Nucleon isovector form factors from domain-wall lattice QCD at the physical mass

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Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_\mu F_1(q^2) - i\sigma_{\mu\lambda} q_\lambda \frac{F_2(q^2)}{2m_N}\right] u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_\mu \gamma_5 F_A(q^2) + \gamma_5 q_\mu \frac{F_P(q^2)}{2m_N}\right] u_n e^{iq\cdot x},$$

Related to

- mean-squared charge radii, $F_1 = F_1(0) - \frac{1}{6}\langle r_E^2\rangle Q^2 + \ldots$
- anomalous magnetic moment, $F_2(0)$,
- $g_A = F_A(0) = 1.2754(13) g_V$ ($g_V = F_1(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$),
- νN scattering,
- μ capture.

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LHP+RBC+UKQCD calculation using RBC+UKQCD 48I ensemble*: 2+1-flavor physical-mass DWF, Iwasaki gauge, $48^3 \times 96$ with $a^{-1} = 1.730(4)$ GeV and $La = 5.4750(14)$ fm.

The ratio of two- and three-point correlators, $\dfrac{C_{3\text{pt}}^{\Gamma,O}(t_{\text{src}}, t, t_{\text{snk}})}{C_{2\text{pt}}(t_{\text{src}}, t_{\text{snk}})}$, with

$$C^{(2)}(t_{\text{src}}, t_{\text{snk}}) = \sum_{\alpha,\beta} \left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta} \langle N_\beta(t_{\text{snk}}) \bar{N}_\alpha(t_{\text{src}})\rangle,$$

$$C^{(3)\Gamma,O}(t_{\text{src}}, t, t_{\text{snk}}) = \sum_{\alpha,\beta} \Gamma_{\alpha\beta} \langle N_\beta(t_{\text{sink}}) O(t) \bar{N}_\alpha(0)\rangle,$$

with appropriate nucleon operator, eg, $N = \epsilon_{abc}(u_a^T C \gamma_5 d_b) u_c$, gives a plateau in t for a lattice bare value $\langle O \rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t) i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

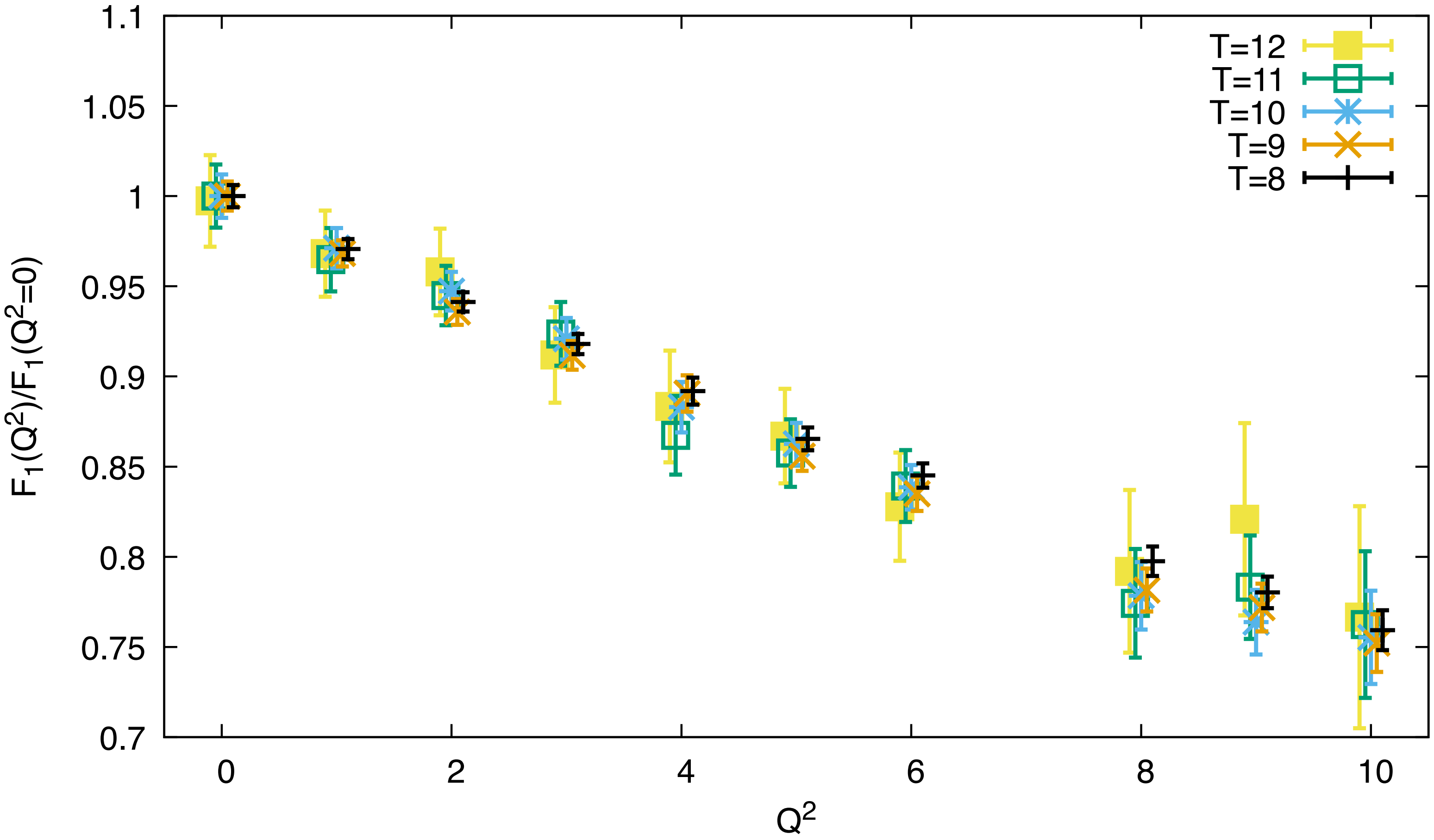
More specifically, for the form factors, ratios such as

