

Status of RBC/UKQCD $g-2$ program

Christoph Lehner
(University of Regensburg)

August 1, 2023 – Lattice 23

The RBC & UKQCD collaborations

[University of Bern & Lund](#)

Dan Hoying

[BNL and BNL/RBRC](#)

Peter Boyle (Edinburgh)

Taku Izubuchi

Yong-Chull Jang

Chulwoo Jung

Christopher Kelly

Meifeng Lin

Nobuyuki Matsumoto

Shigemi Ohta (KEK)

Amarjit Soni

Raza Sufian

Tianle Wang

[CERN](#)

Andreas Jüttner (Southampton)

Tobias Tsang

[Columbia University](#)

Norman Christ

Sarah Fields

Ceran Hu

Yikai Huo

Joseph Karpie (JLab)

Erik Lundstrum

Bob Mawhinney

Bigeng Wang (Kentucky)

[University of Connecticut](#)

Tom Blum

Luchang Jin (RBRC)

Douglas Stewart

Joshua Swaim

Masaaki Tomii

[Edinburgh University](#)

Matteo Di Carlo

Luigi Del Debbio

Felix Erben

Vera Gülpers

Maxwell T. Hansen

Tim Harris

Ryan Hill

Raoul Hodgson

Nelson Lachini

Zi Yan Li

Michael Marshall

Fionn Ó hÓgáin

Antonin Portelli

James Richings

Azusa Yamaguchi

Andrew Z.N. Yong

[Liverpool Hope/Uni. of Liverpool](#)

Nicolas Garron

[LLNL](#)

Aaron Meyer

[University of Milano Bicocca](#)

Mattia Bruno

[Nara Women's University](#)

Hiroshi Ohki

[Peking University](#)

Xu Feng

[University of Regensburg](#)

Davide Giusti

Andreas Hackl

Daniel Knüttel

Christoph Lehner

Sebastian Spiegel

[RIKEN CCS](#)

Yasumichi Aoki

[University of Siegen](#)

Matthew Black

Anastasia Boushmelev

Oliver Witzel

[University of Southampton](#)

Alessandro Barone

Bipasha Chakraborty

Ahmed Elgaziari

Jonathan Flynn

Nikolai Husung

Joe McKeon

Rajnandini Mukherjee

Callum Radley-Scott

Chris Sachrajda

[Stony Brook University](#)

Fangcheng He

Sergey Syritsyn (RBRC)

Hadronic light-by-light contribution to the muon anomaly from lattice QCD with infinite volume QED at physical pion mass

Thomas Blum,^{1,*} Norman Christ,² Masashi Hayakawa,^{3,4} Taku Izubuchi,^{5,6}
Luchang Jin,^{1,6,†} Chulwoo Jung,⁵ Christoph Lehner,⁷ and Cheng Tu¹

(RBC and UKQCD Collaborations)

¹*Physics Department, University of Connecticut, Storrs, CT 06269-3046, USA*

²*Physics Department, Columbia University, New York, NY 10027, USA*

³*Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

⁴*Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan*

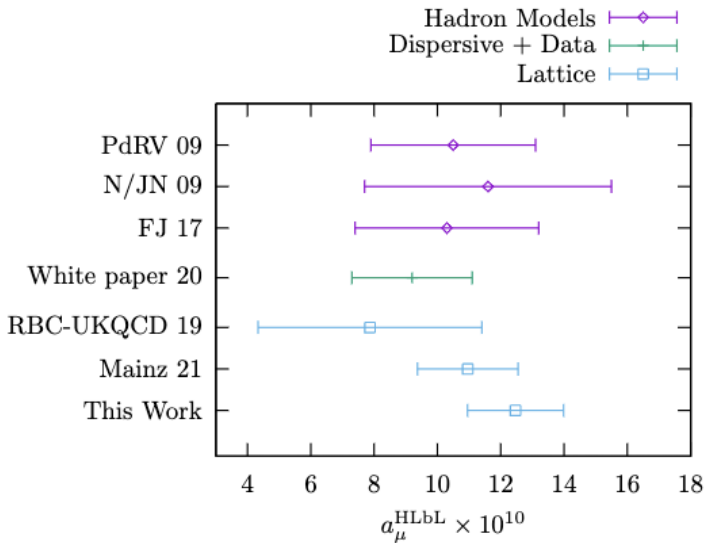
⁵*Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

