Thermal QCD phase transition with dynamical chiral fermions

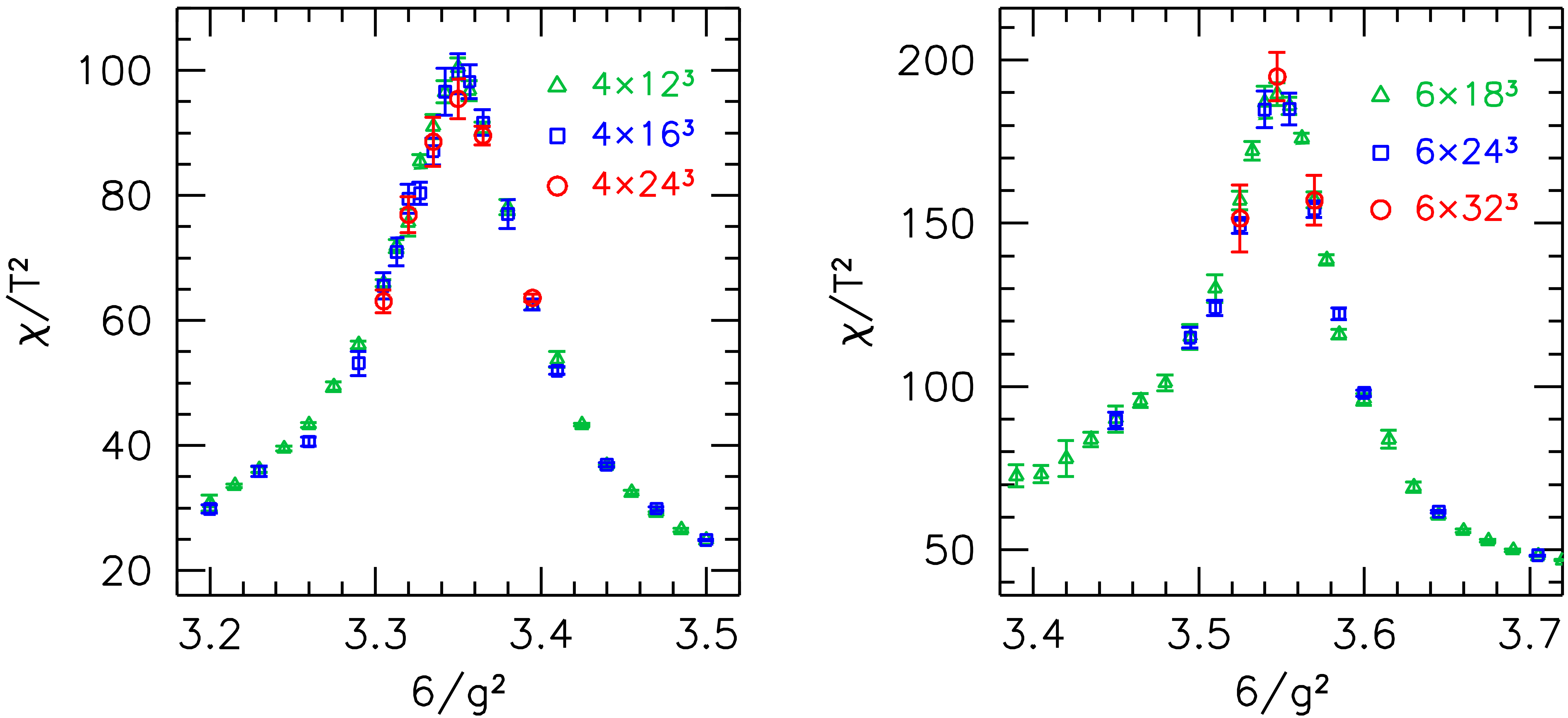
A. Yu. Kotov
in collaboration with **K. Szabo, Z. Fodor**

JÜLICH Forschungszentrum

Lattice 2023

Thermal QCD phase transition

$N_f = 2 + 1$ **stout smeared staggered fermions,** $m_\pi = m_\pi^{(\mathrm{phys})}$**,** $N_t = 4,\ 6$



[Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo, 2006]

Chiral fermions on the lattice

Way to avoid Nielsen-Ninomiya «no-go» theorem

- Continuum chiral symmetry: $\gamma_5 D + D\gamma_5 = 0$
- Ginsparg-Wilson relation: $\gamma_5 D + D\gamma_5 = 2aD\gamma_5 D$
- Overlap fermions: $aD_{\text{ov}} = \dfrac{1}{2}\Big(1 + \gamma_5 \text{sign}(\gamma_5 D_w(-m_w))\Big)$
- Very expensive numerically: require multiple tricks

[H.Neuberger, 1998]

Action details

- Symanzik improved gauge action
- Fermion sector: 2 steps of HEX smeared gauge fields
- $N_f = 2 + 1$ flavours of overlap quarks, physical masses
- 2 flavours of Wilson fermions with mass $-m_W$
- Two boson fields with mass $m_B = 0.54$
- Statistics: $O(2000 - 9000)$

- $a \to 0$: irrelevant
- Keep $Q = \mathrm{const}$ ($Q = 0$)
- Make calculations faster

[H. Fukaya et al., 2006]

Implementing odd number of flavours

- Monte Carlo: determinant of a hermitian operator $H^2 = D_{\mathrm{ov}} D_{\mathrm{ov}}^\dagger$: $N_f = 2$
- To simulate $\underline{N_f = 1}$ (strange quark): need to take the square root
- Chirality projectors: $P_\pm = \dfrac{1 \pm \gamma_5}{2}$, $H_\pm^2 = P_\pm H^2 P_\pm$
- Fixed topology $Q = \mathrm{const}$:
  $\det H^2 \sim \det H_+^2 \det H_-^2 \sim (\det H_+^2)^2 \sim (\det H_-^2)^2$
- Take $\det H_+^2$ or $\det H_-^2$

