Discretization effects on nucleon root-mean-square radii from lattice QCD at the physical point

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for PACS Collaboration

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Introduction

- The conventional studies and this work

RMS radii from lattice QCD

Sources of uncertainties:

- Statistical noise
- Excited states contamination
- Model dependences
- CCF extrapolations

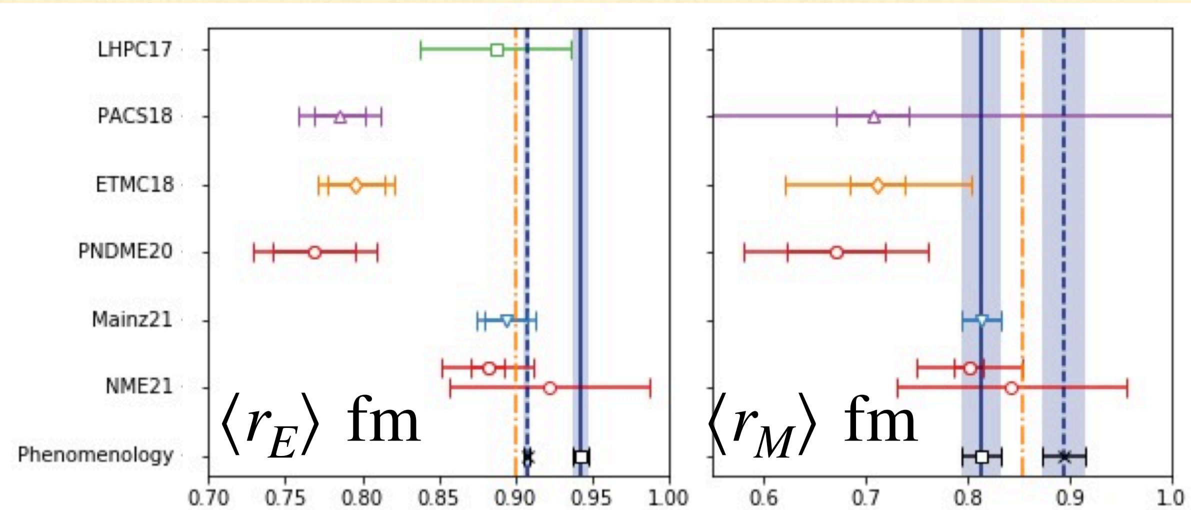
PACS('18) :

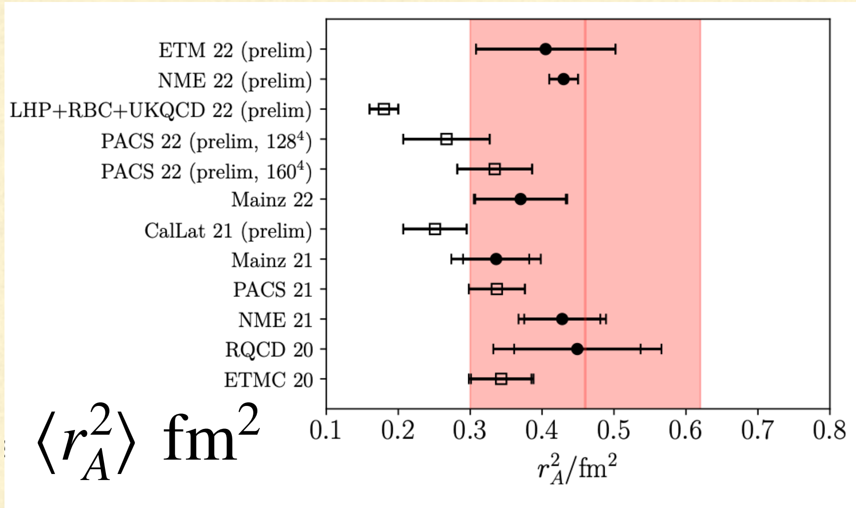
✔All-mode-averaging

✔More than two choice of t_{sep}

✔Conventional + Direct

- Large vol. + Phys. m_π





How large the discretization error on the RMS radii are ? = this work

Chiral
Continuum
Finite size

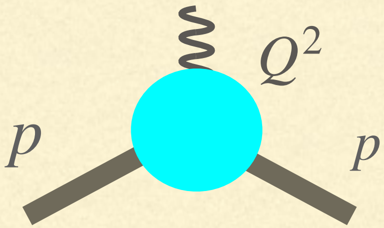
[1] K.-I. Ishikawa et al. for PACS Collaboration, Phys.Rev. D **104,** 074514(2021)
[2] Dalibor Djukanovic ,The 38th International Symposium on lattice field theory
[3] A. S. Meyer et al., arXiv:2301.04616 (2023).

Lattice QCD measurements

- Calculation strategy with Lattice QCD

Calculation strategy using lattice QCD

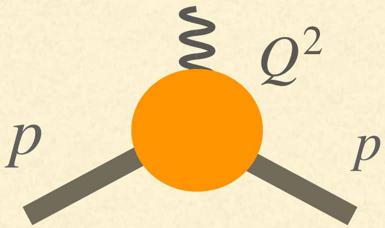
Targets : RMS radius of $G_l(q^2)$: $\sqrt{\langle r_{l^2} \rangle} = -\frac{6}{G_l(0)} \frac{dG_l(q^2)}{dq^2}\bigg|_{q^2 \to 0}$



$$= \bar{u}(p')\left[\frac{(p'+p)^\mu}{2M}\frac{G_E(q^2) - \frac{q^2}{4M^2}G_M(q^2)}{1-\frac{q^2}{4M^2}} + i\sigma^{\mu\nu}\frac{q_\nu}{2M}G_M(q^2)\right]u(p)$$

$\langle N(p')\,|\,\bar{q}\gamma_\mu q\,|\,N(p)\rangle$

$\rightarrow \langle r_E^2 \rangle,\ \mu,\ \langle r_M^2 \rangle$



$$= \bar{u}(p')\left[\gamma_\mu\gamma_5 F_A(q^2) + iq^\mu\gamma_5 F_P(q^2)\right]u(p)$$

$\langle N(p')\,|\,\bar{q}\gamma_\mu\gamma_5 q\,|\,N(p)\rangle$ $\rightarrow \langle r_A^2 \rangle$, gA

*Local current

Determination of the RMS radius $\langle r_l^2 \rangle$: z-expansion today

$$G_l(z) = \sum_{k=0}^{\infty} c_k z^k,\ z = (\sqrt{t_{\rm cut}+q^2} - \sqrt{t_{\rm cut}})/(\sqrt{t_{\rm cut}+q^2} + \sqrt{t_{\rm cut}}) \text{ with } t_{\rm cut} = \begin{cases} 4m_\pi^2 \ (l = E, M) \\ 9m_\pi^2 \ (l = A) \end{cases}$$

Numerical results

- Nucleon renormalized axial charge
- Electromagnetic form factors and RMS radii
- Axial form factors and RMS radius
- Discretization error on RMS radii

Simulation details -PACS10 configuration[1][2]

Eliminate major uncertainties

Finite size effect
Chiral extrapolation

$\otimes$ Low q^2 data are accessible $q^2 = (2\pi/L)^2 \times |\vec{n}^2|$ = PACS10

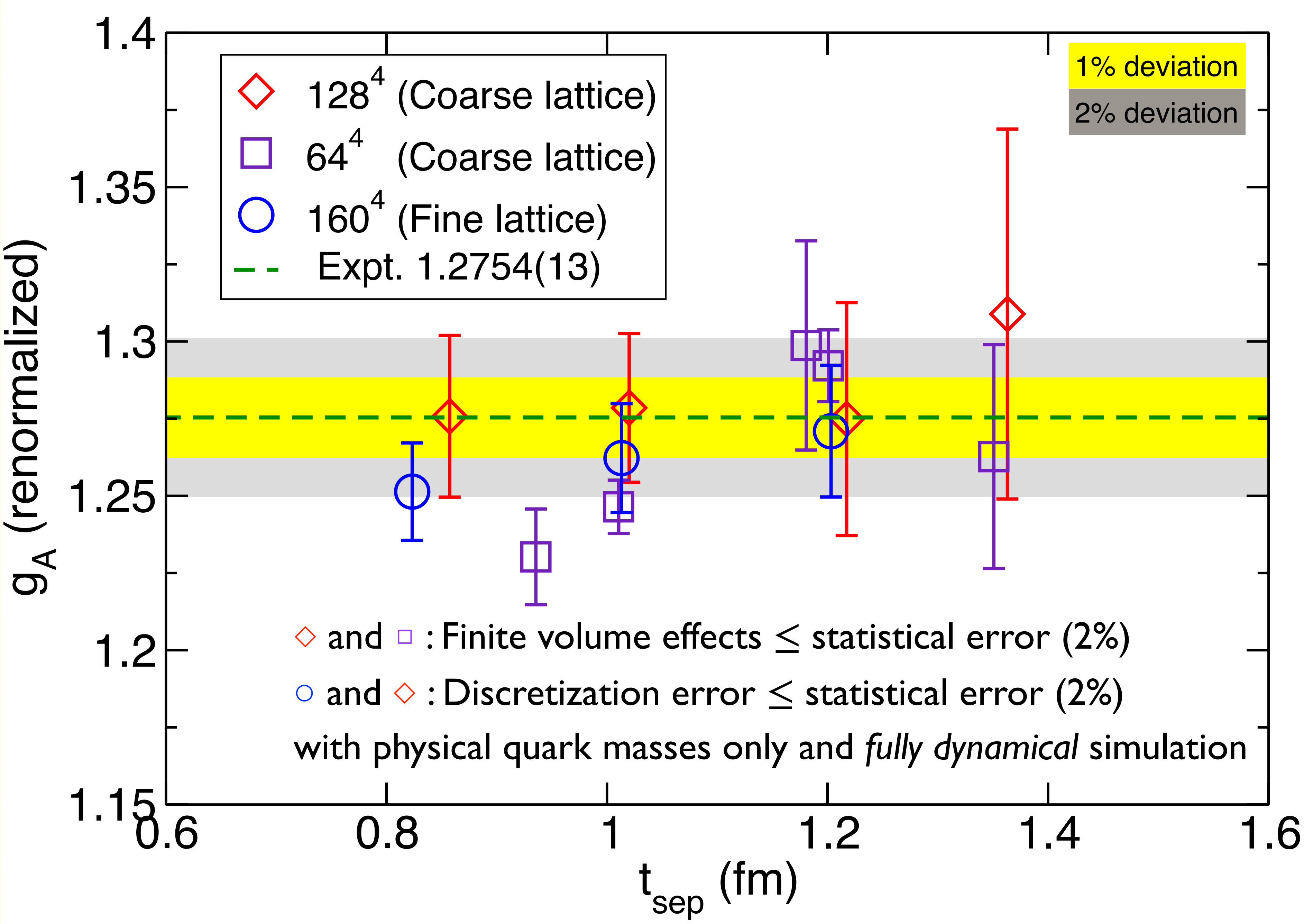
Lattice size	128^4 [1]	160^4 [2]
Spacial vol. ≫ nucleon	$\sim (10.9\ \mathrm{fm})^3$	$\sim (10.1\ \mathrm{fm})^3$
Pion mass $\sim m_\pi^{\mathrm{exp.}}$	135 MeV	138 MeV
Nucleon mass	~ 0.935 GeV	~ 0.947 GeV
|tsink-tsrc|/a	10, 12, 14, 16	13,16,19
Lattice spacing	coarse ~ 0.085 fm	fine ~ 0.063 fm

[1] E. Shintani et al., Phys. Rev. D **99**, 014510(2019), (Erratum; Phys. Rev. D 102 (2020) 019902.)
[2] E. Shintani and Y.Kuramashi, Phys.Rev. D **100**, 034517(2019)

The stout-smeared $O(a)$ improved Wilson fermions and Iwasaki gauge action.

