Variational ansatz inspired by quantum imaginary time evolution

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collaborate with

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[arXiv:2303.11016]

[arXiv:2307.13598]

Lattice conference 2023 - Fermilab

Content

- Background: VQE algorithm and QAOA ansatz
- Quantum Imaginary Time Evolution (QITE)
- Numerical results

Ground state preparation for Ising model

$$H_{\text{Ising}} = -\sum_{\langle i,j \rangle} Z_i Z_j$$

Ground state: $\frac{1}{\sqrt{2}}(\,|000\ldots0\rangle + |111\ldots1\rangle)$

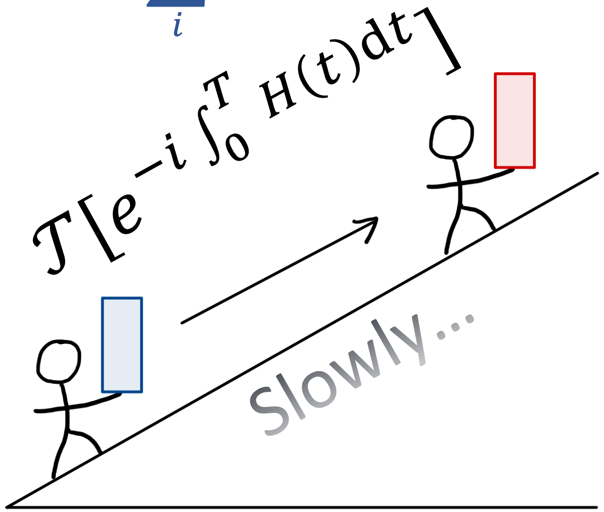
- Prepare ground state with an adiabatic process:

$$H(t) = \frac{t}{T} H(0) + \left(1 - \frac{t}{T}\right) H(T)$$

$$H(0) = -\sum_i X_i \qquad H(T) = -\sum_{\langle i,j \rangle} Z_i Z_j$$

$-\sum_i X_i$ Ground state:

$$\frac{1}{\sqrt{2^N}}(\,|0\rangle + |1\rangle)^{\otimes N} = |+\rangle^{\otimes N}$$



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$$-\sum_i X_i \qquad\qquad -\sum_{\langle i,j \rangle} Z_i Z_j$$

$$\mathcal{T}[e^{-i\int_0^T H(t)\mathrm{d}t}]$$

Initial state: final state:

$$\frac{1}{\sqrt{2^N}}(\,|0\rangle + |1\rangle)^{\otimes N} = |+\rangle^{\otimes N} \Rightarrow e^{i\delta t \sum X} e^{i\delta t \sum ZZ} \ldots e^{i\delta t \sum X} e^{i\delta t \sum ZZ} \Rightarrow \frac{1}{\sqrt{2}}(\,|000\ldots0\rangle + |111\ldots1\rangle)$$

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Initial state: $\frac{1}{\sqrt{2^N}}(\,|0\rangle + |1\rangle)^{\otimes N} = |+\rangle^{\otimes N}$ ➡ $e^{i\delta t\sum X} e^{i\delta t\sum ZZ} \ldots e^{i\delta t\sum X}\, e^{i\delta t\sum ZZ}$ ➡ final state: $\frac{1}{\sqrt{2}}(\,|000\ldots0\rangle + |111\ldots1\rangle)$

$\frac{T}{\delta t}$ layers, too many!

