Chiral fermion on quantum computers

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Introduction

artificial symmetry breaking in quantum computing

device development

theory improvement

gauge symmetry: gauge fixing, dual variable, ...

chiral symmetry: chiral fermion

Introduction

Let's study the chiral fermion in the Hamiltonian formalism

& the application to quantum simulation!

Overlap fermion

Creutz, Horvath, Neuberger (2002)

Hamiltonian

$$H_f = \psi^+ \gamma^0 D \psi$$

$$D = 1 + \frac{D_W}{\sqrt{D_W^+ D_W}}$$

↑ 3-dim. Dirac operator (not classical !)

Overlap fermion

Creutz, Horvath, Neuberger (2002)

Hamiltonian

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"naive" chiral charge

$$Q_{\text{naive}} = \psi^+ \gamma^5 \psi$$

$$\left[H_f, Q_{\text{naive}}\right] \neq 0$$

"conserved" chiral charge

$$Q = \psi^+ \gamma^5 \left(1 - \frac{1}{2} D\right) \psi$$

$$\left[H_f, Q\right] = 0$$

Overlap fermion

$$[H_f, Q] = 0 \longrightarrow \begin{array}{c} H_f|\Psi_n\rangle = \varepsilon_n|\Psi_n\rangle \\ Q|\Psi_n\rangle = q_n|\Psi_n\rangle \end{array} \longrightarrow$$

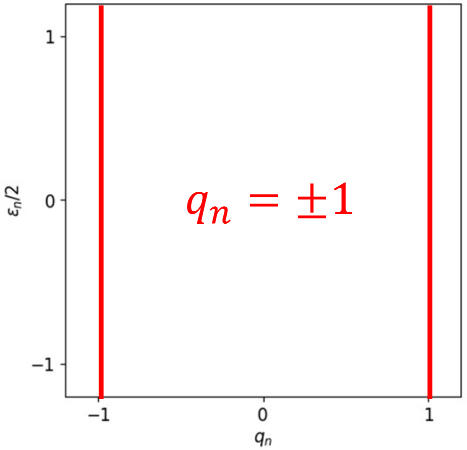
eigenvalue spectrum

ε_n vs q_n

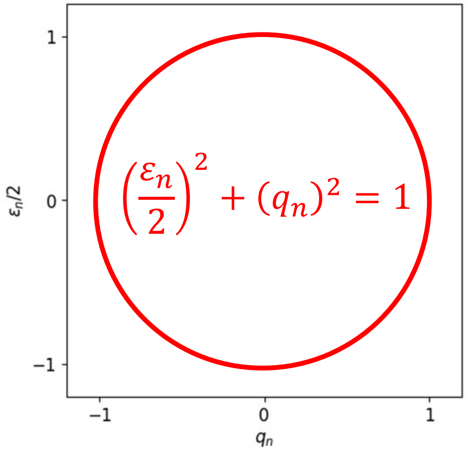
Overlap fermion

$$[H_f, Q] = 0 \longrightarrow \begin{array}{c} H_f|\Psi_n\rangle = \varepsilon_n|\Psi_n\rangle \\ Q|\Psi_n\rangle = q_n|\Psi_n\rangle \end{array} \longrightarrow \begin{array}{c} \text{eigenvalue spectrum} \\ \varepsilon_n \text{ vs } q_n \end{array}$$

