# Hidden Conformal Symmetry from the Lattice

arXiv:2305.03665

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for the LSD Collaboration

Lattice 2023, Fermilab

August 3, 2023

# Outline

# Collaborators

## Lattice Strong Dynamics

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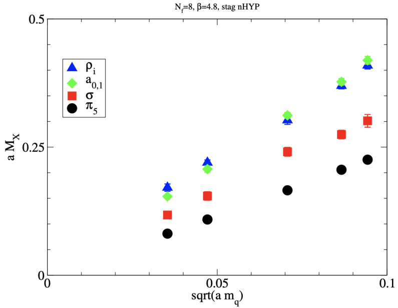
# The $N_f = 8$ Theory

We analyze our latest lattice data for the SU(3) gauge theory with 8 Dirac fermion flavors. The data is presented in 2306.06095:




- This gauge theory is believed to lie close to the boundary of the conformal window.
- The $\sigma$ and $\pi$ are somewhat separated from the $\rho$ in the spectrum.

SU(3) with $N_f = 8$ can be used to build composite Higgs models, e.g PRL **126** (2021) 191804

# Dilaton EFT

Reviewed in Universe 9 (2023) 1, 10 with T. Appelquist and M. Piai.

**Field Content**

i. $N_f^2 - 1$ NGB fields $\pi^a$
$\Sigma = \exp\{2i\pi^a T^a/F_\pi\}$
$\langle\Sigma\rangle = \mathbb{1}$

ii. Dilaton field $\chi$
$\langle\chi\rangle = F_d$

See dilaton EFT of Golterman and Shamir: PRD **94** (2016)

**Symmetries**

**Chiral Symmetry**

$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$
$\Sigma \to L\Sigma R^\dagger$

**Scale Invariance**

Scale $\times$ Poincaré $\to$ Poincaré
$\chi(x) \to e^\lambda \chi(e^\lambda x)$

# Dilaton EFT at Leading Order

## Theory Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] + \frac{m B_\pi f_\pi^2}{2}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] - V(\chi) . \quad (1)$$

- NGB terms are similar to those in chiral Lagrangian.
- Dependence on compensator field $\chi$ is determined by scale invariance.
- Expect $f_\pi \sim f_d$ set by confinement scale.
- Parameter $y$ has been identified with scaling dimension of $\bar{\psi}\psi$ above the confinement scale: Bardeen et al NPB **323**, 493 (1989).

# Dilaton Potential

$$V_\Delta = \frac{m_d^2 \chi^4}{4(4-\Delta) f_d^2} \left[ 1 - \frac{4}{\Delta} \left( \frac{f_d}{\chi} \right)^{4-\Delta} \right]. \tag{2}$$

- Potential contains a scale invariant term ($\sim \chi^4$) and a deformation ($\sim \chi^\Delta$), which explicitly violates scale invariance.
- This potential has a minimum at $\chi = f_d$, and a weak curvature $m_d^2 \ll (4\pi f_d)^2$.
- For $\Delta < 4$, $V_\Delta$ grows as $\chi^4$ for large $\chi$.
- For $\Delta > 4$, $V_\Delta$ grows as $\chi^\Delta$ for large $\chi$.
- Potentials of this form are discussed in e.g: Rattazzi & Zaffaroni JHEP **0104**, 021 (2001), GGS PRL.**100** 111802, (2008), CCT PRD.**100** 095007 (2019).

