Recent Developments of Euclidean Dynamical Triangulations with Non-Trivial Measure Term

Lattice 2023

August 1, 2023

Marc Schiffer, Perimeter Institute

In collaboration with

M. Dai, W. Freeman, J. Laiho, and J. Unmuth-Yockey: **to appear**

see also [Mingwei Dai, today 4:20 pm, WH3NE], [Jack Laiho, today 4:40 pm, WH3NE]

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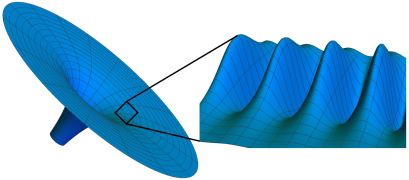
Asymptotically Safe Quantum Gravity



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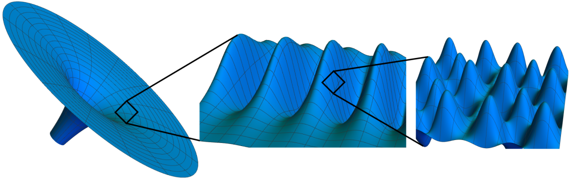
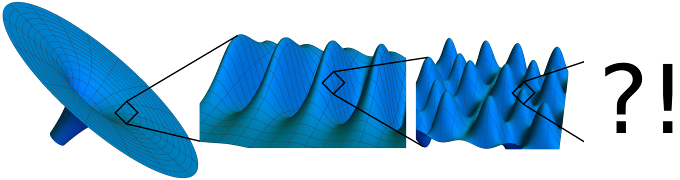


- Perturbative quantum gravity:



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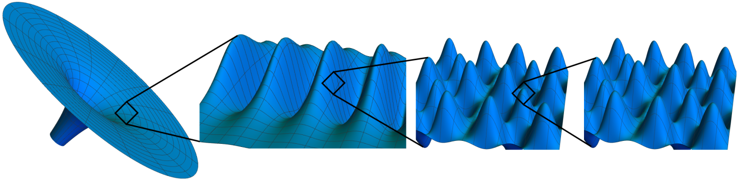

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- Key idea of asymptotic safety:
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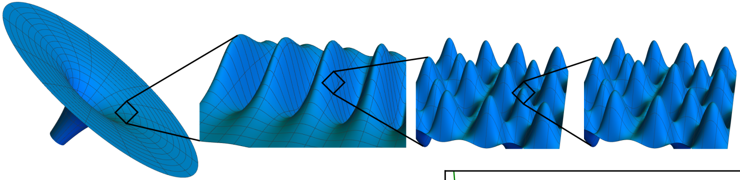
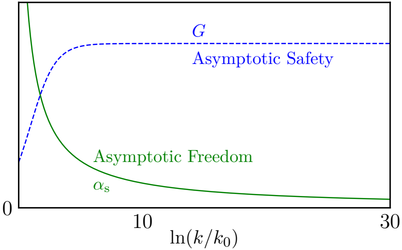


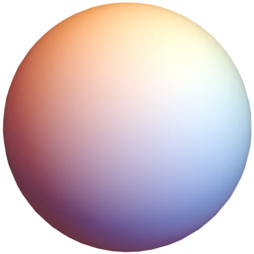
- Perturbative quantum gravity: loss of predictivity
- Key idea of asymptotic safety: Quantum realization of scale symmetry
  - imposes infinitely many conditions on theory space
  - relevant directions: need measurement
  - irrelevant directions: predictions of theory



$$k\partial_k \alpha_s = -\frac{11}{2\pi}\alpha_s^2 + \mathcal{O}(\alpha_s^4)$$

$$k\partial_k G = \epsilon G - \frac{50}{3}G^2 + \mathcal{O}(G^3)$$

Dynamical triangulations



- Discretization of spacetime in terms of triangulations

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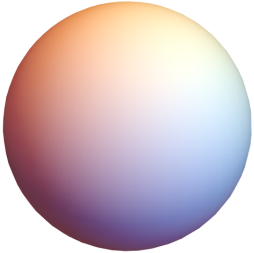