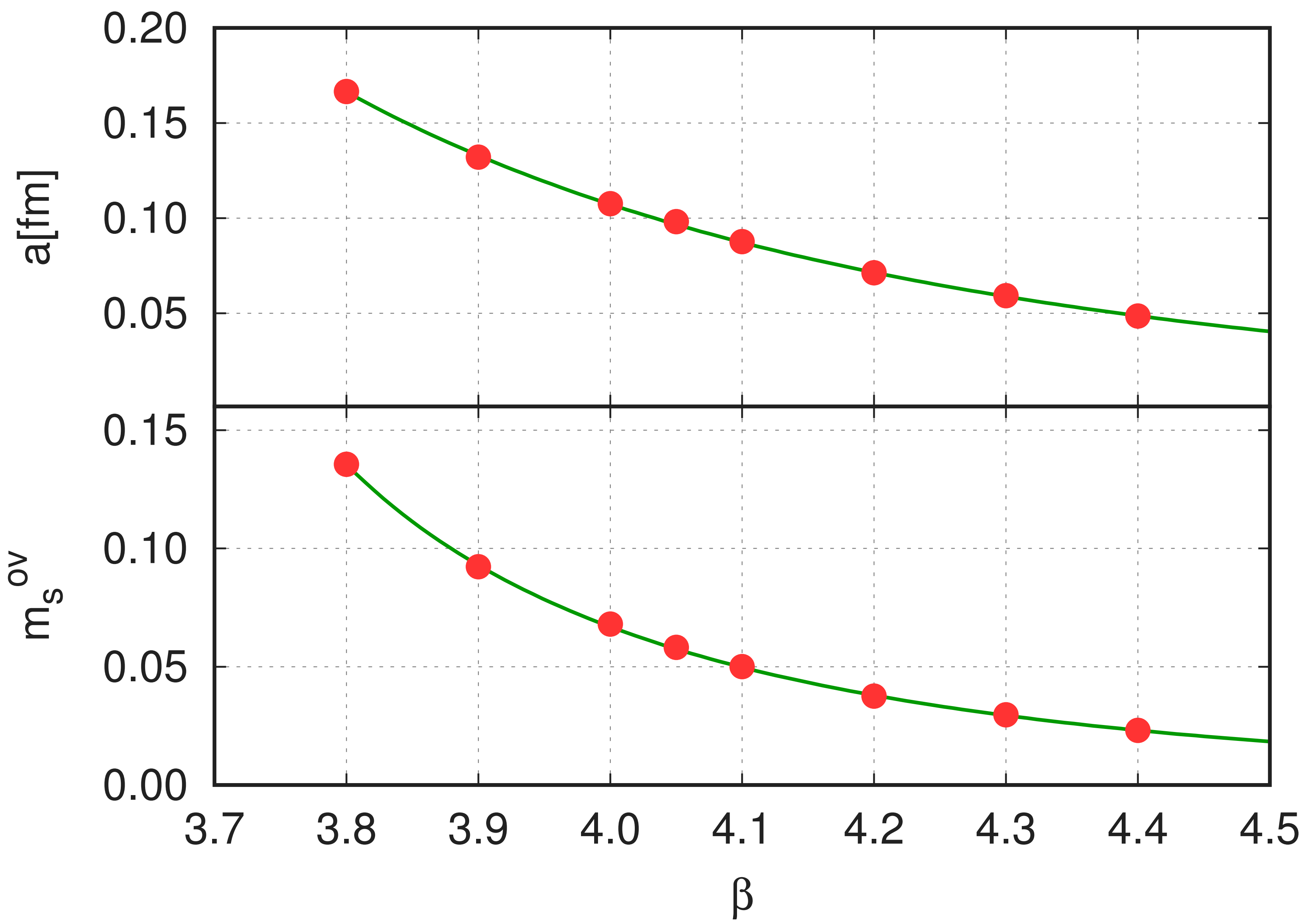
Algorithm details

Overlap: $aD_{\text{ov}} = \frac{1}{2}\left(1 + \gamma_5 \text{sign}(\gamma_5 D_w(-m_w))\right)$

- Standard Hybrid Monte Carlo algorithm
- $\text{sign}X$ implementation:
  - Explicit form on 32-128 low-lying eigenmodes of X
  - Rest of the X spectrum: $\text{sign}X = X/\sqrt{XX^\dagger}$, Chebyshev polynomial approximation for $\sqrt{XX^\dagger}$
- Implicitly restarted Lanczos algorithm for Wilson/overlap eigenvalues
- D_{ov} inversion with FGMRES algorithm and Wilson-Dirac as preconditioner

Lattice details, scale setting

Scale setting from simulations with large m_π

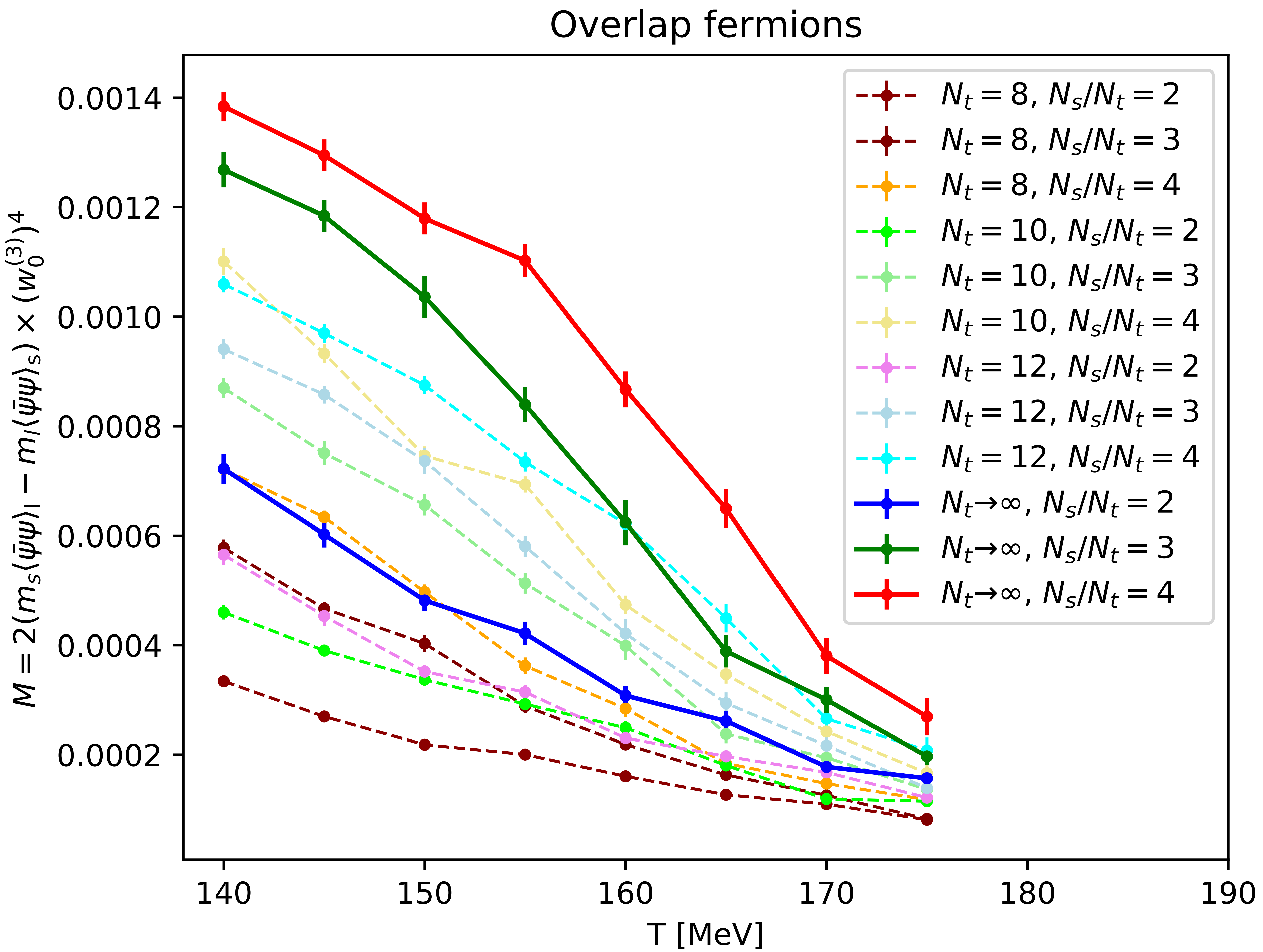


- Simulations are done <u>along the LCP</u>
- Scale setting: require <u>$T = 0$</u> simulations
- $N_f = 3$ staggered simulations, $T = 0$, $w_0^{(3)} = 0.153(1)$ fm, $m_\pi^{(3)} = 712(5)$ MeV
- $N_f = 3$ overlap simulations, $T = 0$, at each β tune $m_s^{\rm ov}$ to have $m_\pi w_0 \equiv m_\pi^{(3)} w_0^{(3)}$
- $N_f = 2 + 1$ overlap simulations, $T \neq 0$: $m_s = m_s^{\rm ov}, \quad m_{ud} = R m_s^{\rm ov}, \quad a = w_0^{(3)}/w_0^{\rm ov}$
- <u>Physical point</u>: $m_{ud} = m_{ud}^{(phys)}, \quad m_s = m_s^{(phys)}$

