Mass effects on the QCD $\beta$-function

M. Costa¹, D. Gavriel¹, H. Panagopoulos³, G. Spanoudes¹
¹Department of Physics, University of Cyprus, 1 Panepistimiou Avenue, 2109 Aglantzia, Cyprus, *Speaker

Abstract
In this study we present lattice results on the QCD $\beta$-function in the presence of quark masses. The $\beta$-function is calculated to three loops in perturbation theory and for improved lattice actions; it is extracted from the renormalization of the coupling constant $Z_g$. The background field method is used to compute $Z_g$, where it is simply related to the background gluon field renormalization constant $Z_0$. We focus on the mass effects in the background gluon propagator, the dependence of the QCD $\beta$-function on the number of colors $N_c$, the number of fermionic flavors $N_f$ and the quark masses, shown explicitly. The perturbative results of the QCD $\beta$-function will be applied to the precise determination of the strong coupling constant, calculated by Monte Carlo simulations removing the mass effects from the nonperturbative Green's functions.

Introduction - Motivation
The renormalized $\beta$-function describes the dependence of the renormalized coupling constant $g$ on the scale inherent in the renormalization scheme [1] (chosen in this work to be the MS scheme). It encodes the underlying dynamics of QCD from low to high momentum regions. Nonperturbative estimations of the strong coupling $\alpha_s$ in several renormalization schemes, through numerical simulations of the corresponding lattice theories, are being studied by a number of groups at present (See Refs. [2, 3], and references therein). The three-loop bare QCD $\beta$-function [4] can be extracted from the two-loop calculation of the renormalization factor $Z_g$, which relates the bare running coupling $\alpha_0$ to the MS renormalized running coupling $\alpha_{\text{MS}}$ (both are MS renormalized scale $\alpha$ and $\lambda$ are the lattice spacing) through:

$$\alpha_0 = Z_g^2(\alpha_0, \mu) \times \alpha_{\text{MS}} \lambda$$

The inclusion of the quark masses makes this calculation even more complicated [5]. Note that we are interested in the discretization errors proportional to the quark mass ($O(a m)$) effects) on the $\beta$-function. For simplicity of notation, we denote all flavor masses by $m$; the case of different flavor masses can be trivially recovered from our results. This new direction is very important due to the fact that the $O(a m)$ effects will be removed from the nonperturbative Green's functions entering the strong coupling, allowing for a more precise determination. Removing $O(a m)$ effects will improve important any quantity that is calculated using Wilson-type fermions [6].

Computational setup and methods used to calculate $Z_g$

The renormalized $\beta$-function and the bare $\beta$-function on the lattice ($\beta_0(y)$) are defined:

$$\beta(\alpha_0) = -\frac{d \ln Z_\beta}{d \ln \alpha_0} \bigg|_{\alpha_0} \quad \beta(y) = -\frac{d \ln Z_\beta}{d \ln y} \bigg|_{\alpha_0}$$

In the asymptotic limit, one can write the expansion of Eq. (2) in powers of $g_0$:

$$\beta(y) = -2g_0 - 2\ln g_0 - \frac{g_0^2}{3} - \frac{g_0^3}{5} - \cdots - (3)$$

The coefficients $b_k$ in the universal expansion of logarithm independent for the massless case; $b_k$ (which is $(2 \geq k)$) has to be calculated perturbatively. $\beta(y)$ and $\beta_0(y)$ can be related using the renormalization function $Z_\beta$, and:

$$\beta(y) = \left( 1 - \frac{2}{3} \ln \frac{Z_\beta(\alpha_0, \mu)}{\alpha_0} \right) \beta_0(y)$$

We employ the clover action for fermions and the Symanzik improved action for gluons.

The most convenient and economical way to proceed with the calculation of $Z_g(q, \mu)$ is to use the Background Field (BF) technique [7, 8], in which the relation is:

$$Z_g(q, \mu) = Z_0(q, \mu)$$

$Z_0$ is the MS renormalization factor. In the lattice version of the BF technique, the link variable takes the form: $U_i(x) = e^{-i q \cdot x} Q_0(x)$, where $Q_0$ is the quantum field, $A_0$ is the background field. In this framework, instead of calculating $Z_0$, it suffices to compute $Z_0$. For the above lattice calculation, we consider the 2-pt BF 1PI Green's function ($A_{(x)} A_{(y)}$), which is associated with the 2-point Green's function ($A_{(x)} A_{(y)}$)) is the sum of these two Feynman diagrams:

One-loop Results

$\beta_0(y)$ and $\beta(y)$ are connected through the following relation:

$$\frac{d \ln Z_\beta}{d \ln y} \bigg|_{\alpha_0} = \beta(y) = \beta_0(y) + \frac{1}{\alpha_0} \frac{d \ln Z_\beta}{d \ln \alpha_0} \bigg|_{\alpha_0} \quad (\alpha_0 = Z_\beta(\alpha_0, \mu) \alpha_{\text{MS}} \lambda)$$

Large-mass expansion of the background field coupling

At one loop order, expressing $g_B(q, \mu, m)$ in terms of renormalized quantities ($O(a m) = \mu = 0$), and taking the limit $\lambda \rightarrow \infty$, we get:

$$\lim_{\lambda \rightarrow \infty} g_B(q, m) = \frac{4\pi}{q^2 + M(q, m)}$$

Where $M(q, m)$ plays the role of the logarithmic mass dependence of the heavy quarks in the continuum limit.

Two-loop Calculations

The calculation of the two-loop Feynman diagrams is currently underway.

Two-loop results of the 2-pt lattice Green's function is the sum of the following two Feynman diagrams:

Since we are interested in the $O(a m)$ corrections, we use the relation for the level fermion propagator in momentum space: $O(q^2) = \frac{1}{q^2 + M(q, 0)} + O(a^2 m^2)$)

One-loop results

$$A_{(x)} A_{(y)} = g_0 \left( \mathbb{1} + \frac{1}{12} O(g_0^2) \right)$$

$$g_B(q, m) = \frac{g_0^2 f_0(q, m) + f_0(q, m) + f_0(q, m) + f_0(q, m)}{f_0(q, m) + f_0(q, m) + f_0(q, m) + f_0(q, m)}$$

$$\frac{d \ln Z_\beta}{d \ln y} \bigg|_{\alpha_0} = \beta(y) = \beta_0(y) + \frac{1}{\alpha_0} \frac{d \ln Z_\beta}{d \ln \alpha_0} \bigg|_{\alpha_0} \quad (\alpha_0 = Z_\beta(\alpha_0, \mu) \alpha_{\text{MS}} \lambda)$$

References