continuous Dirac fermion



overlap fermion

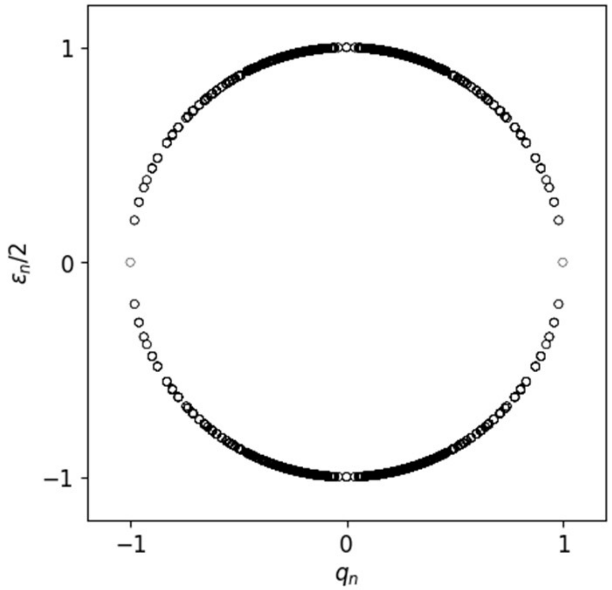


Overlap fermion

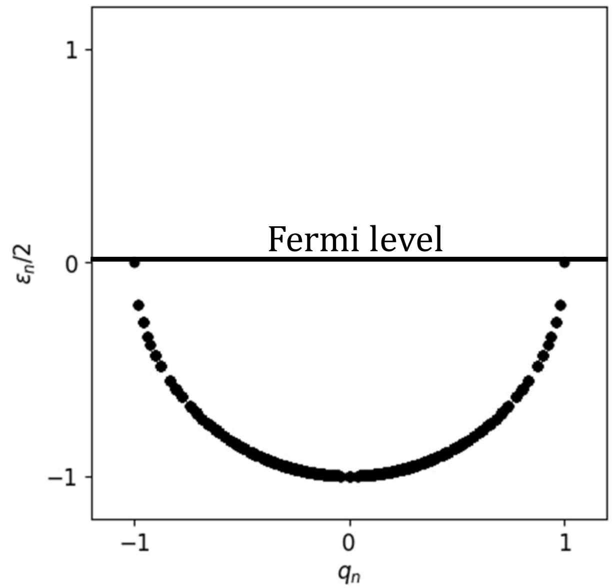
Hayata, Nakayama, Yamamoto (2023)

eigenvalue spectra of 3D overlap fermion

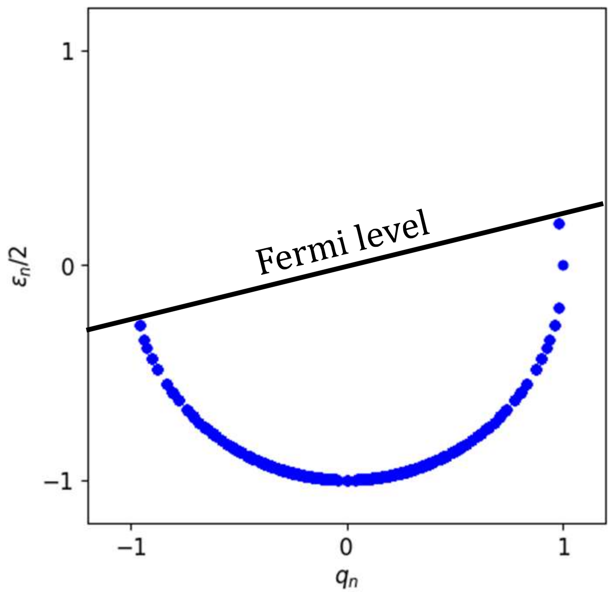
1-particle spectrum



vacuum



chiral chemical potential

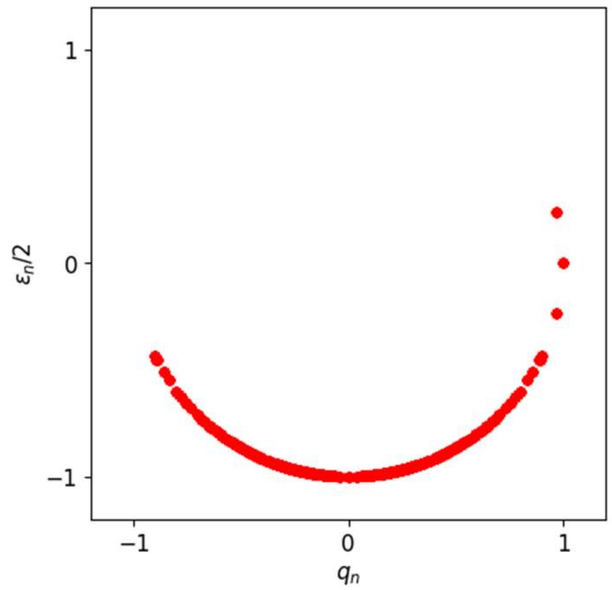


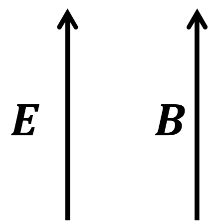
Overlap fermion

Hayata, Nakayama, Yamamoto (2023)

eigenvalue spectra of 3D overlap fermion

external gauge field





"would-be" chiral anomaly

Quantum simulation

- ✓ fermion + classical (external) gauge field

  only one-particle state $\quad$ cost $\sim \mathrm{poly}(V)$

- ✓ fermion + quantum gauge field

  full Hilbert space $\quad$ cost $\sim 2^{\mathrm{poly}(V)}$ $\longrightarrow$ quantum computer