Preliminary

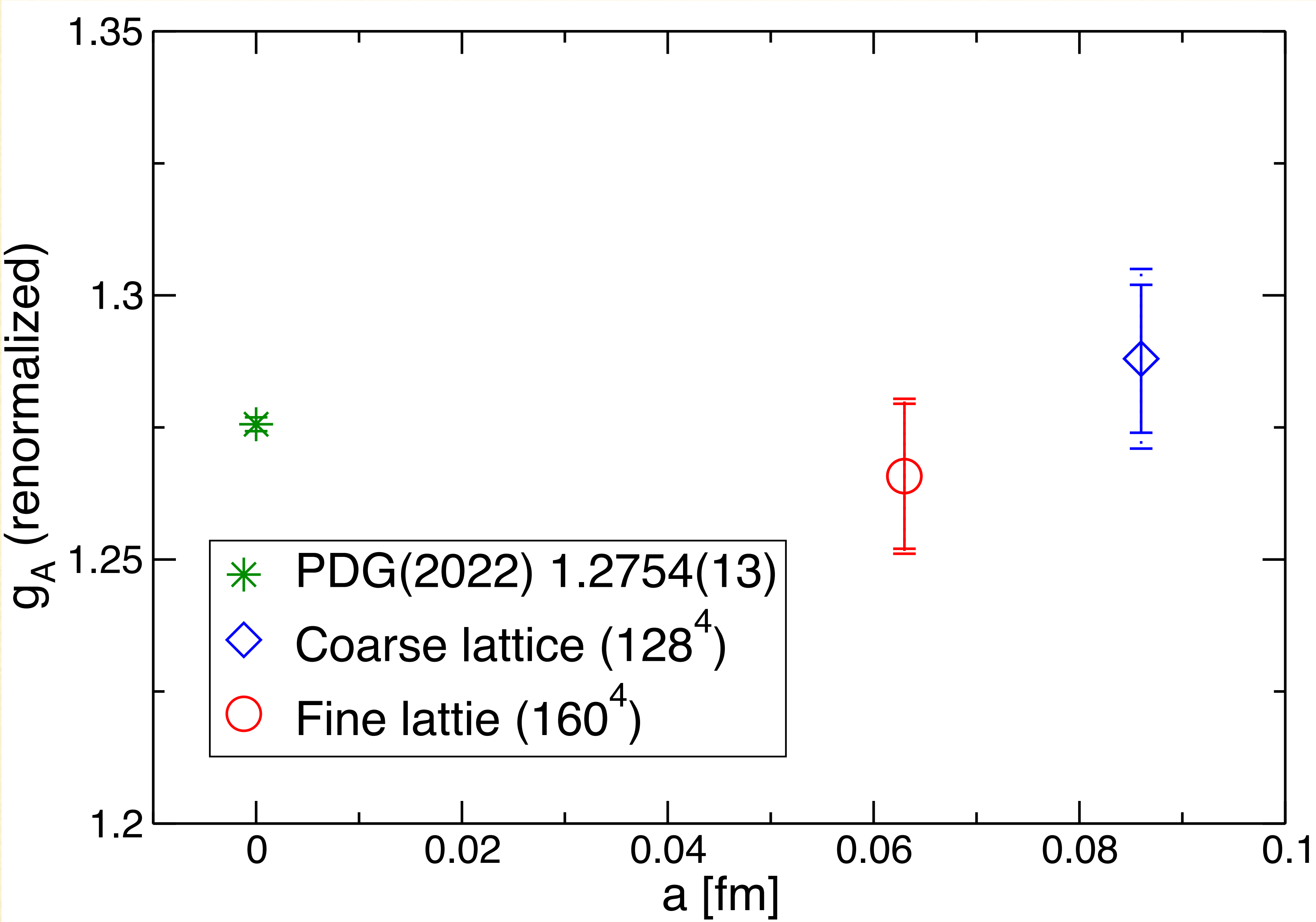
Nucleon axial charge



Preliminary

Nucleon axial charge

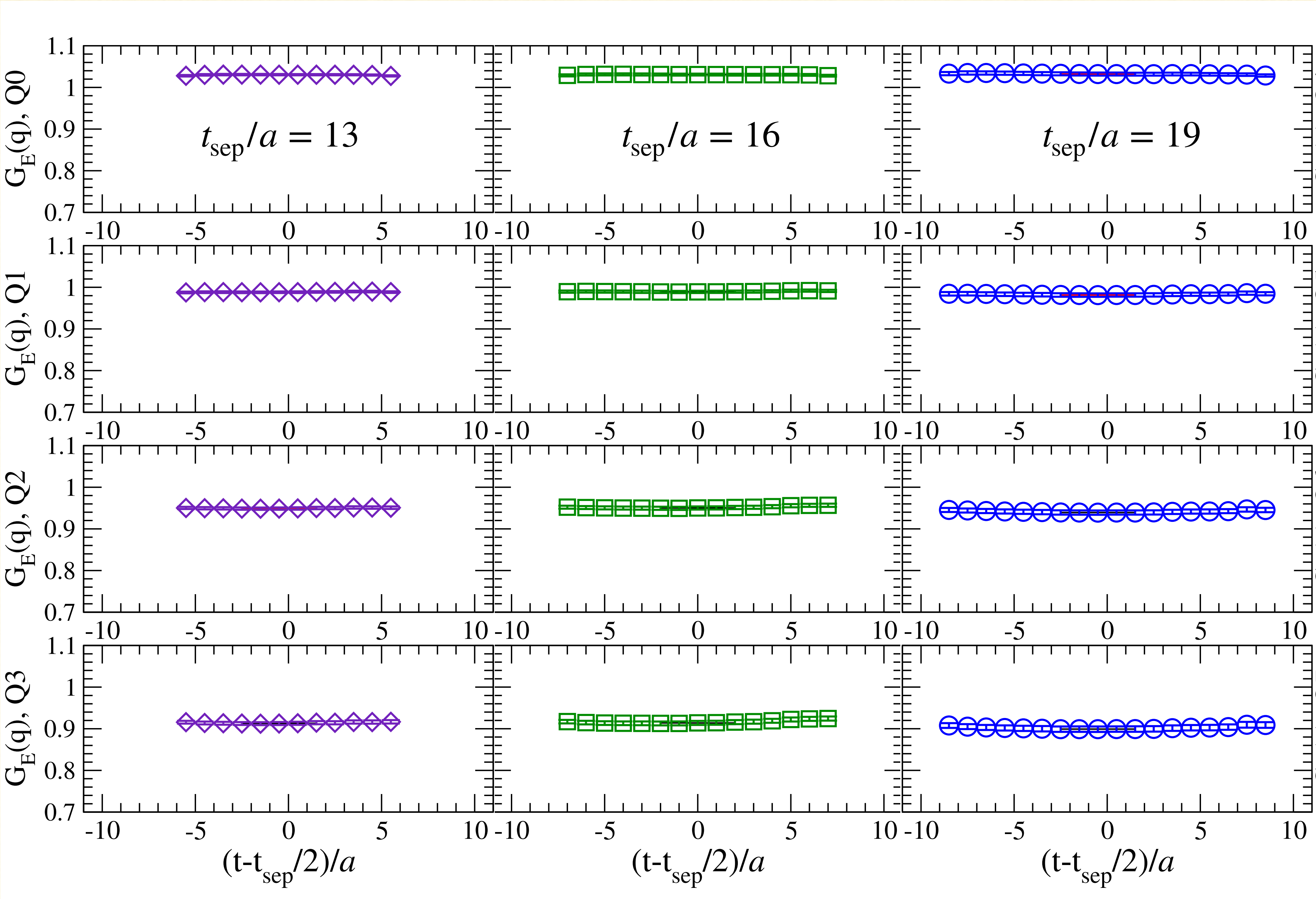
$$\text{Disc. err.} \equiv \frac{|(\text{coarse}) - (\text{fine})|}{(\text{coarse})}$$