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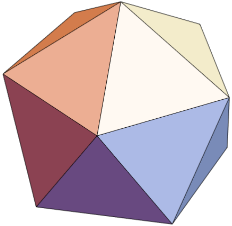

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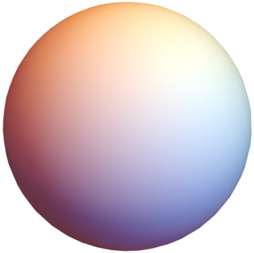

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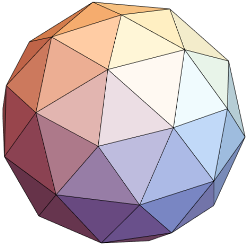

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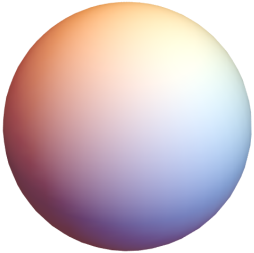

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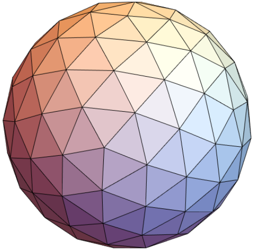

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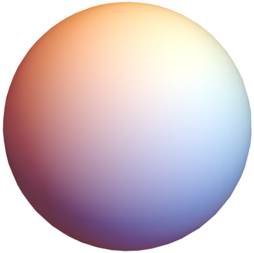

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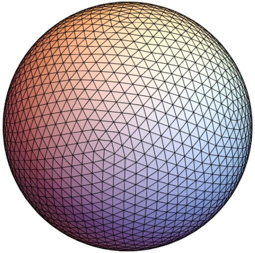

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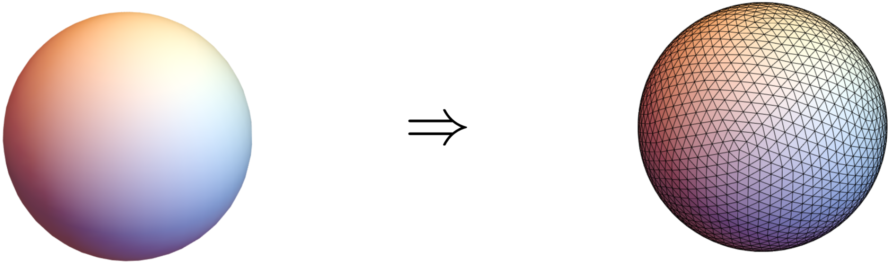

- Discretization of spacetime in terms of triangulations
  [Ambjørn and Jurkiewicz, 1992], [Agishtein and Migdal, 1992], . . .

$$\int \mathcal{D}g\, e^{-S[g]} \to \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}}\, e^{-S_{\mathrm{ER}}}$$

with Euclidean Einstein-Regge action $S_{\mathrm{ER}} = -\kappa_2 N_2 + \kappa_4 N_4$ [Regge, 1961]

Lattice quantum gravity in $d = 4$

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- in $d = 4$: no physical phase, no indications for higher-order transition in κ_2-κ_4 - space

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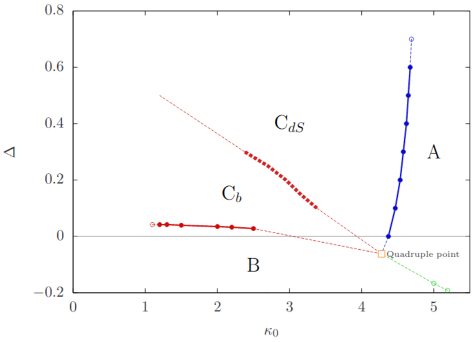

Taken from [Loll, 2020]

Lattice quantum gravity in $d = 4$

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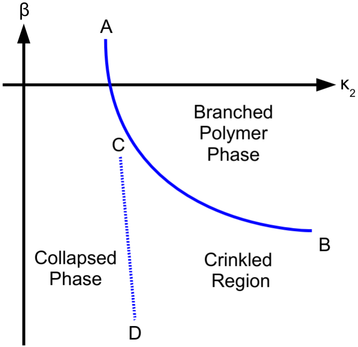