Quantum simulation

time evolution

$$|\Psi(t)\rangle = e^{-i\left(H_g + H_f\right)t}|\Psi(0)\rangle$$

overlap fermion
- pro: conserved chirality
- con: large computational cost

Quantum simulation

time evolution

$$|\Psi(t)\rangle = e^{-i(H_g+H_f)t}|\Psi(0)\rangle$$

↑

Wilson fermion circuit

just use $\langle Q\rangle$ instead of $\langle Q_{\mathrm{naive}}\rangle$

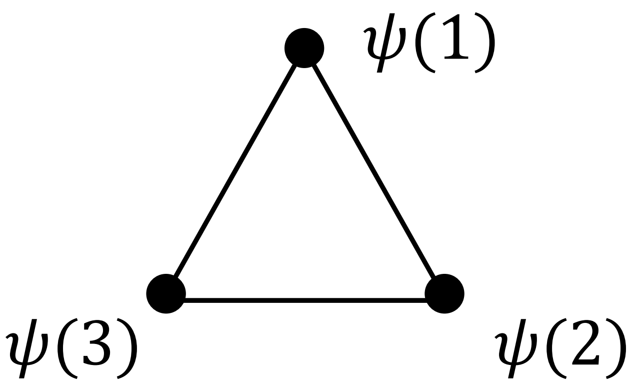
overlap fermion

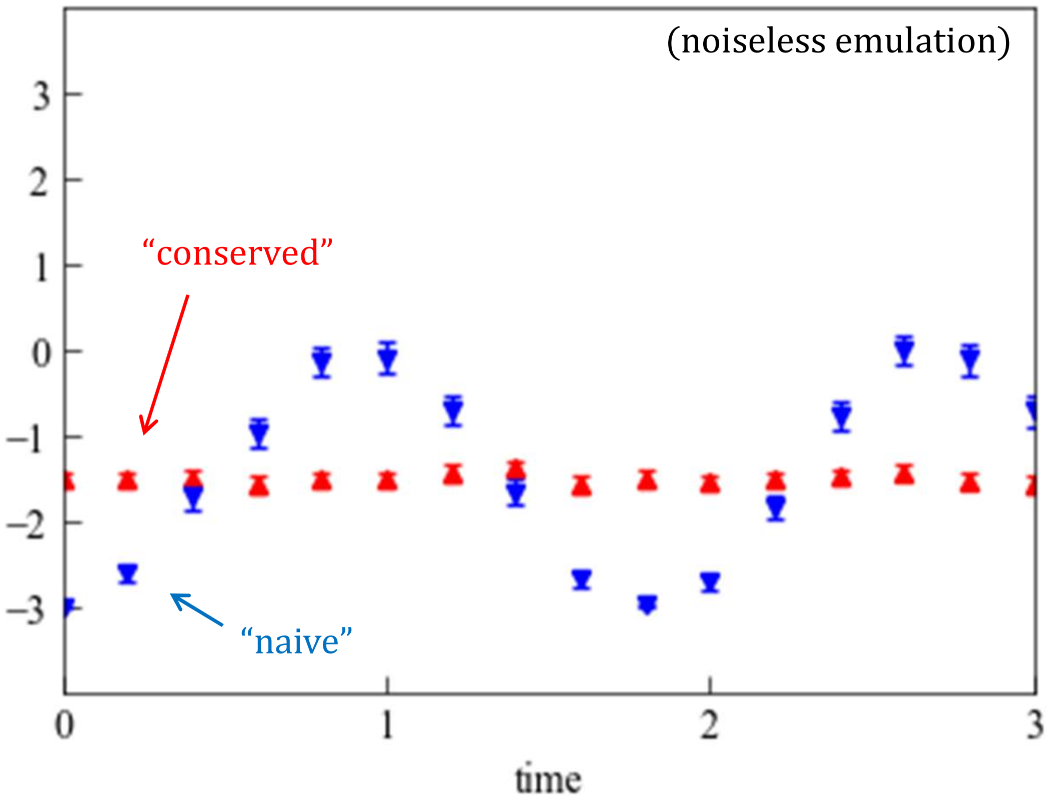
- pro: conserved chirality
- con: ~~large computational cost~~

