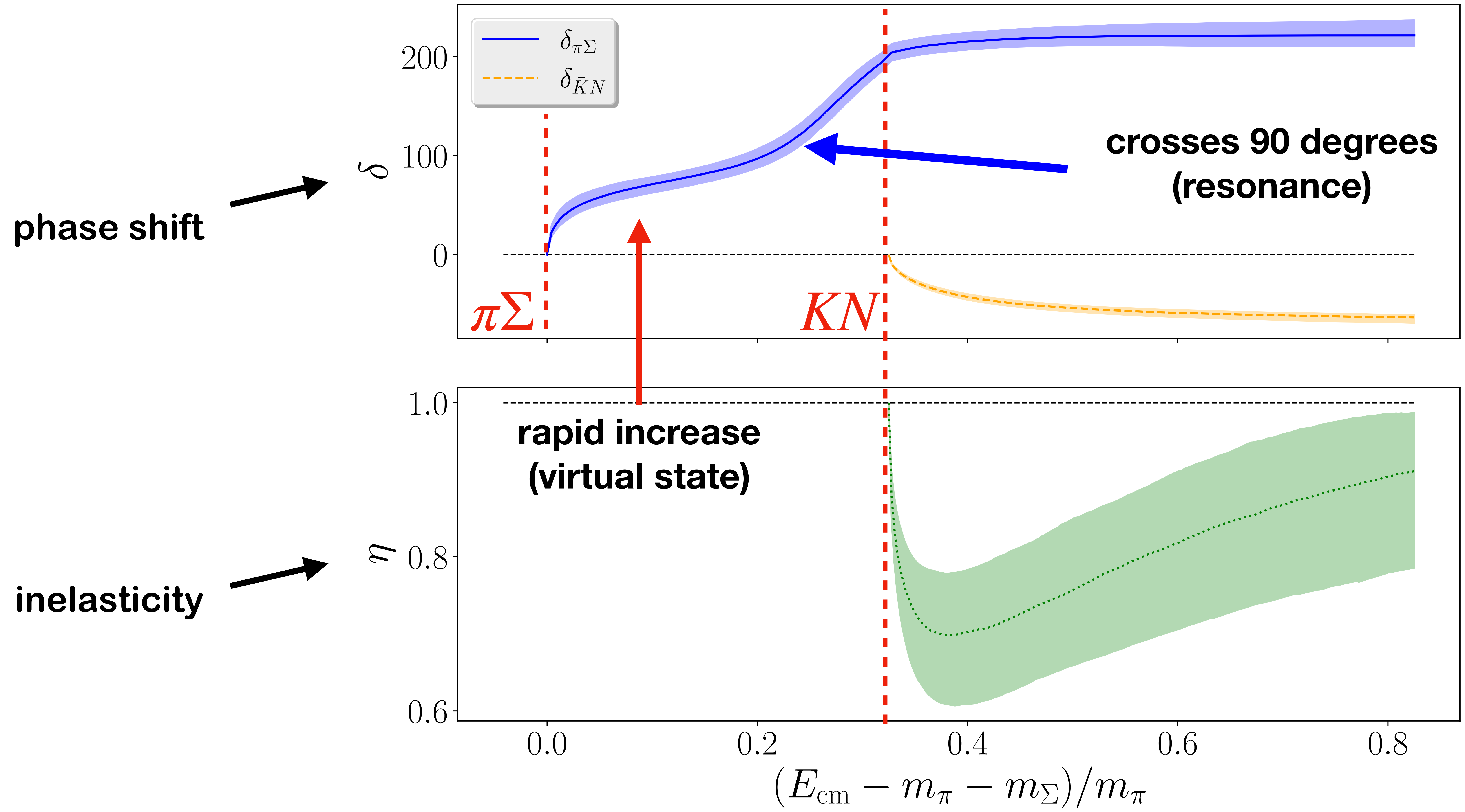
$$\text{Re } E_1 = 1325 - 1380 \text{ MeV}$$

$$\text{Re } E_2 = 1421 - 1434 \text{ MeV}$$

- Poles are at slightly larger energies
- Lower pole on the real axis
  - Unphysical pion mass effect?