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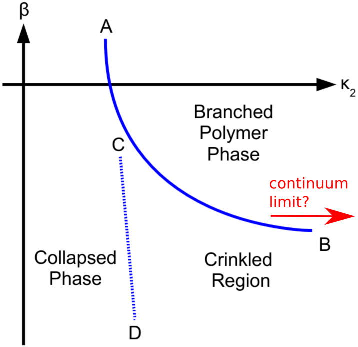

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Phase diagram of EDT

- Idea:

  follow AB-line towards large κ_2

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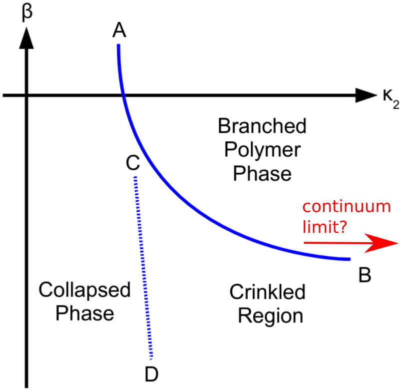

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Phase diagram of EDT

- Idea:
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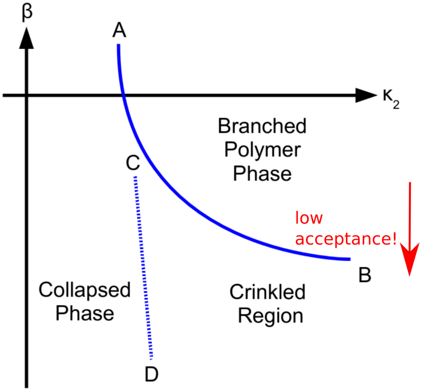
- Challenge: Metropolis acceptance rate p drops:

$$\kappa_2 = 1.7\,,\quad p \sim 10^{-3};$$
$$\kappa_2 = 2.45\,, p \sim 10^{-4};$$
$$\kappa_2 = 3.0\,,\quad p \sim 3 \cdot 10^{-5};$$
$$\kappa_2 = 3.8\,,\quad p \sim 5 \cdot 10^{-6};$$
$$\kappa_2 = 4.5\,,\quad p \sim 1 \cdot 10^{-6};$$



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Need efficient algorithm for low acceptance rates.

Rejection-free algorithm

- Used in studies of dynamical systems (e.g., growth of crystals)
  [Norman, Cannon, 1972], [Bortz, Kalos, Lebowitz, 1975], [Gillespie, 1976], [Schulze, 2004], . . .
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     - 1.1 Compute probability $p_{A\to B}$ at each lattice site i, save them in a list $p_i = p_{A\to B}(i)$.
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Problem: $S_{\mathrm{ER}} = -\kappa_2 N_2 + \kappa_4 N_4$, purely global!

Detailed balance and probabilities

- Including local measure term: $S_{\text{tot}} = S_{\text{loc}} + S_{\text{glob}}$
- Detailed balance:

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Monte-Carlo time from weight ("dwell time"): $\omega = \frac{1}{\sum_{i=1}^{N} \mathcal{P}_{A\to B_i}}$.

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  [Beale; 1996]

$$P_k(T) = \frac{g_k\, e^{-\frac{4\,k\,J}{T}}}{\sum_{k=0}^{N} g_k\, e^{-\frac{4\,k\,J}{T}}}$$

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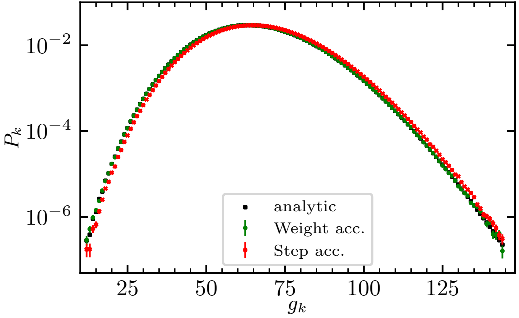