$$\frac{C_{\text{GG}}^{(3)\Gamma,O}(t_{\text{src}}, t, t_{\text{snk}}, \vec{p}_{\text{src}}, \vec{p}_{\text{snk}})}{C_{\text{GG}}^{(2)}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}}, \vec{p}_{\text{snk}})} \times \sqrt{\frac{C_{\text{LG}}^{(2)}(t, t_{\text{snk}}, \vec{p}_{\text{src}})) C_{\text{GG}}^{(2)}(t_{\text{src}}, t, \vec{p}_{\text{snk}})) C_{\text{LG}}^{(2)}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{snk}}))}{C_{\text{LG}}^{(2)}(t, t_{\text{snk}}, \vec{p}_{\text{snk}})) C_{\text{GG}}^{(2)}(t_{\text{src}}, t, \vec{p}_{\text{src}})) C_{\text{LG}}^{(2)}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}}))}}$$

with point (L) or Gaussian (G) smearings, give plateaux dependent only on momentum transfer.

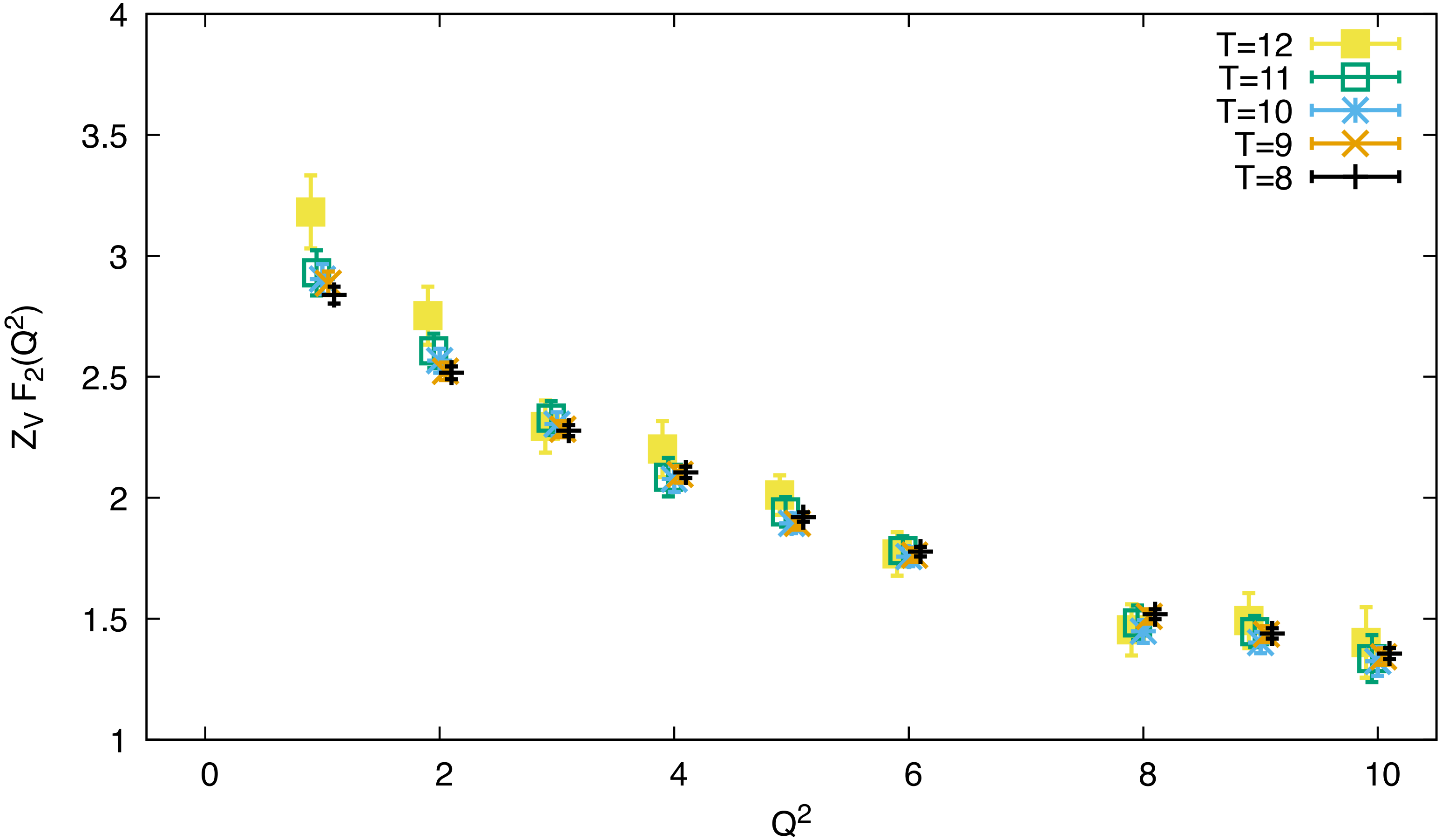
*T. Blum *et al.* [RBC and UKQCD], Phys. Rev. D **93**, no.7, 074505 (2016) doi:10.1103/PhysRevD.93.074505 [arXiv:1411.7017 [hep-lat]].