# Dilaton Potential

Special case: The SM Higgs potential $\Delta = 2$.

$$V(\chi) = \frac{m_d^2}{8f_d^2}\left(\chi^2 - f_d^2\right)^2 \qquad (3)$$

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Special case: Near marginal deformation $\Delta \to 4$.

$$V(\chi) = \frac{m_d^2}{16f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right) \tag{4}$$

# Scalar Decay Constant

## Measured by LatKMI in PRD 96 (2017) 014508

Define scalar decay constant using the matrix element

$$\langle 0| J_S(x) |\chi(p)\rangle \equiv F_S M_d^2 e^{-p\cdot x}\,, \tag{5}$$

where

$$J_S(x) \equiv m \sum_{i=1}^{N_f} \bar{\psi}_i \psi_i\,. \tag{6}$$

1. $F_S$ can be extracted from lattice measurement of correlator $\langle J_S(x) J_S(0)\rangle$, which is used already to measure $M_d$.
2. It is a true decay constant: It would control the decay rate of the dilaton if there was a heavy scalar mediator coupled to $\bar{\psi}\psi$ along with light states. Analogous to $f_\pi$ for the QCD pion decaying to leptons via $W^\pm$.

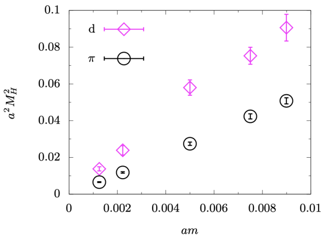
# Scalar Decay Constant

This quantity can also be calculated in dilaton EFT:

$$|F_S| = \frac{yN_f M_\pi^2 F_\pi}{2M_d^2}\frac{f_\pi}{f_d}. \tag{7}$$

- Incorporating Eq. (7) into our EFT fit provides a direct test of the coupling between the light scalar and the fermion mass, treated as an external source.

# Lattice Data



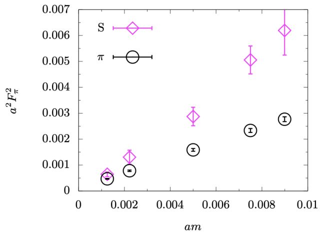




Figure: Lattice data for $M_\pi^2$, $M_d^2$, $F_\pi^2$ and $F_S^2$ from LSD 2306.06095. The lattice spacing is denoted by $a$.

We also include data for the $\pi$–$\pi$ scattering length in the I=2, $\ell = 0$ channel from LSD PRD **105** (2022) 034505

# Result Of Global Fit to dEFT

| Parameter | Value and Uncertainty |
| --- | --- |
| $y$ | 2.091(32) |
| $aB_\pi$ | 2.45(13) |
| $\Delta$ | 3.06(41) |
| $a^2 f_\pi^2$ | $6.1(3.2) \times 10^{-5}$ |
| $f_\pi^2/f_d^2$ | 0.1023(35) |
| $m_d^2/f_d^2$ | 1.94(65) |
| $\chi^2$/dof | 21.3/19=1.12 |

Table: Central values of fit parameters obtained in a six parameter global fit to LSD data for $M^2_{\pi,d}$, $F^2_{\pi,S}$ and scattering length.

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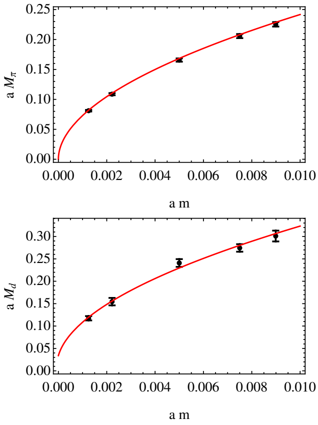

# Scaling Relations at Leading Order

We also want to test the alternate possibility - that the $N_f = 8$ theory is *inside* the conformal window.

Assuming the gauge coupling $g$ has reached its fixed point value $g^\star$, physical quantities may be fitted to scaling relations Zwicky, del Debbio PLB **700** (2011)

$$M_X = C_X\, m^{[1/(1+\gamma^\star)]}\,, \tag{8}$$
$$F_Y = C_Y\, m^{[1/(1+\gamma^\star)]}\,, \tag{9}$$
$$1/a_0^{(2)} = C_a\, m^{[1/(1+\gamma^\star)]}\,. \tag{10}$$

Following approach of Appelquist et al PRD **84** (2011) 054501.