Both coarse & fine reproduce PDG within statistical errors

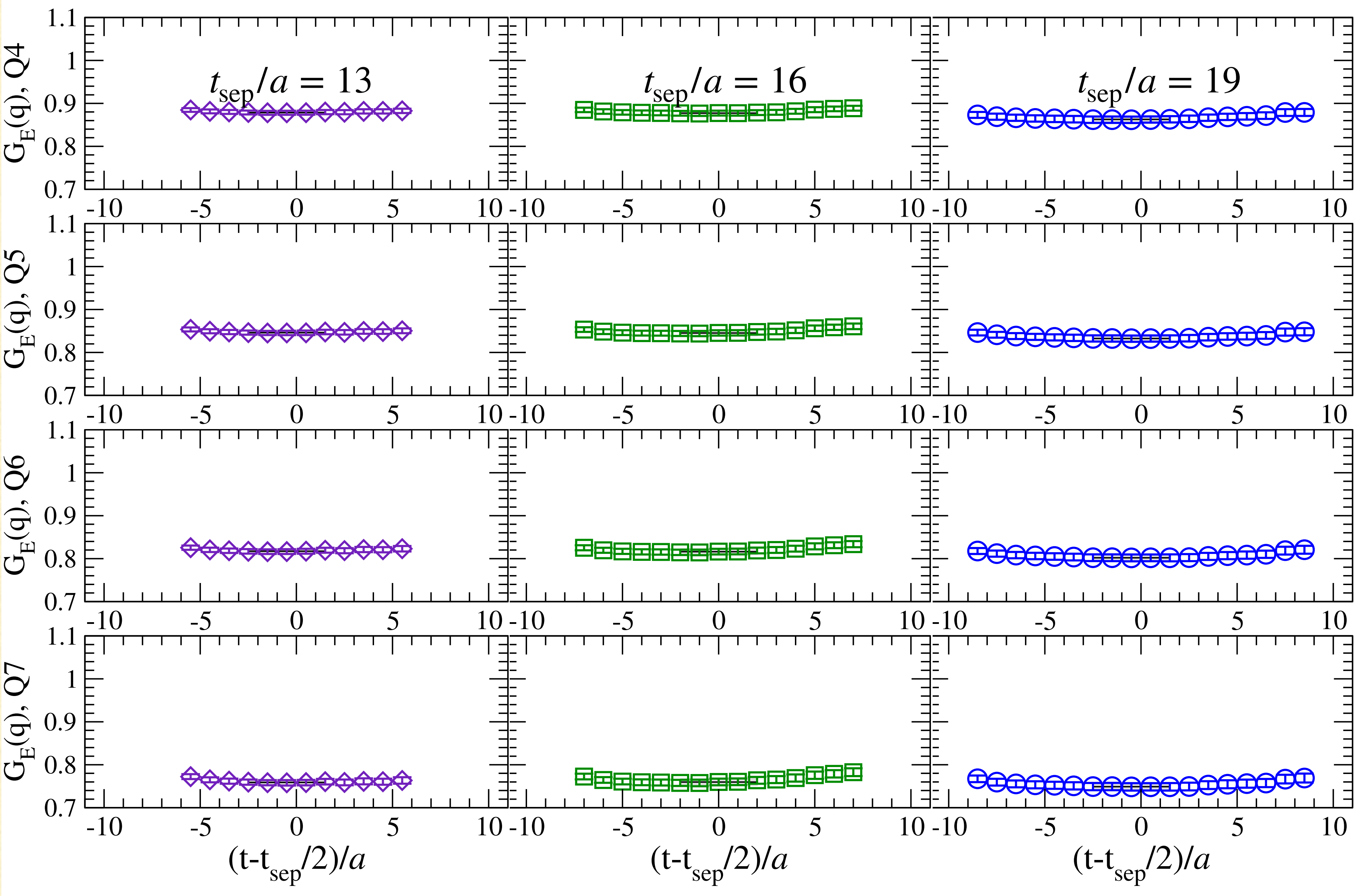
Discretization error (1.6%) $\lesssim$ Statistical error (1.9%)