Ground state preparation for Ising model

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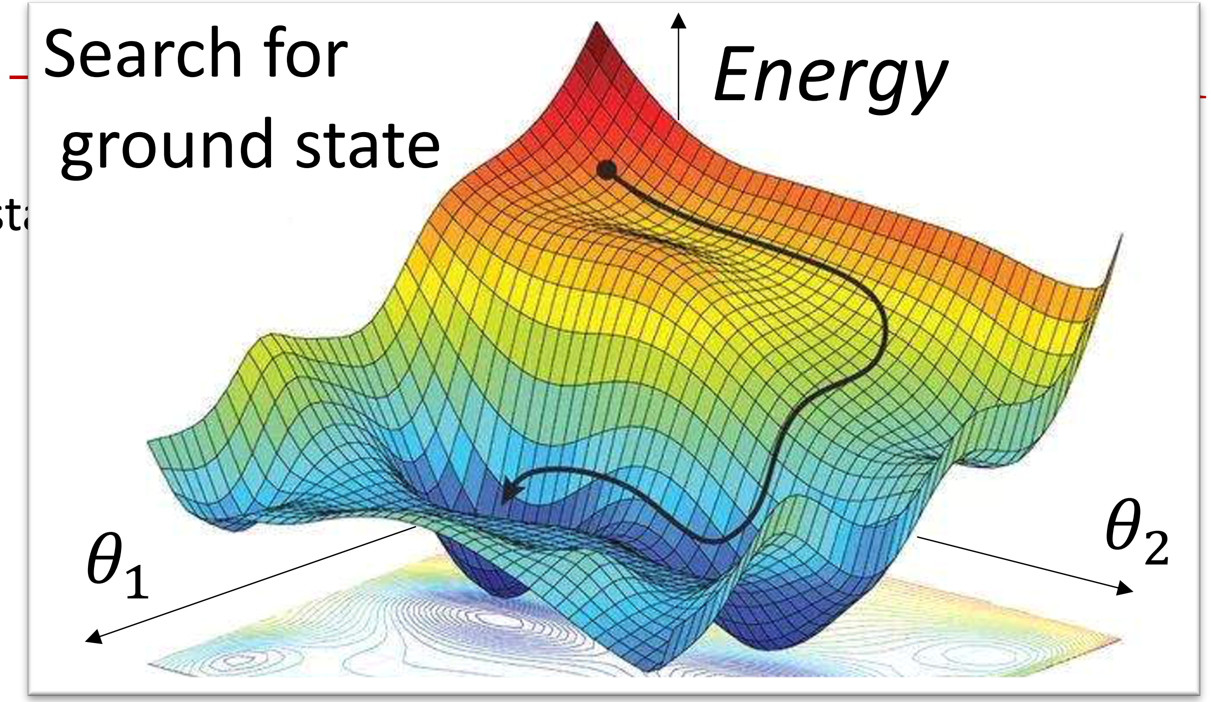
Initial state: $\frac{1}{\sqrt{2^N}}(|0\rangle + |1\rangle)^{\otimes N} = |+\rangle^{\otimes N}$ ⇨ $e^{i\theta_1 \sum X} e^{i\theta_2 \sum ZZ} e^{i\theta_3 \sum X} e^{i\theta_4 \sum ZZ}$ ⇨ final state: $\frac{1}{\sqrt{2}}(|000\ldots0\rangle + |111\ldots1\rangle)$

Turn the evolution time into arbitrary rotation angles
$\theta_1, \theta_2, \theta_3, \theta_4$

Ground state preparation for Ising model

$H_{\text{Ising}} = -$ … $11 \ldots 1\rangle)$

- Prepare ground st…



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final state:

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Variational Quantum Eigensolver (VQE) with QAOA ansatz [E. Farhi et al. 2014]

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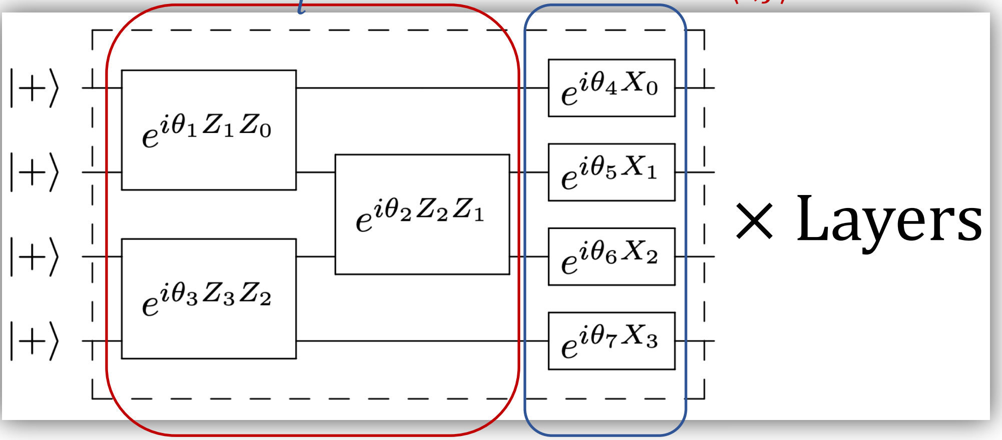

final state:

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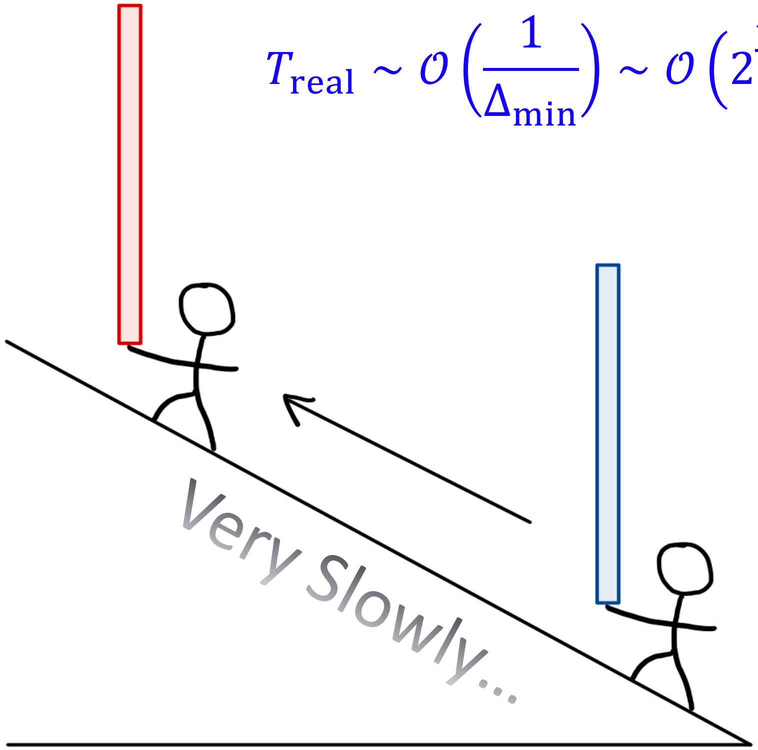
QAOA ansatz [E. Farhi et al. 2014]

Adiabatic process is too slow!

