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# Local Polyakov-loop fluctuation and center domains in quark-gluon plasma with many colors

**Yuto NAKAJIMA** [A], **Hideo SUGANUMA** [B]

Faculty of Science, Kyoto University [A]

Department of Physics, Kyoto University [B]

Based on **YN** and H. Suganuma, arXiv 2212.13874 [hep-th] (2022)

Introduction

**YN** and H. Suganuma, arXiv 2212.13874 [hep-th] (2022)

# Deconfinement vacuum breaks $\mathbb{Z}_N$ symmetry

- In this talk, we consider **finite-temperature $SU(N)$ QCD without fermions**
- Plaquette action is invariant under $\mathbb{Z}_N$-transf. (Center symmetry)

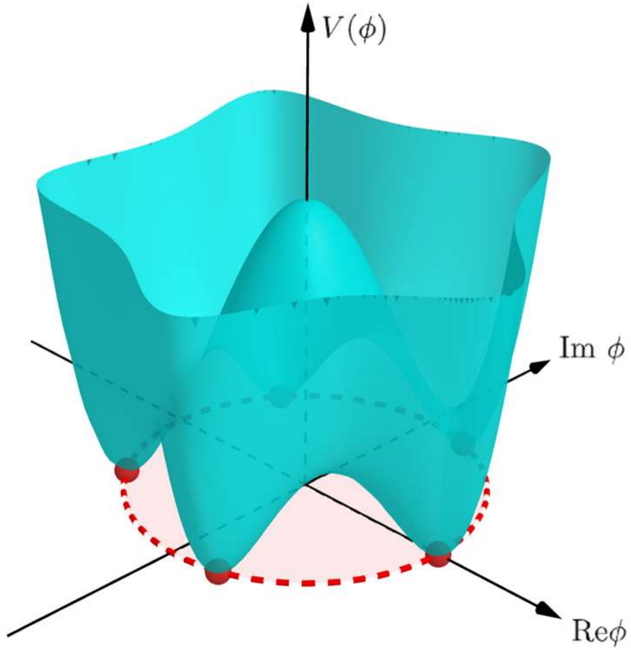
Center symmetry is spontaneously broken in deconfinement phase:

$$\langle \phi \rangle \xrightarrow{\mathbb{Z}_N \text{ transf.}} z \langle \phi \rangle \quad (z \in \mathbb{Z}_N \subset \mathrm{SU}(N))$$

$$\left( \text{Polyakov loop: } \phi = \frac{1}{N} \mathrm{Tr}(L) = \mathrm{Tr}\left[ \prod_\tau U_\tau(\tau, \boldsymbol{s}) \right] \right)$$

Due to this SSB, quark-gluon plasma (w/o quarks) has $N$ different degenerated vacua:

$$\boxed{\langle \phi \rangle \sim e^{2\pi i k/N} \; (k = 0, 1, \ldots, N)}$$




e.g.) Traced Polyakov loop effective potential for $N = 5$

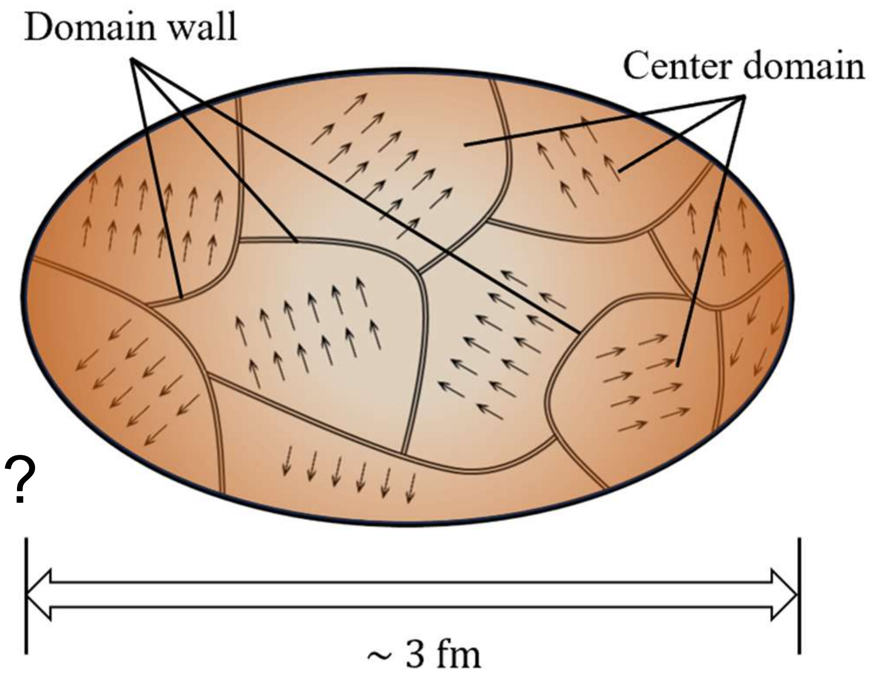
**YN** and H. Suganuma, arXiv 2212.13874 [hep-th] (2022)