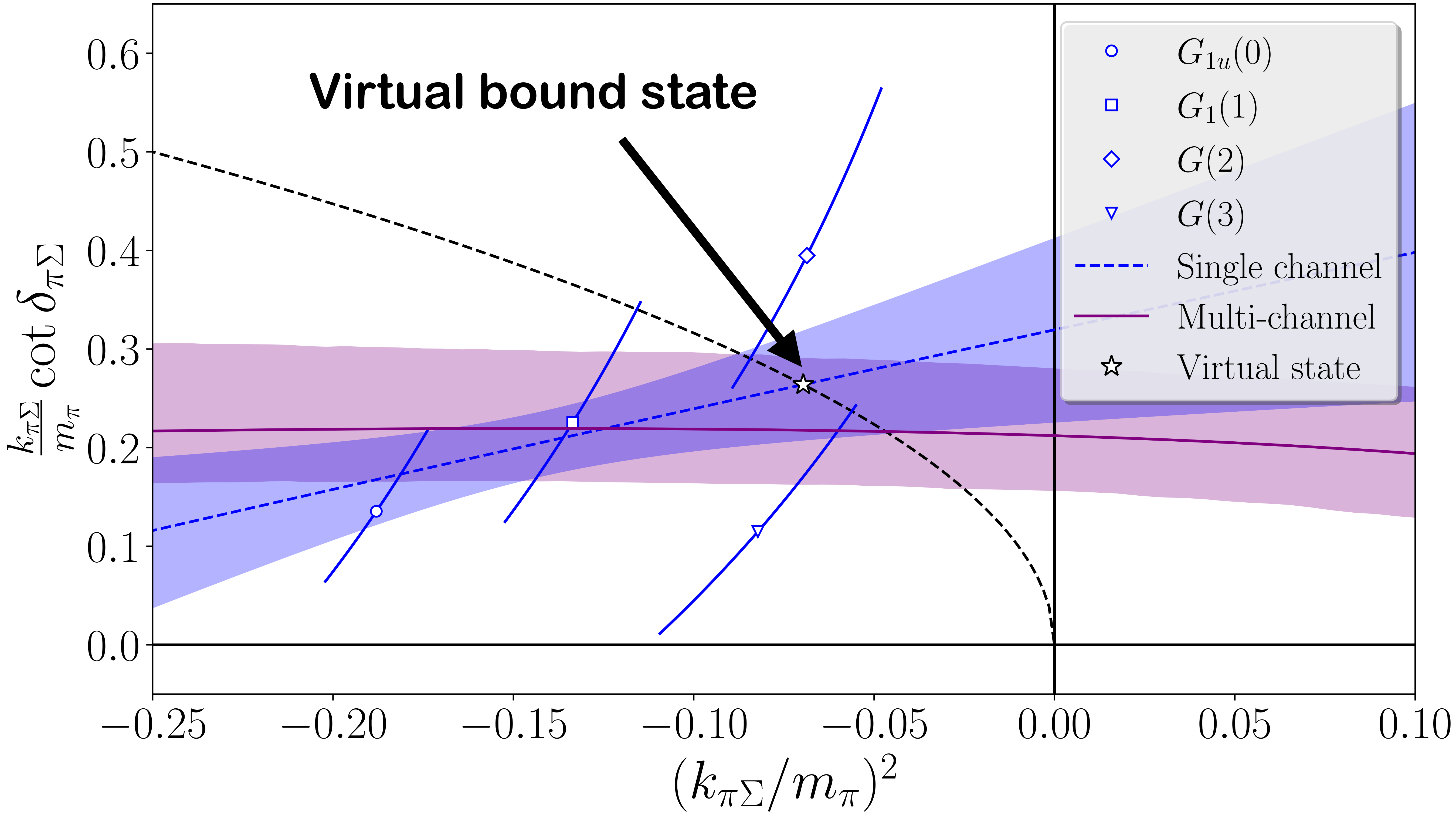
Phase and inelasticity

- Phase shifts and inelasticities provide an alternative visualization of the results:



Single-channel analysis

- Single-channel Luscher formalism valid around the $\pi\Sigma$ threshold
- ☑ Agreement with multi-channel analysis
- ☑ Supports existence of lower pole



Conclusion & Outlook

- ☑ First Lattice QCD study of coupled $\pi\Sigma - NK$ scattering in the $\Lambda(1405)$ energy region

- ☑ We find that at $m_\pi \sim 200$ MeV there is a virtual bound state and a resonance

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- ☑ Two-pole picture remains robust under variations of the parametrization of the amplitudes
  - Every used parametrization finds two poles

- ☑ Qualitative picture in agreement with chiral unitarity models.
  - Unphysical quark mass effect: poles at larger energies

- ○ Outlook: physical quark masses, discretization effects, other meson-baryon resonances

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Thanks!