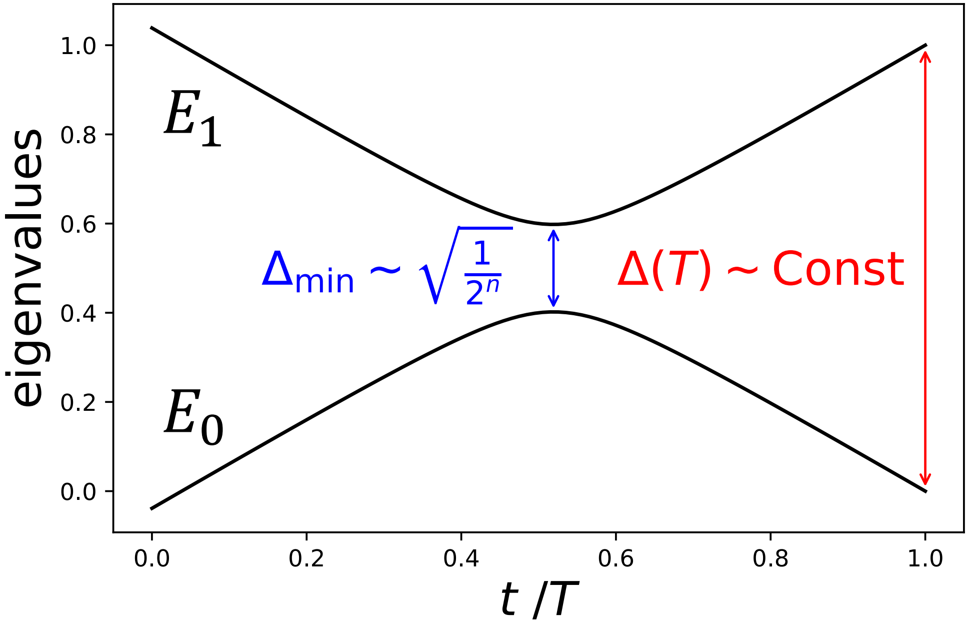
- In some models H_P [E. Farhi et al. 2000], the evolution time T_{real} grows exponentially with the system size n

$$T_{\text{real}} \sim \mathcal{O}\left(\frac{1}{\Delta_{\min}}\right) \sim \mathcal{O}\left(2^{\frac{n}{2}}\right) \sim \text{Layers}$$



$$H(t) = \frac{t}{T} H(0) + \left(1 - \frac{t}{T}\right) H_P$$

Illustration of $H(t)$ mass gap
$\Delta(t) = E_1(t) - E_0(t)$



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$$e^{-T_{\text{imag}} H_P}$$

- In contrast, the **imaginary time evolution** needs evolution time:

$$T_{\text{imag}} \sim \mathcal{O}\left(\frac{1}{\Delta(T)}\right) \sim \mathcal{O}(\text{Const}) \sim \text{Layers}$$

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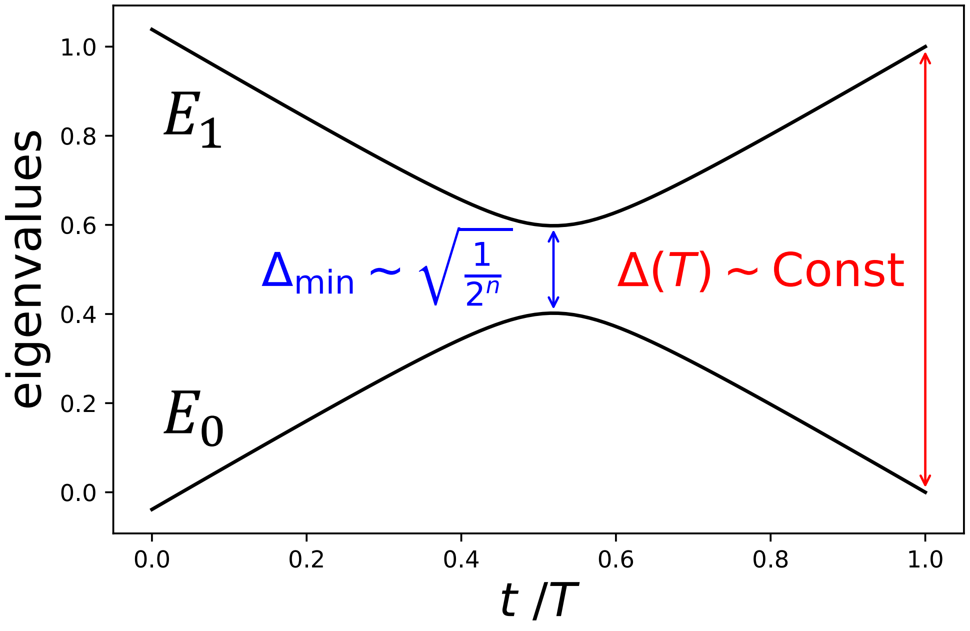