## Introduction

# Center domains in deconfinement vacuum

S. Borsányi *et al.*, J. Phys. Conf. Ser. **312**, 012005 (2010)
M. Asakawa *et al.*, Phys. Rev. Lett. **110**, 202301 (2013)

As a plausible possibility, quark-gluon plasma has several center domains separated by potential walls.




**Points**:

- Can fluctuations change the local expectation value $\langle \phi \rangle$ in center domains?
- How large is typical volume of stable center domains?

**YN** and H. Suganuma, arXiv 2212.13874 [hep-th] (2022)

# Polyakov loop model from plaquette action

J. Polonyi, Phys. Lett. B **110**, 395 (1982)
F. Green and F. Karsch, Nucl. Phys. B **238**, 297 (1984)

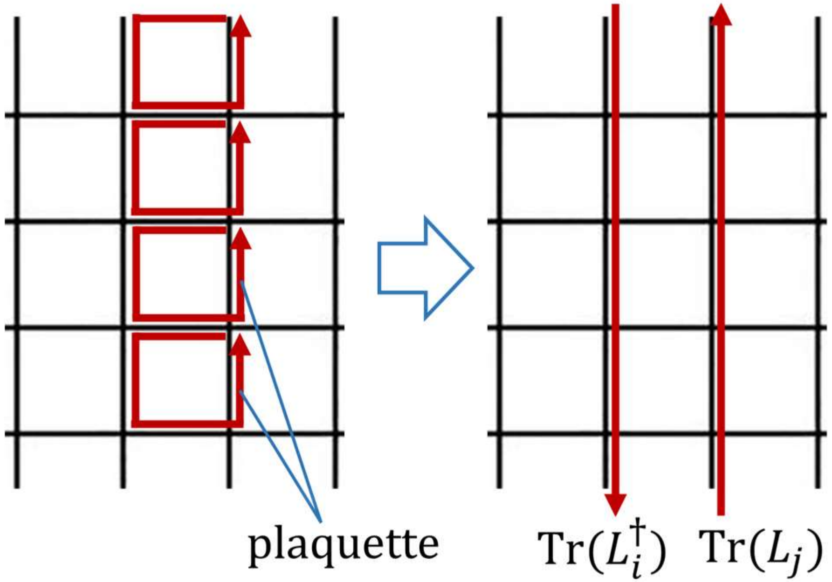
Reduced plaquette action (without quarks)

$$Z_{\rm YM} = \int \mathscr{D}L \exp\left[ e^{-\beta\sigma a} \sum_{\langle i,j\rangle} \mathrm{Tr}(L_i^\dagger)\, \mathrm{Tr}(L_j) \right]$$

$$= \int \mathscr{D}\phi \exp\left[ N^2 e^{-\beta\sigma a} \sum_{\langle i,j\rangle} \phi_i^* \phi_j + \sum_i \ln \boxed{\mathscr{H}^{(N)}(\phi_i)} \right]$$

Jacobian

$i, j$: spatial coordinates
$a$: lattice spacing, $\beta = 1/T$, $\sigma = a^{-2} \ln(g^2 N)$




When $N = 2$ or $N = 3$, the Jacobian is precisely known:

$$\mathscr{H}^{(2)}(\phi) = 1 - \phi^2$$

$$\mathscr{H}^{(3)}(\phi) = 1 - 6|\phi|^2 - 3|\phi|^4 + 8\mathrm{Re}\, \phi^3$$

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# $\mathbb{Z}_N$-invariant Polyakov loop effective action