F_1 shape does not seem to depend on source-sink separation, T:

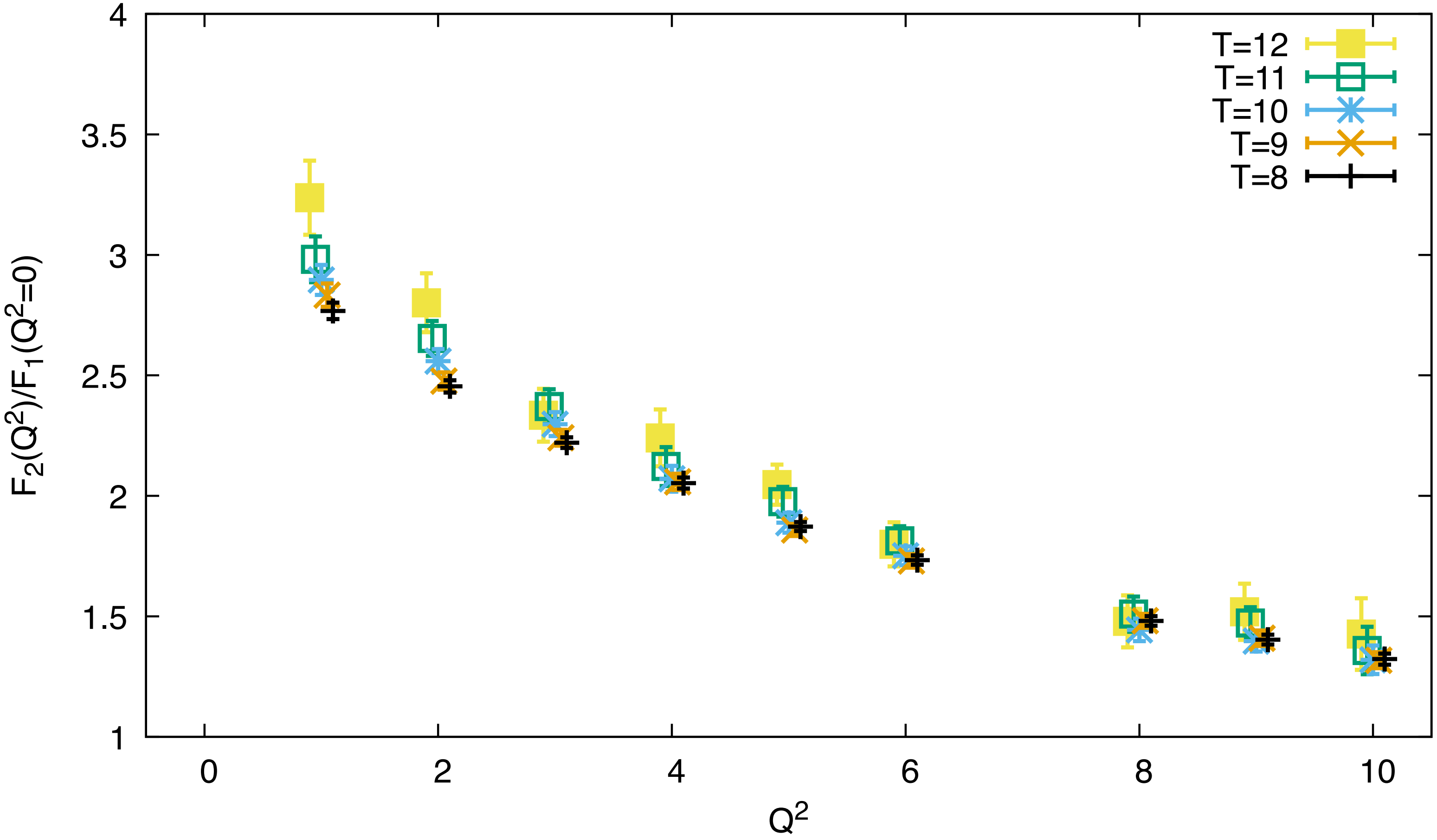


Form factors from $T = 8$, 9, and 10 are informative: no need for more statistics?

F_2 may not be affected by excited states either.



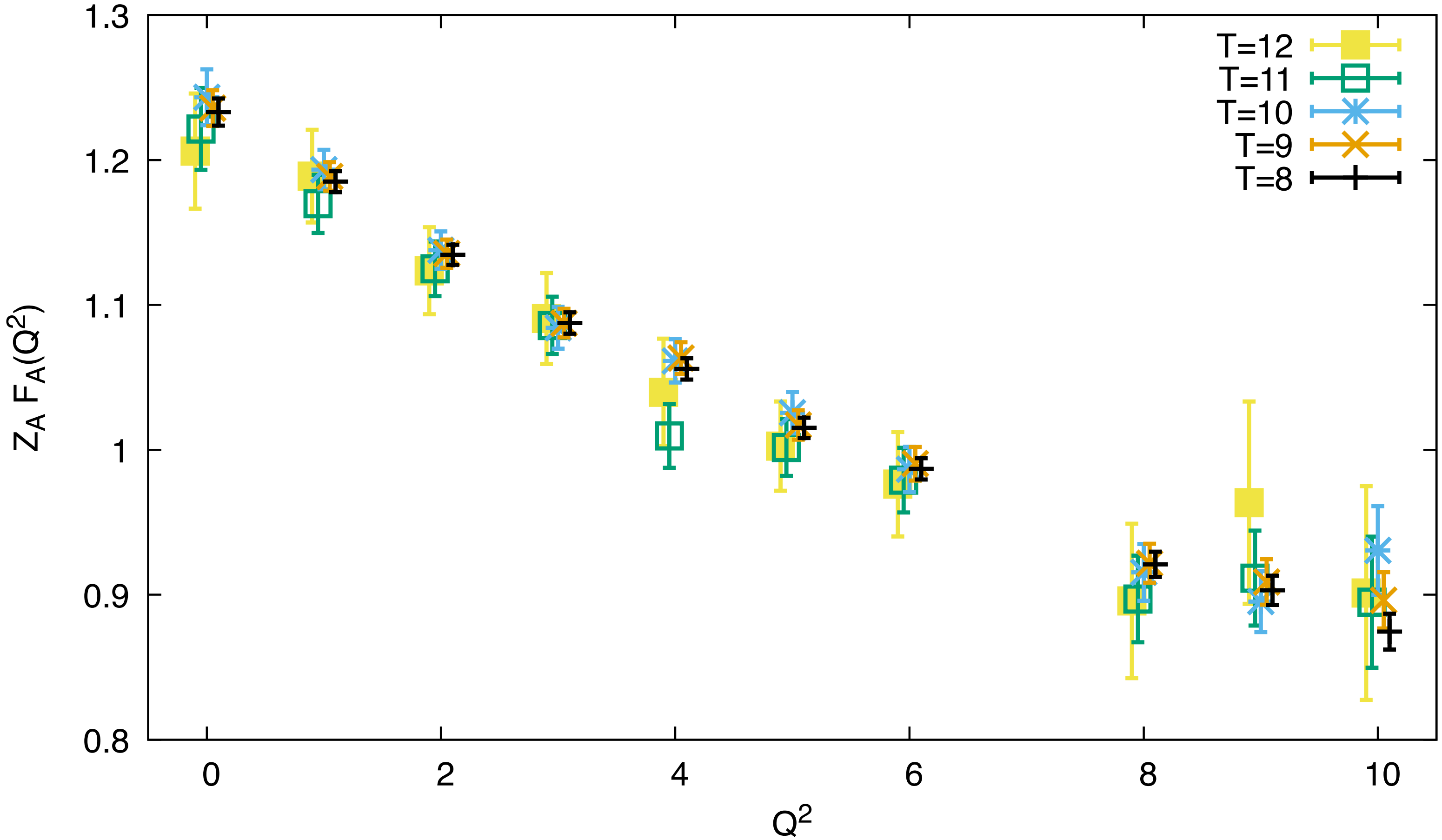
Or maybe?