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Imaginary time evolution on quantum computer

- Euclidean correlation function:

$$\lim_{T\to\infty}\frac{1}{Z_T}\mathrm{Tr}\left[e^{-(T-t)H}O_2 e^{-tH}O_1\right] = \sum_n \langle 0|O_2|n\rangle\langle n|O_1|0\rangle e^{-tE_n}$$

- Quantum Imaginary Time Evolution (QITE) [M. Motta et al. 2020]:

$$\frac{e^{-\Delta\tau H}}{\sqrt{\langle\psi|e^{-2\Delta\tau H}|\psi\rangle}}|\psi\rangle = e^{i\Delta\tau\sum_i a_i\sigma_i}|\psi\rangle = \prod_i e^{i\Delta\tau a_i\sigma_i}|\psi\rangle + \mathcal{O}(\Delta\tau^2)$$

Decomposed into CNOT, R_X, R_Y, R_Z quantum gates

Completeness of Pauli-basis

Trotter decomposition

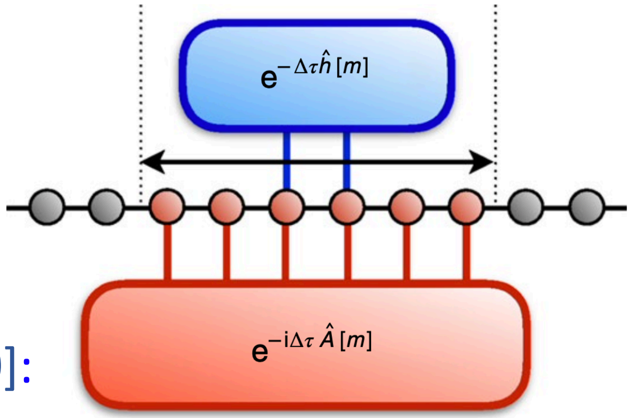
4^n: Too many Paulis!

eg: $\sigma_i = X \otimes Y \otimes Z \otimes I$

Imaginary time evolution on quantum computer

Number of Paulis is a constant if:
- H is **local**
- $|\psi\rangle$ is **finite correlated**

[M. Motta et al. 2020]



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$$e^{i\Delta\tau a_i \sigma_i} \rightarrow e^{i\theta_i \sigma_i},$$
$$\forall\, a_i \neq 0$$

Which $e^{i\Delta\tau a_i \sigma_i}$ has non-zero a_i ?
Use **symmetry** [1] !

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[1] X. Wang, Y. Chai, M. Demidik, X. Feng, K. Jansen, C. Tüysüz. arXiv:2307.13598

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Use **symmetry** [1] !

$$H_{\text{Ising}} = \sum_l Z_l Z_{l+1}$$

$$e^{-\epsilon ZZ} \propto e^{ia_1\epsilon II} \times e^{ia_2\epsilon IX} \times e^{ia_3\epsilon IY} \times e^{ia_4\epsilon IZ}$$
$$\times e^{ia_5\epsilon XI} \times e^{ia_6\epsilon XX} \times e^{ia_7\epsilon XY} \times e^{ia_8\epsilon XZ}$$
$$\times e^{ia_9\epsilon YI} \times e^{ia_{10}\epsilon YX} \times e^{ia_{11}\epsilon YY} \times e^{ia_{12}\epsilon YZ}$$
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:Time-reversal symmetry

:$\mathbb{Z}_2$ symmetry, $\left[H_{\text{Ising}}, \prod X_i\right] = 0$

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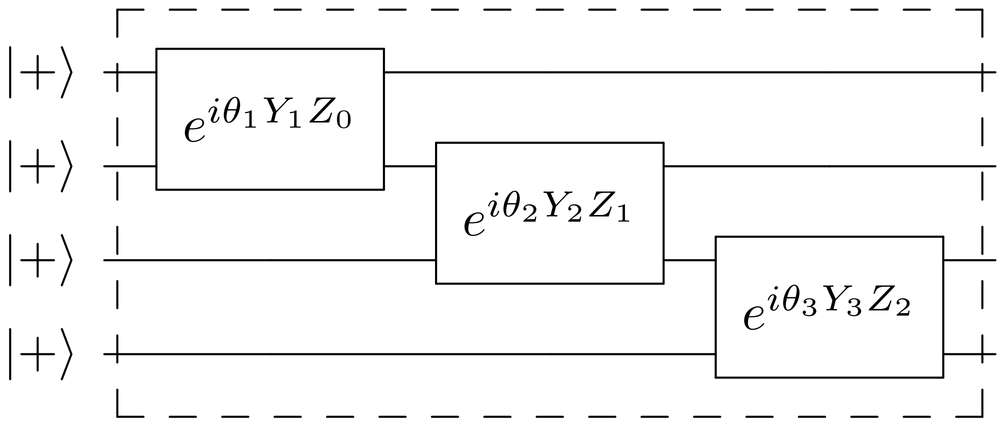
QITE-inspired ansatz for Ising model