Introduce $\mathbb{Z}_N$-invariant Jacobian as an ansatz:

$$\mathscr{H}^{(N)}(\phi) = 1 - b_2|\phi|^2 - b_4|\phi|^4 + b_N \mathrm{Re}\, \phi^N$$

F. Sannino, Phys. Rev. D **72** (2005)

Then, obtain the **effective action** for Polyakov loop field:

$$Z_{\mathrm{YM}} = \int \mathscr{D}\phi \exp\left[N^2 e^{-\beta\sigma a} \sum_{\langle i,j\rangle} \phi_i^* \phi_j + \sum_i \ln \boxed{\mathscr{H}^{(N)}(\phi_i)}\right]$$

- respects $\mathbb{Z}_N$ symmetry
- yields the $N$-fold degeneracy of vacua

→To focus on **the fluctuation of $\boldsymbol{\phi_i}$ around one of the potential minima**

**Coarse graining**

$$S_{\mathrm{YM}}[\phi(x)] = C \int d^3x \left( |\nabla\phi(x)|^2 - \frac{6}{a^2}|\phi(x)|^2 - \frac{e^{\beta\sigma a}}{N^2 a^2} \ln \mathscr{H}^{(N)}(\phi(x)) \right)$$

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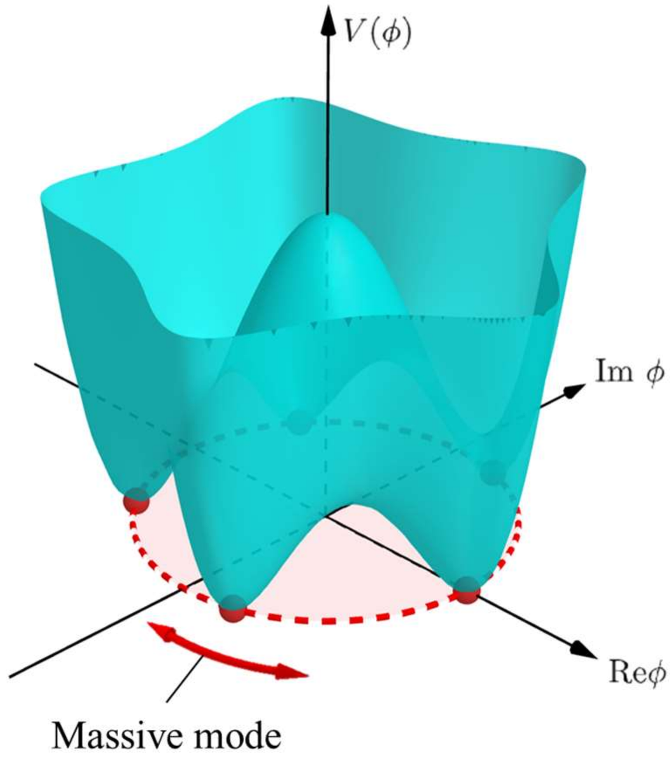
# Correlation length of Polyakov loop phase

Freeze the amplitudes $|\phi|$ to focus on **the phase of Polyakov loop field $\boldsymbol{\phi(x)}$:**

$$S_{\mathrm{YM}}[\phi(x)] = C \int \mathrm{d}^3x \left( |\nabla \phi(x)|^2 - \frac{6}{a^2}|\phi(x)|^2 - \frac{e^{\beta\sigma a}}{N^2 a^2} \ln \mathscr{H}^{(N)}(\phi(x)) \right)$$

$$\xrightarrow{\boxed{\phi(\boldsymbol{x}) = l_0 e^{i\theta(\boldsymbol{x})/l_0}}} \quad 2C \int \mathrm{d}^3x \left[ \frac{1}{2}(\nabla\theta)^2 + V_{\mathrm{YM}}(\theta) \right]$$




Spatial phase correlation function:

$$\langle \theta(\boldsymbol{x})\theta(0) \rangle \propto \frac{1}{|\boldsymbol{x}|} e^{-m_{\mathrm{YM}}|\boldsymbol{x}|} \qquad \left( m_{\mathrm{YM}} \equiv \sqrt{\frac{e^{\beta\sigma a}}{2a^2} \frac{b_N l_0^{N-2}}{b}} \right)$$

which **vanishes in the large-$N$ limit**

$$\boxed{\lim_{N\to\infty} m_{\mathrm{YM}} = 0}$$

$$\left( \text{A typical case:} \quad m_{\mathrm{YM}} \sim 2\ \mathrm{GeV} \quad (N = 3, T = 400\ \mathrm{MeV}, a = 0.4\ \mathrm{fm}) \right)$$