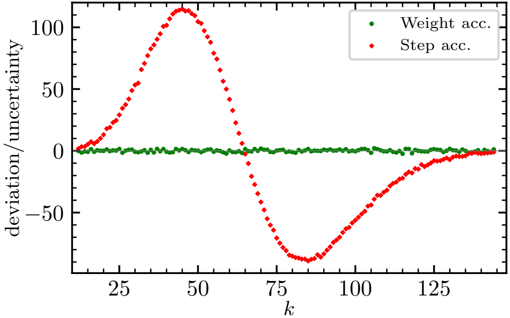

Weight-accumulating Rejection-free algorithm reproduces correct distribution!

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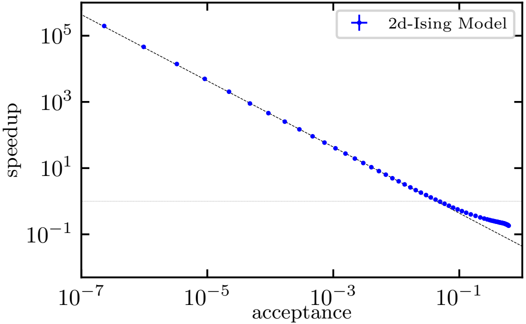

At low acceptance rate: significant speedup!

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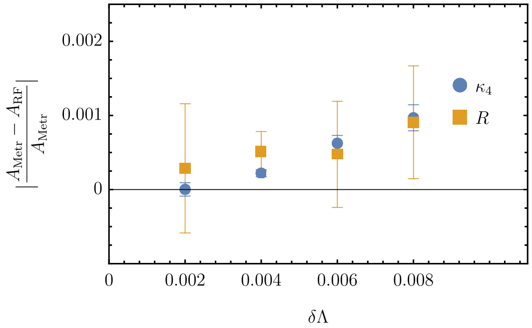

Metropolis and Rejection free algorithms seem to agree in physical limit $\delta\Lambda \to 0$

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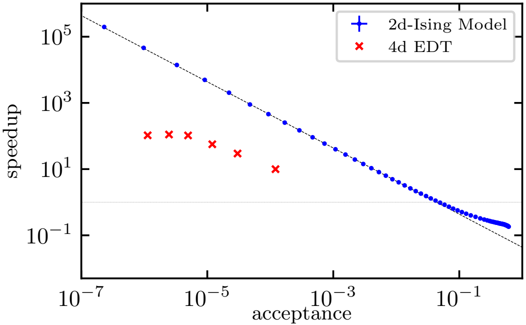

Some speedup, but plateau; Reason: higher connectivity.

Key object: the shelling function

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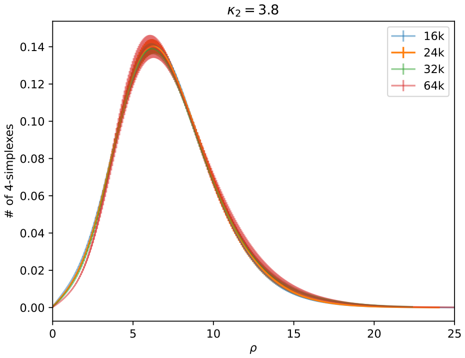

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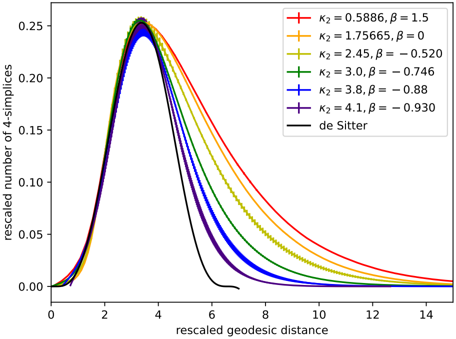
  $\Rightarrow$ approximate de Sitter profile better for finer lattices