Plateaus of G_E -$q^2 = 0.0 \sim 0.044\ \mathrm{GeV}^2$



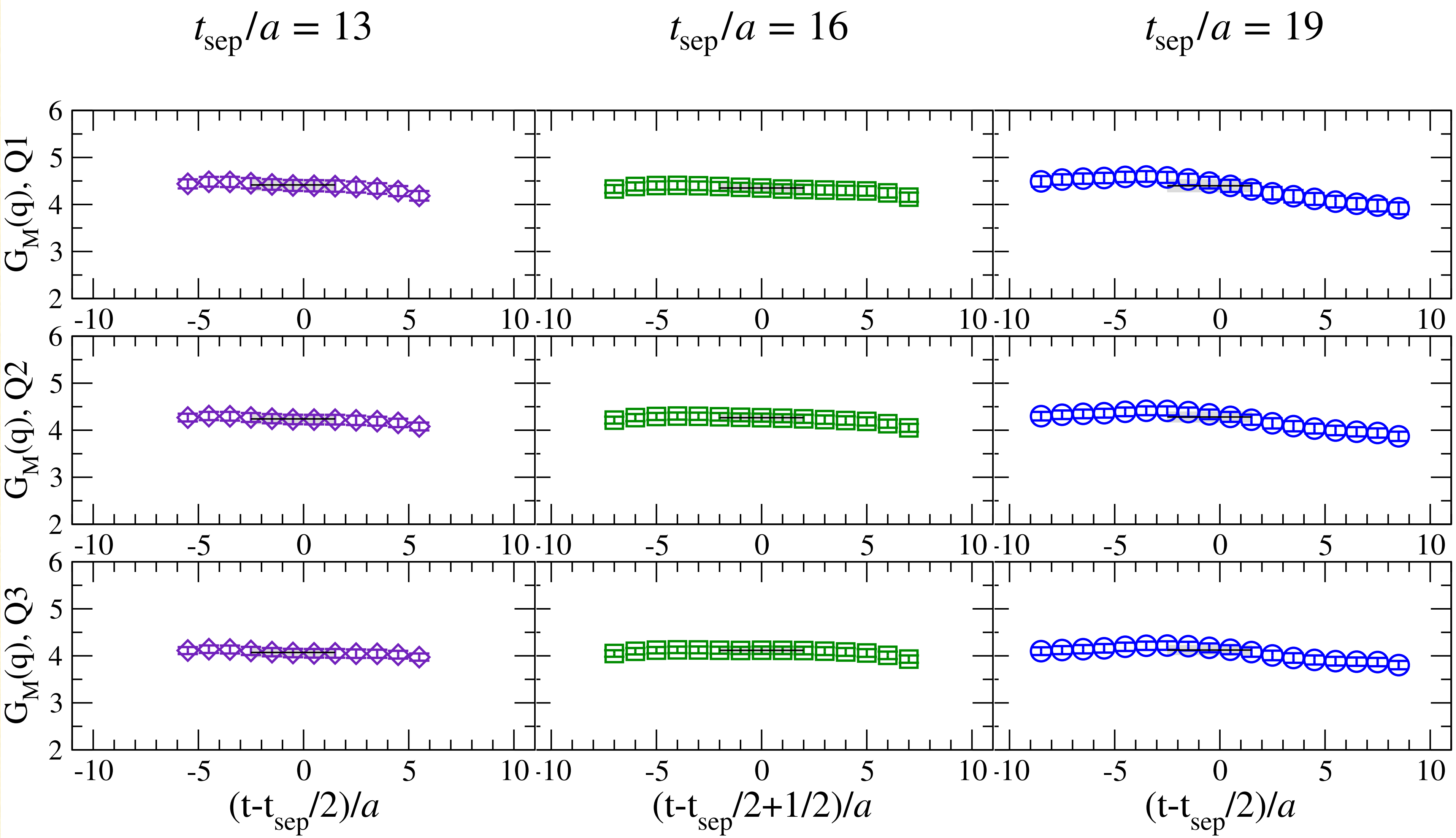
Preliminary

Plateaus of G_E -$q^2 = 0.059 \sim 0.116$ GeV^2



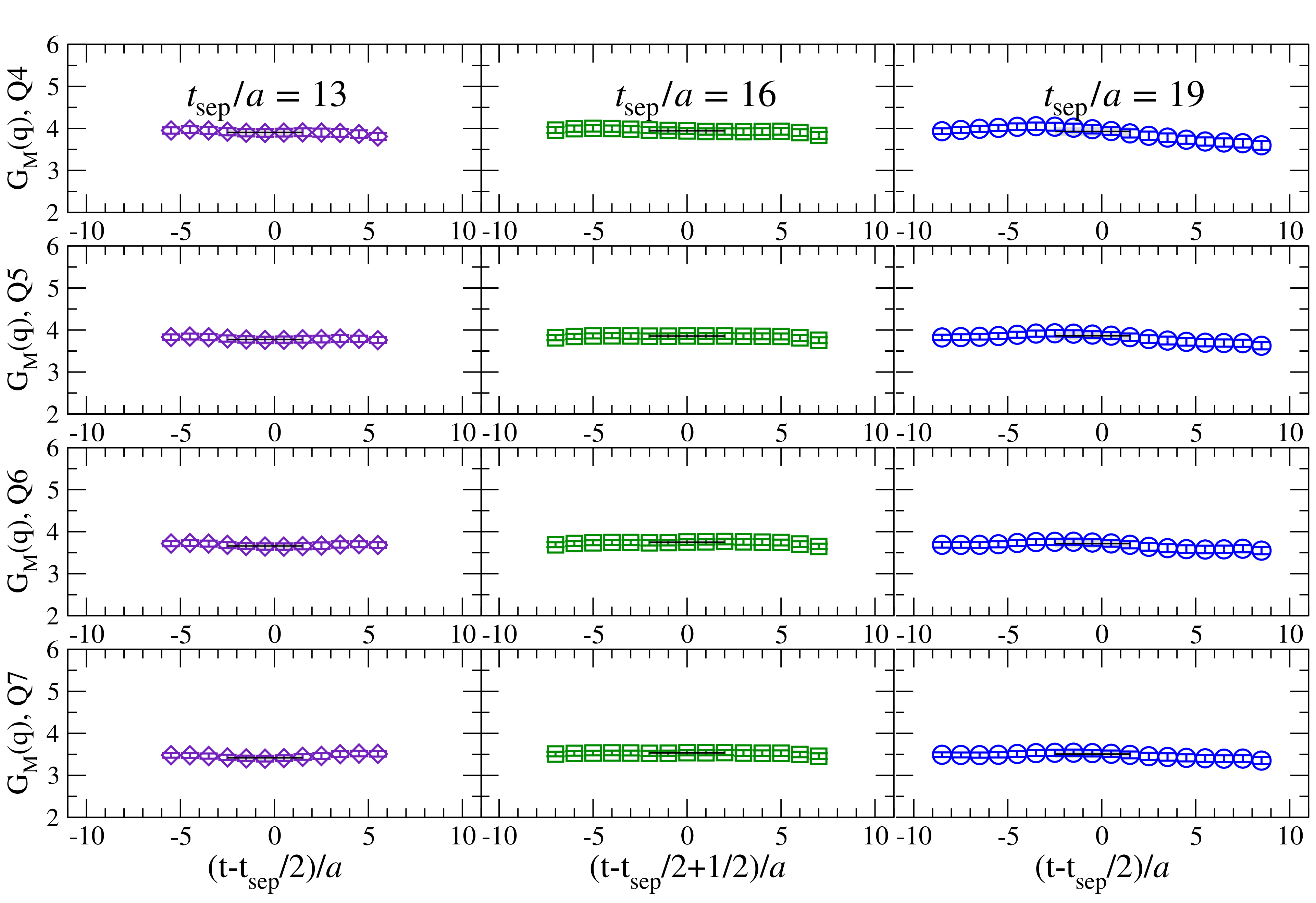
Preliminary

Plateaus of G_M - $q^2 = 0.0 \sim 0.044$ GeV^2



Preliminary

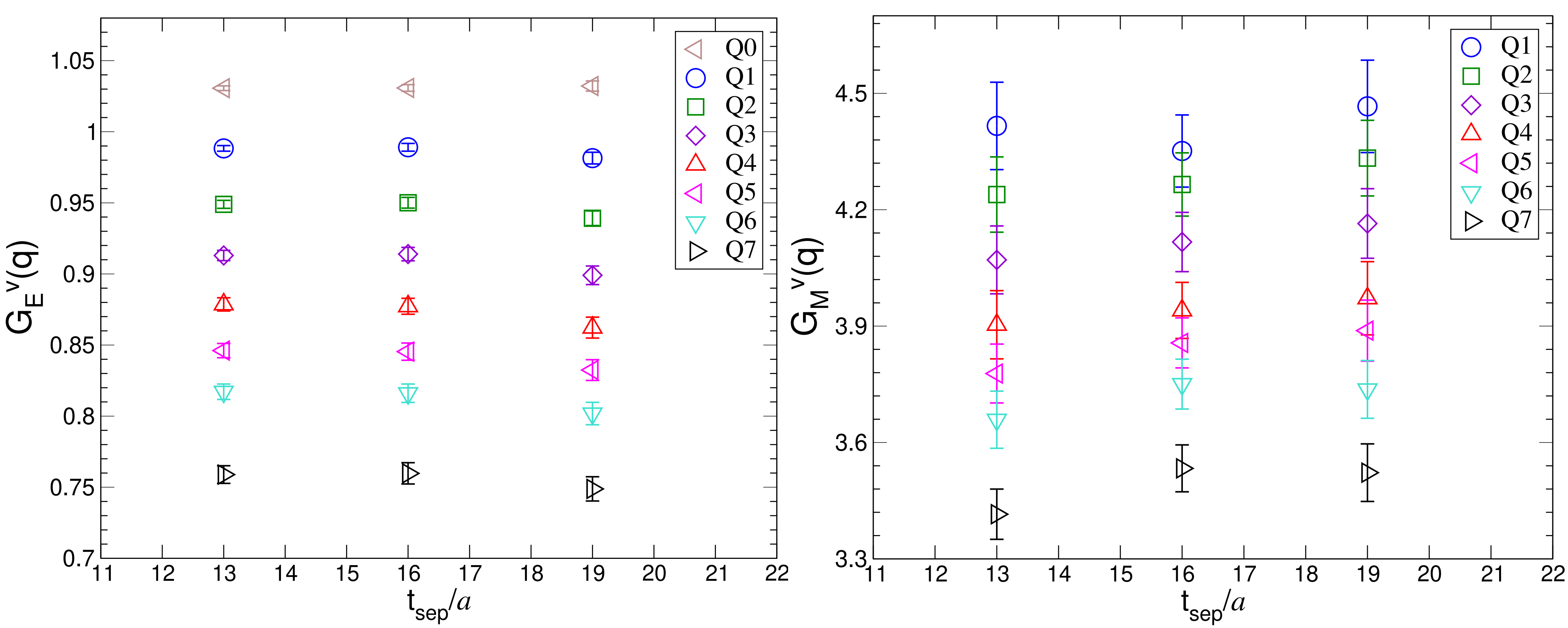
Plateaus of G_M -$q^2 = 0.059 \sim 0.116\ \mathrm{GeV}^2$



Preliminary

t_{sep}-dependences G_E & G_M

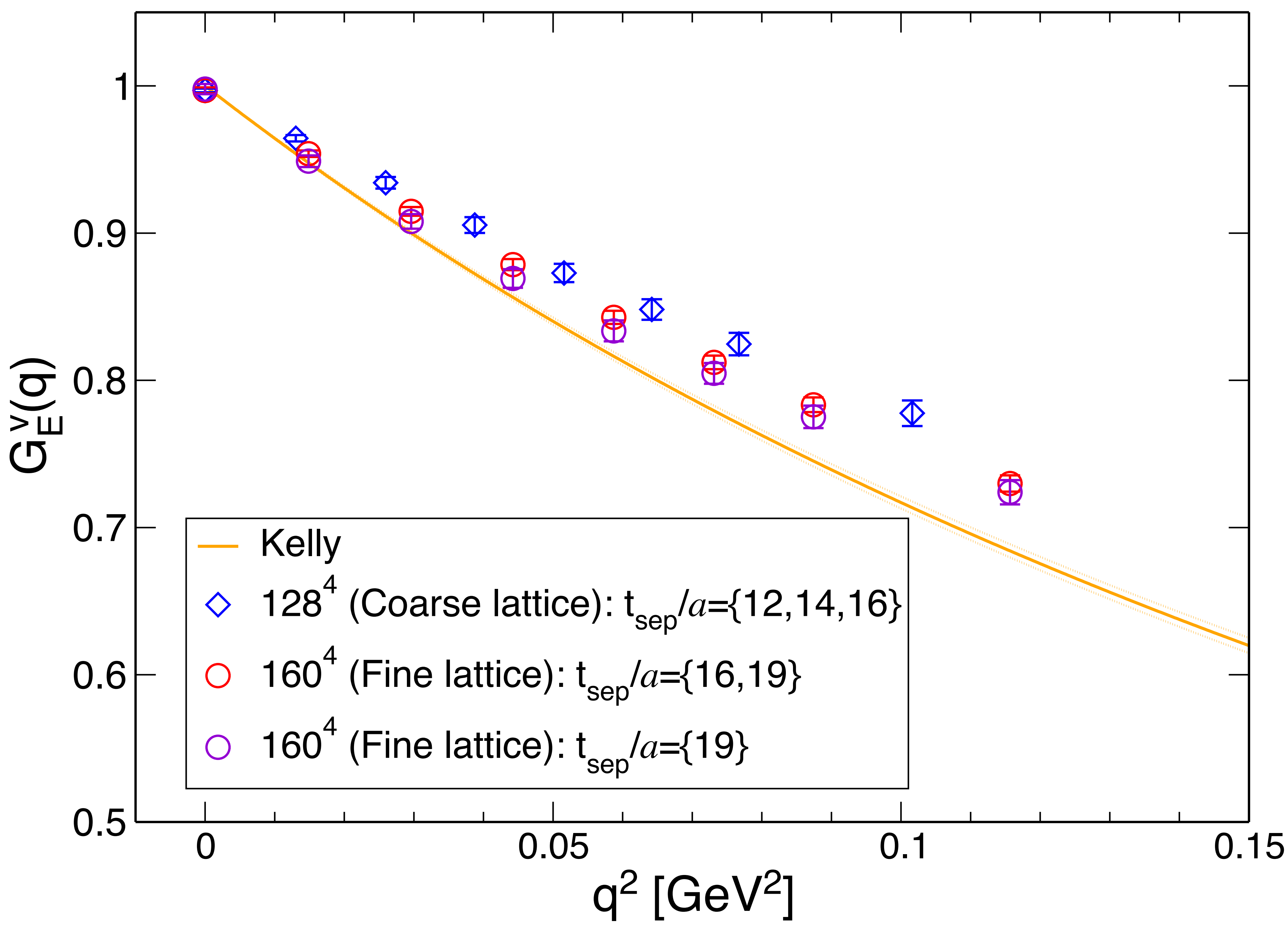
$$\langle N(p')\,|\,V_\mu(q)\,|\,N(p)\rangle = \bar{u}(p')\left[\frac{(p'+p)^\mu}{2M}\frac{G_E(q^2)-\frac{q^2}{4M^2}G_M(q^2)}{1-\frac{q^2}{4M^2}} + i\sigma^{\mu\nu}\frac{q_\nu}{2M}G_M(q^2)\right]u(p)$$



