Unveiling Generalized Parton Distributions through the Pseudo-Distribution Approach

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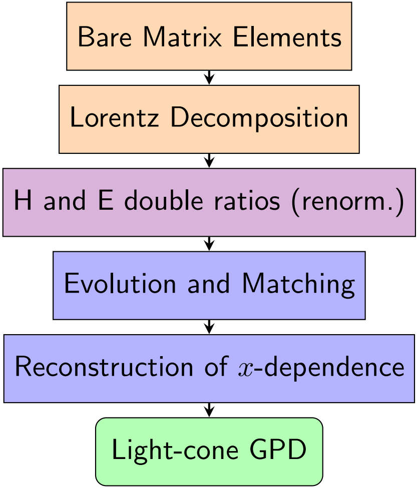
Motivation: Understanding the internal structure of nucleons is crucial to comprehend the strong interaction in Quantum Chromodynamics (QCD).

The cool stuff you can do with Generalized Parton Distributions (GPDs):

- Insights into the spatial distribution of partons within nucleons (3D image).
- Allows us to quantify the mechanical properties of hadrons.
- GPDs allow us to compute form factors!
- GPDs are generalizations of PDFs. (At least for the H-GPD).
- GPDs give us spin information (Sum rules)

Pseudo[1] Workmap to GPDs

Introduction



Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks

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Continuum limit of parton distribution functions from the pseudo-distribution approach on the lattice

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This work:

Combine, and apply!

[1] A. Radyushkin (2017)

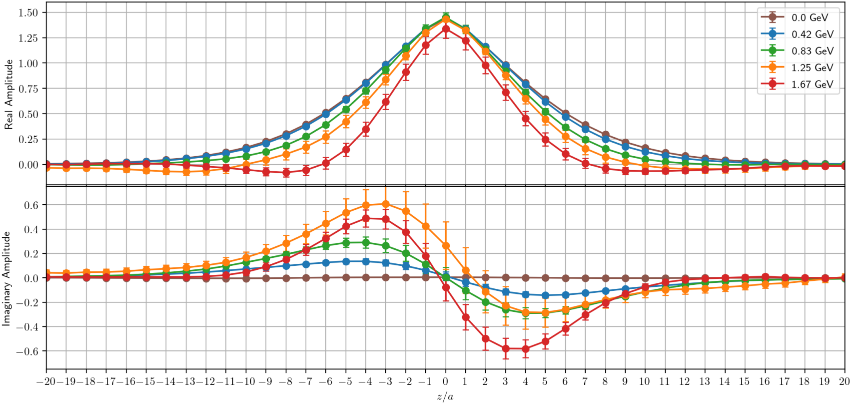
- Fermions: $N_f = 2 + 1 + 1$ twisted mass fermions + clover term
- Lattice spacing: $a \approx 0.093\,\mathrm{fm}$
- Lattice dimensions: $64 \cdot 32^3$
- Unphysical pion mass: $260\,\mathrm{MeV}$
- One momentum transfer $t = -0.64\,\mathrm{GeV}^2$
- Zero skewness $\xi = 0$
- $u - d$ isovector combination

P_3 [GeV]	N_{confs}	$N_{kinematics}$	$N_{sourcepos.}$	N_{meas}
0.0	404	1	8	3,232
0.42	100	8	8	6,400
0.83	100	8	8	6,400
1.25	269	8	8	17,216
1.67	404	8	32	103,424

