Flattening of the quantum effective potential in fermionic theories

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in collaboration with
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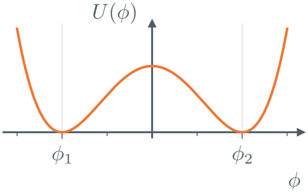
Spontaneous symmetry breaking

Consider example theory

$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + U(\phi^2)$



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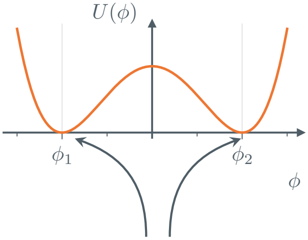

classically broken symmetry

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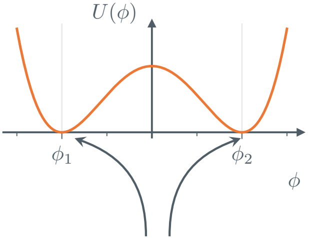

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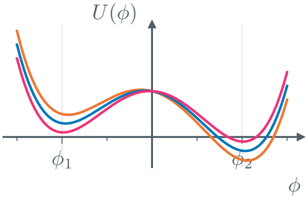
Standard procedure in lattice simulations

- Tilt potential with term $\propto \phi J$
- Calculate $\lim_{J\to 0}\lim_{V\to\infty}\langle\phi\rangle$



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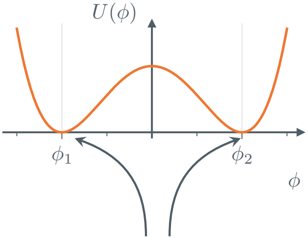

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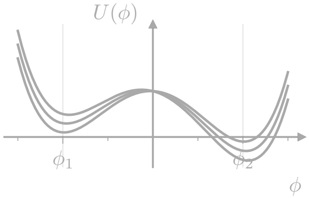
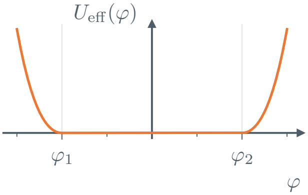


Quantum effective potential of $\varphi = \langle\phi\rangle$

- Convex, flat region when symmetry broken
- Edge of flat region corresponds to realized $\langle\phi\rangle$



Accessing the effective potential (on the lattice)

- Consider constrained path integral

$$\mathcal{Z}_{\bar{\phi}} = \int \mathcal{D}\phi \, \mathrm{e}^{-S[\phi]} \, \delta\left(\textstyle\int \phi - \bar{\phi}\right) = \mathrm{e}^{-V\Omega(\bar{\phi})}$$

- $\Omega(\bar{\phi})$ agrees with $U_{\text{eff}}(\langle\phi\rangle)$ in the infinite volume limit [L. O'Raifeartaigh *et al.*, *Nucl. Phys. B.* **271** (1986)]

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- One can construct algorithms that satisfy the constraint in Monte Carlo simulations
  [Z. Fodor *et al.*, (2007)]
- Also possible with cont. symmetries, e.g., $O(2)$ symmetric ϕ^4 in $3D$
  [G. Endrődi *et al.*, *Phys. Rev. Lett.* **127** (2021)]

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  [G. Endrődi *et al.*, *Phys. Rev. Lett.* **127** (2021)]
- Investigating SSB via constrained approach can be more precise and controlled than the double limit $\lim_{J\to 0} \lim_{V\to\infty} \langle\phi\rangle$

Constraining fermionic condensates

- Constraining bosonic degrees of freedom is understood ... what about fermionic condensates?
- Goal: Develop strategy to constrain fermionic condensates such as $\bar{\psi}\psi$
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- Use low-dimensional four-fermion models in the large-N limit as a test bed
- Start with the Gross-Neveu model in $2D$

Gross-Neveu model in $2D$

- Bosonized action:

$$S[\bar{\psi},\psi] = \int \mathrm{d}^2x \left[\bar{\psi}\partial\!\!\!/\psi - \frac{\lambda}{2N_f}(\bar{\psi}\psi)^2\right] \xrightarrow{\text{H.S. trafo}} S_{\text{bos}}[\sigma,\bar{\psi},\psi] = \int \mathrm{d}^2x \left[\bar{\psi}(\partial\!\!\!/ + \sigma)\psi + \frac{N_f\sigma^2}{2\lambda}\right]$$