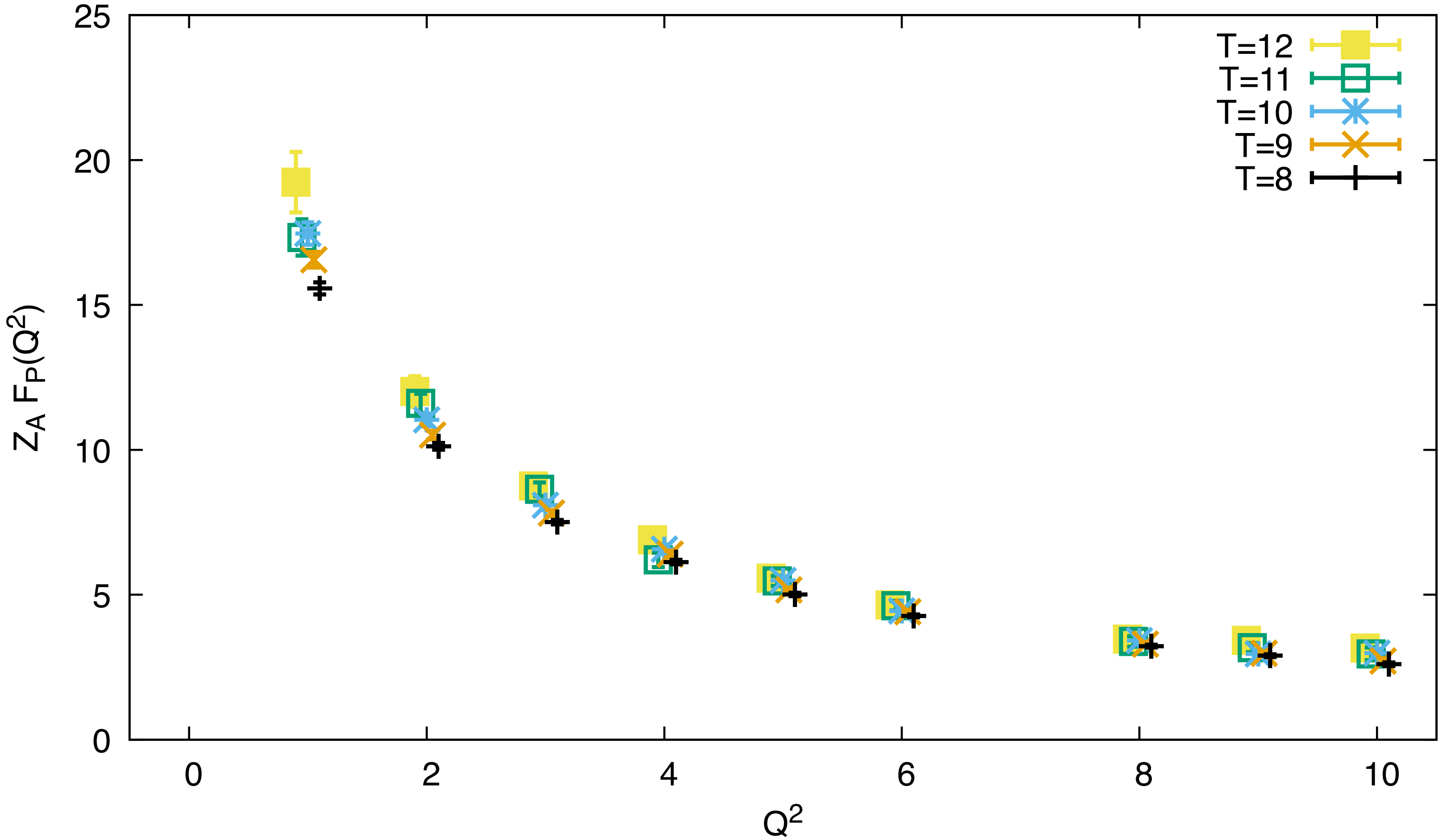
Extrapolate to $\sim 3.4(2)\mu_N$. Experiment: 2.7928473446(8) + 1.9130427(5) - 1 = 3.705874(5).

More statistics desired at larger T.