[Sz. Borsanyi et al., 2016]

Chiral condensate

- $M = 2\left(m_s\langle\bar{\psi}\psi\rangle_l - m_l\langle\bar{\psi}\psi\rangle_s\right)$
- Large cutoff effect and FV effects
- $T_{pc} \approx 163(6)\ \mathrm{MeV}$



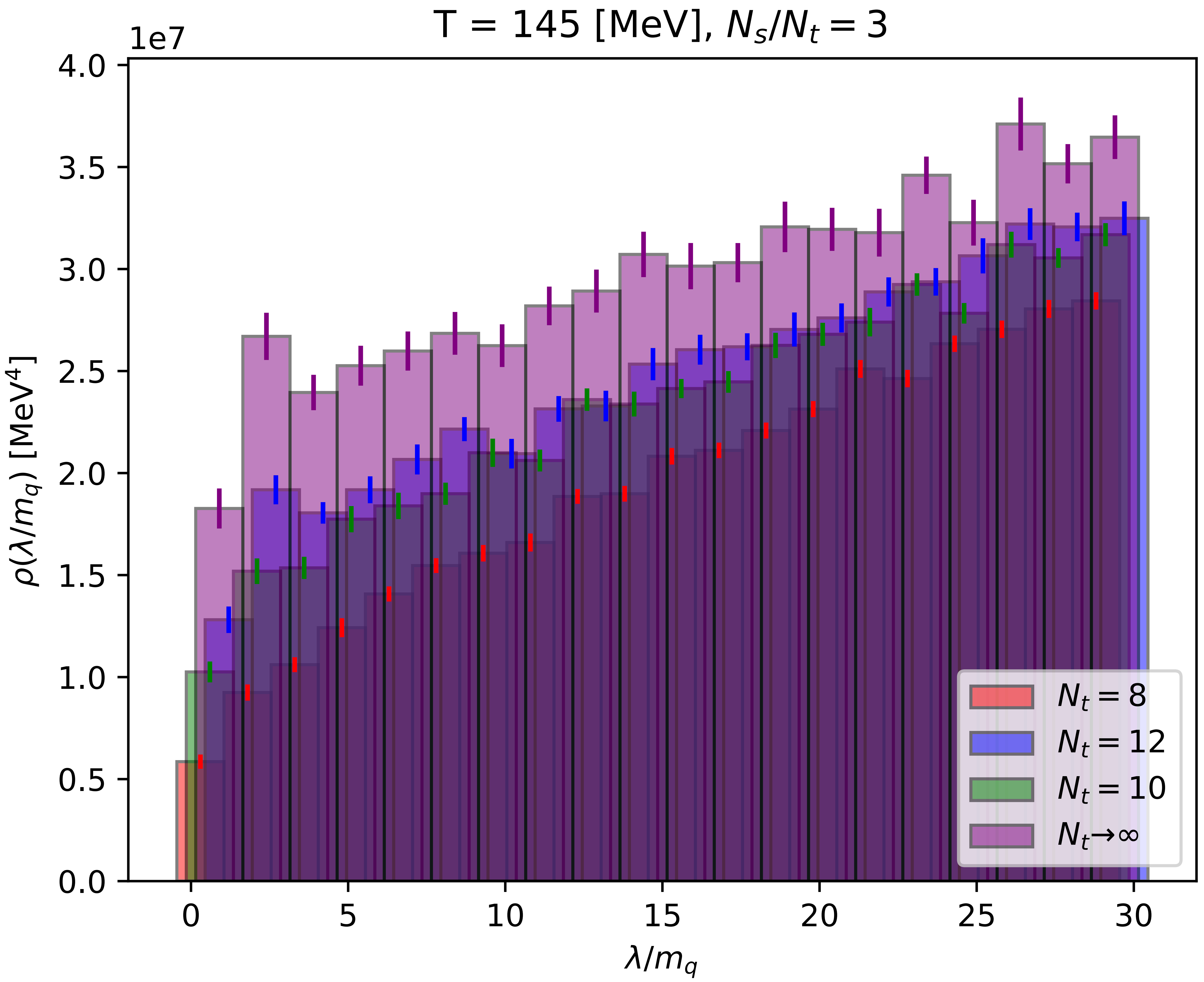
Chiral susceptibility

$$\chi_M = m\partial_m M$$

$$T_{pc} \sim 160 \text{ MeV}$$

Eigenvalue spectrum

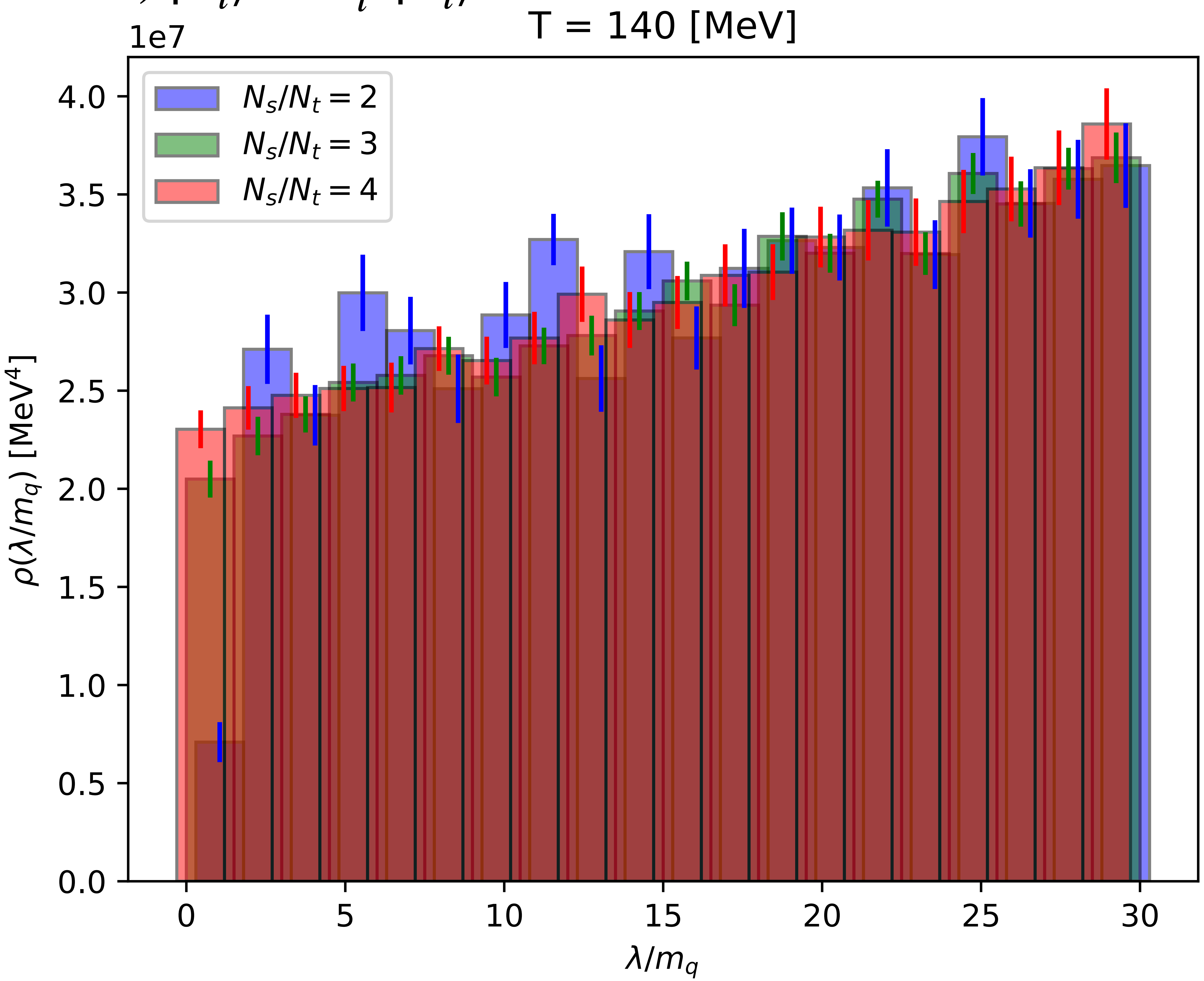
$$D_{\mathrm{ov}}^{\dagger}(m=0)D_{\mathrm{ov}}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$