t_{sep}-dependences < statistical err→ ground-state saturation

$G_E(q^2; t_{sep}/a = 19)$ has slightly small values→Not solved yet

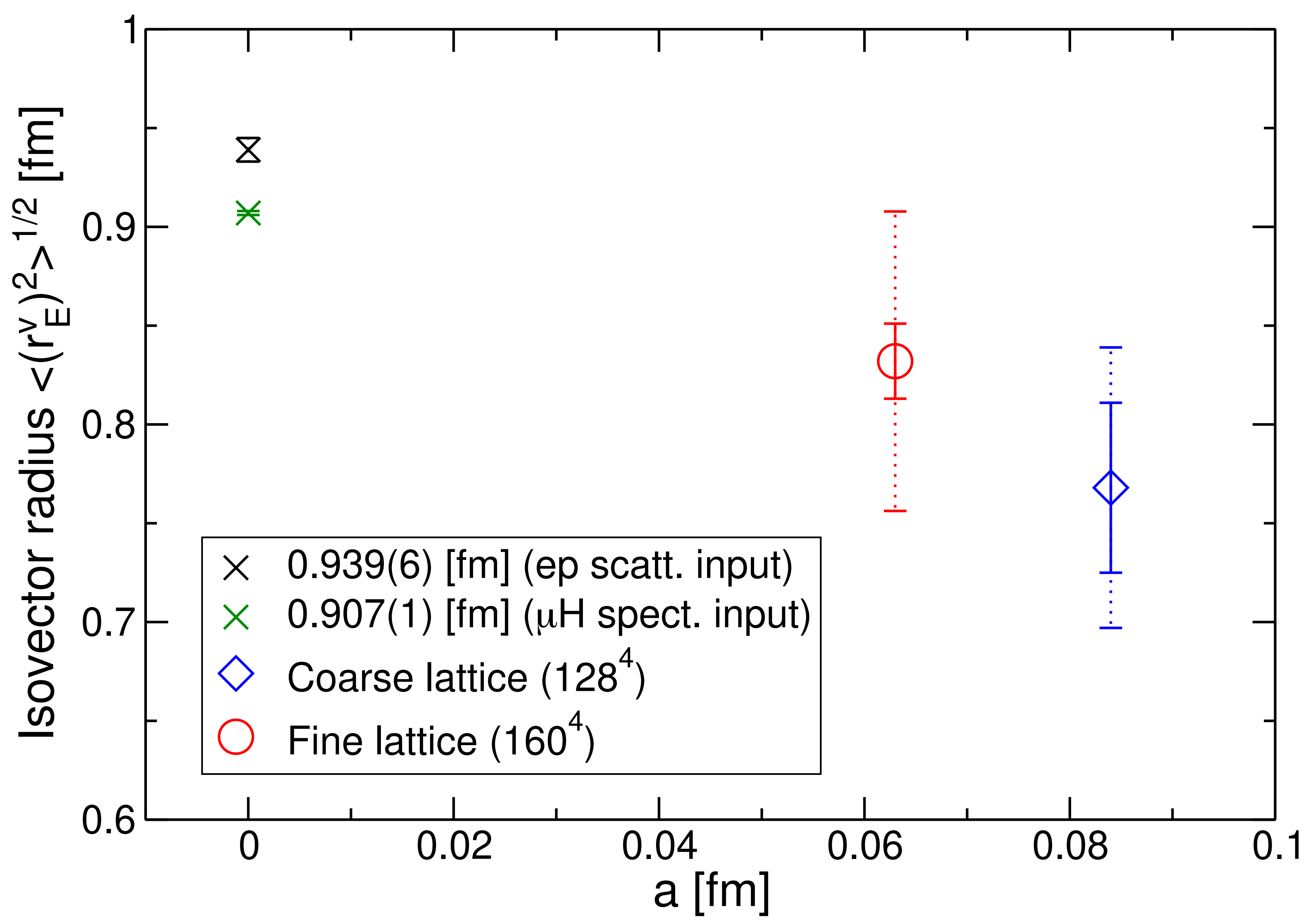
Electric form factor



Preliminary

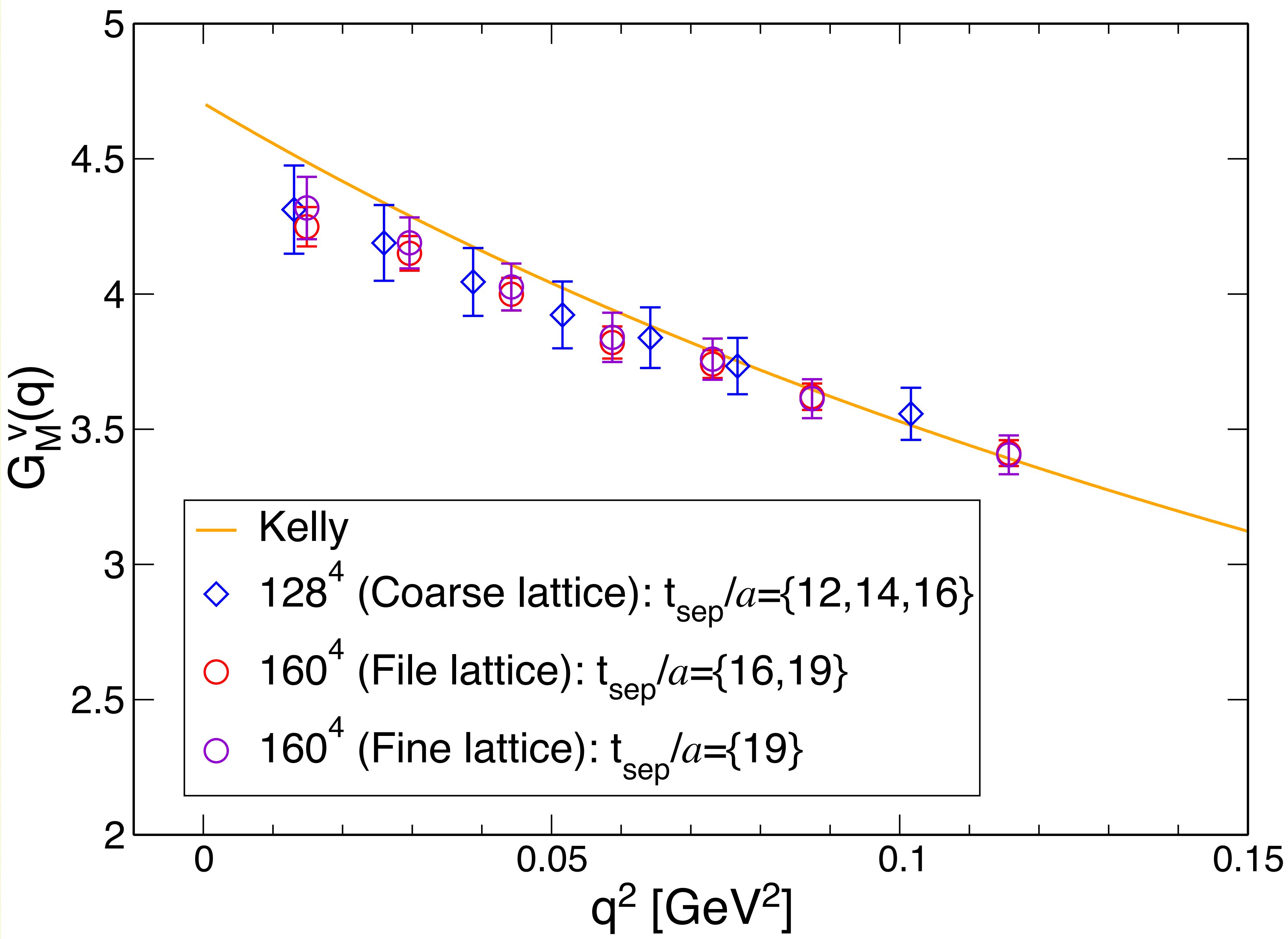
Electric radius

$$\text{Disc. err.} \equiv \frac{|(\text{coarse}) - (\text{fine})|}{(\text{coarse})}$$



Statistical error (5.6%) $\lesssim$ Discretization error (8.3%)

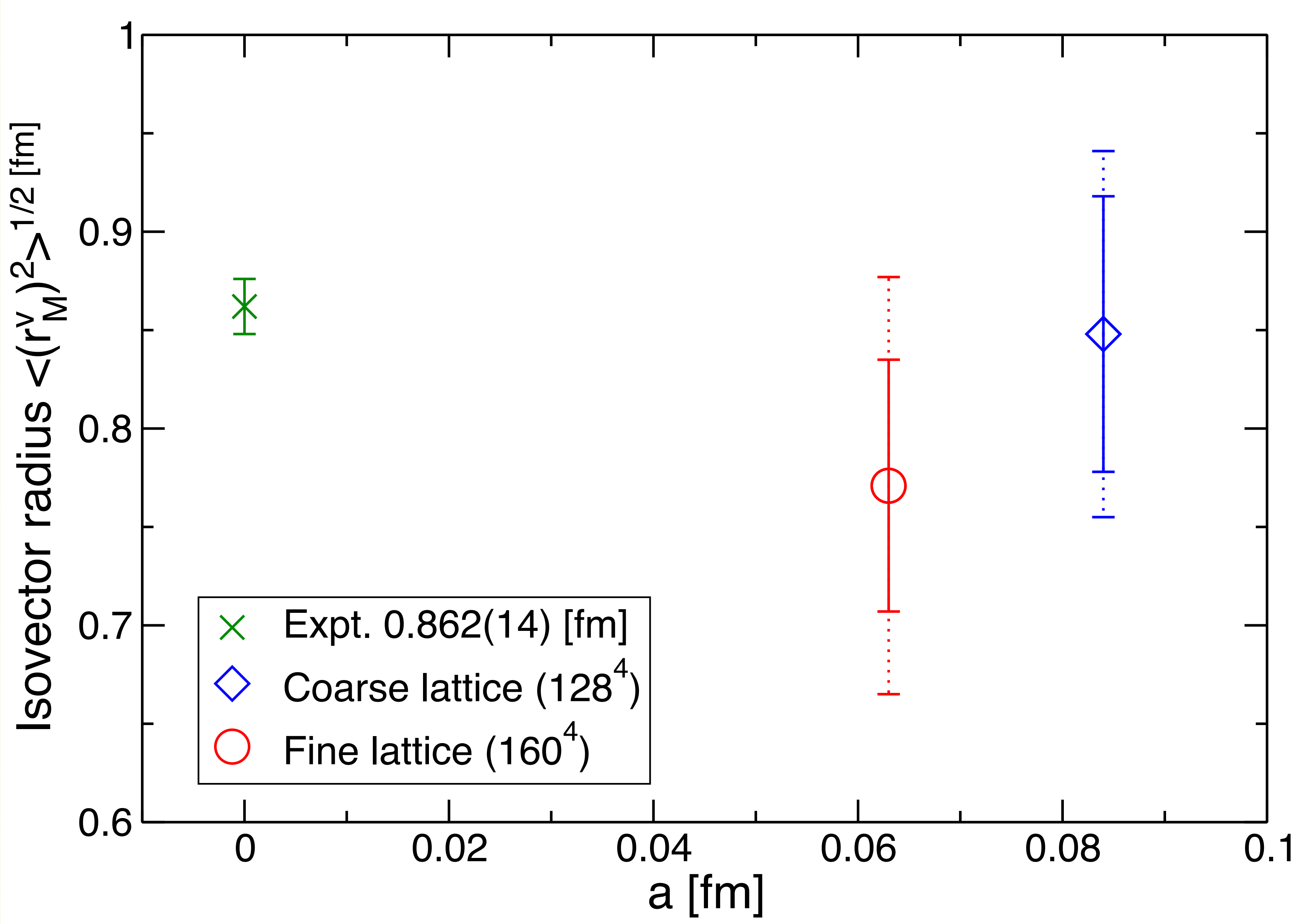
Magnetic form factor



Preliminary

Magnetic radius

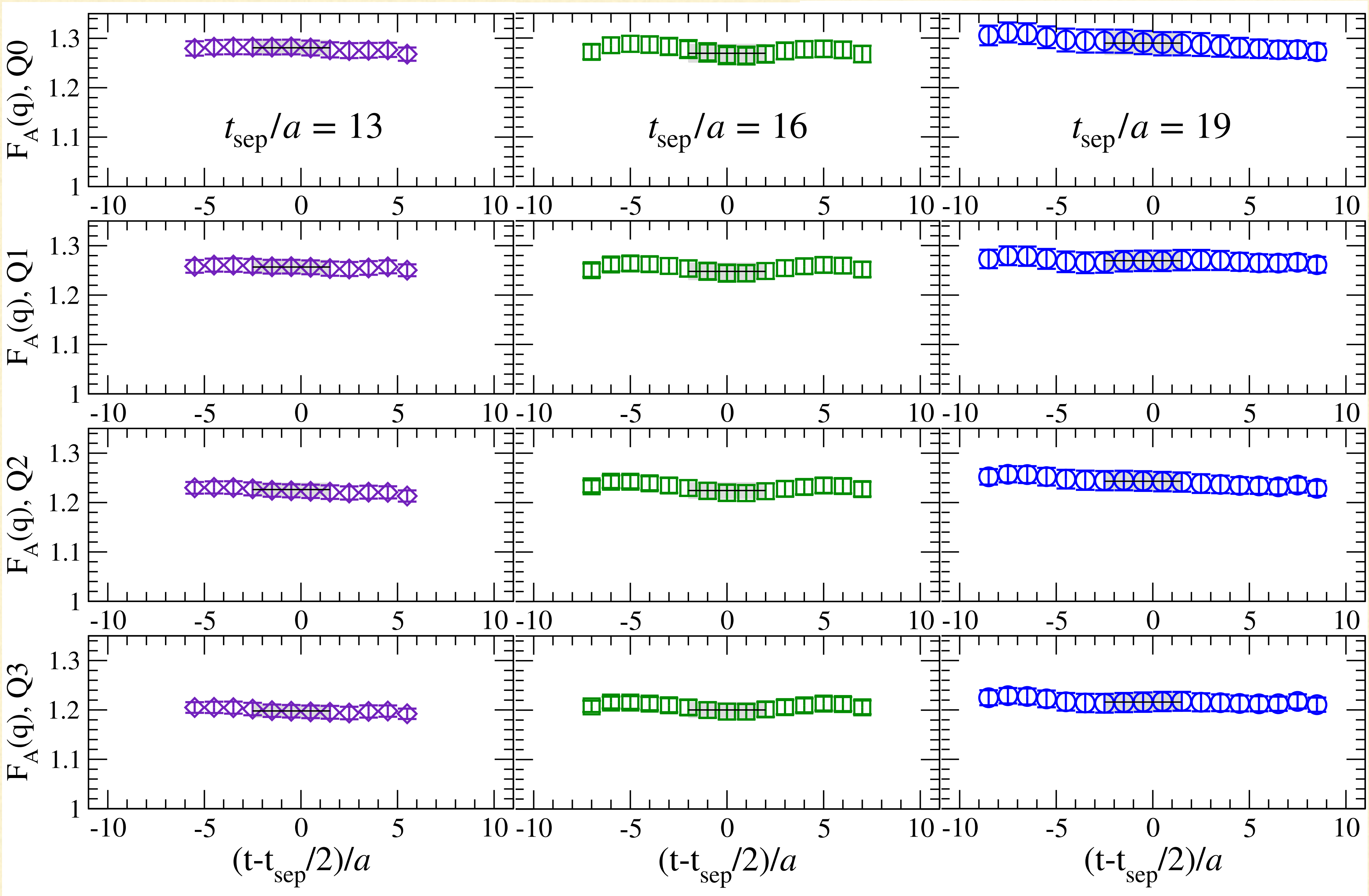
$$\text{Disc. err.} \equiv \frac{|(\text{coarse}) - (\text{fine})|}{(\text{coarse})}$$