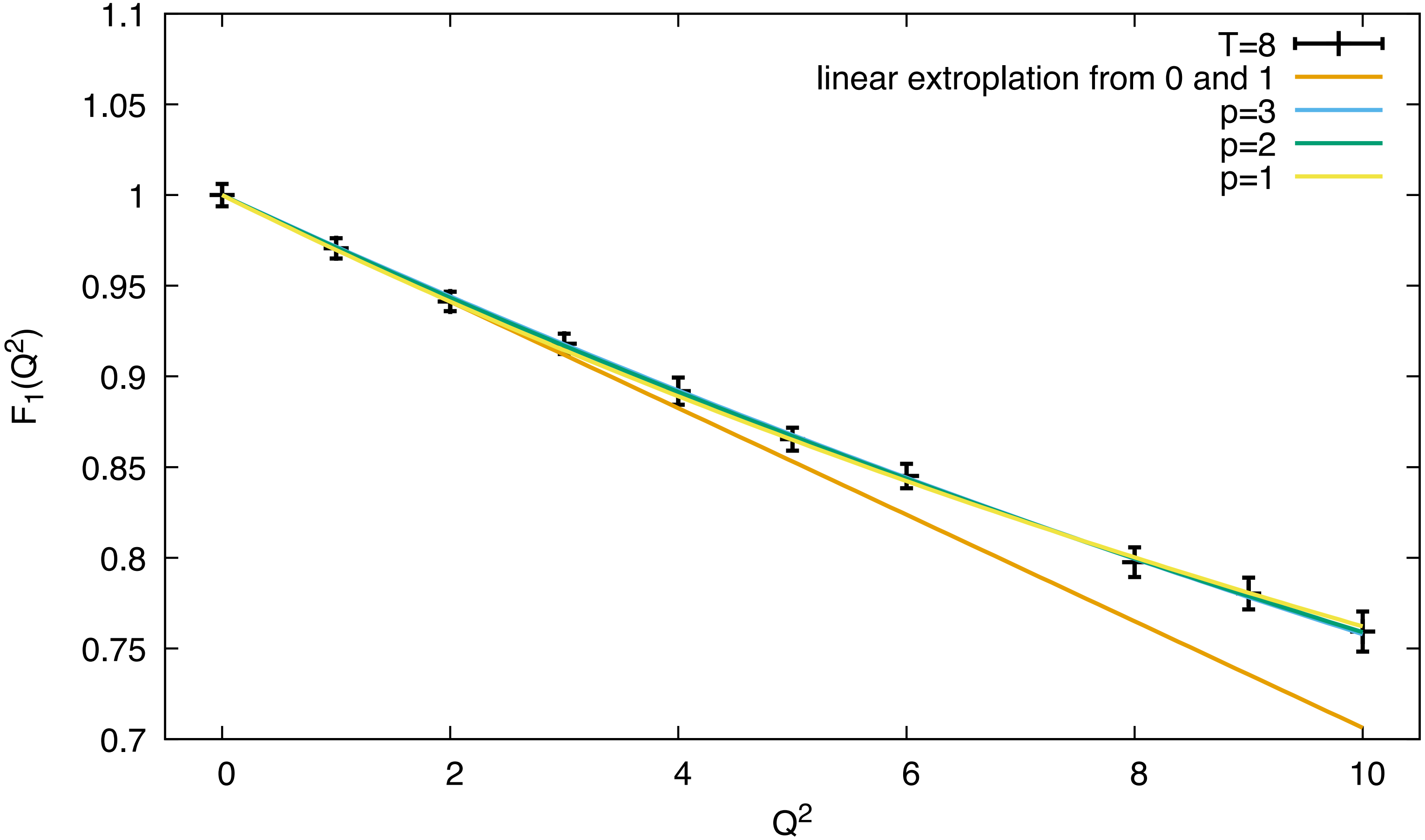
F_A:



F_P:

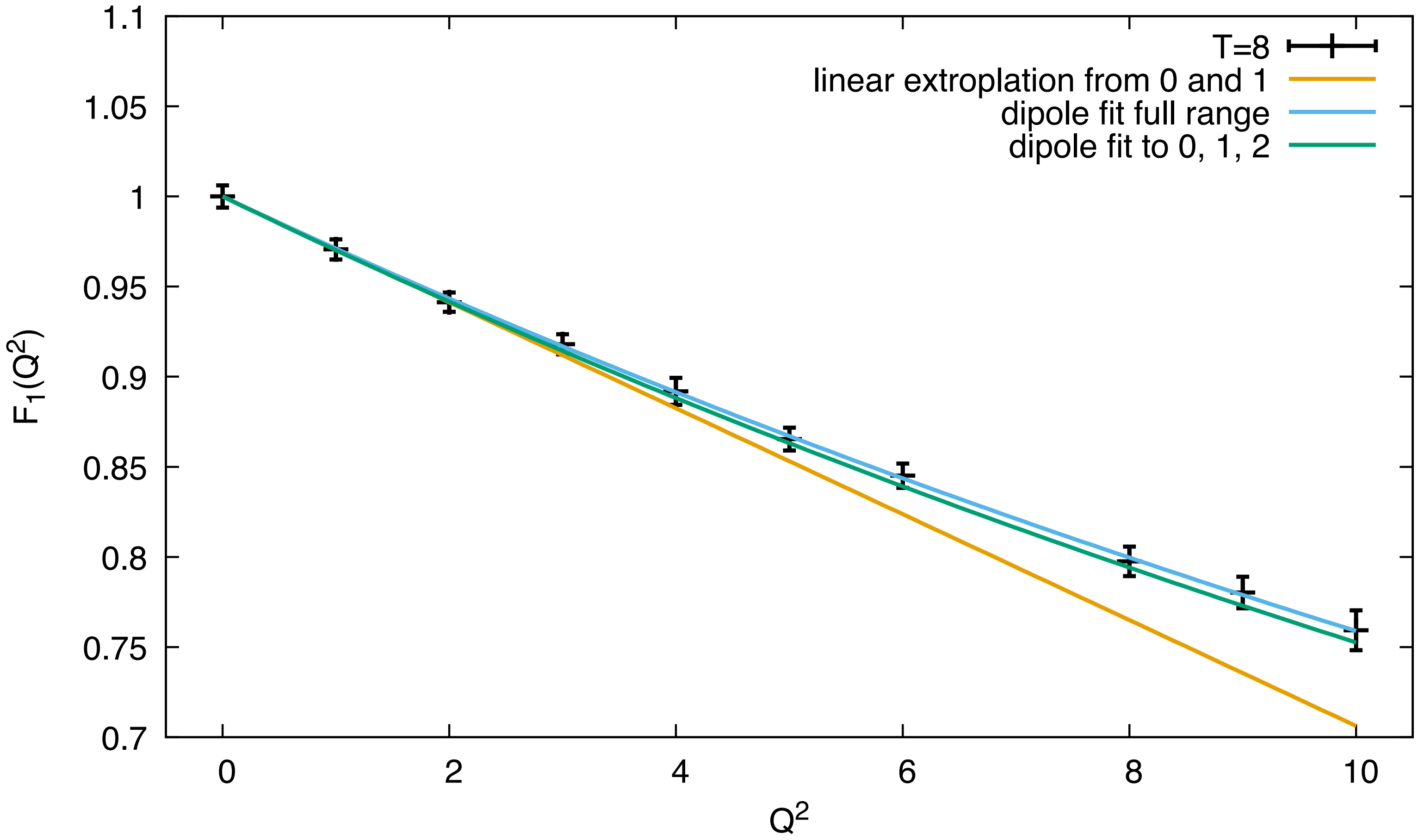


Extrapolations: "multipole" fits, to $F(Q^2) \sim F(0) \left(1 + \frac{Q^2}{M_p^2}\right)^{-p}$, with $p = 1$, 2, and 3, appear to work well:



with similar $\langle r_1^2 \rangle = 6p/M_p^2 \sim 0.14~\mathrm{fm}^2$ estimates.

Multipole fits do not change much with the fitting range:



and they agree with the linear estimate for the smallest Q^2 pair.

Dubious if much more can be squeezed for $\langle r^2_{1,A} \rangle$ or $F_{2,P}(0)$, Extrapolations by "linear" using the smallest two Q^2, and "dipole," $F(Q^2) \sim F(0)\left(1+\frac{Q^2}{M^2_{\rm dipole}}\right)^{-2}$:

		$T=8$	9	10	11	12	experiment
$\langle r_1^2 \rangle$	linear	0.134(14)	0.14(2)	0.13(3)	0.16(5)	0.13(8)	0.868(3) fm^2
	dipole	0.135(6)	0.143(8)	0.142(13)	0.14(2)	0.13(3)	
$F_2(0)$	linear	3.159(4)	3.250(6)	3.242(8)	3.252(13)	3.61(2)	3.705874(5)μ_N
	dipole	3.10(5)	3.15(6)	3.22(8)	3.24(11)	3.5(2)	
$\langle r_A^2 \rangle$	linear	0.177(2)	0.174(2)	0.182(4)	0.192(5)	0.066(8)	0.5(2)†fm^2
	dipole	0.177(7)	0.174(10)	0.176(14)	0.18(2)	0.15(3)	
$F_P(0)$	linear	21.01(3)	22.61(5)	23.90(7)	23.04(11)	26.5(2)	–
	dipole	23(2)	25(2)	26(2)	26(2)	30(2)	

"Linear" and "dipole" fits agree with each other but do not agree with experiments.
So other fitting ansatze such as "bounded-z" polynomials should not differ much.

†T. Cai *et al.* [MINERvA], Nature **614**, no.7946, 48-53 (2023) doi:10.1038/s41586-022-05478-3.

So other fitting ansatze such as "bounded-z" polynomials should not differ much. Is it so?

Bounded-z parameter[‡]:

$$z(q^2) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}},$$

maps $t = q^2 = -Q^2$ to within the unit disk $|z| \leq 1$.

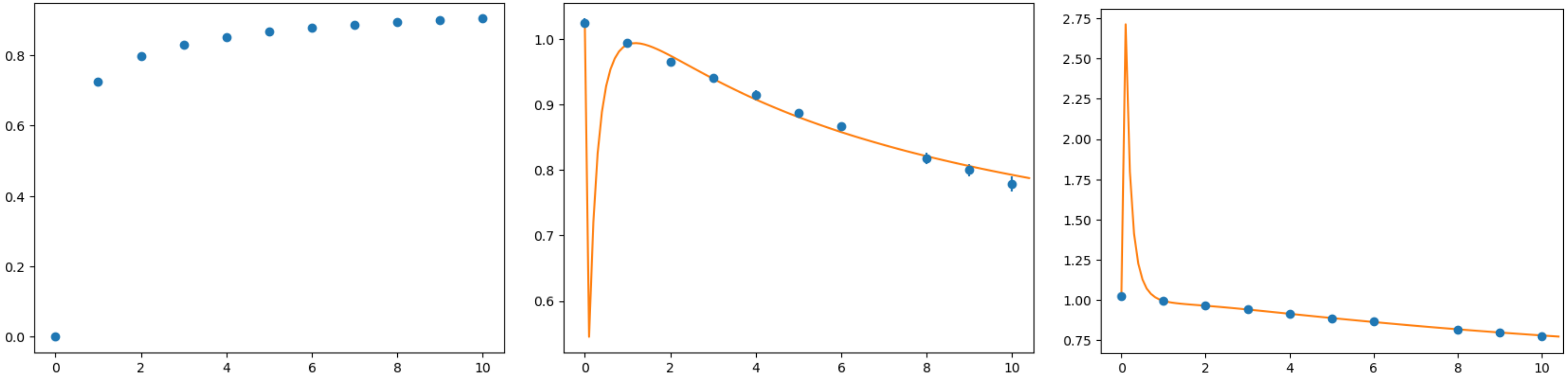
Form factors are expanded by polynomials of z