- discrete Chiral Symmetry $\mathbb{Z}_2$: $\psi \to \gamma_5\psi\,, \quad \bar{\psi} \to -\gamma_5\bar{\psi}\,, \quad \sigma \to -\sigma$
- Ward identity: $\langle\bar{\psi}\psi\rangle = \frac{-N_f}{\lambda}\langle\sigma\rangle$

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- Use naive fermion discretization
- Consider single coupling $\lambda = 0.4920$ and various lattice sizes $V = L^2$

Constraining mechanisms

Bosonic constraint

- Constrain bosonic field and with it implicitly $\bar{\psi}\psi$

$$\mathcal{Z}_{\bar{\phi}} = \int \mathcal{D}\sigma\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\, \mathrm{e}^{-S_{\mathrm{bos}}}\, \delta\left(\bar{\phi} - \tfrac{1}{V}\int_V \sigma\right)$$

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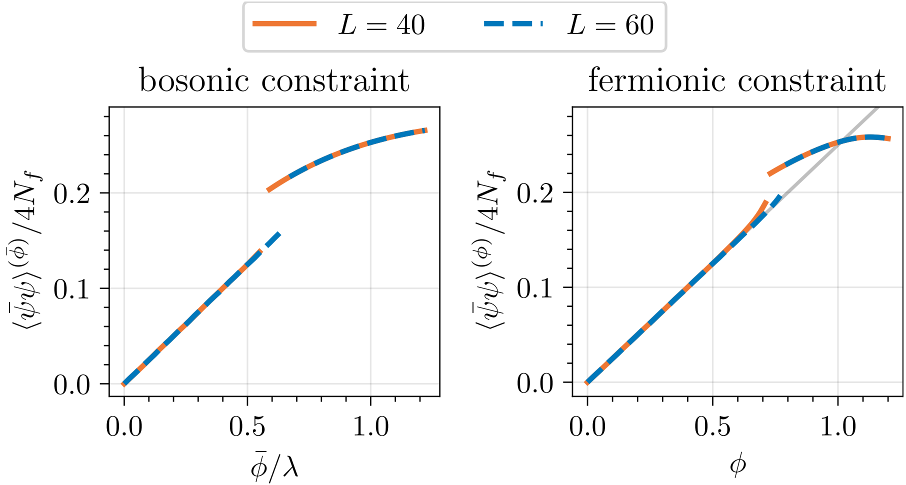
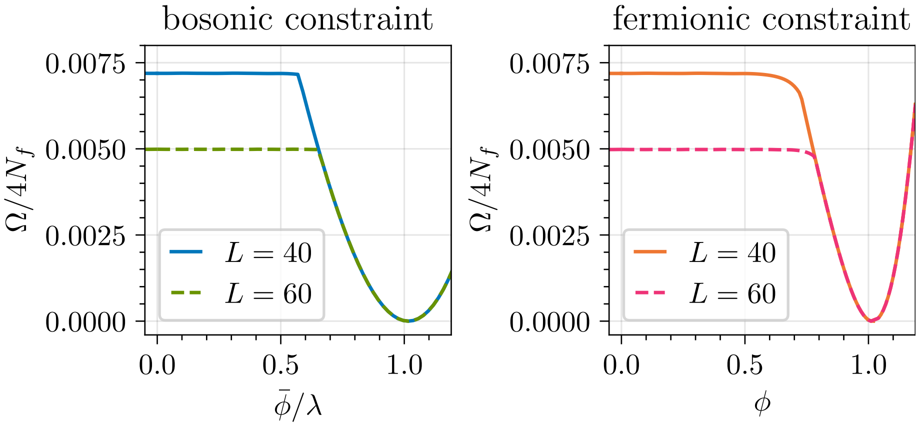
- Type of constraint that would be applicable in QCD