Statistical error (8.3%) $\lesssim$ Discretization error (9.0%)

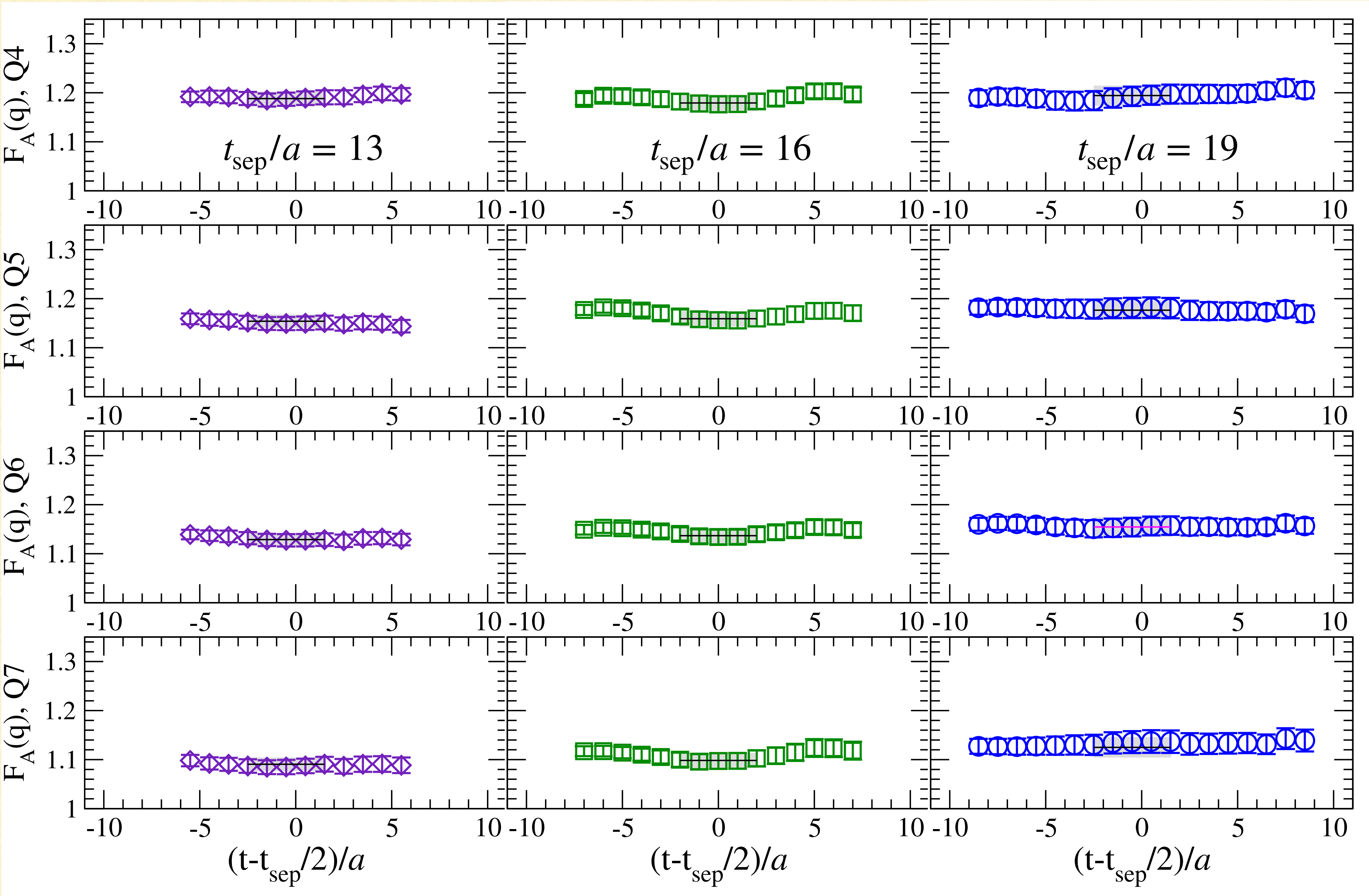
Preliminary

Plateaus of F_A -$q^2 = 0.0 \sim 0.044\ \mathrm{GeV}^2$



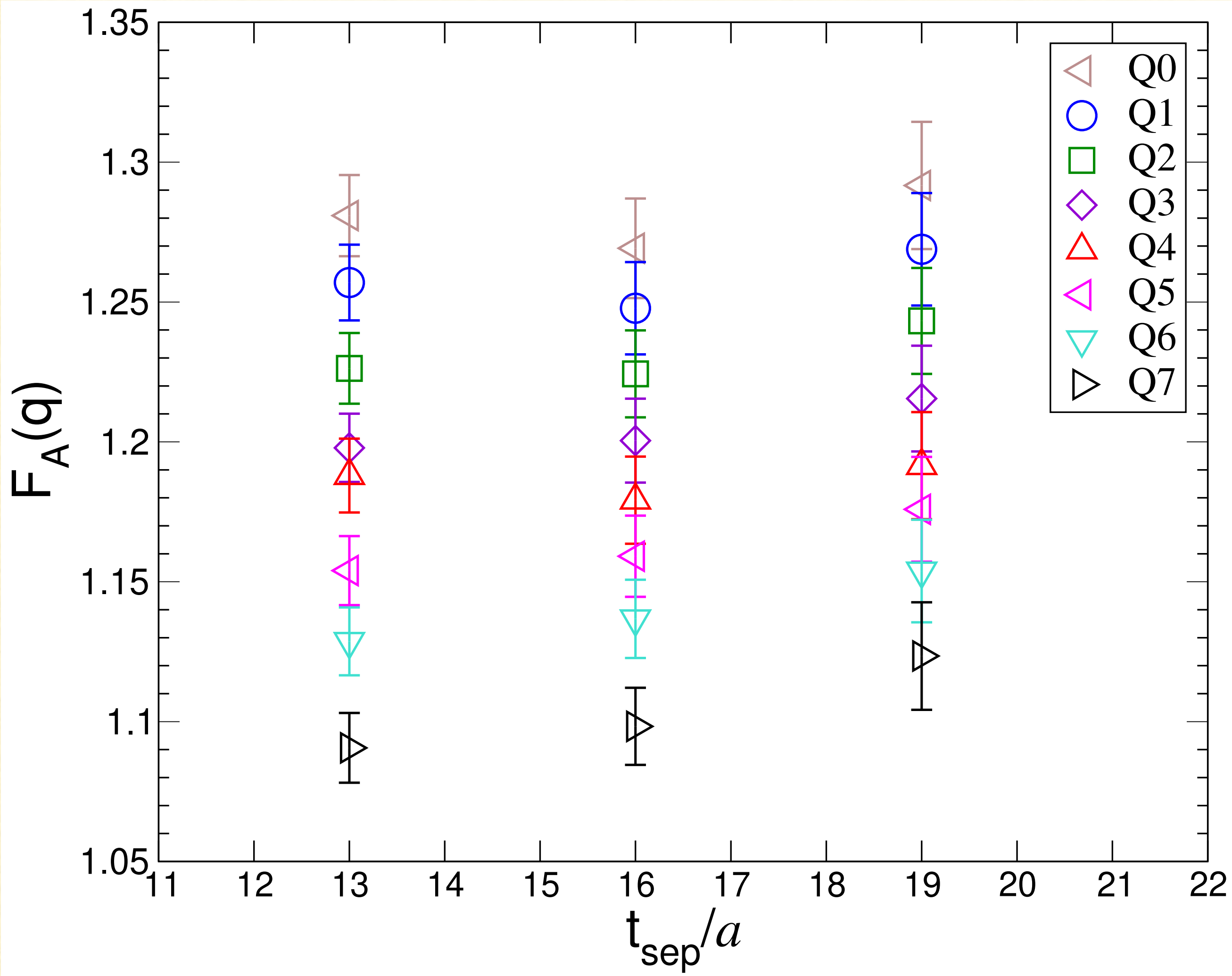
Preliminary

Plateaus of F_A $-q^2 = 0.059 \sim 0.116\ \mathrm{GeV}^2$



t_{sep}-dependences F_A

$$\langle N(p')\,|\,A_\mu(q)\,|\,N(p)\rangle = \bar{u}(p')\left[\gamma_\mu F_A(q^2) + iq^\mu F_P(q^2)\right]u(p)$$