$$F(Q^2) = \sum_{k=0}^{k_{\rm max}} F_k z(q^2)^k,$$

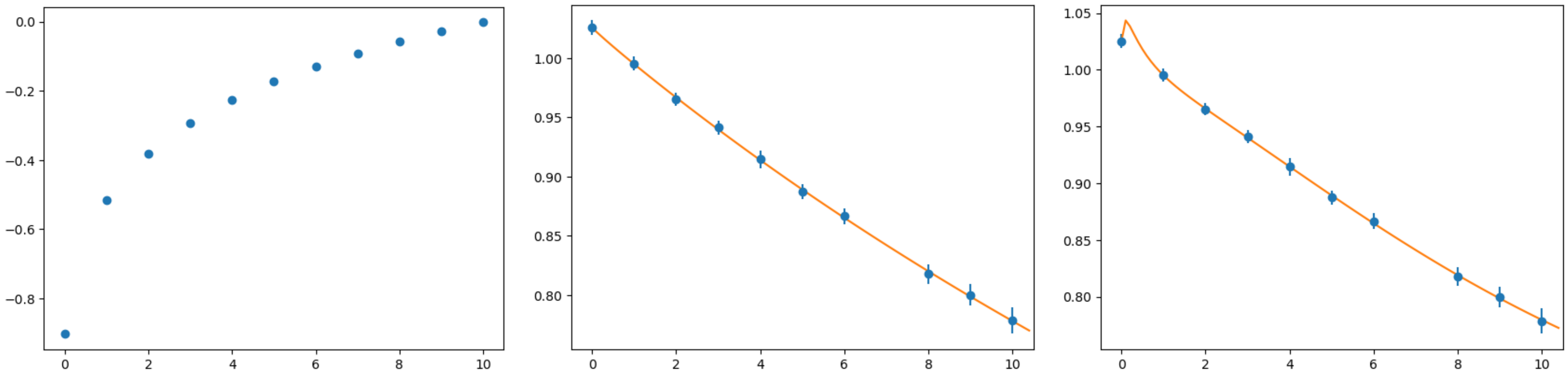
with appropriate and $t_{\rm cut} = 4m_\pi^2$ for vector and $t_{\rm cut} = 9m_\pi^2$ for axialvector currents.

[‡]G. Lee, J. R. Arrington and R. J. Hill, Phys. Rev. D **92**, no.1, 013013 (2015) doi:10.1103/PhysRevD.92.013013 [arXiv:1505.01489 [hep-ph]].

t_0 adjusts $Q^2 \mapsto z$ mapping. Naive fits to the present F_1 for $t_0 = 0$ and $k_{\text{max}} = 3$ and 4:



With $t_0 = -Q^2_{\text{max}}$ and $k_{\text{max}} = 4$ (left) gives a somewhat improved estimate of $\langle r_1^2 \rangle \sim (0.4\text{fm})^2$,



but with $k_{\text{max}} = 5$ (right) the kink from the narrowed gap in z returns and changes sign of $\langle r_1^2 \rangle$

At large Q^2 the form factors should fall at least as fast as $1/Q^4$ [§]:

$$Q^n F(Q^2) \to 0,$$

for $n = 0$, 1, 2, and 3. Since $\lim_{Q^2\to\infty} z = 1$, these are equivalent with