Eigenvalue spectrum

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165$ MeV

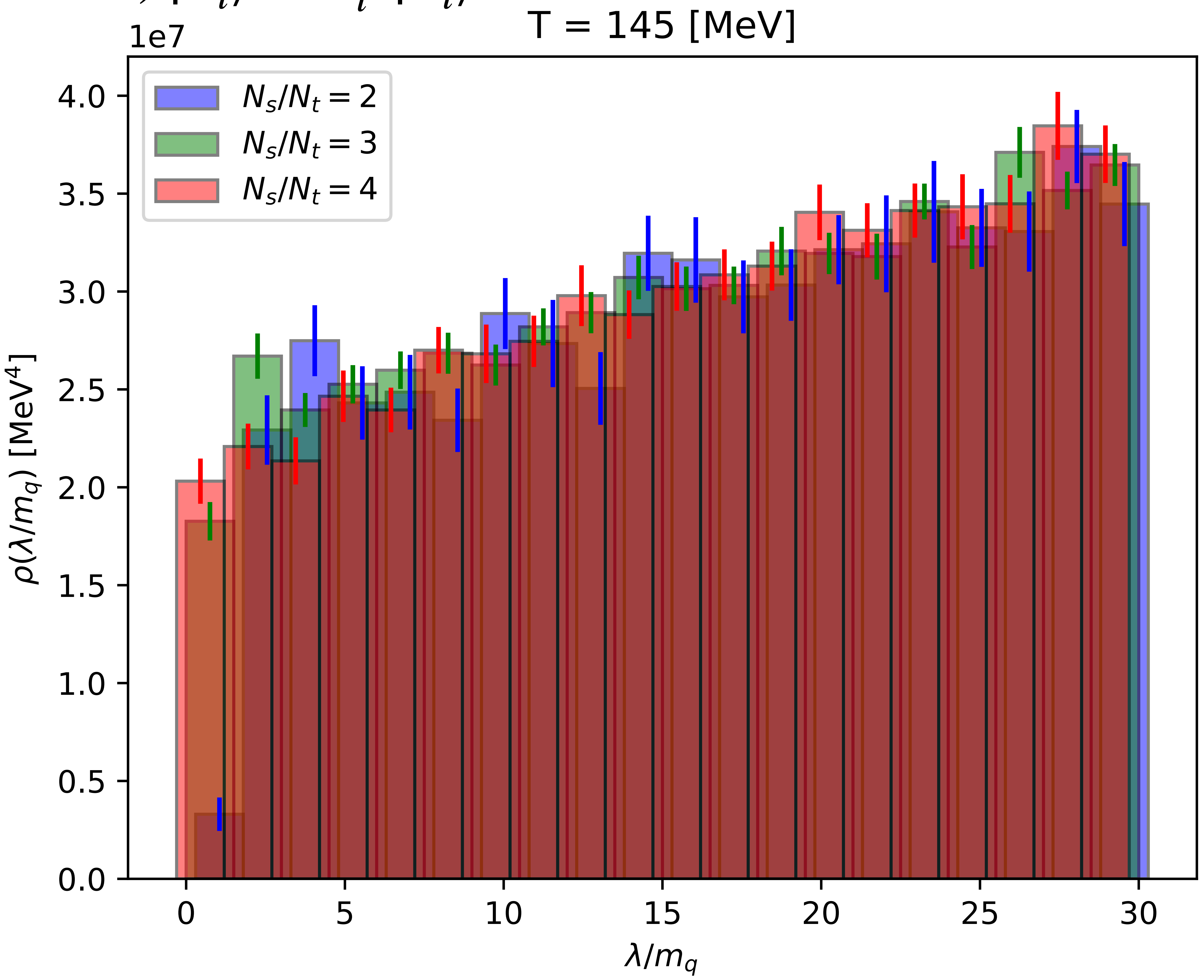
$$D_{\mathrm{ov}}^{\dagger}(m=0)D_{\mathrm{ov}}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$



Eigenvalue spectrum

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165$ MeV

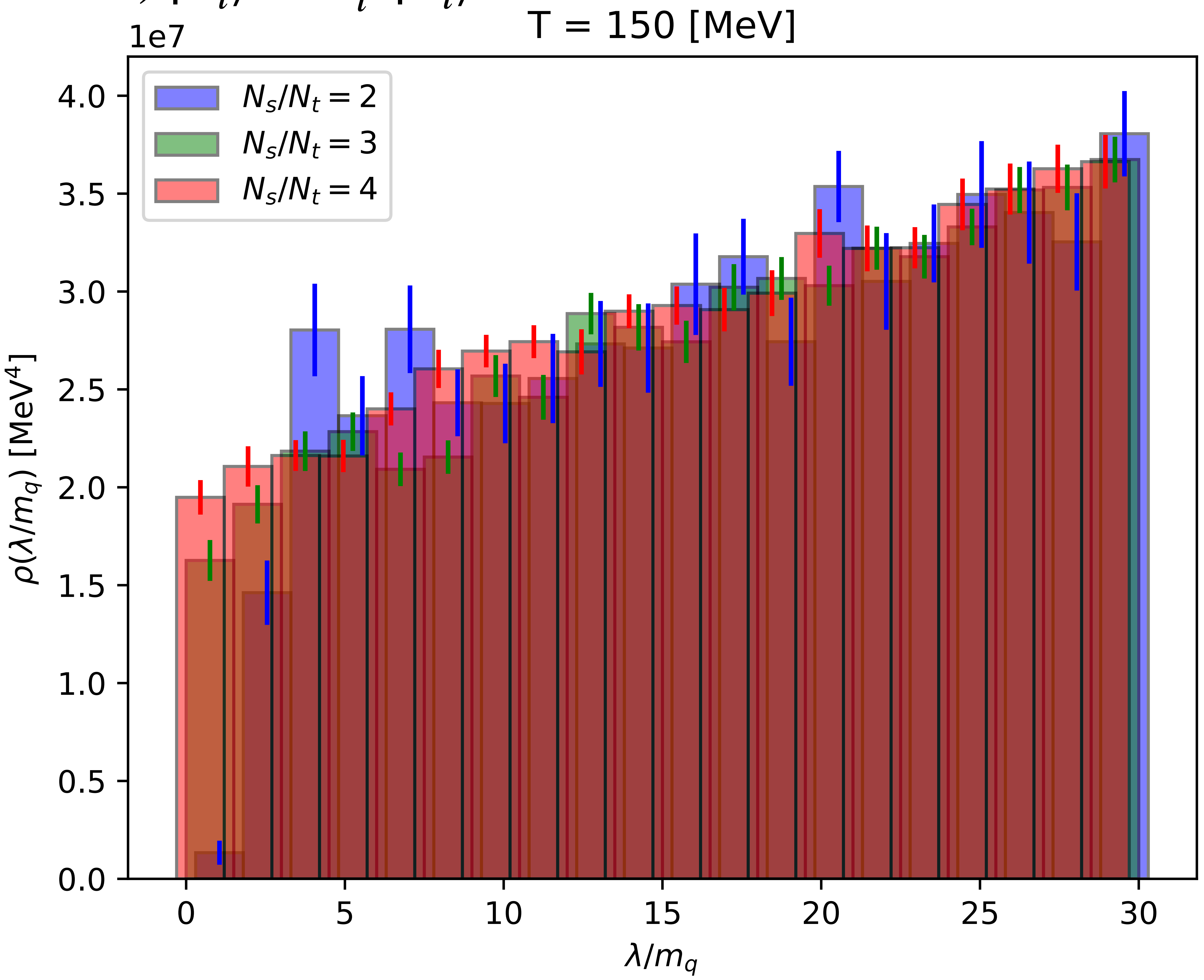
$$D^{\dagger}_{\rm ov}(m=0)D_{\rm ov}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$