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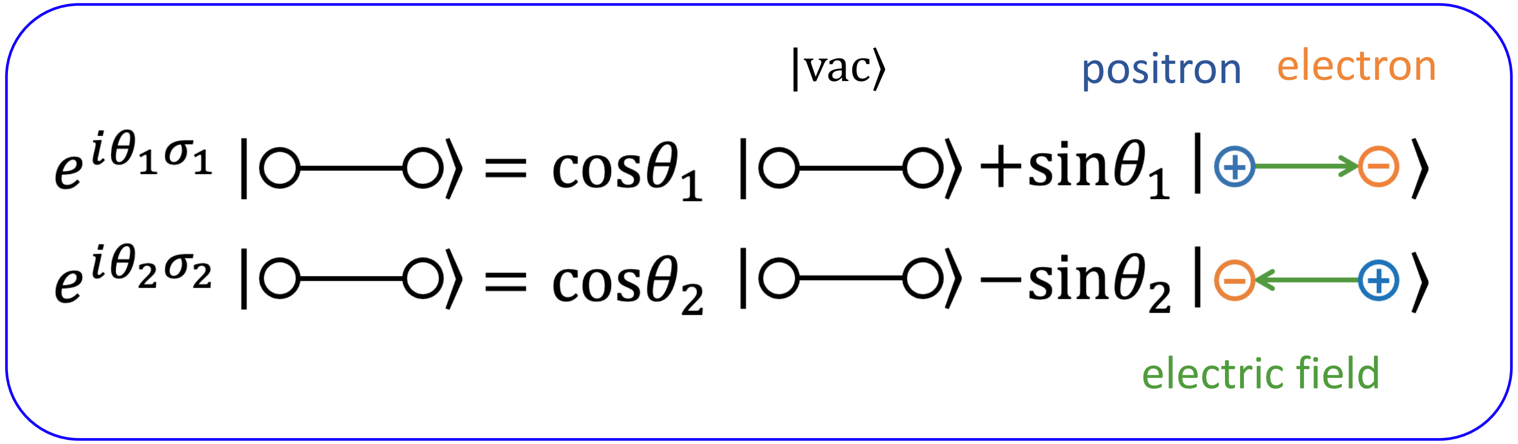
Local U(1) symmetry ➡ **Gauss law** is kept in Schwinger model (No penalty terms)



Electric field line emitted from the positron and absorbed by the electron

[1] X. Wang, Y. Chai, M. Demidik, X. Feng, K. Jansen, C. Tüysüz. arXiv:2307.13598

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Gibbs state preparation on QITE-inspired ansatz

$$H_{\text{Ising}} = -\sum_{\langle i,j \rangle} Z_i Z_j$$

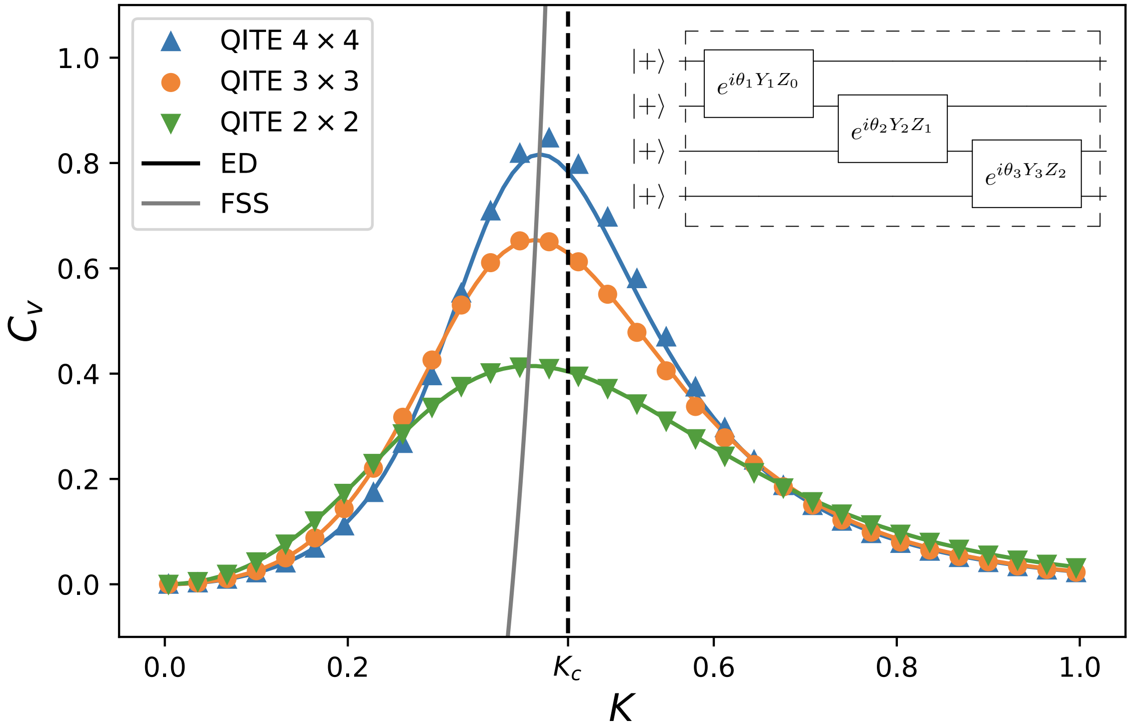
Gibbs state:

$$\rho = \frac{e^{-KH_{\text{Ising}}}}{Z}, \qquad \langle \hat{O} \rangle = \text{Tr}(\rho \hat{O})$$

Calculate the heat capacity C_v:

$$C_v = \frac{K^2}{|\Lambda|} \left(\langle H^2 \rangle - \langle H \rangle^2 \right)$$

- Heat capacity as a function of inverse temperature K. [1]



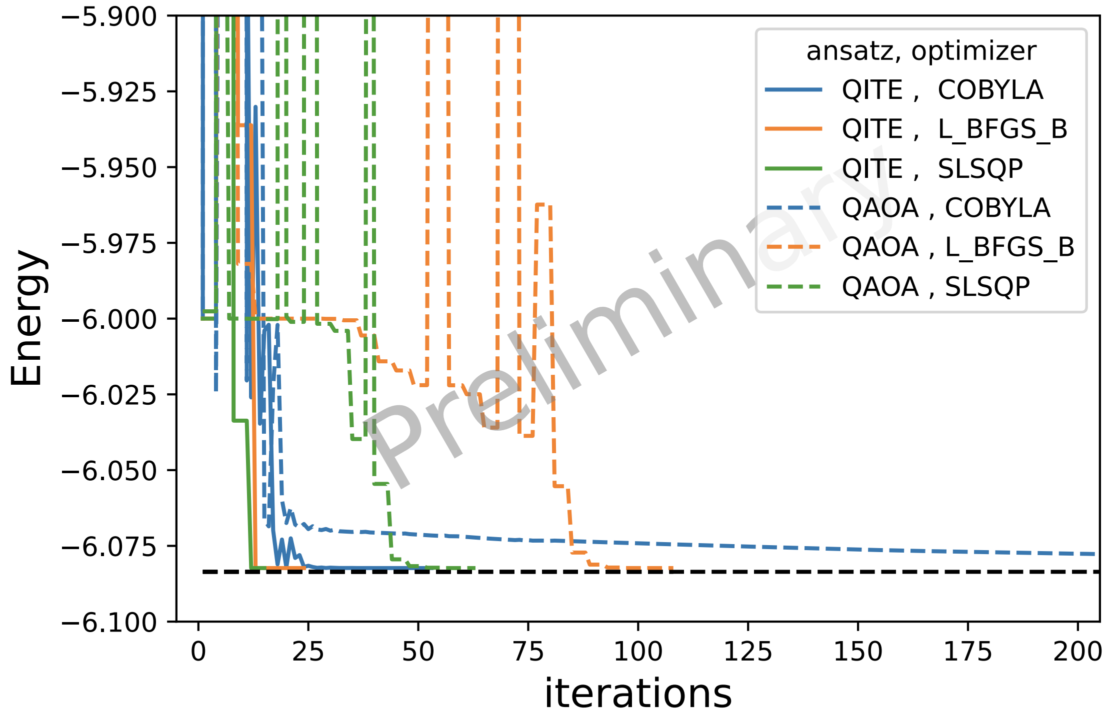
[1] X. Wang, X. Feng, T. Hartung, K. Jansen, P. Stornati. arXiv: 2302.14279

