Local Polyakov-loop fluctuation and center domains in quark-gluon plasma with many colors

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Based on YN and H. Suganuma, arXiv 2212.13874 [hep-th] (2022)
Introduction

Deconfinement vacuum breaks $\mathbb{Z}_N$ symmetry

- In this talk, we consider finite-temperature $SU(N)$ QCD without fermions
- Plaquette action is invariant under $\mathbb{Z}_N$-transf. (Center symmetry)

Center symmetry is spontaneously broken in deconfinement phase:

\[
\langle \phi \rangle \xrightarrow{\mathbb{Z}_N \text{ transf.}} z \langle \phi \rangle \quad (z \in \mathbb{Z}_N \subset SU(N))
\]

(Polyakov loop: $\phi = \frac{1}{N} \text{Tr}(L) = \text{Tr} \left[ \prod_\tau U_\tau(\tau, s) \right]$)

Due to this SSB, quark-gluon plasma (w/o quarks) has $N$ different degenerated vacua:

\[
\langle \phi \rangle \sim e^{2\pi ik/N} \quad (k = 0, 1, \ldots, N)
\]

e.g.) Traced Polyakov loop effective potential for $N = 5$
As a plausible possibility, quark-gluon plasma has several center domains separated by potential walls.

**Points:**
- Can fluctuations change the local expectation value $\langle \phi \rangle$ in center domains?
- How large is typical volume of stable center domains?


Polyakov loop model from plaquette action


Reduced plaquette action (without quarks)

\[ Z_{YM} = \int \mathcal{D}L \exp \left[ e^{-\beta \sigma a} \sum_{\langle i,j \rangle} \text{Tr}(L_i^\dagger) \text{Tr}(L_j) \right] \]
\[ = \int \mathcal{D}\phi \exp \left[ N^2 e^{-\beta \sigma a} \sum_{\langle i,j \rangle} \phi_i^* \phi_j + \sum_i \ln \mathcal{H}^{(N)}(\phi_i) \right] \]

\( i, j \): spatial coordinates
\( a \): lattice spacing, \( \beta = 1/T, \sigma = a^{-2} \ln(g^2 N) \)

When \( N = 2 \) or \( N = 3 \), the Jacobian is precisely known:

\[ \mathcal{H}^{(2)}(\phi) = 1 - \phi^2 \]
\[ \mathcal{H}^{(3)}(\phi) = 1 - 6|\phi|^2 - 3|\phi|^4 + 8\text{Re} \phi^3 \]
\( \mathbb{Z}_N \)-invariant Polyakov loop effective action

Introduce \( \mathbb{Z}_N \)-invariant Jacobian as an ansatz:

\[
\mathcal{H}^{(N)}(\phi) = 1 - b_2|\phi|^2 - b_4|\phi|^4 + b_N \text{Re} \phi^N
\]


Then, obtain the effective action for Polyakov loop field:

\[
Z_{YM} = \int \mathcal{D}\phi \exp \left[ N^2 e^{-\beta \sigma} \sum_{\langle i,j \rangle} \phi_i^* \phi_j + \sum_i \ln \mathcal{H}^{(N)}(\phi_i) \right]
\]

- respects \( \mathbb{Z}_N \) symmetry
- yields the \( N \)-fold degeneracy of vacua

→ To focus on the fluctuation of \( \phi_i \) around one of the potential minima

\[
S_{YM}[\phi(x)] = C \int d^3x \left( |\nabla \phi(x)|^2 - \frac{6}{a^2} |\phi(x)|^2 - \frac{e^{\beta \sigma}}{N^2 a^2} \ln \mathcal{H}^{(N)}(\phi(x)) \right)
\]
Correlation length of Polyakov loop phase