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# Surface tension between adjacent domains

$$S_{\mathrm{YM}}[\phi(x)] \sim \int \mathrm{d}^3x \left[\frac{1}{2}(\nabla\theta)^2 + V_{\mathrm{YM}}(\theta)\right]$$

→ Surface tension is evaluated:

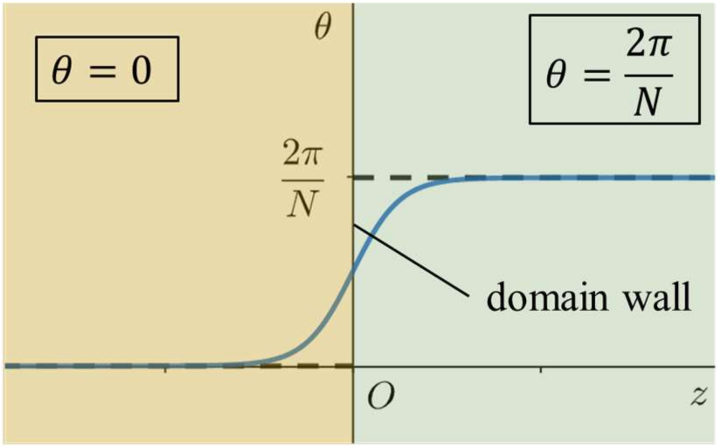
$$\boxed{\alpha = \frac{N}{2a^3}\frac{b_N}{b} l_0^N}$$

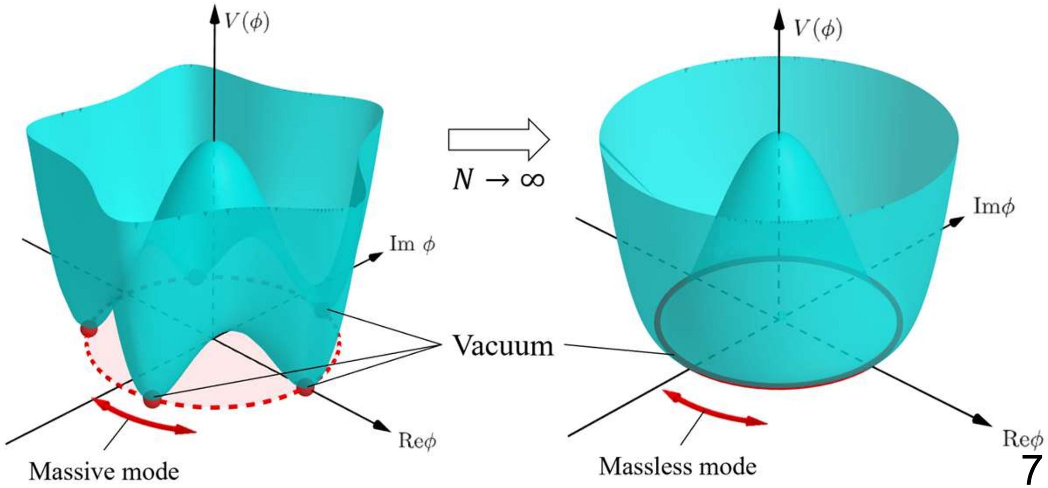
which vanishes in the large-$N$ limit

$$\lim_{N\to\infty} \alpha = 0$$

The large-$N$ limit
- generates a massless mode
- vanishes the surface tension

Consistent to $\mathbb{Z}_N \xrightarrow{N\to\infty} \mathrm{U}(1)$.