Schwinger model

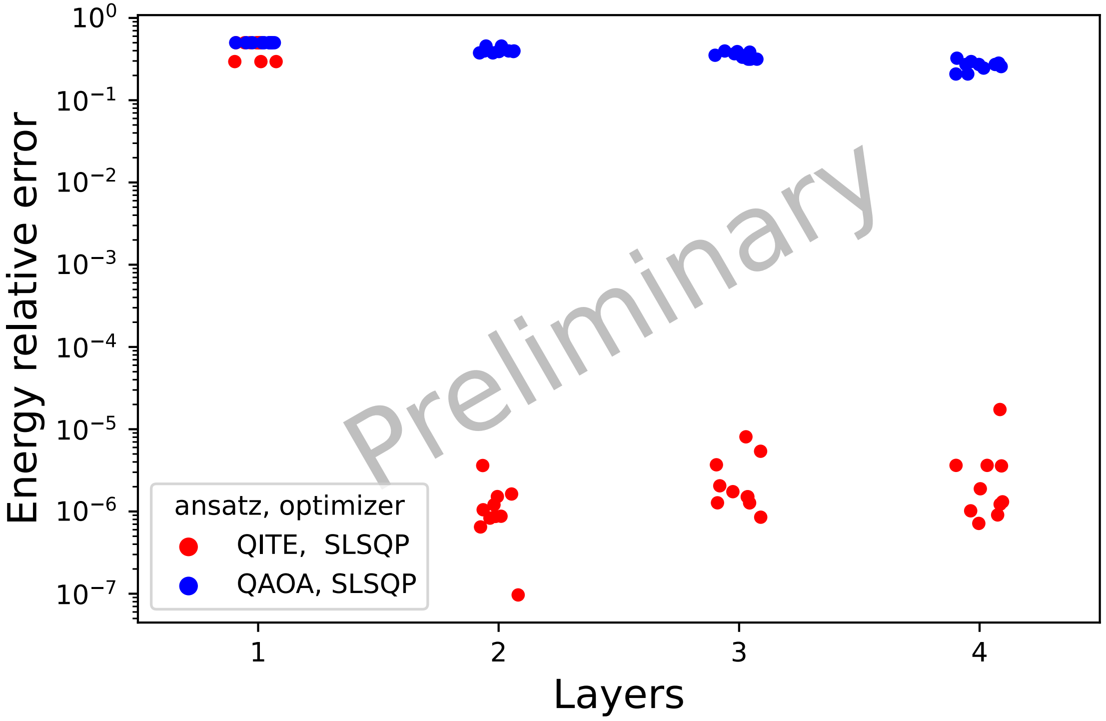
Hamiltonian with Wilson fermion [1]

$$W = ix(r-1)\sum_{n=1}^{N-1}\left(\sigma_{2n}^{-}\sigma_{2n+1}^{+} - \sigma_{2n}^{+}\sigma_{2n+1}^{-}\right) + ix(r+1)\sum_{n=1}^{N-1}\left(\sigma_{2n-1}^{+}\sigma_{2n}^{z}\sigma_{2n+1}^{z}\sigma_{2n+2}^{-} - \sigma_{2n-1}^{-}\sigma_{2n}^{z}\sigma_{2n+1}^{z}\sigma_{2n+2}^{+}\right) + \sum_{n=1}^{N-1}\left(l_0 + \sum_{k=1}^{n} Q_k\right)^2 + 2i\left(\frac{m_{\mathrm{lat}}}{g}\sqrt{x} + xr\right)\sum_{n=1}^{N}\left(\sigma_{2n-1}^{-}\sigma_{2n}^{+} - \sigma_{2n-1}^{+}\sigma_{2n}^{-}\right).$$

- **QITE-inspired ansatz** converges faster to the ground state



- **QITE-inspired ansatz** requires less layers:



[1] T. Angelides, L. Funcke, K. Jansen, S. Kühn. arXiv:2303.11016

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$$
\begin{aligned}
W = {} & ix(r-1)\sum_{n=1}^{N-1}\left(\sigma^-_{2n}\sigma^+_{2n+1} - \sigma^+_{2n}\sigma^-_{2n+1}\right) \\
& + ix(r+1)\sum_{n=1}^{N-1}\left(\sigma^+_{2n-1}\sigma^z_{2n}\sigma^z_{2n+1}\sigma^-_{2n+2} - \sigma^-_{2n-1}\sigma^z_{2n}\sigma^z_{2n+1}\sigma^+_{2n+2}\right) \\
& + \sum_{n=1}^{N-1}\left(l_0 + \sum_{k=1}^{n} Q_k\right)^2 + 2i\left(\frac{m_{\text{lat}}}{g}\sqrt{x} + xr\right)\sum_{n=1}^{N}\left(\sigma^-_{2n-1}\sigma^+_{2n} - \sigma^+_{2n-1}\sigma^-_{2n}\right).
\end{aligned}
$$