Freeze the amplitudes $|\phi|$ to focus on the phase of Polyakov loop field $\phi(x)$:

$$S_{YM}[\phi(x)] = C \int d^3x \left( |\nabla \phi(x)|^2 - \frac{6}{a^2} |\phi(x)|^2 - \frac{e^{\beta \sigma a}}{N^2 a^2} \ln \mathcal{H}^{(N)}(\phi(x)) \right)$$

$$\phi(x) = l_0 e^{i \theta(x)/l_0}$$

$$2C \int d^3x \left[ \frac{1}{2} (\nabla \theta)^2 + V_{YM}(\theta) \right]$$

Spatial phase correlation function:

$$\langle \theta(x)\theta(0) \rangle \propto \frac{1}{|x|} e^{-m_{YM}|x|}$$

$$m_{YM} = \sqrt{\frac{e^{\beta \sigma a} b N l_0^{N-2}}{2a^2 b}}$$

which vanishes in the large-$N$ limit

$$\lim_{N \to \infty} m_{YM} = 0.$$
Surface tension between adjacent domains

\[ S_{YM}[\phi(x)] \sim \int d^3x \left[ \frac{1}{2} (\nabla \theta)^2 + V_{YM}(\theta) \right] \]

→ Surface tension is evaluated:

\[ \alpha = \frac{N}{2\alpha^3} \frac{b_N}{b} l_0^N \]

which vanishes in the large-\(N\) limit

\[ \lim_{N \to \infty} \alpha = 0 \]

The large-\(N\) limit

• generates a massless mode
• vanishes the surface tension

Consistent to \( \mathbb{Z}_N \overset{N \to \infty}{\longrightarrow} U(1) \).
To evaluate vacuum-to-vacuum transition

Assume that the phase of Polyakov loop \( \theta(x) \) is uniform in each center domain.

\[
S_{YM}[\phi(x)] \sim \int d^3x \left[ \frac{1}{2} (\nabla \theta)^2 + V_{YM}(\theta) \right] \quad \rightarrow \quad \frac{V}{a} \int d\tau \left[ \frac{1}{2} \left( \frac{\partial \theta}{\partial \tau} \right)^2 + V_{YM}(\theta) \right] \quad (V: \text{domain volume})
\]

→ Describe transition of the phase in a center domain

\[
\theta(\tau) = 0 \quad \rightarrow \quad \theta(\tau) = \frac{2\pi}{N}
\]

\[
V_{YM}(\theta) \equiv \frac{m_{YM}^2 l_0^2}{N^2} \left( 1 - \cos \left( \frac{N}{l_0} \theta \right) \right)
\]
Vacuum lifetime can be calculated considering thermal fluctuations and quantum tunneling effects, the lifetime of domain $\tau_v$ (average duration of staying in one configuration) is evaluated:

A typical result ($T = 400$ MeV, $a = 0.4$ fm)

larger $N$: closer to a massless mode $\rightarrow$ shorter lifetime (unstable)

larger $V$: larger potential barrier $\rightarrow$ longer lifetime (stable)
Stable domains have lower bounds of $V$

The lifetime ($N$: fixed, $V$: variable) increases rapidly at a specific volume.

The **threshold volume** that separates stable domain region and unstable domain region.

A domain with smaller $V$ *(unstable)*

A domain with larger $V$ *(stable)*
Conclusion

Domain properties largely depend on $N$

Investigated center domains in finite-temperature QCD

$$S_{\text{YM}}[\phi(x)] = C \int d^3x \left( |\nabla \phi(x)|^2 - \frac{6}{a^2} |\phi(x)|^2 - \frac{e^{\beta \sigma_u}}{N^2 a^2} \ln \mathcal{H}^{(N)}(\phi(x)) \right)$$

Symmetry conversion $\mathbb{Z}_N \rightarrow U(1)$ in the large-$N$ limit

- A massless mode (spatial long-range correlation) emerges
  \[
  \lim_{N \rightarrow \infty} m_{\text{YM}} = 0
  \]

- Surface tension of domain walls vanishes
  \[
  \lim_{N \rightarrow \infty} \alpha = 0
  \]

Evaluation of center domain volume

- There exists volume lower bound of stable center domain

Outlook

What if dynamical fermions exist?

- The present work is at a quenched level
  - The further analysis of the system including fermions
  - Phenomenological (experimental) properties of domain walls

- Beyond strong-coupling expansion
  - Monte-Carlo lattice QCD simulation
  - To demonstrate the quark-gluon plasma properties numerically

Thank you for your attention!