⁶*RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA*

⁷*Fakultät für Physik, Universität Regensburg, Universitätsstraße 31, 93040 Regensburg, Germany*

(Dated: April 11, 2023)

The hadronic light-by-light scattering contribution to the muon anomalous magnetic moment, $(g-2)/2$, is computed in the infinite volume QED framework with lattice QCD. We report $a_{\mu}^{\text{HLbL}} = 12.47(1.15)(0.99) \times 10^{-10}$ where the first error is statistical and the second systematic. The result is mainly based on the 2+1 flavor Möbius domain wall fermion ensemble with inverse lattice spacing $a^{-1} = 1.73$ GeV, lattice size $L = 5.5$ fm, and $m_{\pi} = 139$ MeV, generated by the RBC-UKQCD collaborations. The leading systematic error of this result comes from the lattice discretization. This result is consistent with previous determinations.



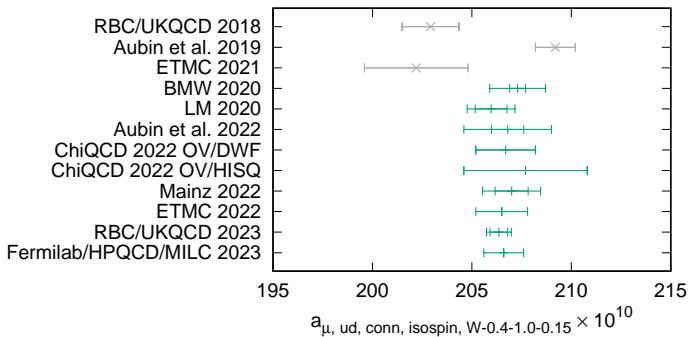
HVP short-distance and intermediate-distance window update (2301.08696)

An update of Euclidean windows of the hadronic vacuum polarization

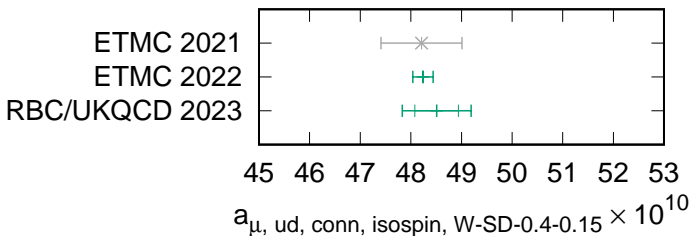
T. Blum,¹ P. A. Boyle,^{2,3} M. Bruno,^{4,5} D. Giusti,⁶ V. Gülpers,³ R. C. Hill,³
T. Izubuchi,^{2,7} Y.-C. Jang,^{8,9} L. Jin,^{1,7} C. Jung,² A. Jüttner,^{10,11} C. Kelly,¹²
C. Lehner,^{6,*} N. Matsumoto,⁷ R. D. Mawhinney,⁹ A. S. Meyer,^{13,14} and J. T. Tsang^{10,15}
(RBC and UKQCD Collaborations)

We compute the standard Euclidean window of the hadronic vacuum polarization using multiple independent blinded analyses. We improve the continuum and infinite-volume extrapolations of the dominant quark-connected light-quark isospin-symmetric contribution and address additional sub-leading systematic effects from sea-charm quarks and residual chiral-symmetry breaking from first principles. We find $a_\mu^W = 235.56(65)(50) \times 10^{-10}$, which is in 3.8σ tension with the recently published dispersive result of $a_\mu^W = 229.4(1.4) \times 10^{-10}$ [1] and in agreement with other recent lattice determinations. We also provide a result for the standard short-distance window. The results reported here are unchanged compared to our presentation at the Edinburgh workshop of the g-2 Theory Initiative in 2022 [2].