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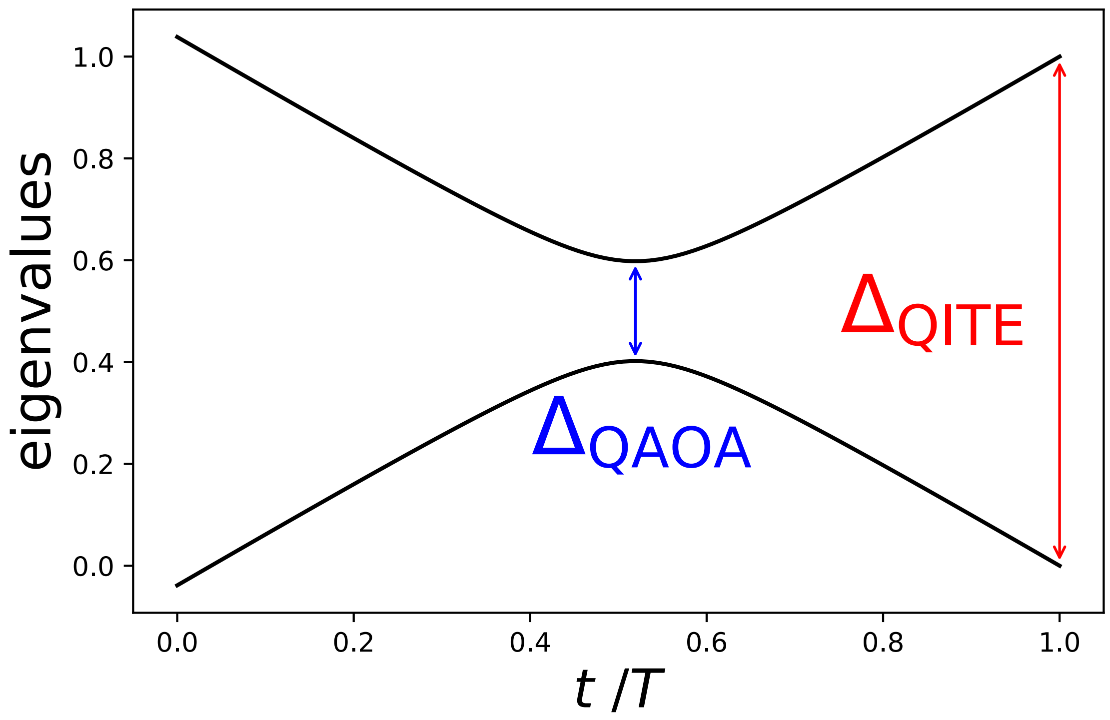
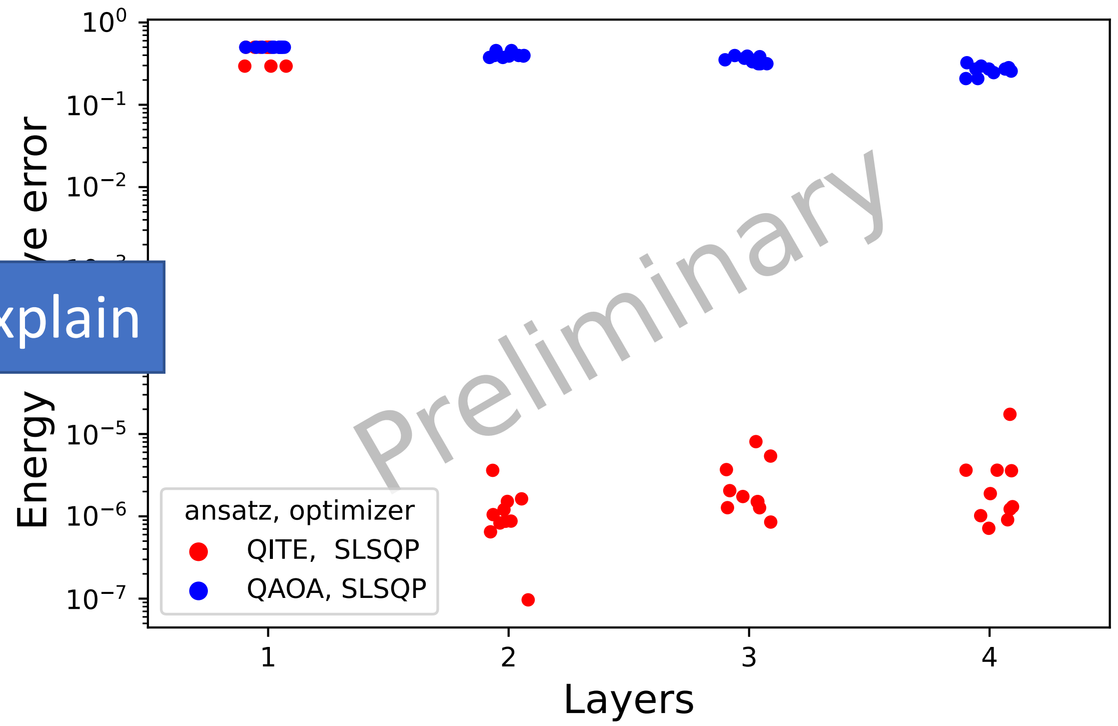


explain



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$$e^{i\Delta\tau a_i \sigma_i} \to e^{i\theta_i \sigma_i},$$
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Advantages of **QITE-inspired ansatz**:

1. symmetry preserving
2. good at Gibbs state preparation (better than hardware-efficient ansatz [1])
3. converging faster to the ground state
4. requiring fewer layers than QAOA

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Conclusion

- We propose an ansatz design strategy inspired by quantum imaginary time evolution.
- The ansatz is suitable for **Gibbs state preparation**, and performs better than the hardware-efficient ansatz.
- The ansatz could perform better than the QAOA ansatz in the **ground state preparation**.

Thanks for listening!