Back-up

Other fits family 1 & 2

1. An effective range expansion (ERE) of the form

$$\widetilde{K}_{ij} = \frac{m_\pi}{E_{\rm cm}}\Big(A_{ij} + B_{ij}\Delta_{\pi\Sigma}(E_{\rm cm})\Big). \qquad (13)$$

2. A variation of the first parametrization without the factor of $m_\pi/E_{\rm cm}$:

$$\widetilde{K}_{ij} = \widehat{A}_{ij} + \widehat{B}_{ij}\Delta_{\pi\Sigma}(E_{\rm cm}). \qquad (14)$$

TABLE VIII. Fit results for $\widetilde{K}$ parametrization class 1 shown in Eq. (13). Errors are propagated through the derivative method. Empty entries indicate parameters set to zero in a fit. AIC refers to Akaike Information Criterion.

Fit	A_{00}	A_{11}	A_{01}	B_{00}	B_{11}	B_{01}	χ^2/dof	AIC
a	1.5(1.4)	−8.78(72)	8.30(65)				15.68/(15 − 3)	−8.32
b	4.1(1.2)	−10.5(1.1)	10.3(1.3)			−29(15)	10.52/(15 − 4)	−11.48
c	2.3(1.3)	−8.62(58)	7.60(80)		−18(11)		12.29/(15 − 4)	−9.71
d	15.1(5.3)	−11.8(1.3)	7.6(1.3)	−56(19)			11.48/(15 − 4)	−10.52
e	9.6(6.2)	−12.7(3.4)	11.1(2.8)	−23(26)	18(31)	−37(29)	9.70/(15 − 6)	−8.30

TABLE IX. Fit results for $\widetilde{K}$ parametrization class 2 shown in Eq. (14). Errors are propagated through the derivative method. Empty entries indicate parameters set to zero in a fit. AIC refers to Akaike Information Criterion.

Fit	$\widehat{A}_{00}$	$\widehat{A}_{11}$	$\widehat{A}_{01}$	$\widehat{B}_{00}$	$\widehat{B}_{11}$	$\widehat{B}_{01}$	χ^2/dof	AIC
a	0.16(19)	−1.229(91)	1.140(88)				15.44/(15 − 3)	−8.56
b	0.52(18)	−1.45(15)	1.42(18)			−3.9(2.0)	10.73/(15 − 4)	−11.27

Other fits family 3 & 4

3. An ERE of $\widetilde{K}^{-1}$ of the form

$$\widetilde{K}^{-1}_{ij} = \frac{E_{\rm cm}}{m_\pi}\left(\widetilde{A}_{ij} + \widetilde{B}_{ij}\Delta_{\pi\Sigma}(E_{\rm cm})\right). \tag{15}$$

4. A Blatt-Biederharn [84] parametrization:

$$\widetilde{K} = C\ F\ C^{-1}, \tag{16}$$

where

$$C = \begin{pmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \end{pmatrix}, \tag{17}$$

$$F = \begin{pmatrix} f_0(E_{\rm cm}) & 0 \\ 0 & f_1(E_{\rm cm}) \end{pmatrix}, \tag{18}$$

and

$$f_i(E_{\rm cm}) = \frac{m_\pi}{E_{\rm cm}}\frac{a_i + b_i\Delta_{\pi\Sigma}(E_{\rm cm})}{1 + c_i\Delta_{\pi\Sigma}(E_{\rm cm})}. \tag{19}$$

TABLE X. Fit results for $\widetilde{K}$ parametrization class 3 shown in Eq. (15). Errors are propagated through the derivative method. Empty entries indicate parameters set to zero in a fit. AIC refers to Akaike Information Criterion.

Fit	$\widetilde{A}_{00}$	$\widetilde{A}_{11}$	$\widetilde{A}_{01}$	$\widetilde{B}_{00}$	$\widetilde{B}_{11}$	$\widetilde{B}_{01}$	χ^2/dof	AIC
a	0.092(21)	−0.036(15)	0.082(20)	0.28(15)			11.73/(15 − 4)	−10.27
b	0.114(25)	−0.041(24)	0.096(19)		0.19(16)		14.57/(15 − 4)	−7.43
c	0.137(33)	−0.019(14)	0.119(21)			−0.142(85)	13.10/(15 − 4)	−8.90

TABLE XI. Fit results for $\widetilde{K}$ parametrization class 4 shown in Eq. (16). Errors are propagated through the derivative method. Empty entries indicate parameters set to zero in a fit. AIC refers to Akaike Information Criterion.