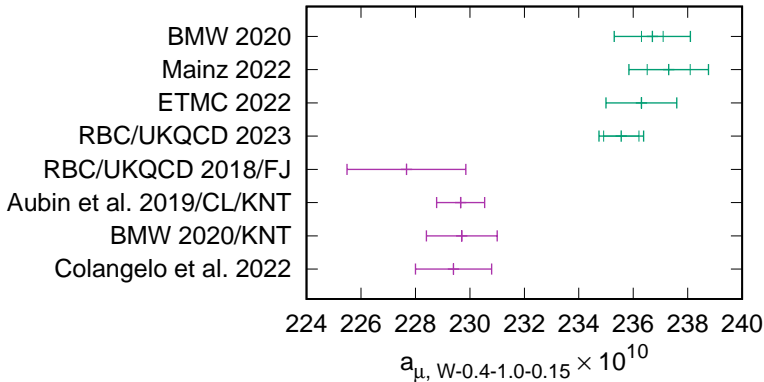
HVP short-distance and intermediate-distance window update (2301.08696)



HVP short-distance and intermediate-distance window update (2301.08696)

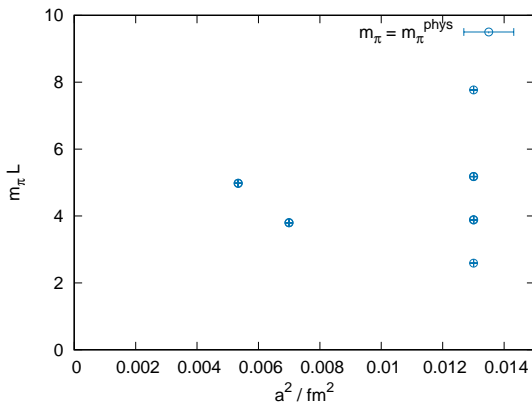


HVP short-distance and intermediate-distance window update (2301.08696)



Next step: long-distance window

- ▶ In progress: blind analysis with 5 analysis groups
- ▶ Blind all vector currents with group-specific blinding factor
- ▶ Ensembles used for this analysis:



- ▶ Follow our improved bounding method strategy of [1910.11745](#); distillation data with 60 laplace eigenmodes for $m_\pi L \approx 4$ ensembles and 200 laplace eigenmodes for $m_\pi L \approx 5$ ensembles

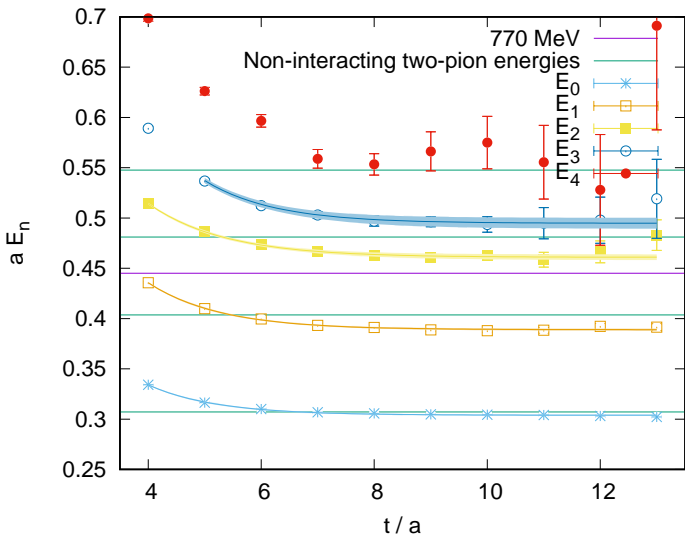
A quick look at group A

- ▶ In following show blind results for group A for single ensemble ($a^{-1} = 1.73$ GeV, $m_\pi L \approx 4$, $m_\pi = m_\pi^{\text{phys}}$) to illustrate method