# Result of Global Fit to Mass–Deformed CFT

Fitting to the same set of lattice data as in the dilaton case, we find:

| Parameter | Value and Uncertainty |
|---|---|
| $C_{M_\pi}$ | 2.121(78) |
| $C_{F_\pi}$ | 0.522(19) |
| $C_{M_d}$ | 2.97(12) |
| $C_{F_S}$ | 0.706(33) |
| $C_a$ | -5.88(22) |
| $\gamma^*$ | 1.073(28) |
| $\chi^2/\text{dof}$ | $48.1/19 = 2.53$ |

The $\chi^2/\text{dof}$ is larger than for the dEFT fit, while the number of fit parameters is the same. This indicates a lower quality fit.

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- Assuming the gauge theory is inside the conformal window, we fit the same set of lattice data to mass–deformed CFT scaling relations. This fit is of lesser quality.
- The worse mdCFT fit could be a consequence of $g \not\approx g^\star$.
- Adding particular NLO corrections can improve the AIC for both kinds of fit. The required NLO corrections are large in the mdCFT case.

Thank you!

# Lattice Action

- Our numerical calculations use improved nHYP smeared staggered fermions with smearing parameters $\alpha = (0.5, 0.5, 0.4)$. [LSD PRD 99(2019)014509]
- $\beta_A/\beta_F = -0.25$ where $\beta_F = 4.8$.
- After taste splitting, only $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ flavor symmetry preserved in massless theory (3 exact NGBs).
- Spectral study has revealed that the taste splitting of the 63-plet masses are on the order of 20–30%. [LSD PRD 99(2019)014509]

# Summary of Improvements to Lattice Dataset

## Presented in 2306.06095

Since the previous LSD study of the $N_f = 8$ theory PRD **99** (2019) 014509, we have made some changes.

1. We have data for a new observable: The scalar decay constant $F_S$.
2. We have extrapolated the quantities $M_\pi$, $F_\pi$, $M_\sigma$ (and also $F_S$) to the infinite volume limit.
3. We have improved our estimates of systematic uncertainties using Bayesian Model Averaging Jay, Neil PRD **103** (2021) 114502

The $N_f = 8$ spectrum has also been calculated before in LatKMI PRD **96** (2017) 014508

# $I = 2$ Interpolating Operators

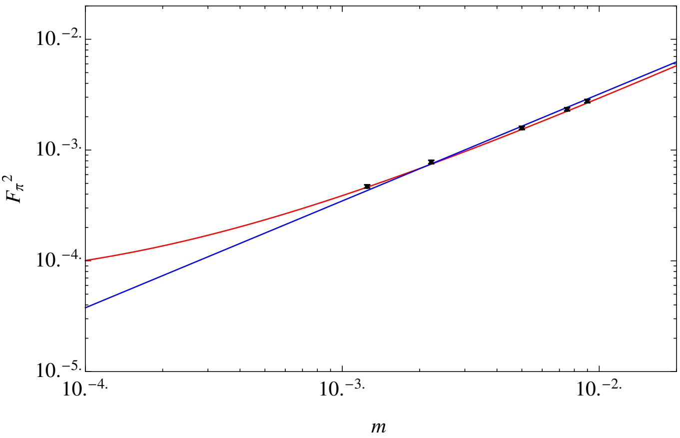
$$\pi^+(t) = \sum_{\vec{x}} \bar{\chi}_2(x)\epsilon(x)\chi_1(x), \text{ where } \epsilon(x) = (-1)^{x+y+z+t} \tag{11}$$

$$\mathcal{O}_{I=2}(t) = \pi^+(t)\pi^+(t+1) \tag{12}$$

$$\begin{aligned} C_{I=2}(t, t_0) &= \langle \mathcal{O}_{I=2}(t)\mathcal{O}_{I=2}(t_0)^\dagger \rangle \\ &= \sum_{\vec{x}_1,\cdots,\vec{x}_4} \langle \pi^+(t_4, \vec{x}_4)\pi^+(t_3, \vec{x}_3)\pi^+(t_2, \vec{x}_2)^\dagger \pi^+(t_1, \vec{x}_1)^\dagger \rangle \end{aligned} \tag{13}$$

Wall sources used - moving wall method.