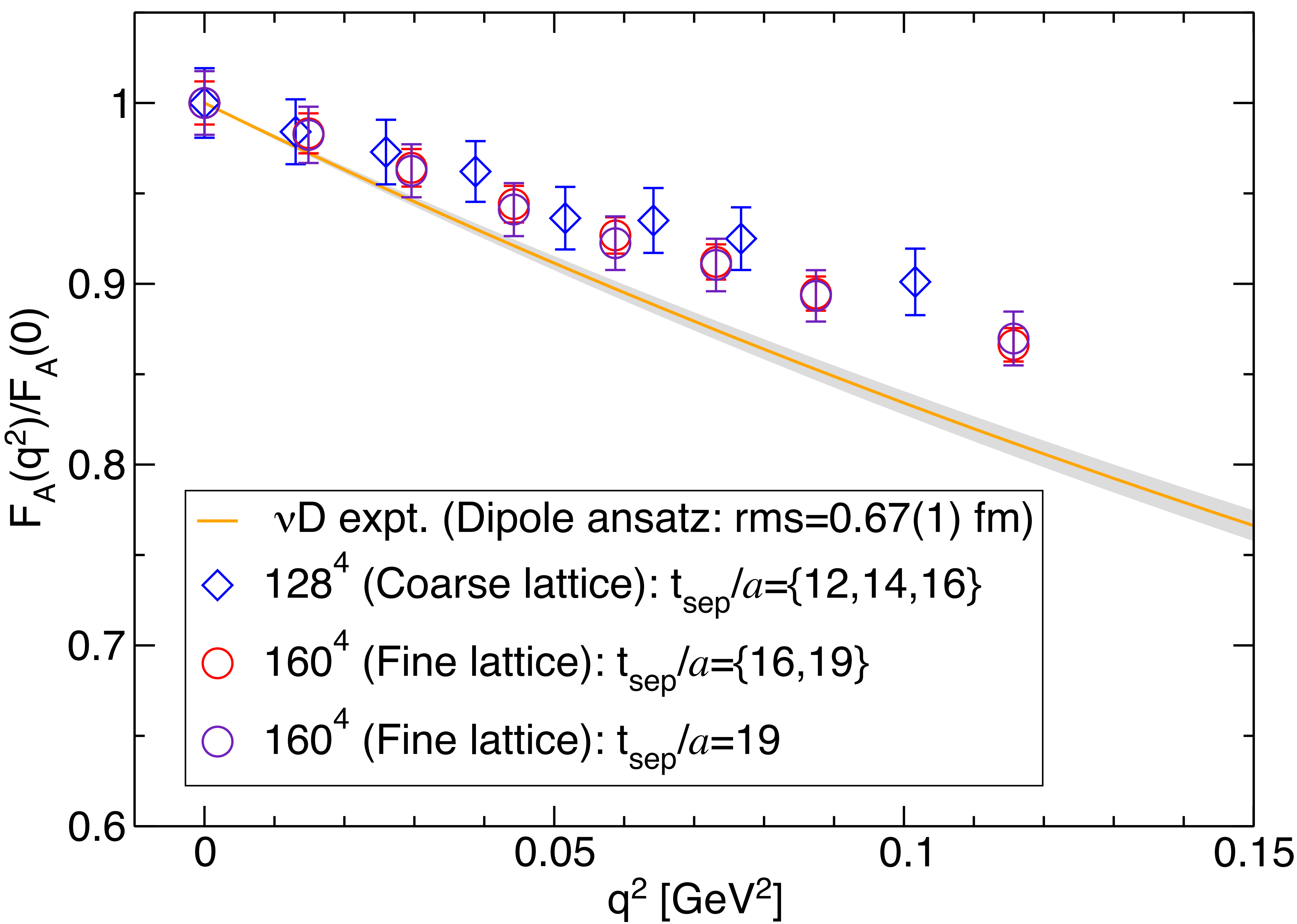


t_{sep}-dependences < statistical err → ground-state saturation

Excited-states could be unproblematic in our precision at low Q^2.

Preliminary

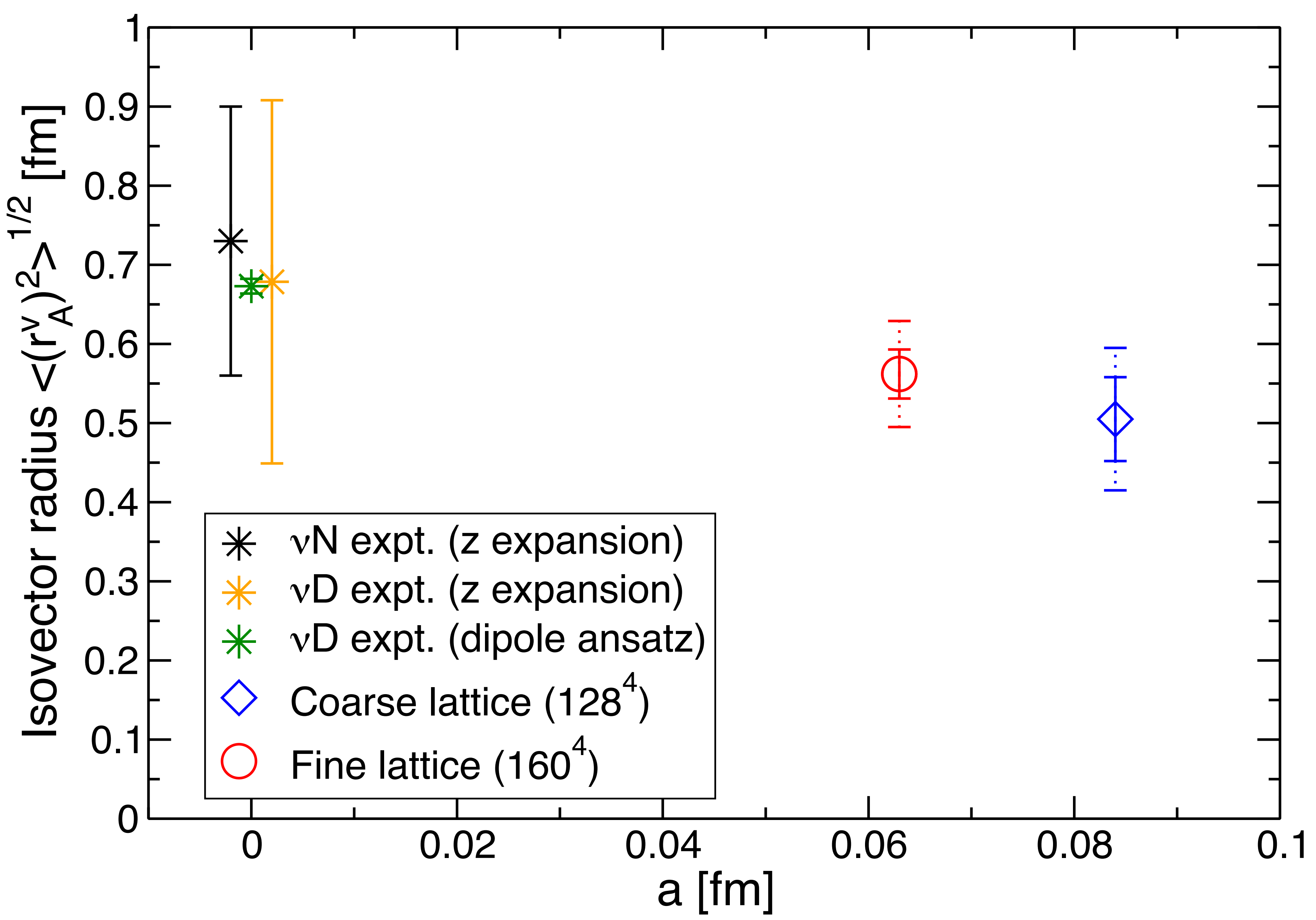
Axial form factor



Preliminary

Axial radius

$$\text{Disc. err.} \equiv \frac{|(\text{coarse}) - (\text{fine})|}{(\text{coarse})}$$



Statistical error (10.5%) $\lesssim$ Discretization error (11.3%)

Discretization error

Error budget	gA	$\sqrt{\langle (r_E^v)^2 \rangle}$	$\sqrt{\langle (r_M^v)^2 \rangle}$	$\sqrt{\langle (r_A^v)^2 \rangle}$
Statistical:	1.9%	5.6%	8.3%	10.5%
Discretization:	1.6%	8.3%	9.0%	11.3%

Statistical error $\lesssim$ Discretization error

Check

1. Dispersion relation of nucleon

2. $O(a)$ improved current $A_\alpha^{\mathrm{imp}} = A_\alpha + c_A a \partial_\alpha P$ → PCAC relation

$$m_{\mathrm{PCAC}} \equiv \frac{m_\pi^2 f_\pi}{2\langle 0 | P^+(0) | \pi \rangle} \longleftrightarrow m_{\mathrm{AWTI}}^{\mathrm{PCAC}} \equiv \frac{\langle N_{\mathrm{snk}} \partial_\mu A_\mu(x) \overline{N}_{\mathrm{src}} \rangle}{2\langle N_{\mathrm{snk}} P(x) \overline{N}_{\mathrm{src}} \rangle}$$

- Pion 2-pt function
- Zero momentum
- Improvement is helpless

$m_{\mathrm{PCAC}} = (m_{\mathrm{PCAC}})^{\mathrm{imp}} \sim (m_{\mathrm{AWTI}}^{\mathrm{PCAC}})^{\mathrm{imp}}$

$\bar{c}_A$ s.t. $m_{\mathrm{PCAC}} \sim (m_{\mathrm{AWTI}}^{\mathrm{PCAC}})^{\mathrm{imp}}$

$\rightarrow \bar{c}_A \propto m_{\mathrm{AWTI}}^{\mathrm{PCAC}} - (m_{\mathrm{AWTI}}^{\mathrm{PCAC}})^{\mathrm{imp}}$

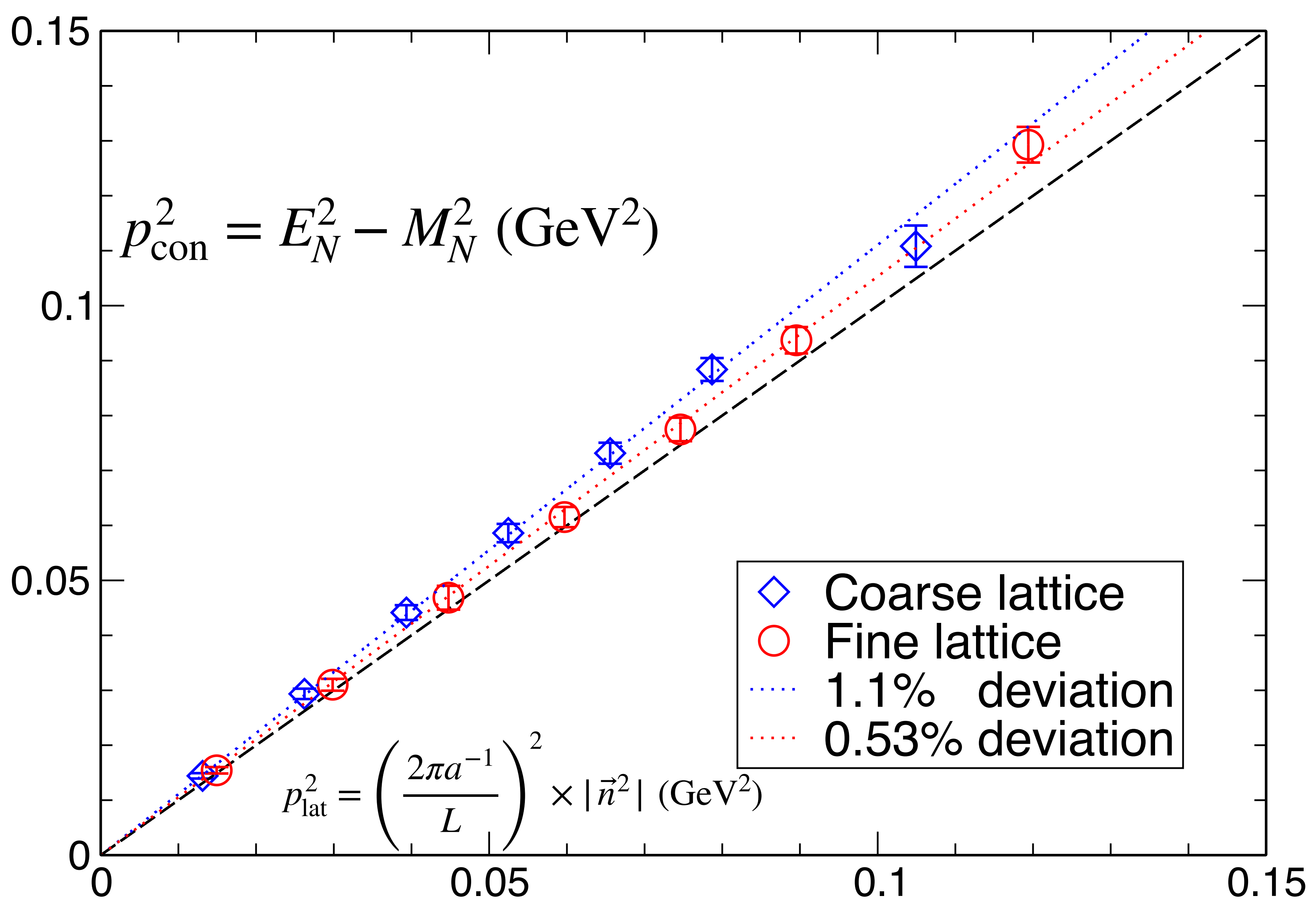
$\sim m_{\mathrm{AWTI}}^{\mathrm{PCAC}} - m_{\mathrm{PCAC}}$