- ▶ All results still preliminary

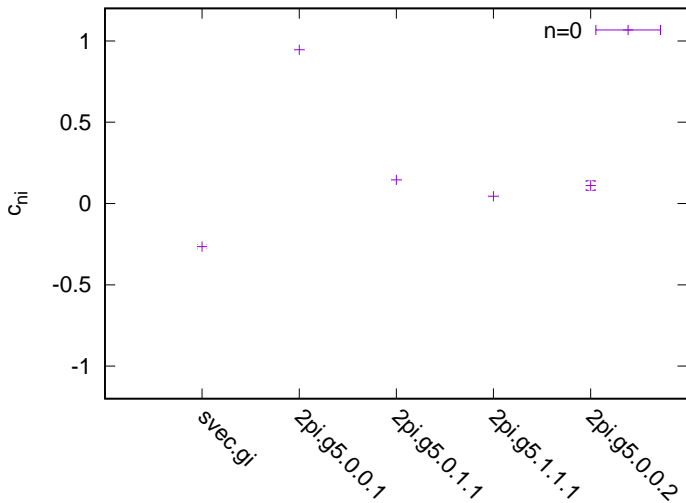
The $l = 1$ spectrum (group A, blind, preliminary)

Using smeared vector current and two-pion operators up to relative momentum of $\vec{p}^2 = 4(2\pi/L)^2$



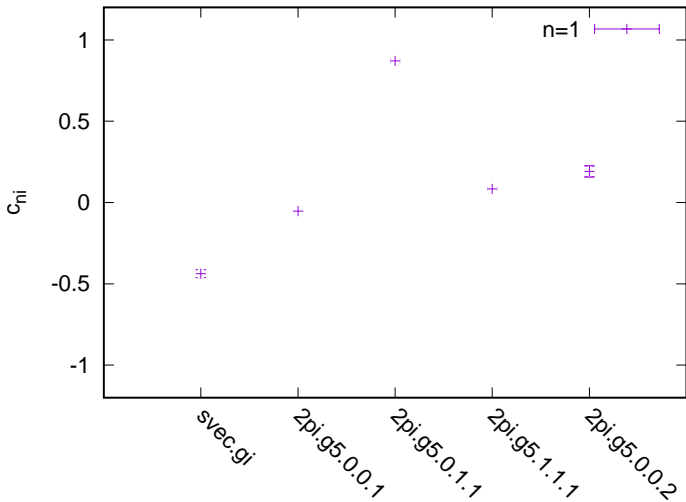
Optimized operators (group A, blind, preliminary)

Out of 5 operator basis, construct operators that project out single finite-volume state:



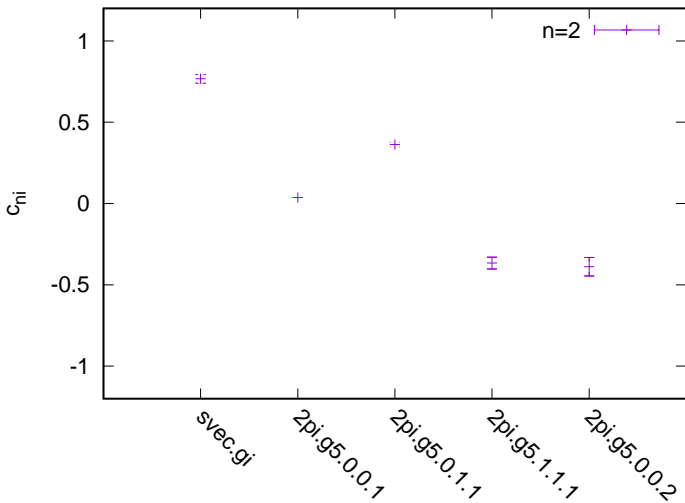
Optimized operators (group A, blind, preliminary)

Out of 5 operator basis, construct operators that project out single finite-volume state:



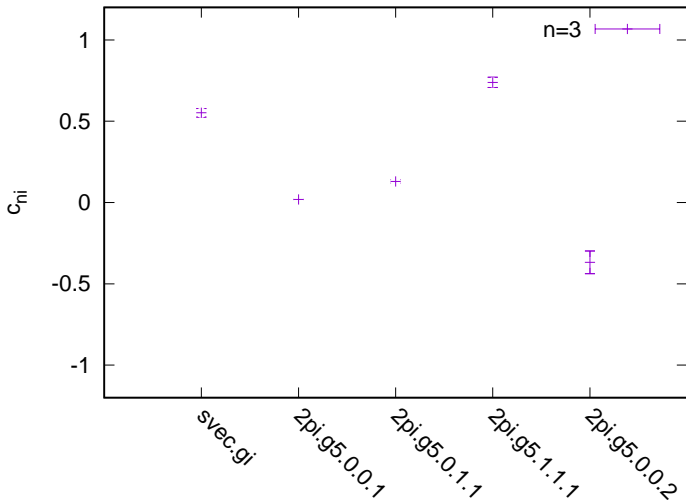
Optimized operators (group A, blind, preliminary)

Out of 5 operator basis, construct operators that project out single finite-volume state:



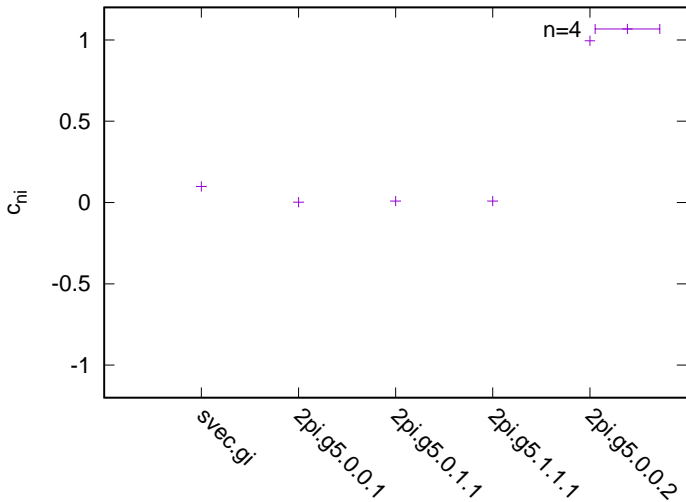
Optimized operators (group A, blind, preliminary)

Out of 5 operator basis, construct operators that project out single finite-volume state:



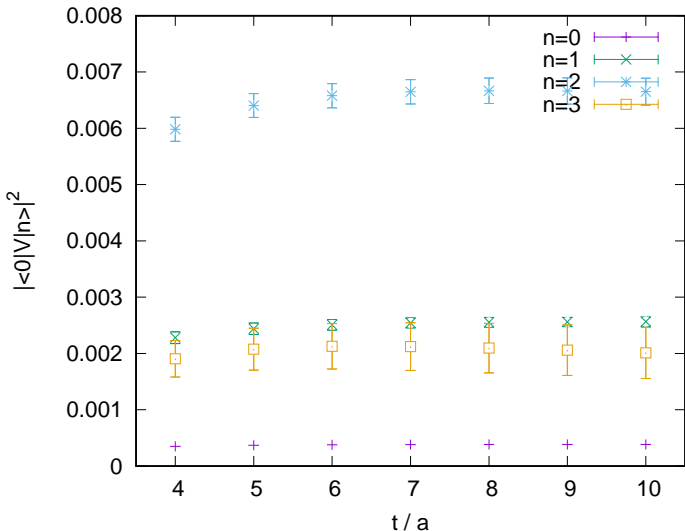
Optimized operators (group A, blind, preliminary)

Out of 5 operator basis, construct operators that project out single finite-volume state:



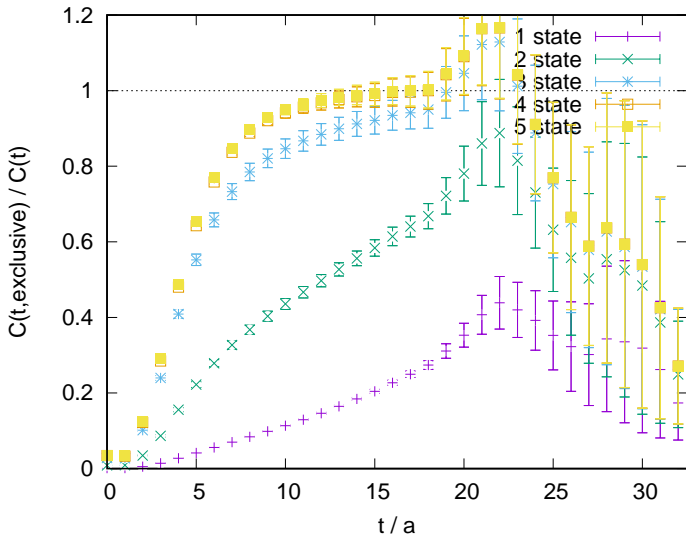
Overlap with local vector current (group A, blind, preliminary)

Next, study correlators of single-state operators with local vector current to obtain $\langle 0|V_i|n\rangle$ matrix elements:



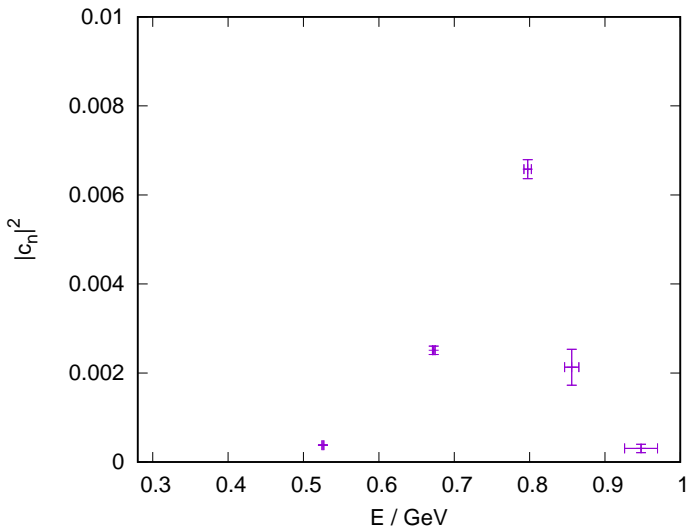
Exclusive-state reconstruction (group A, blind, preliminary)

Next, reconstruct correlator from lowest n states:



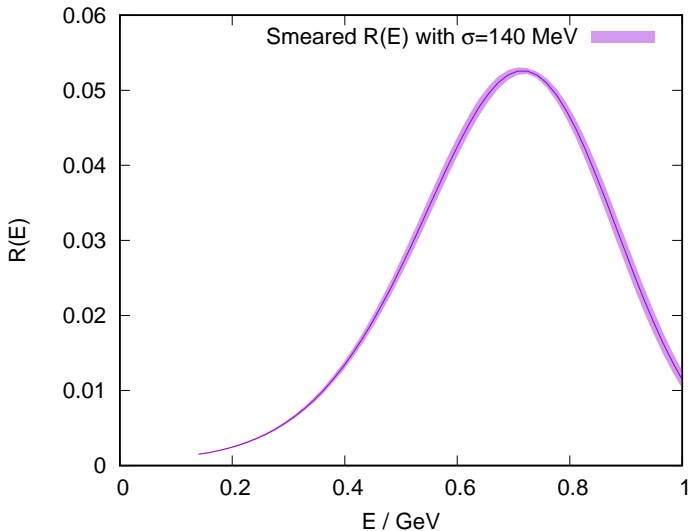
Finite-volume states (group A, blind, preliminary)

Nice to plot this as $|c_n|^2 = |\langle 0|V_j|n\rangle|^2$ versus energy of finite-volume state E_n :