# Extrapolation Of $F_\pi^2$



# Corrections to Scaling in mdCFT

If we continue to assume that $g \approx g^\star$, we can also expect corrections to scaling relations that are polynomial in $m$.

Adding the next-to-leading corrections yields

$$M_X = C_X m^{[1/(1+\gamma^\star)]} + D_X m\,, \tag{14}$$
$$F_Y = C_Y m^{[1/(1+\gamma^\star)]} + D_Y m\,, \tag{15}$$
$$1/a_0^{(2)} = C_a m^{[1/(1+\gamma^\star)]} + D_a m\,. \tag{16}$$

To compare the quality of fits to models with different numbers of free parameters, we use the Akaike Information Criterion (AIC). Models with lower AIC are more probable in a Bayesian framework.

# NLO Fits to mdCFT

We have only added the correction terms which minimize the AIC.

| Parameter | LO | NLO 1 | NLO 2 |
|---|---|---|---|
| $C_{M_\pi}$ | 2.121(78) | 1.56(11) | 1.57(12) |
| $C_{F_\pi}$ | 0.522(19) | 0.445(21) | 0.448(23) |
| $C_{M_d}$ | 2.97(12) | 2.53(12) | 2.55(13) |
| $C_{F_S}$ | 0.706(33) | 0.599(33) | 0.459(63) |
| $C_a$ | -5.88(22) | -5.05(24) | -5.86(53) |
| $\gamma^*$ | 1.073(28) | 1.207(41) | 1.200(44) |
| $D_{M_\pi}$ | — | 4.80(87) | 4.71(90) |
| $D_{F_S}$ | — | — | 2.77(98) |
| $D_a$ | — | — | 12.9(5.8) |
| $\chi^2$/dof | 48.1/19 | 20.9/18 | 6.90/16 |
| AIC | 60.1 | 34.9 | 24.9 |

NLO correction to $F_S$ grows to 46% of LO contribution size.

# NLO Corrections in dEFT

We do not have complete NLO calculations for all our observables in dEFT.

Some of these corrections will likely come suppressed by $M_\pi^2/(4\pi F_\pi)^2$.

Lets take a phenomenological approach and add a contribution to the observable that shows the largest tension in the fit:

$$M_\pi a_0^{(2)} = \frac{-M_\pi^2}{16\pi F_\pi^2}\left(1-(y-2)^2\frac{f_\pi^2}{f_d^2}\frac{M_\pi^2}{M_d^2}+\frac{l_a M_\pi^2}{(4\pi F_\pi)^2}\right), \qquad (17)$$

We neglect potential chiral logs.

# NLO Fit in dEFT

| Parameter | LO | NLO |
|---|---|---|
| $y$ | 2.091(32) | 2.069(32) |
| $B_\pi$ | 2.45(13) | 2.46(13) |
| $\Delta$ | 3.06(41) | 2.88(49) |
| $f_\pi^2$ | $6.1(3.2) \times 10^{-5}$ | $5.8(3.4) \times 10^{-5}$ |
| $f_\pi^2 / f_d^2$ | 0.1023(35) | 0.1089(41) |
| $m_d^2 / f_d^2$ | 1.94(65) | 2.24(80) |
| $l_a$ | — | 0.78(27) |
| $\chi^2$/dof | 21.3/19 | 10.3/18 |
| AIC | 33.3 | 24.3 |

The AIC is reduced by adding the NLO correction to a level below the AIC in the NLO mdCFT case.

Correction is small - under 10%.