Fit	a_0	a_1	b_0	b_1	c_0	c_1	ϵ	χ^2/dof	AIC
a	5.7(1.2)	−11.4(1.2)		−27(15)			0.451(56)	13.27/(15 − 4)	−8.73
b	13.7(4.1)	−14.06(86)	−37(17)				0.349(75)	10.63/(15 − 4)	−11.37
c	5.8(1.2)	−11.8(1.1)				−1.62(95)	0.468(48)	13.54/(15 − 4)	−8.46
d	12.2(3.4)	−14.06(87)			5.8(3.2)		0.360(82)	11.13/(15 − 4)	−10.87

Higher partial waves

- Check for effect of higher partial waves using leves in nontrivial irreps
- Parametrize p-wave K-matrix with simple form

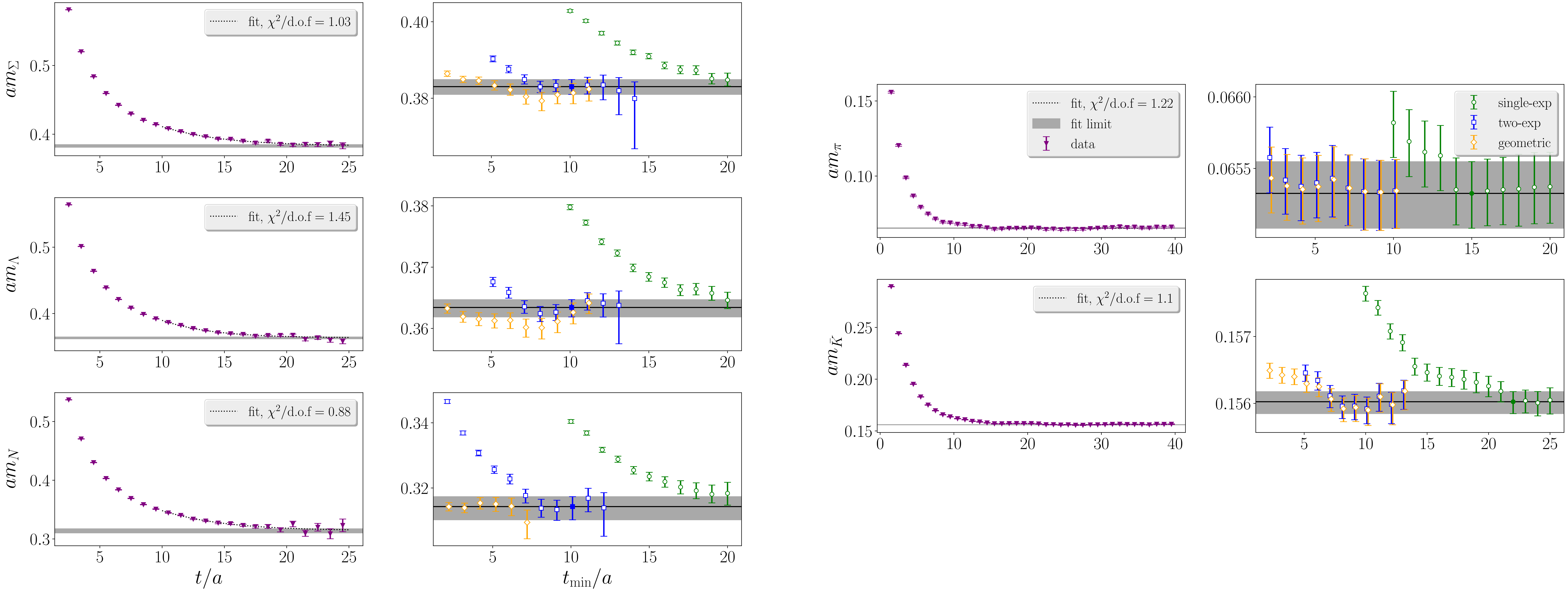
$$\tilde{K}^{J^P} = \mathrm{diag}\left(A_{00}^{J^P}, A_{11}^{J^P}\right).$$

- Impact on s-wave parameters is negligible

TABLE XII. Fit results for $\widetilde{K}$ parametrization class 1 shown in Eq. (13) for the $J^P = 1/2^-$ wave, and Eq. (32) for the $J^P = 1/2^+, 3/2^+$ waves using $\ell_{\max} = 1$. Errors are propagated through the derivative method. Empty entries indicate parameters set to zero in a fit. AIC refers to Akaike Information Criterion.