Eigenvalue spectrum

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165$ MeV

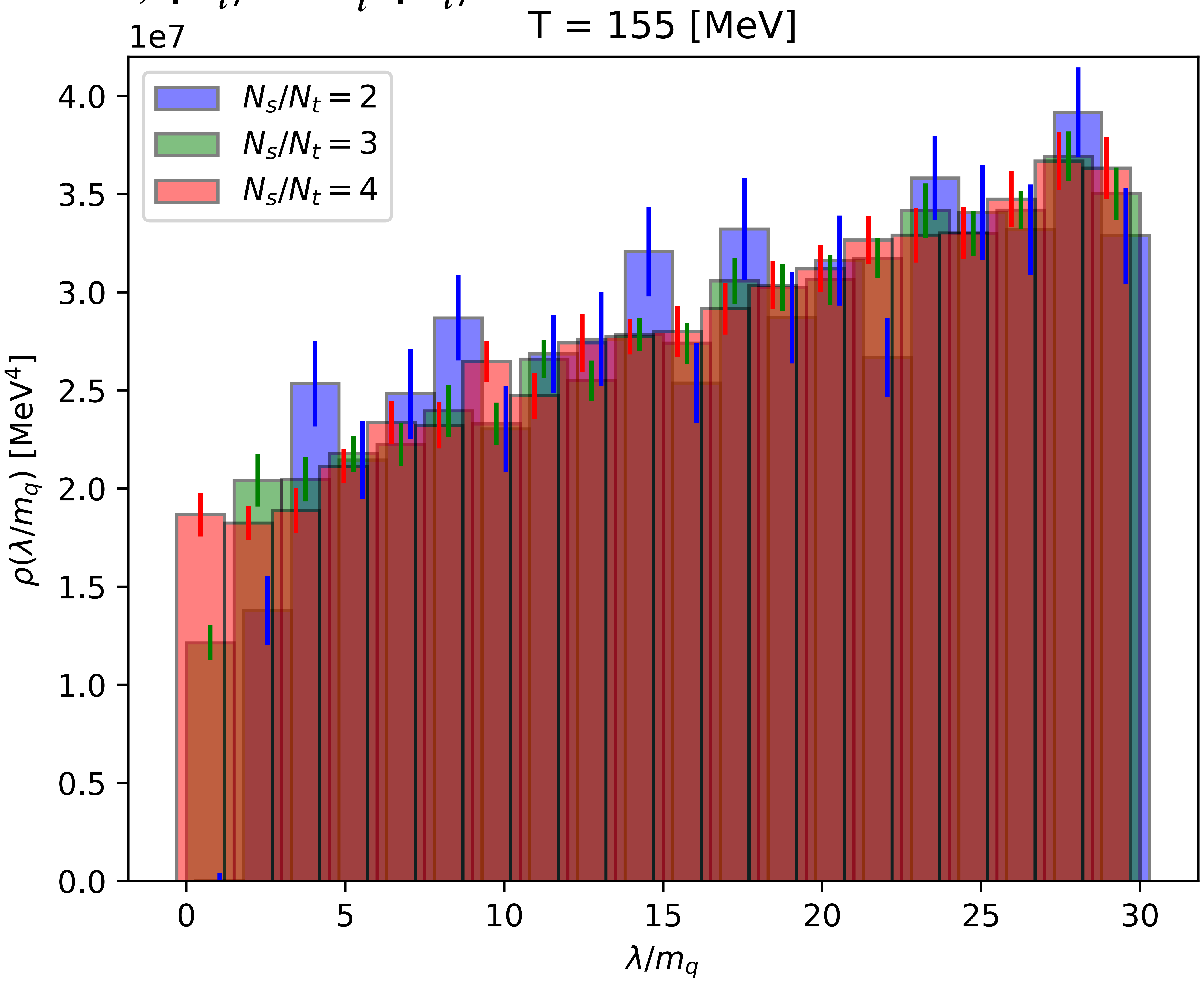
$$D_{\rm ov}^{\dagger}(m=0)D_{\rm ov}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$