$$\left.\frac{d^n F(z)}{dz^n}\right|_{z=1} = 0,$$

for $n = 0$, 1, 2, and 3. These constrain the polynomial form to

$$(1-z)^4 B(z)$$

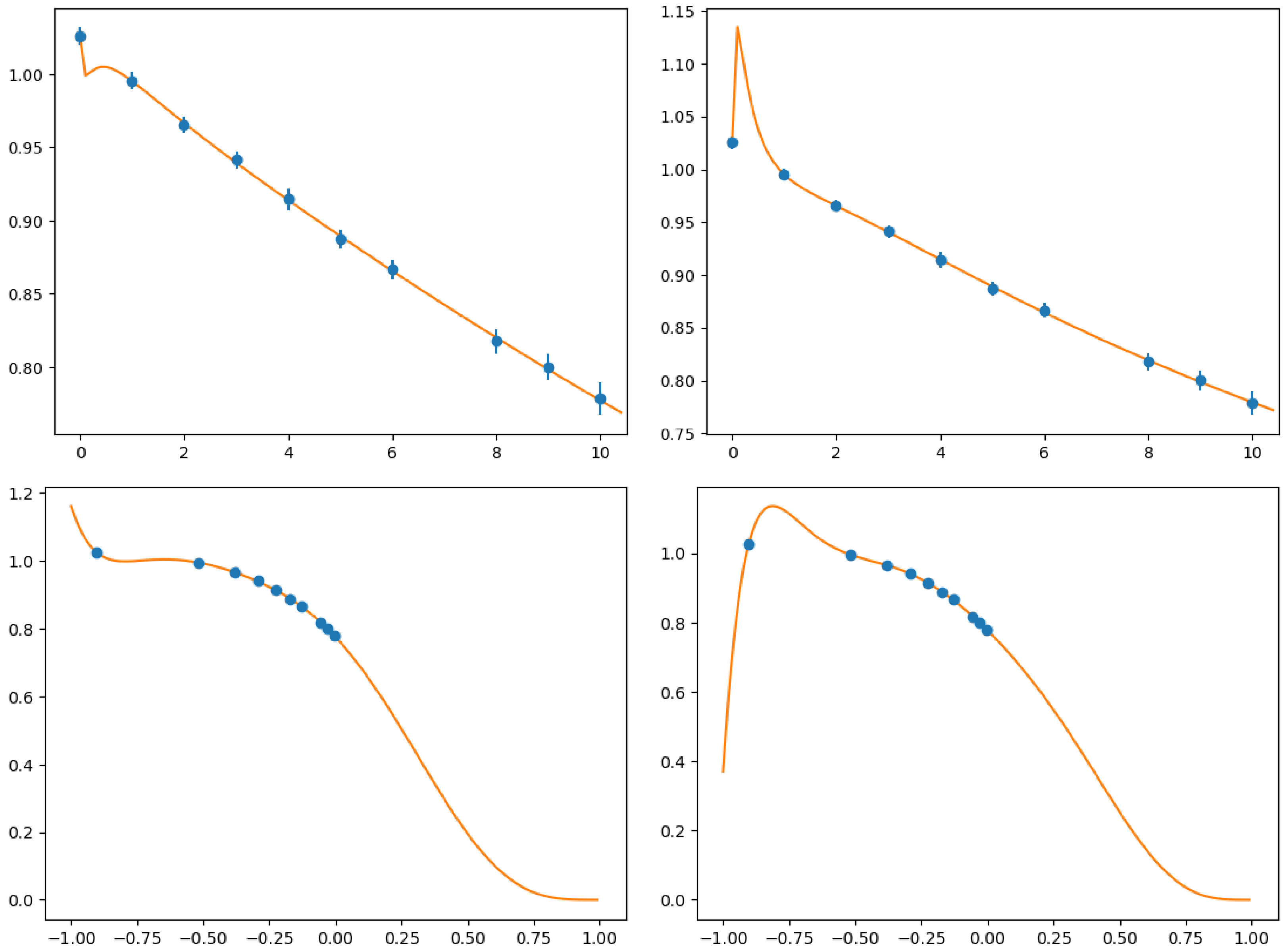
with arbitrary polynomial $B(z)$, as

$$\begin{aligned}
n = 0 \text{ leads to } F(z) &= (1-z)E(z),\\
n = 1 \text{ leads to } E(z) &= (1-z)D(z),\\
n = 2 \text{ leads to } D(z) &= (1-z)C(z),\\
n = 3 \text{ leads to } C(z) &= (1-z)B(z).
\end{aligned}$$

Equivalent to the 'sum rules.'

[§]S. J. Brodsky and G. P. Lepage, AIP Conf. Proc. **74**, 214-239 (1981) doi:10.1063/1.33098

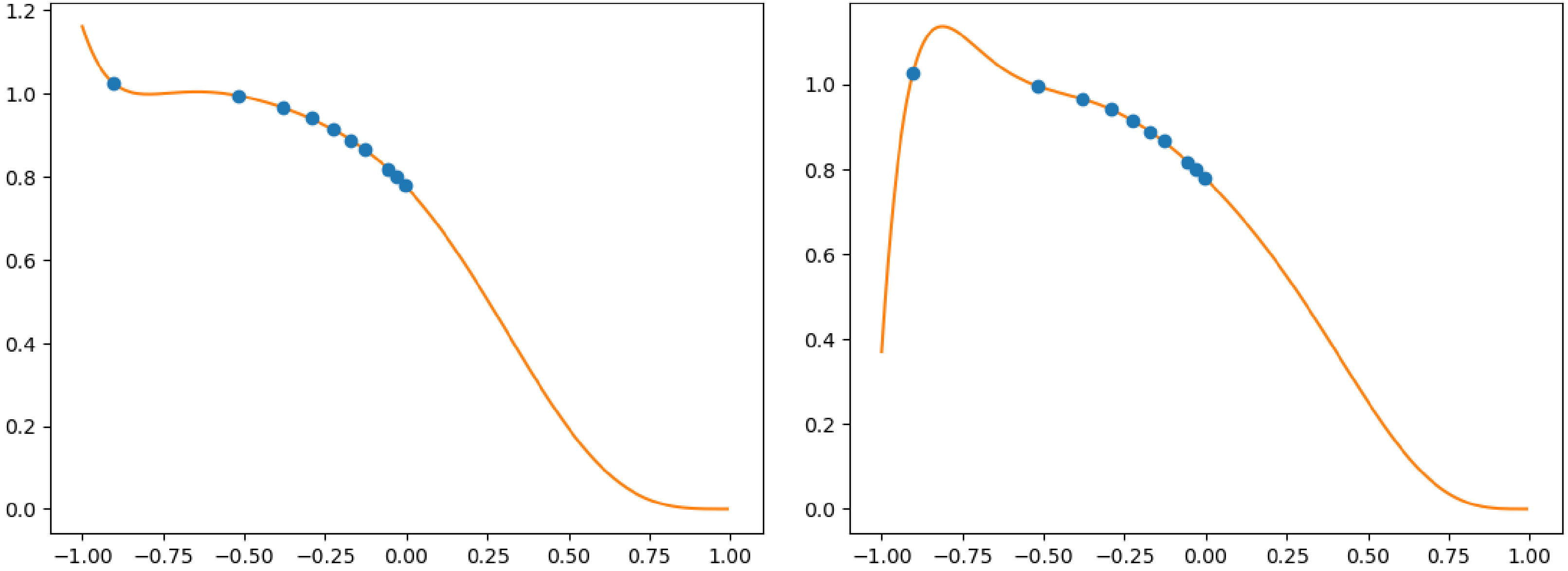
Good fit ($\chi^2/dof < 1$) usually requires $(1-z)^4\times$ fourth-order or higher polynomials:



However those with $k_{\max} = 8$ (left) and 9 (right) differ in $\langle r_1^2 \rangle$ signs.

Bayesian priors may help, such as a well-motivated[¶] or a helpful[‖] one.

Yet, if the prior's function is to thin the influence from large Q^2,



just using the smallest two Q^2 points for linear extrapolations may be better.

[¶] J. M. Flynn, A. Jüttner and J. T. Tsang, [arXiv:2303.11285 [hep-ph]].

[‖] C. Alexandrou, et al, Phys. Rev. D **97**, 094504 (2018) doi:10.1103/PhysRevD.97.094504 [arXiv:1801.09581 [hep-lat]].

However, while linear extrapolations using the two smallest available Q^2 tend to agree with multipole fits,"
$F(Q^2) \sim F(0)\left(1+\frac{Q^2}{M_\mathrm{p}^2}\right)^{-p}$, $1 \le p \le 7$:

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They do not seem to agree well with experiments, in particular, small $\langle r_1^2 \rangle$ compared with the experiment.

Lattice ensembles with smaller momentum transfer units are desired, either

- by larger volume, La, or
- by a twisted boundary condition.

T. Cai *et al.* [MINERvA], Nature **614, no.7946, 48-53 (2023) doi:10.1038/s41586-022-05478-3.