J^P partial waves	A_{00}	A_{11}	A_{01}	B_{01}	$A_{00}^{1/2^+}$	$A_{11}^{1/2^+}$	$A_{00}^{3/2^+}$	$A_{11}^{3/2^+}$	χ^2/dof	AIC
$1/2^-$	4.1(1.2)	−10.5(1.1)	10.3(1.3)	−29(15)					10.52/(15-4)	−11.48
$1/2^-$ and $1/2^+$	4.1(1.2)	−10.5(1.1)	10.3(1.3)	−30(15)	0.0088(39)	0.031(15)			10.52/(17-6)	−11.48
$1/2^-$ and $3/2^+$	4.1(1.1)	−10.9(1.1)	10.4(1.3)	−32(15)			0.0172(48)	0.0218(48)	14.10/(21-6)	−15.90

Single hadron masses



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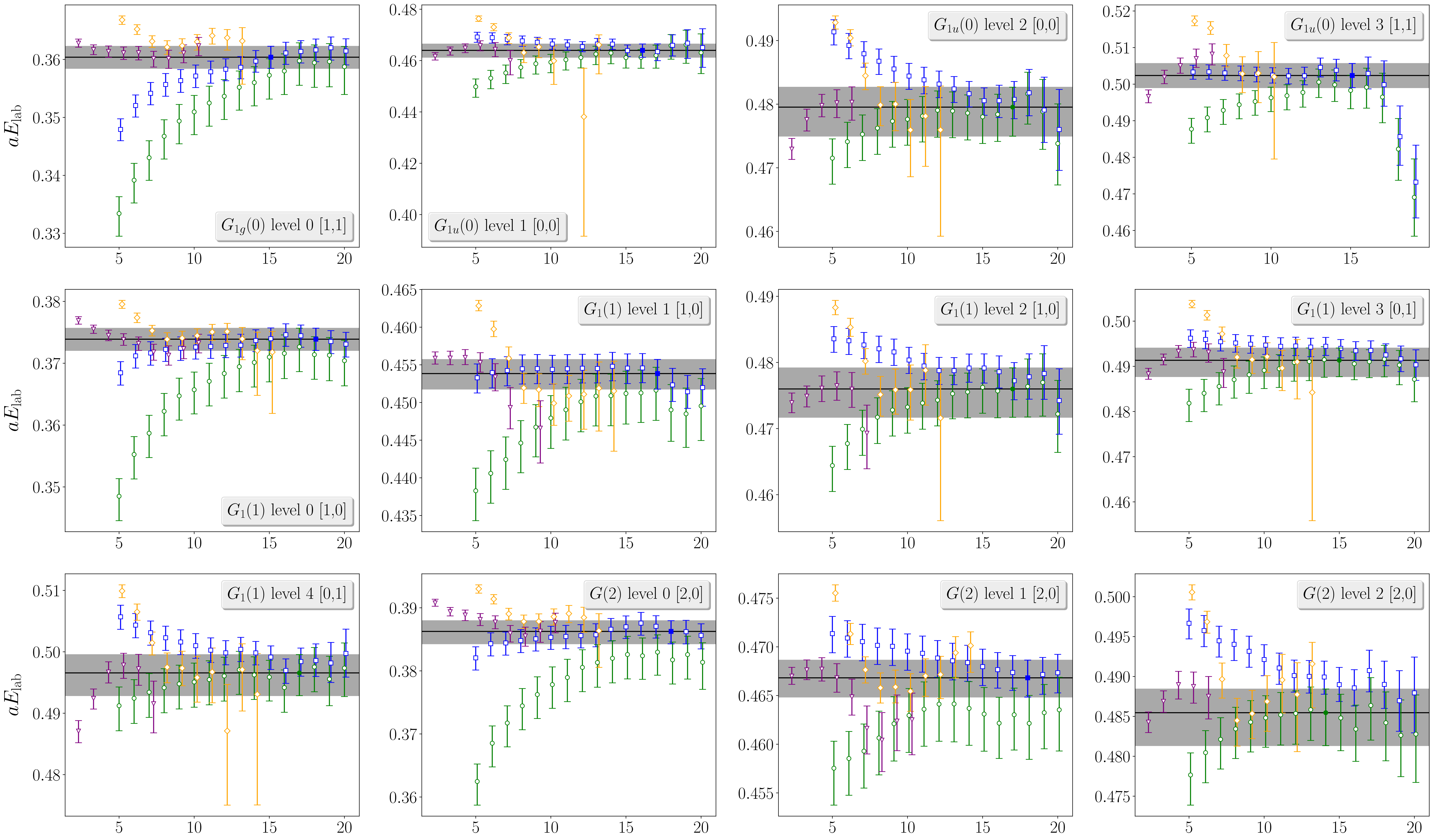
Some t-min plots