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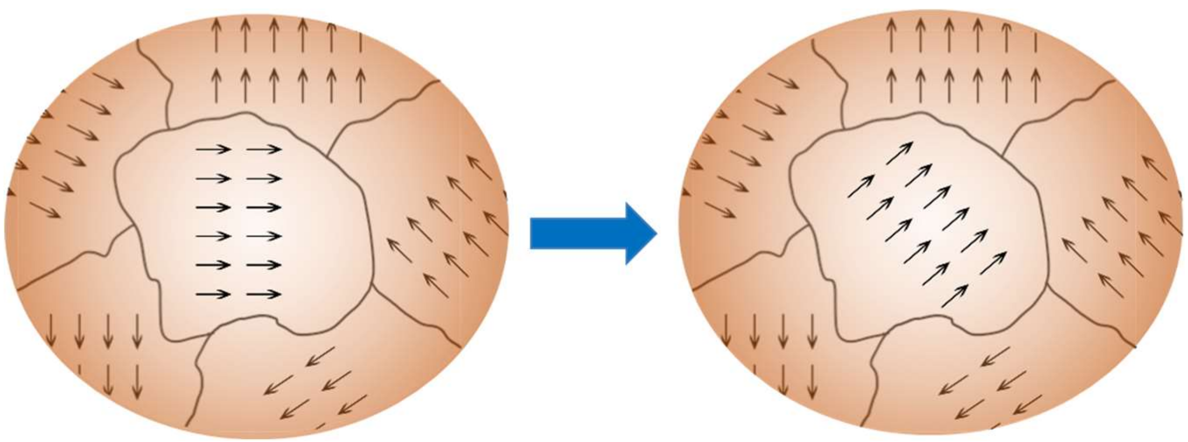
# To evaluate vacuum-to-vacuum transition

Assume that **phase of Polyakov loop $\boldsymbol{\theta(x)}$** is uniform in each center domain

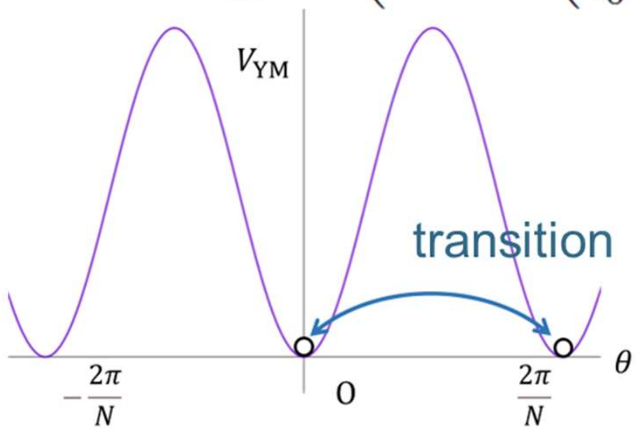
$$S_{\mathrm{YM}}[\phi(x)] \sim \int \mathrm{d}^3x \left[\frac{1}{2}(\nabla\theta)^2 + V_{\mathrm{YM}}(\theta)\right] \Longrightarrow \frac{V}{a}\int \mathrm{d}\tau \left[\frac{1}{2}\left(\frac{\partial\theta}{\partial\tau}\right)^2 + V_{\mathrm{YM}}(\theta)\right]$$ ($V$: domain volume)

→ Describe transition of the phase in a center domain

$\theta(\tau) = 0$ $\qquad\qquad$ $\theta(\tau) = \dfrac{2\pi}{N}$



$$V_{\mathrm{YM}}(\theta) \equiv \frac{m_{\mathrm{YM}}^2 l_0^2}{N^2}\left(1 - \cos\left(\frac{N}{l_0}\theta\right)\right)$$