Eigenvalue spectrum

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165$ MeV

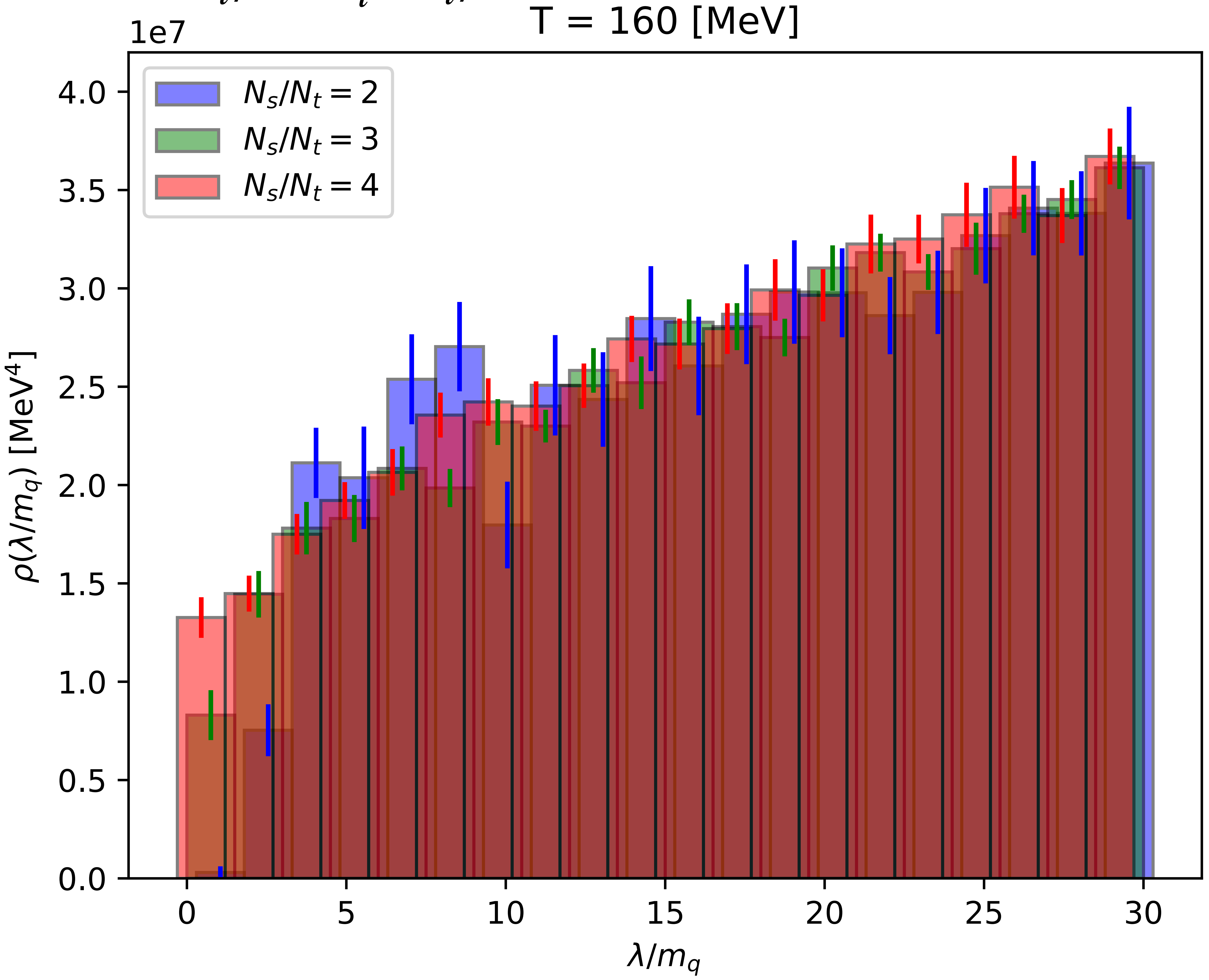
$$D^{\dagger}_{\mathrm{ov}}(m=0)D_{\mathrm{ov}}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$



Eigenvalue spectrum

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165$ MeV

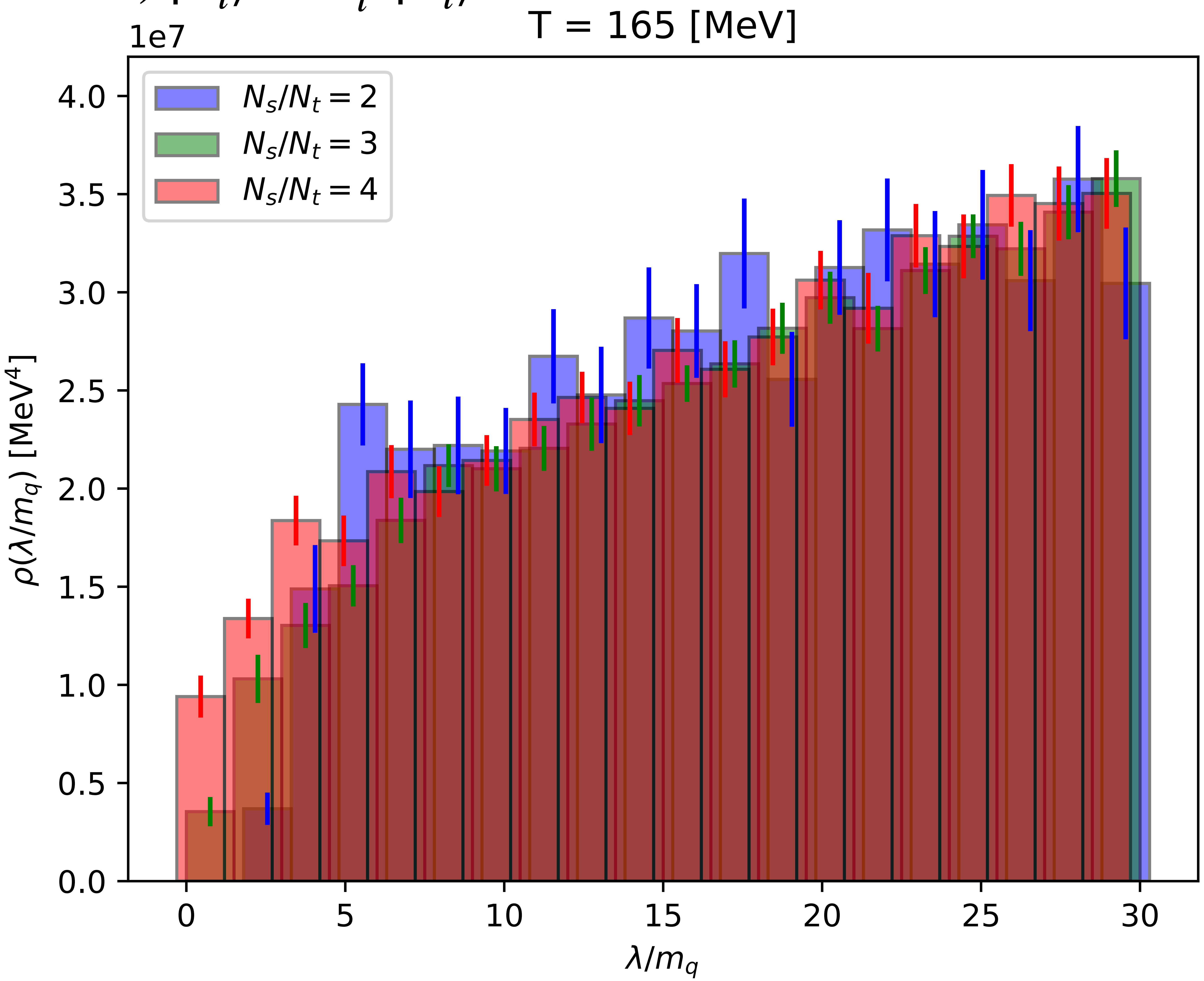
$$D_{\rm ov}^{\dagger}(m=0)D_{\rm ov}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$