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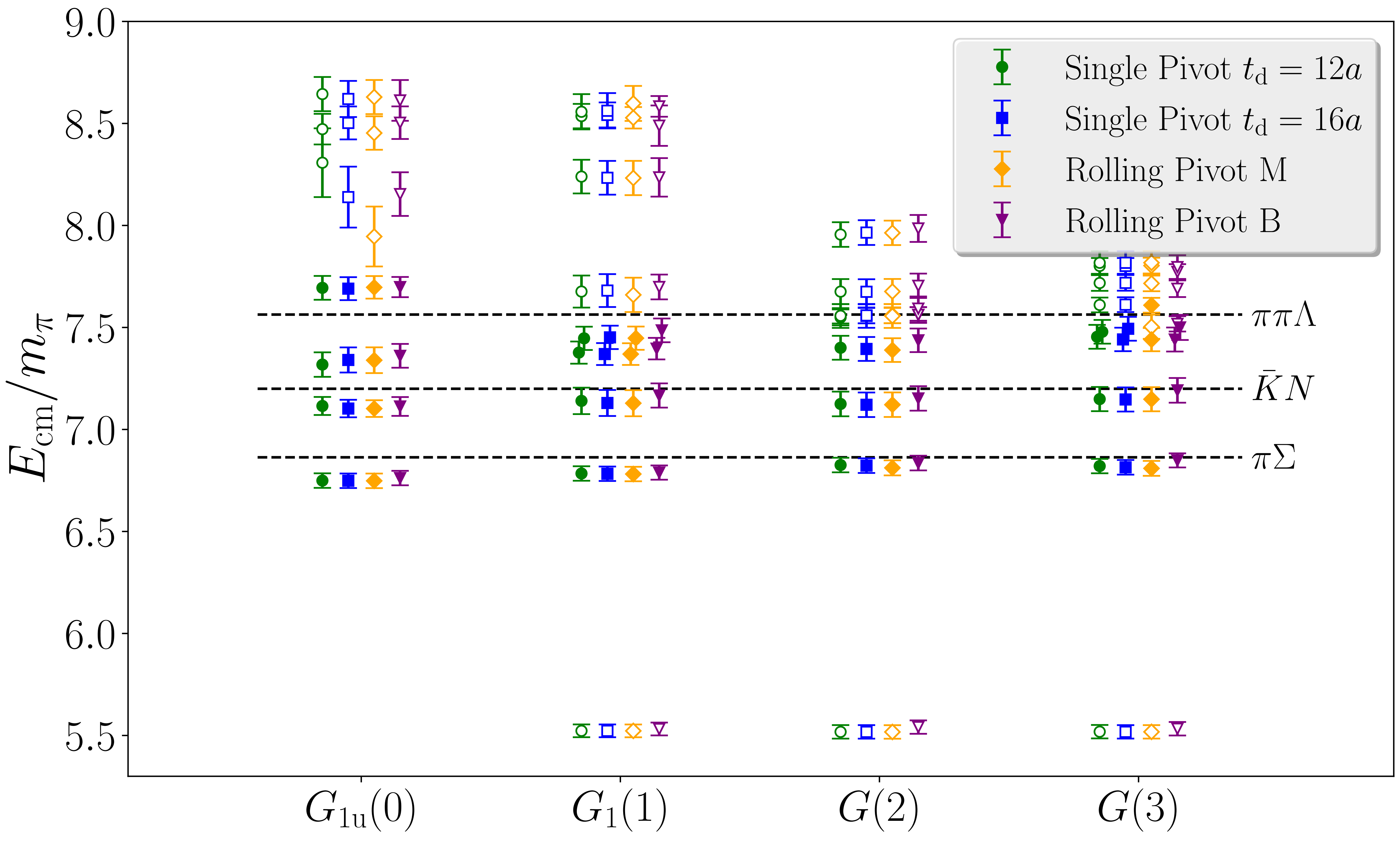
GEVP stability