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EDT lattice spacings: a/ℓ

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- Example: "rather-fine" lattices:
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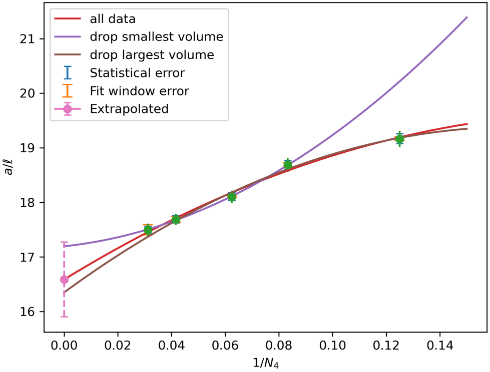

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Determining absolute and relative lattice spacings

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$$\langle N_4 \rangle \simeq \frac{k^2}{4(\kappa_4 - \kappa_4^c)^2} \Rightarrow k = |\kappa_4 - \kappa_4^c| \sqrt{N_4}\,.$$

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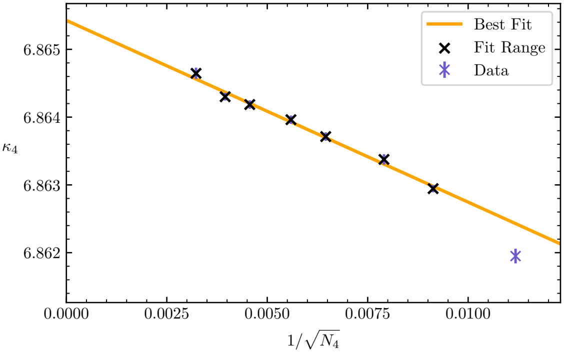
  Extract G from lattice data:

$$\frac{G}{\ell_{\text{fid}}^2} \sim \left(\frac{a}{\ell}\right)^2 \frac{\ell_{\text{rel}}^2}{|s|}\,,$$

Numerical result: finite volume scaling

Example at $\beta = -0.575$, $\kappa_2 = 2.245$:



Extract slope s: $\kappa_4(N_4) = A + s\frac{1}{\sqrt{N_4}}$

Fit result: $s = -0.268 \pm 0.011$

$\chi^2/\mathrm{d.o.f} = 0.82$.

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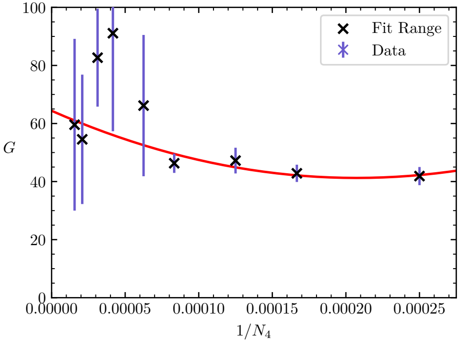

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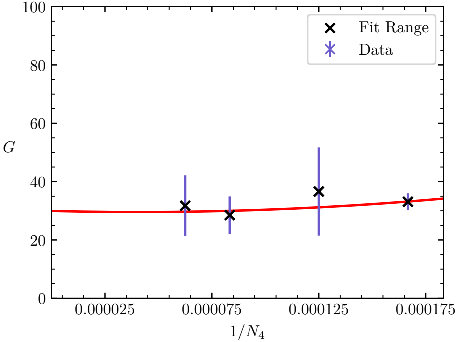

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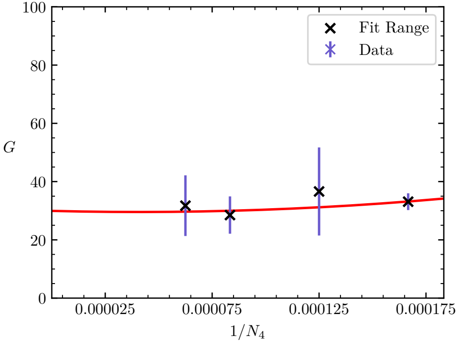

ℓ_{rel} for finer lattices:
WIP; computationally intense;

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- New algorithm: more and finer lattices
- Lattice geometries: resemble semi-classical de Sitter space with $d_{\mathrm{H}} \approx 4$
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[Mingwei Dai, today 4:20 pm, WH3NE]

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Thank you for your attention!