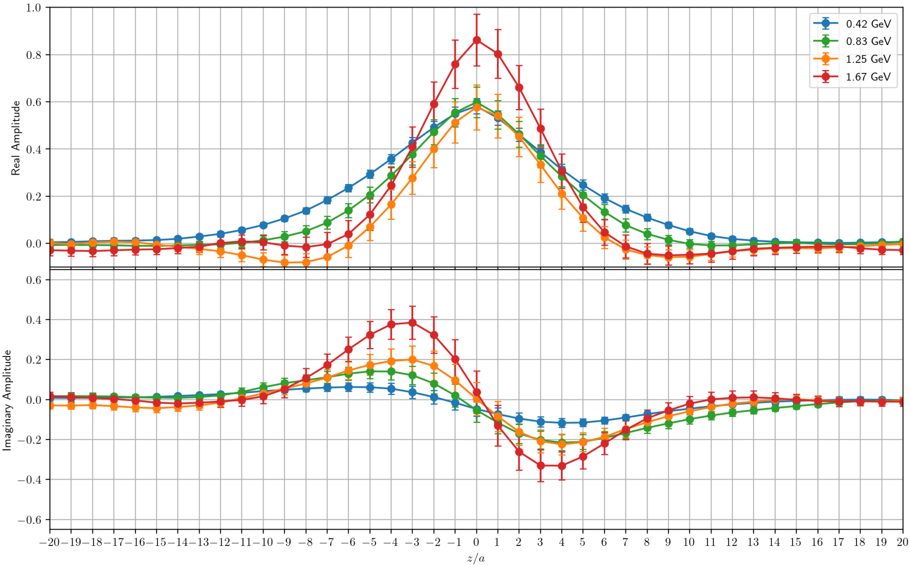
Bare matrix elements with $t = 0$ (Unpolarized PDF)

General form of the matrix elements:

$$h^{\mu}(\Gamma_{\kappa}, z, p_f, p_i) = \langle N(p_f)|\bar{\psi}(z)\gamma_j W(0,z)\psi(0)|N(p_i)\rangle, \quad \mu, \kappa : 0, 1, 2, 3$$



Bare (unpol.) matrix elements with $t = -0.64\,\mathrm{GeV}^2$



Double Ratio (Reduced ITD) Definitions

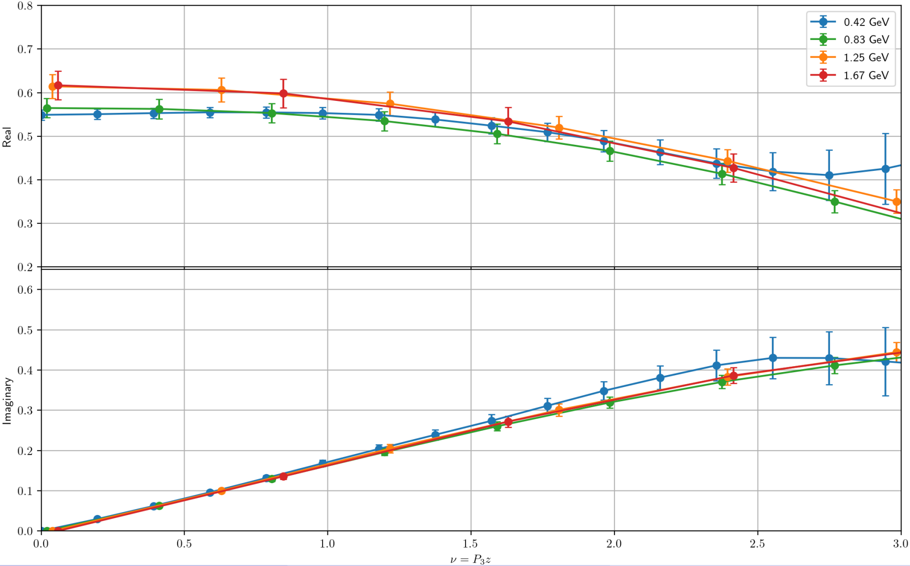
The 16 matrix elements generated by different gamma-projections and gamma-insertions can be disentangled into Lorentz invariant amplitudes:

$$\begin{aligned}\mathrm{GPD}_H &= A_1 \\ \mathrm{GPD}_E &= -A_1 + 2A_5 + 2zP_3A_6.\end{aligned}$$