Eigenvalue spectrum

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165\ \mathrm{MeV}$

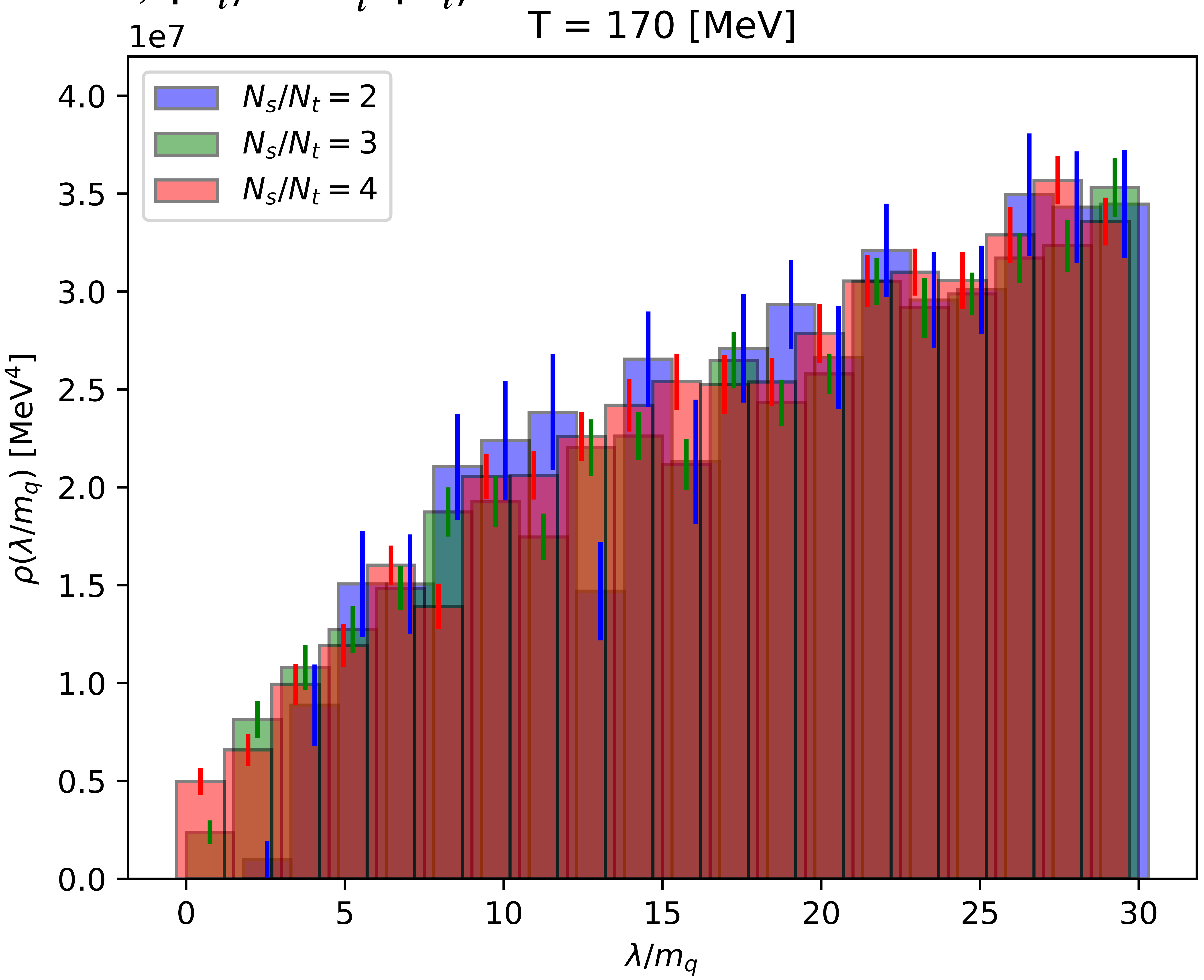
$$D_{\mathrm{ov}}^{\dagger}(m=0)D_{\mathrm{ov}}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$



Eigenvalue spectrum

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165$ MeV

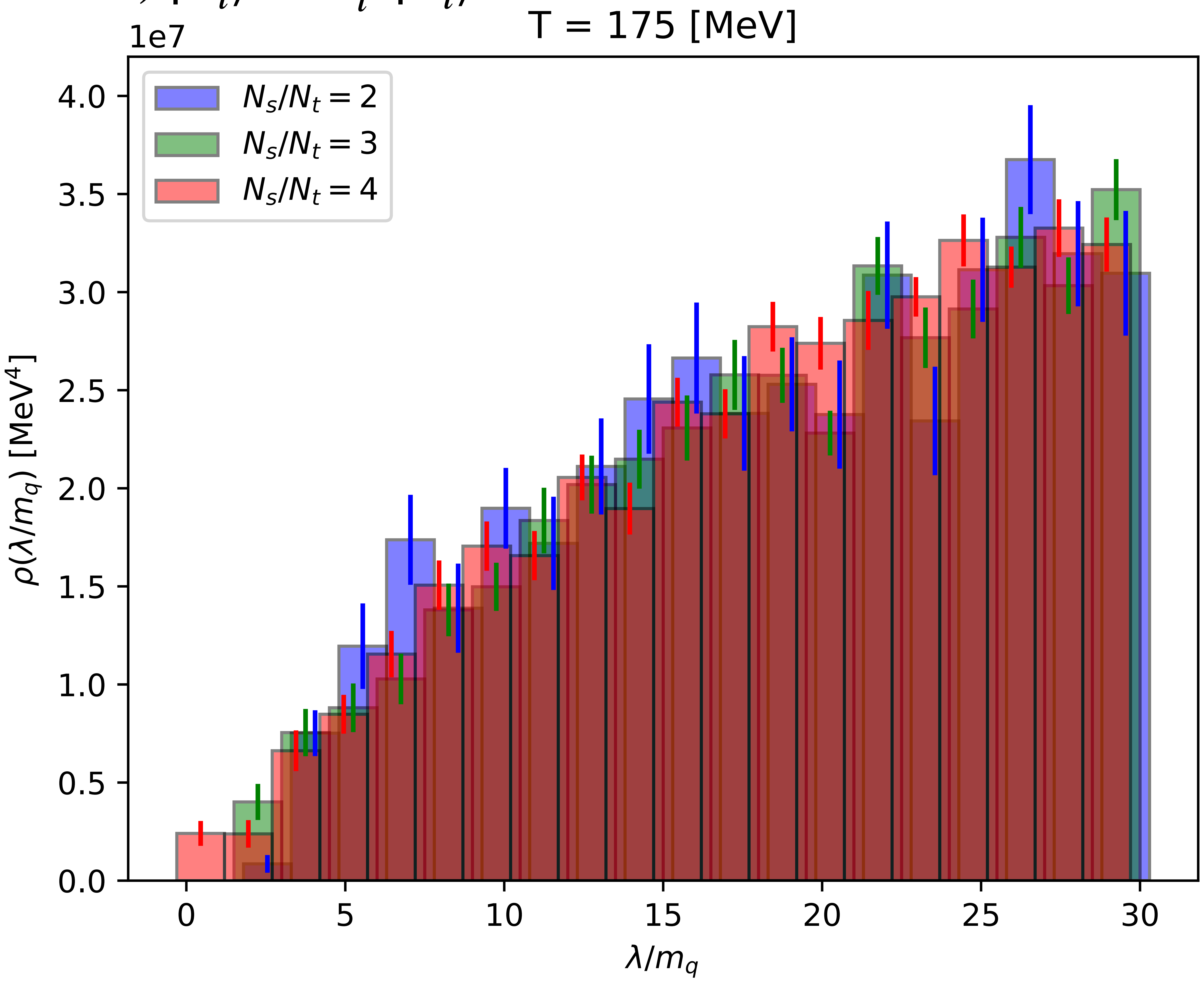
$$D^\dagger_{\rm ov}(m=0)D_{\rm ov}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$