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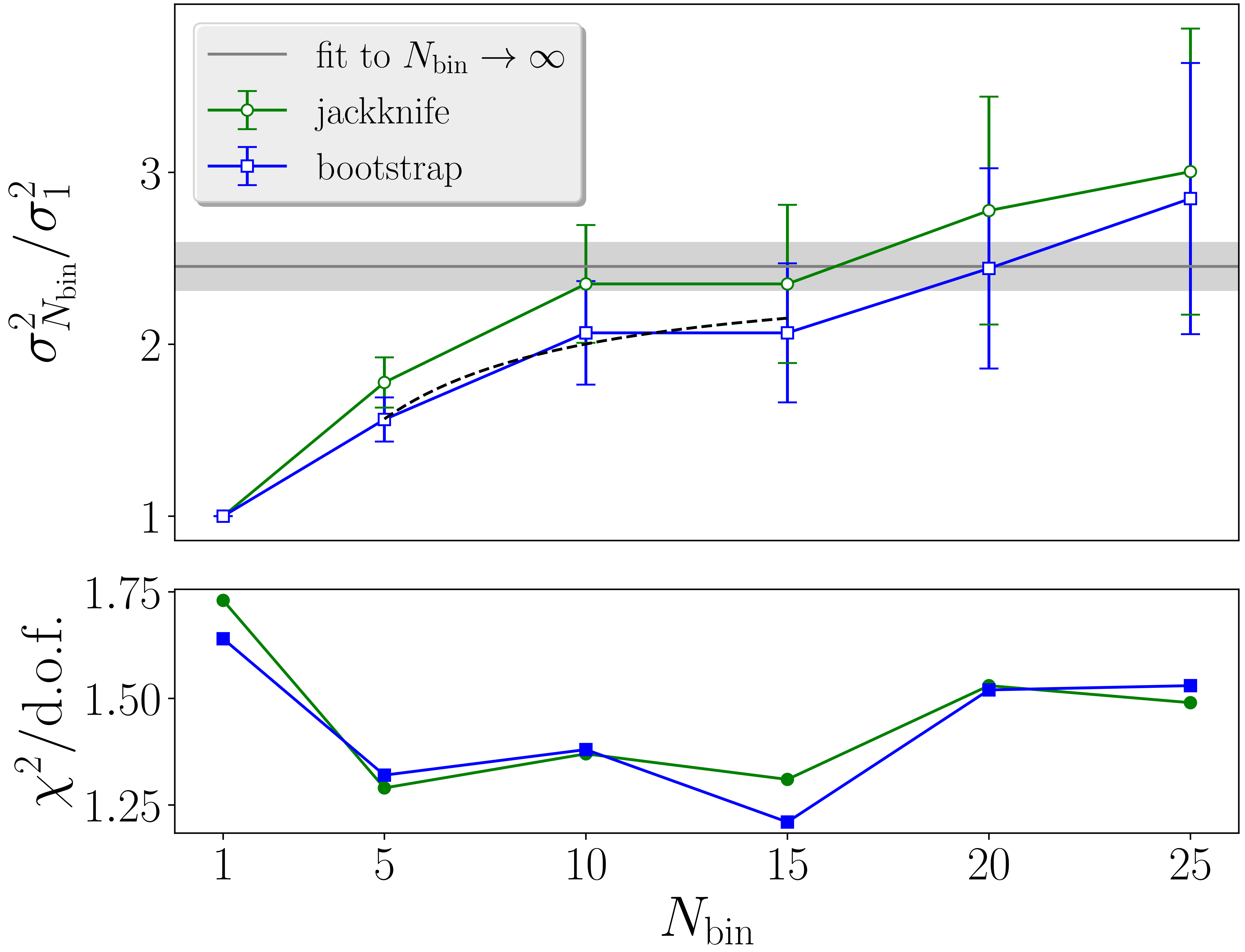
Rebin analysis



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