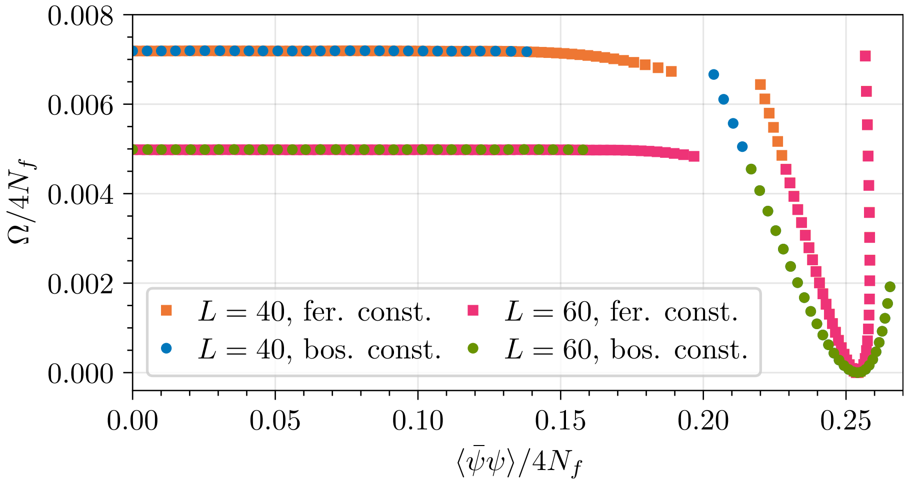
Chiral condensate



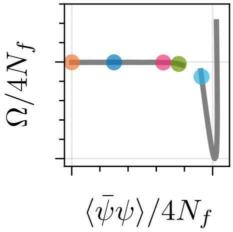
Constrained potential (I)

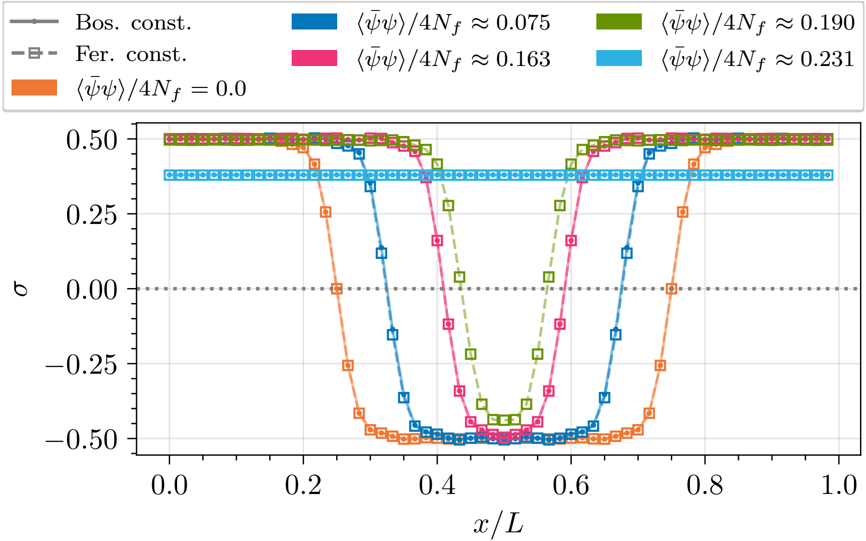


Constrained potential (II)



Field configurations

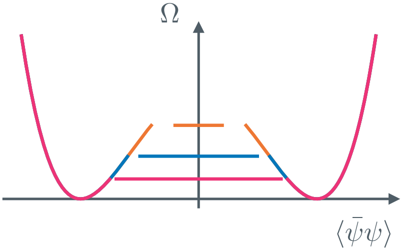




Summary and outlook

Summary:

- Developed a method to constrain fermionic condensates
- Tested in a discrete symmetry model
- Observed the flattening of the constrained potential
- Inhomogeneous configurations in flat region



Outlook:

- Constrained calculations of chiral Gross-Neveu model with continuous symmetry (calculations with bosonic constraint already done)
- Perform Monte-Carlo simulations of chiral Gross-Neveu with both constraints in $2+1$ dimensions