- Nucleon 3-pt function
- Nonzero momentum
- Improvement works

$(m_{\mathrm{AWTI}}^{\mathrm{PCAC}})^{\mathrm{imp}} = m_{\mathrm{AWTI}}^{\mathrm{PCAC}} - a c_A q^2/2$

Dispersion relation

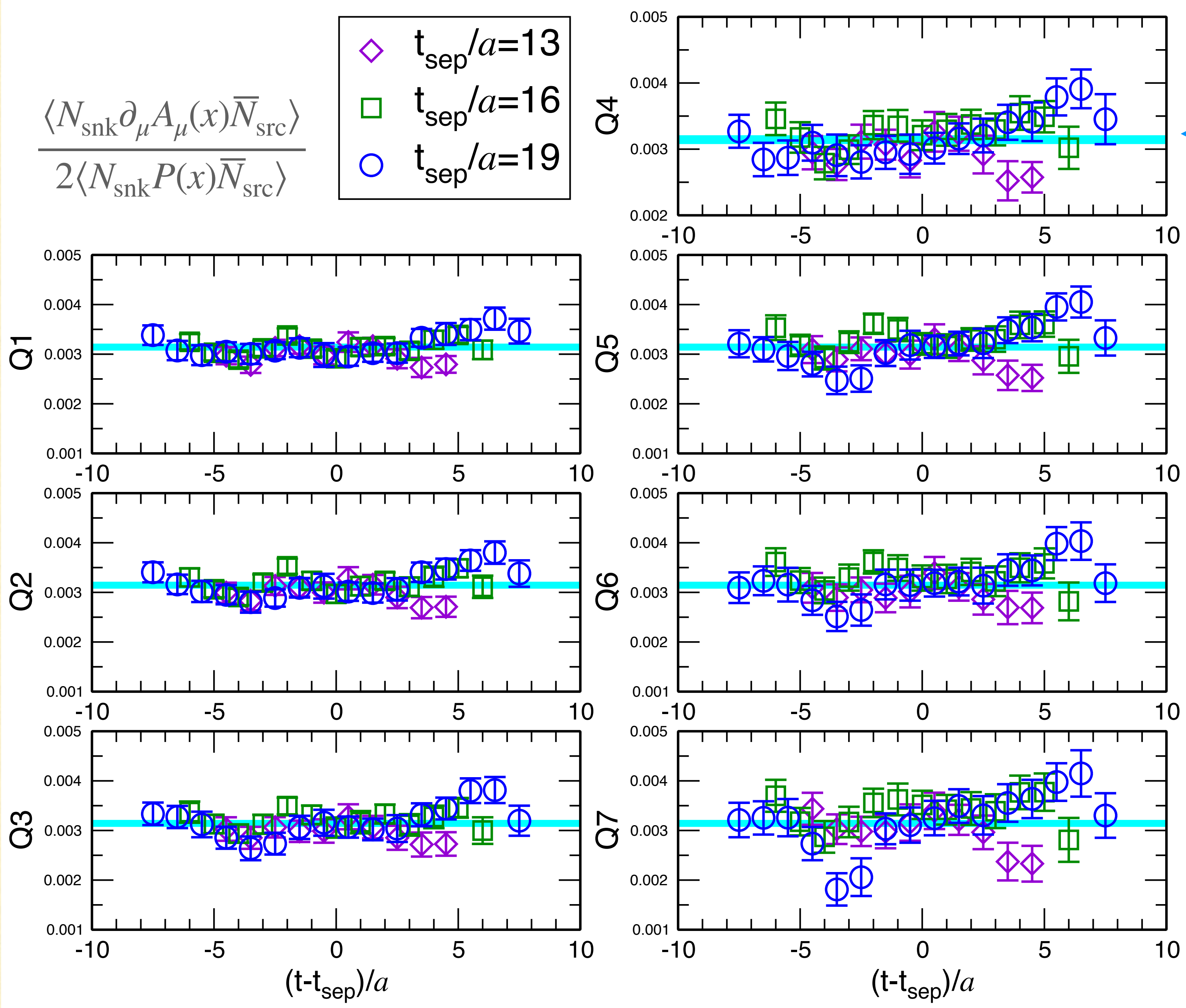
Discretization effect is small



[1] E. Shintani et al., Phys. Rev. D **99**, 014510(2019), (Erratum; Phys. Rev. D 102 (2020) 019902.)

G. S. Bali et al., Phys. Lett. B 789, 666-674 (2019).

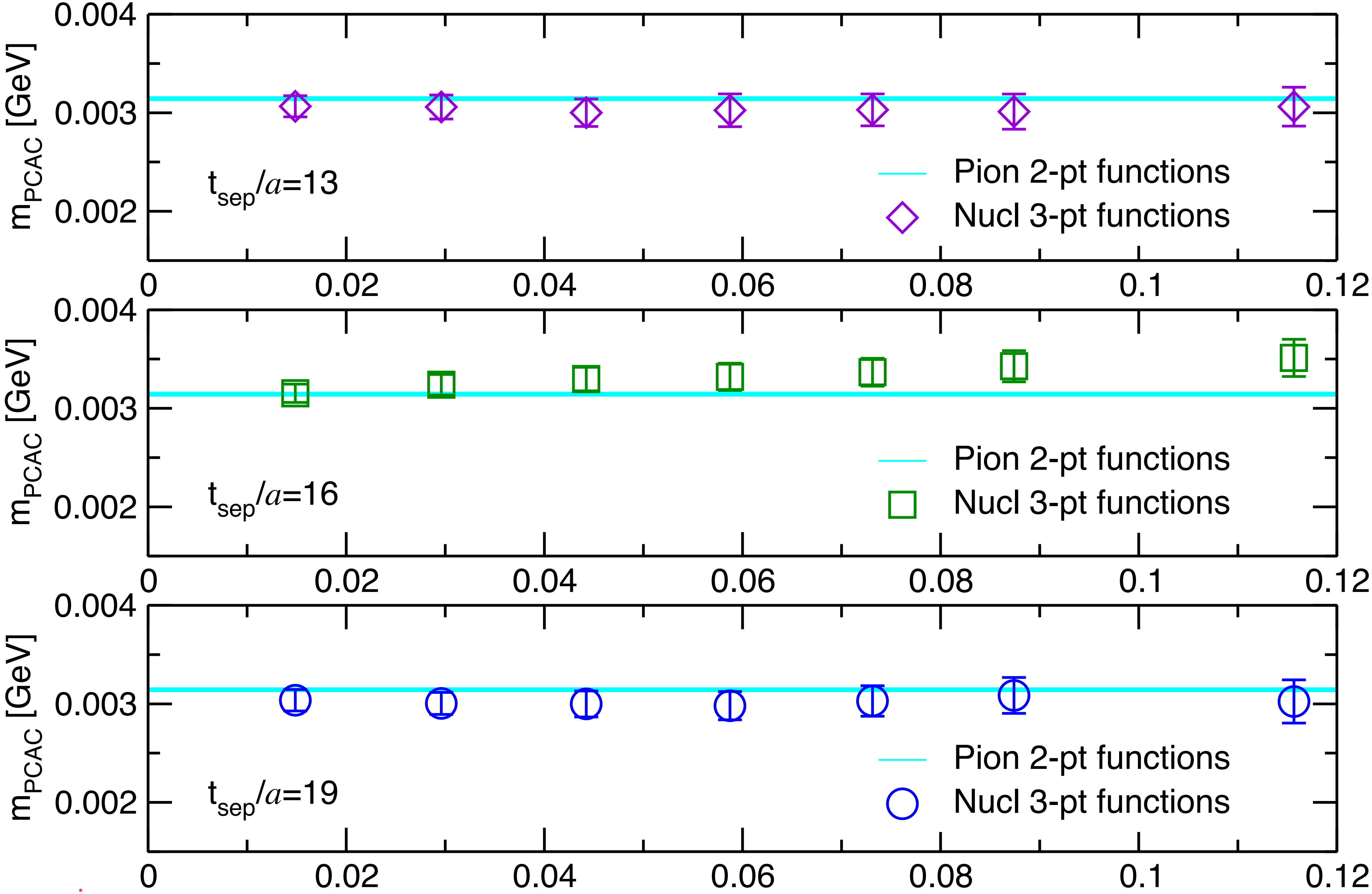
PCAC satisfying correlation functions



m_{PCAC}

G. S. Bali et al., Phys. Lett. B 789, 666-674 (2019). Y. C. Jang et al., arXiv:2305.11330.

PCAC satisfying correlation functions



$A_\alpha^{\rm imp} = A_\alpha + c_A a \partial_\alpha P$ effect is small

Summary of our form factor studies

- Conclusion of this talk

Summary of form factor studies

PACS Collaboration for Nucleon projects:

- AMA technique → High statistical precision
- Physical point → No chiral extrapolations
- Large physical volume ($\sim 10^4\ \mathrm{fm}^4$) → Low Q^2 information
- *Fully dynamical lattice QCD simulations* towards **continuum limit**

Our preliminary results:

- For g_A, both coarse & fine reproduce PDG within stat. err. (2%).
- Large discretization error appears on the radii compared to the error on the dispersion relation, and it would not be resolved by the $O(a)$ improved current.

Error budget	g_A	$\sqrt{\langle (r_E^v)^2 \rangle}$	$\sqrt{\langle (r_M^v)^2 \rangle}$	$\sqrt{\langle (r_A^v)^2 \rangle}$
Statistical:	1.9%	5.6%	8.3%	10.5%
Discretization:	1.6%	8.3%	9.0%	11.3%