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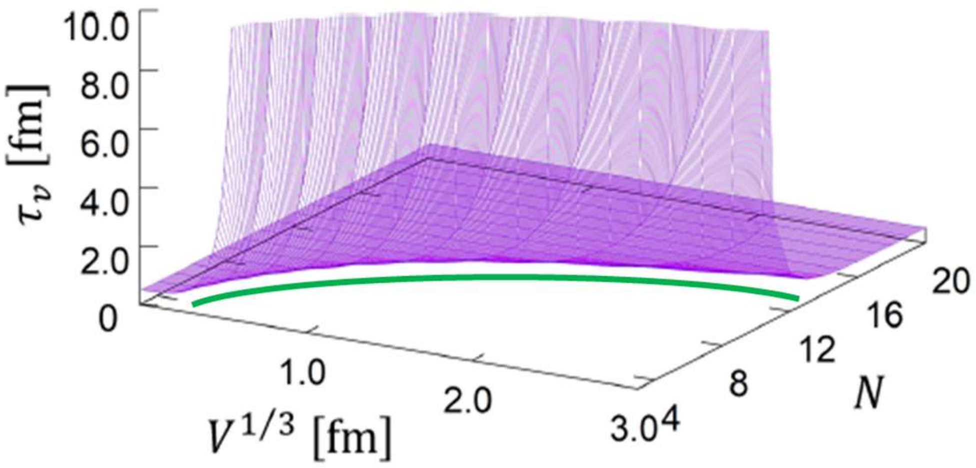
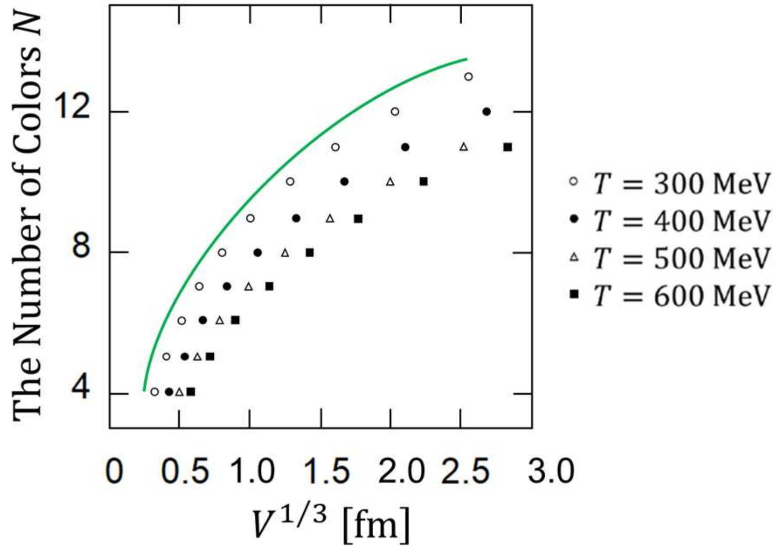
# Vacuum lifetime can be calculated

Considering thermal fluctuations and quantum tunneling effects,
**the** **lifetime** **of domain** $\boldsymbol{\tau_v}$ (average duration of staying in one configuration) is evaluated:

**A typical result** $(T = 400\ \mathrm{MeV}, a = 0.4\ \mathrm{fm})$




**larger** $\boldsymbol{N}$: closer to a massless mode →shorter lifetime (unstable)
**larger** $\boldsymbol{V}$: larger potential barrier →longer lifetime (stable)

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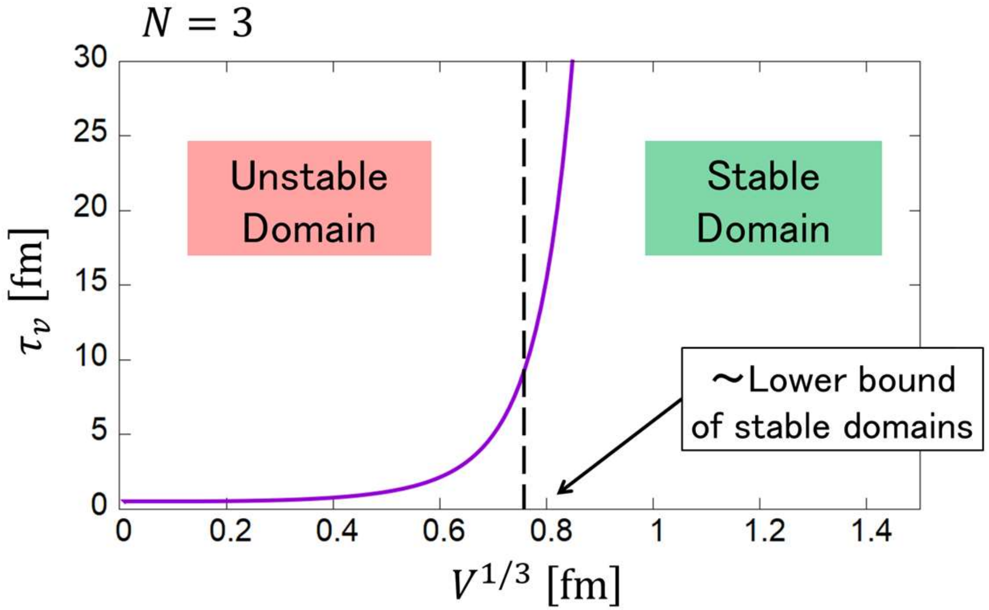
# Stable domains have lower bounds of $V$