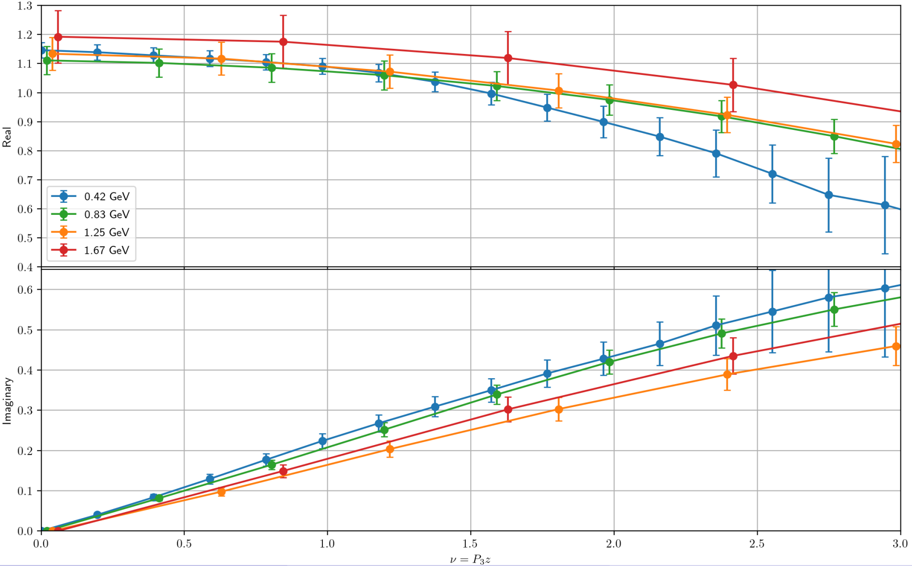
where the amplitudes A_i are solved from the Lorentz decompositions. These GPDs are renormalized using the double ratio - scheme:

$$\mathcal{M}_{H/E}(\nu, z) = \frac{\mathrm{GPD}_{H/E}(\nu, z)/\mathrm{PDF}(\nu, 0)}{\mathrm{PDF}(0, z)/\mathrm{PDF}(0, 0)}$$

H - Double Ratio (Reduced ITD)



E - Double Ratio (Reduced ITD)



Evolution and Matching (1-Loop)[2] [3] [4]

The renormalized pseudo-ITDs ($\mathcal{M}_{H/E}$) can be evolved and matched to the common scale, and to the light-cone using:

$$Q_{H/E}(\nu, z) = \mathcal{M}_{H/E}(\nu, \mu) - \frac{\alpha_s}{\pi} \int_0^1 du C(u, z, \mu)(\mathcal{M}_{H/E}(u\nu, \mu) - \mathcal{M}_{H/E}(\nu, \mu))$$

where,

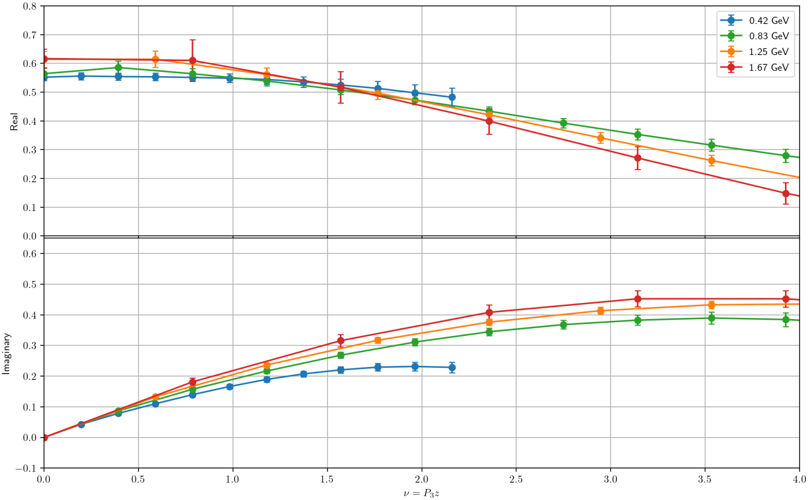
$$C(\mu, z, \nu) = \frac{C_F}{2} \left(L(u) + B(u) \ln \frac{z^2 \mu^2 e^{2\gamma_E + 1}}{4} \right),$$