1D overlap fermion = 1D Wilson fermion

Quantum simulation

free Wilson fermion





Quantum simulation

1D Wilson fermion + Z_2 gauge field

$$Q = \frac{1}{4}\sum_x \psi^+(x)\gamma^0(1-\gamma^1)\sigma_3(x)\psi(x+1) + \cdots$$

link variable operator

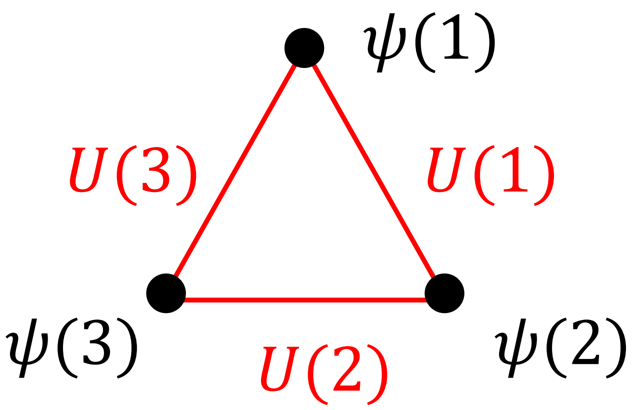
$$H_f = -\frac{1}{2}\sum_x \psi^+(x)\gamma^0(1-\gamma^1)\sigma_3(x)\psi(x+1) + \cdots \qquad [H_f, Q] = 0$$

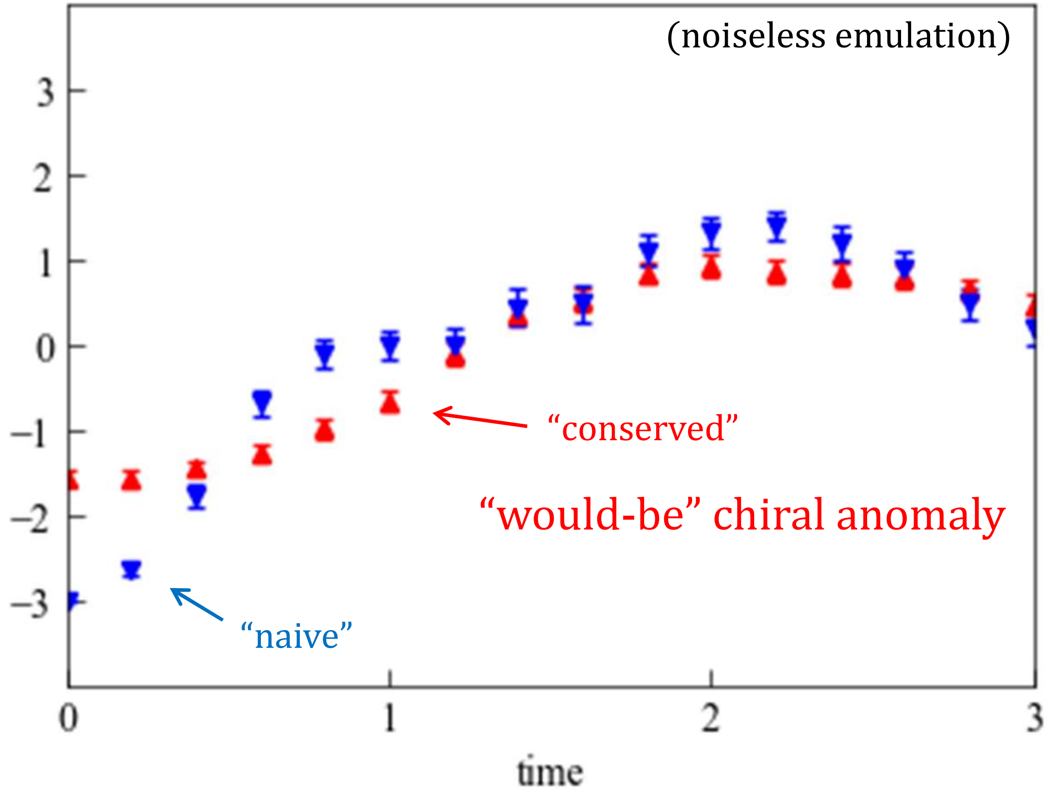
$$H_g = -\sum_x \sigma_1(x) \qquad [H_g, Q] \neq 0$$

electric field operator

Quantum simulation

Wilson fermion + Z_2 gauge field





Summary

- ✓ chiral fermion in the Hamiltonian formalism
- ✓ eigenvalue spectrum of energy vs chiral charge
- ✓ quantum simulation of dynamical chirality generation in 1D