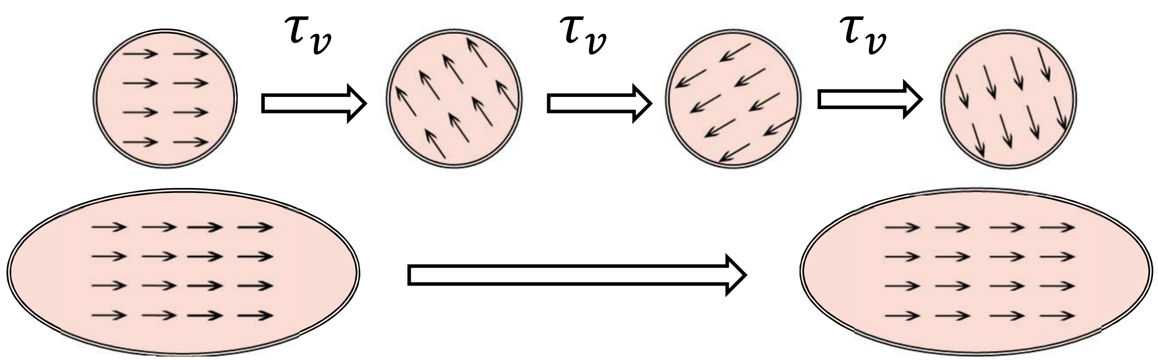



The lifetime ($N$: fixed, $V$: variable) increases rapidly at a specific volume

The **threshold volume** that separates **stable domain region** and **unstable domain region**

A domain with smaller $V$ **(unstable)**

A domain with larger $V$ **(stable)**



Conclusion

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# Domain properties largely depend on $N$

## Investigated center domains in finite-temperature QCD

$$S_{\mathrm{YM}}[\phi(x)] = C\int \mathrm{d}^3x \left( |\nabla\phi(x)|^2 - \frac{6}{a^2}|\phi(x)|^2 - \frac{e^{\beta\sigma a}}{N^2a^2}\ln \mathscr{H}^{(N)}(\phi(x)) \right)$$

## Symmetry conversion $\mathbb{Z}_N \to \mathrm{U}(1)$ in the large-$N$ limit

- **A massless mode** (spatial long-range correlation) emerges

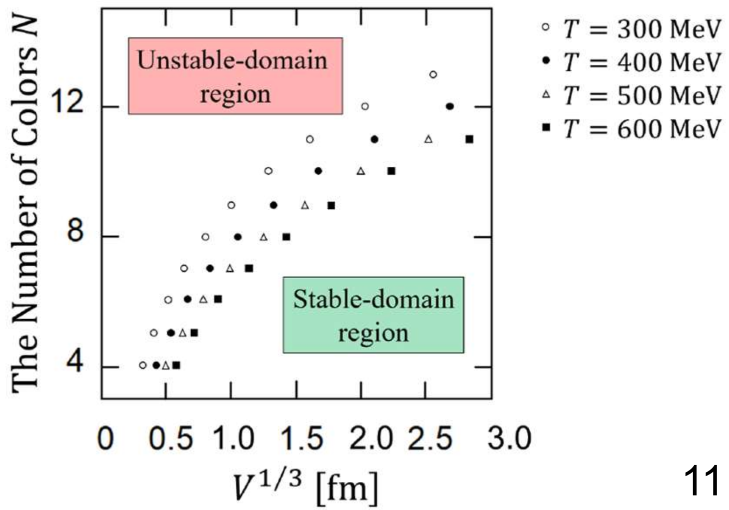
$$\lim_{N\to\infty} m_{\mathrm{YM}} = 0$$

- **Surface tension** of domain walls vanishes

$$\lim_{N\to\infty} \alpha = 0$$

## Evaluation of center domain volume

- There exists **volume lower bound** of stable center domain



Outlook

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# What if dynamical fermions exist?

- **The present work is at a quenched level**
  - The further analysis of the system including fermions
  - Phenomenological (experimental) properties of domain walls

- **Beyond strong-coupling expansion**
  - Monte-Carlo lattice QCD simulation
  - To demonstrate the quark-gluon plasma properties numerically

**Thank you for your attention!**