$$B(u) = \frac{1 + u^2}{u - 1}, \text{"The evolver"}$$

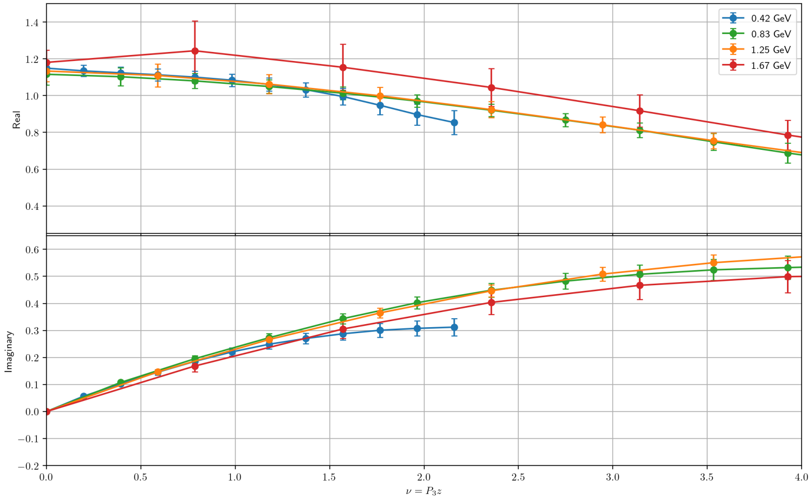
$$L(u) = 4 \frac{\ln(1 - u)}{u - 1} - 2(u - 1). \text{"The matcher"}$$

[2] A. Radyushkin, (2018) 1801.02427.
[3] J.-H. Zhang, et al., (2018) 1801.03023.
[4] T. Izubuchi et al., (2018) 1801.03917.

Matched H-ITD

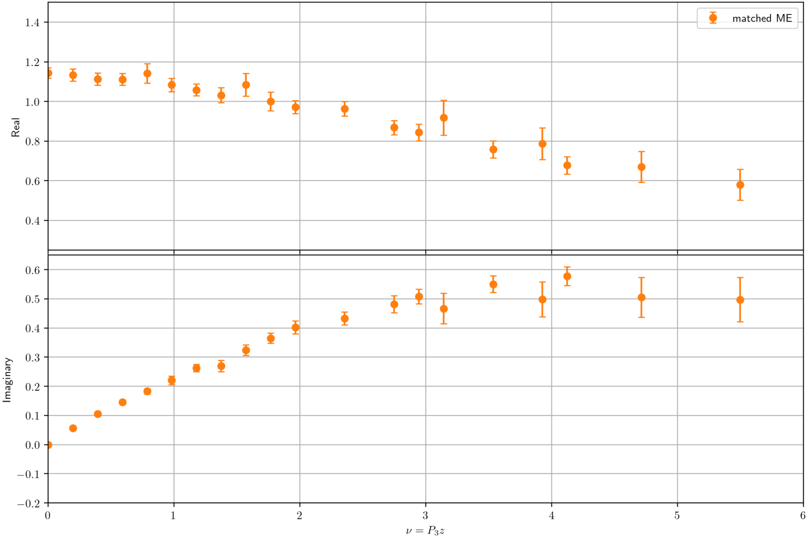


Matched E-ITD



Ioffe - averaging example

(E - function $z_{max} = 0.651\,\mathrm{fm}$)



Definitions and the fitting ansatz

Reconstruction

The reconstruction is done for the real and imaginary part separately, using

$$\mathrm{Re}Q(\nu,\mu) = \int_0^1 \mathrm{d}x \cos(\nu x) q_v(x,\mu)$$
$$\mathrm{Im}Q(\nu,\mu) = \int_0^1 \mathrm{d}x \sin(\nu x) q_{v2s}(x,\mu),$$

where a general fitting ansatz is used for the distribution:

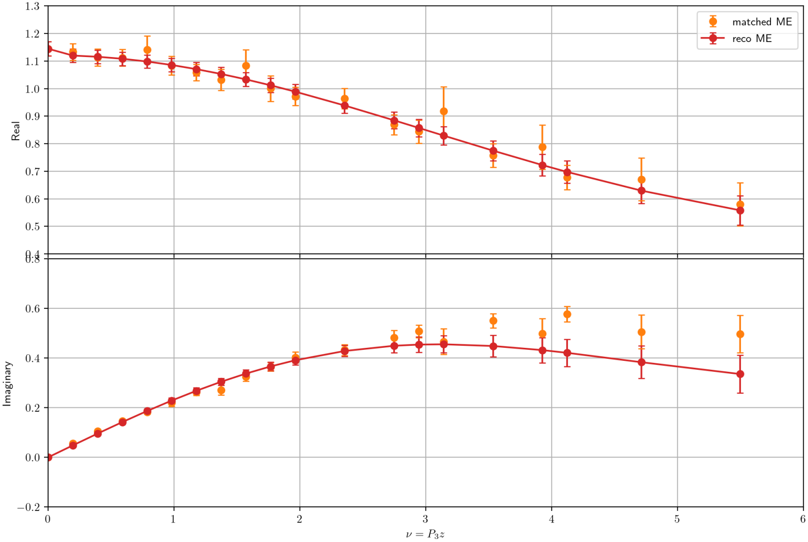
$$q(x) = N x^a (1-x)^b,$$

where a and b are fitting parameters, and N is the normalization factor. N is different for the real and imaginary parts:

- Real: gained from the double ratio at $z = 0$ averaged over all sink momenta.
- Imaginary: as a fitting parameter

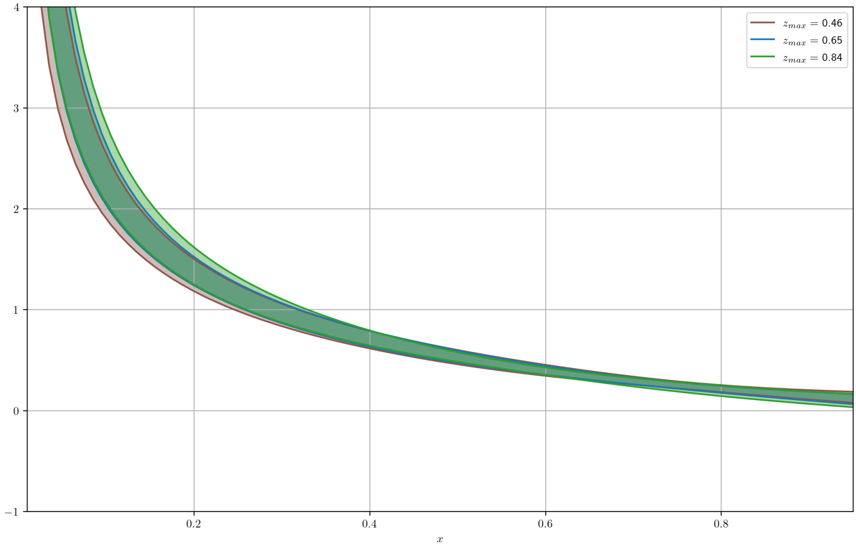
Reconstruction fit example

(E - function $z_{max} = 0.651\,\mathrm{fm}$)



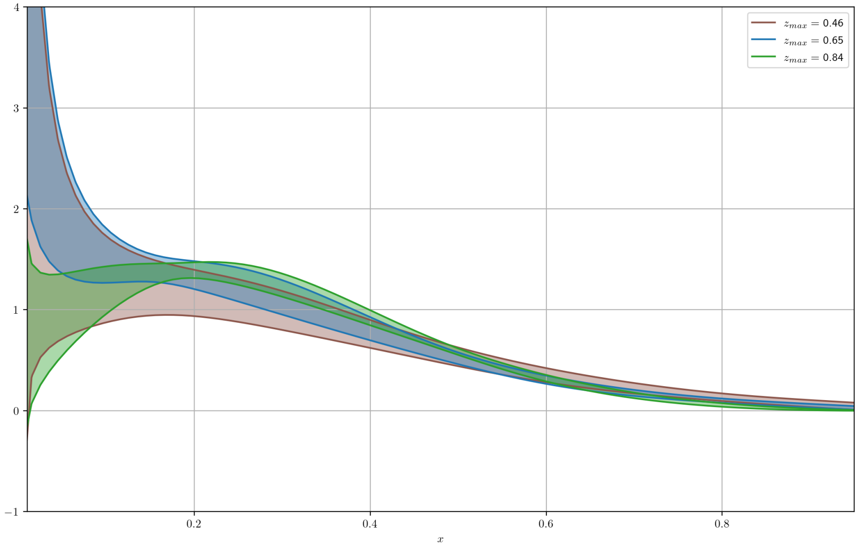
Plot of $q_E(x)$

Plotting the distribution average $q(x) = (q_v + q_{v2s})/2$



Plot of $q_H(x)$

Plotting the distribution average $q(x) = (q_v + q_{v2s})/2$



Prospects

In this talk, we presented our preliminary result for the unpolarized proton H and E-GPDs using asymmetric frames, Lorentz decomposition and pseudo distributions.

To-do list:

- Run with different ensembles.
- Investigation of systematics: z_{max} dependance, fitting dependance
- Generation of more momenta transfers t.

Thank you for your attention!
Questions?