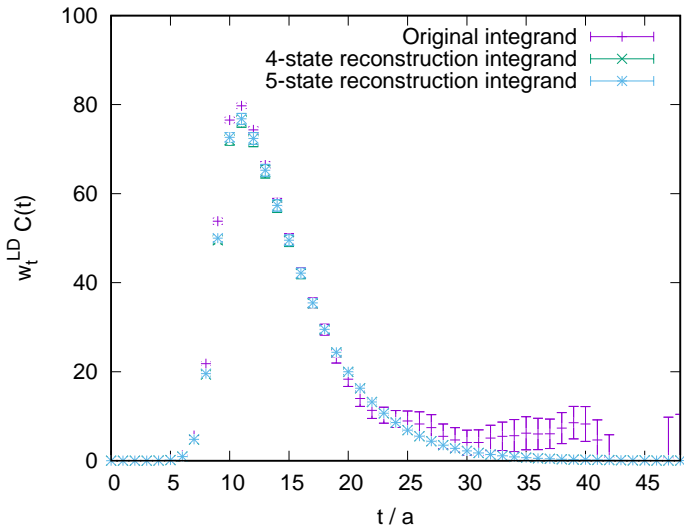
Side-stepping inverse Laplace problem (group A, blind, preliminary)

As noticed in [2012.11488](#), this allows for a nice way to side-step the inverse Laplace problem. Demonstrate this here with $\sigma = 140$ MeV Gaussian-kernel smeared R -ratio $R(E)$:



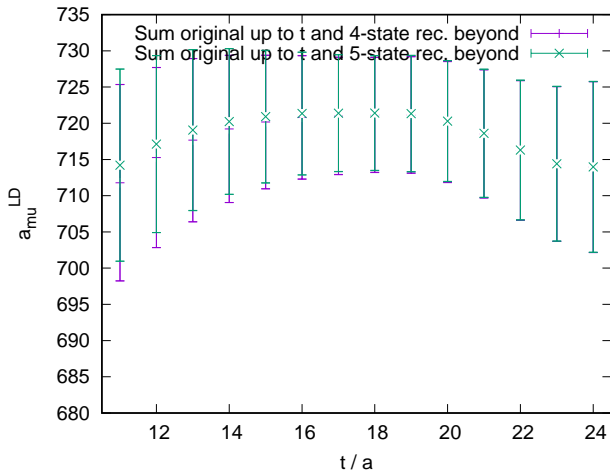
Long-distance integrand (group A, blind, preliminary)

Obtain integrand from original vector-vector data and reconstruction:



Long-distance window estimator (group A, blind, preliminary)

Sum vector-vector for times up to t and reconstructed integrand above:

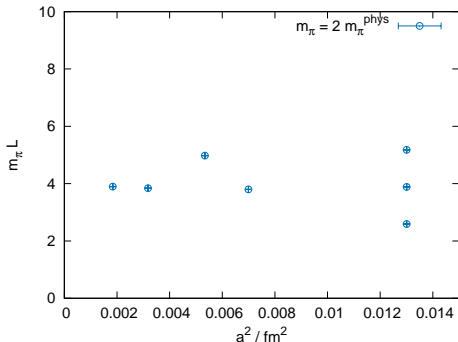


Vector current is blinded allowing for a factor of 4 variation. Aim for error of $O(4 \times 10^{-10})$.

Improved bounding method can be used to further refine the estimator.

Further steps to complete program

- ▶ Re-use of HLbL data sets for update of full QED+SIB corrections in progress
- ▶ Additional ensembles at twice m_π^{phys} have been generated to further consolidate continuum limit (see my GPT talk for scripts to generate them):



- ▶ Additional ensembles to study sea charm, m_{res} , and quark-mass dependence available
- ▶ Will update our 2018 strange, charm, and disconnected results early next year

Summary

- ▶ Update of HLbL with QED_∞ out: [2304.04423](#)
- ▶ Short-distance and intermediate-window results for HVP at desired precision out: [2301.08696](#) (blind analysis, 5 groups)
- ▶ Long-distance window analysis in progress. Still blind, 5 groups.
- ▶ Aim at better than 0.5% precision for total a_μ^{HVP} in 2024.