Eigenvalue spectrum

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165$ MeV

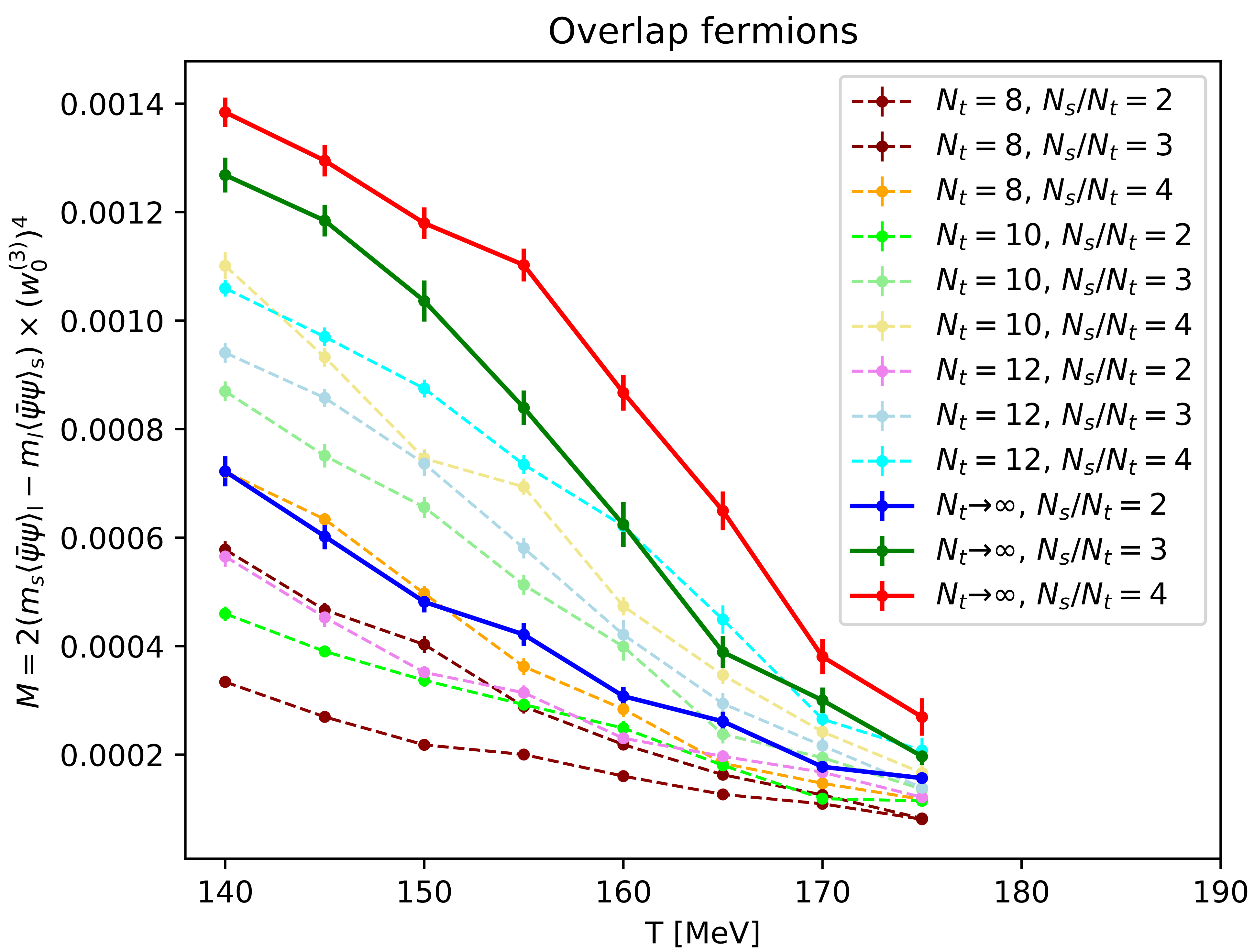
$$D_{\text{ov}}^{\dagger}(m=0)D_{\text{ov}}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$

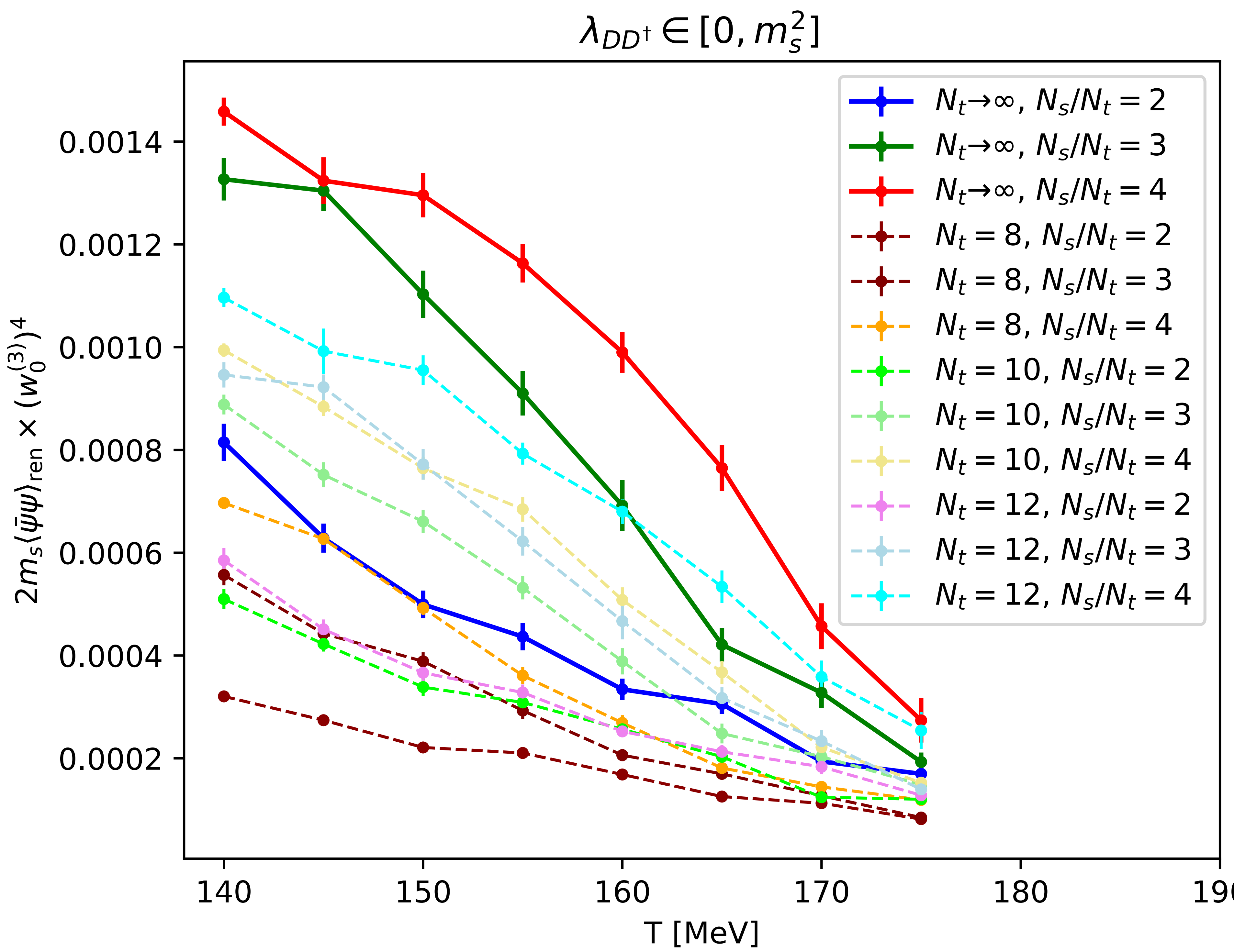


Renormalised condensate from Dirac spectrum

$$D^\dagger_{\rm ov}(m=0)D_{\rm ov}(m=0)\,|\,e_i\rangle = \lambda_i^2\,|\,e_i\rangle$$

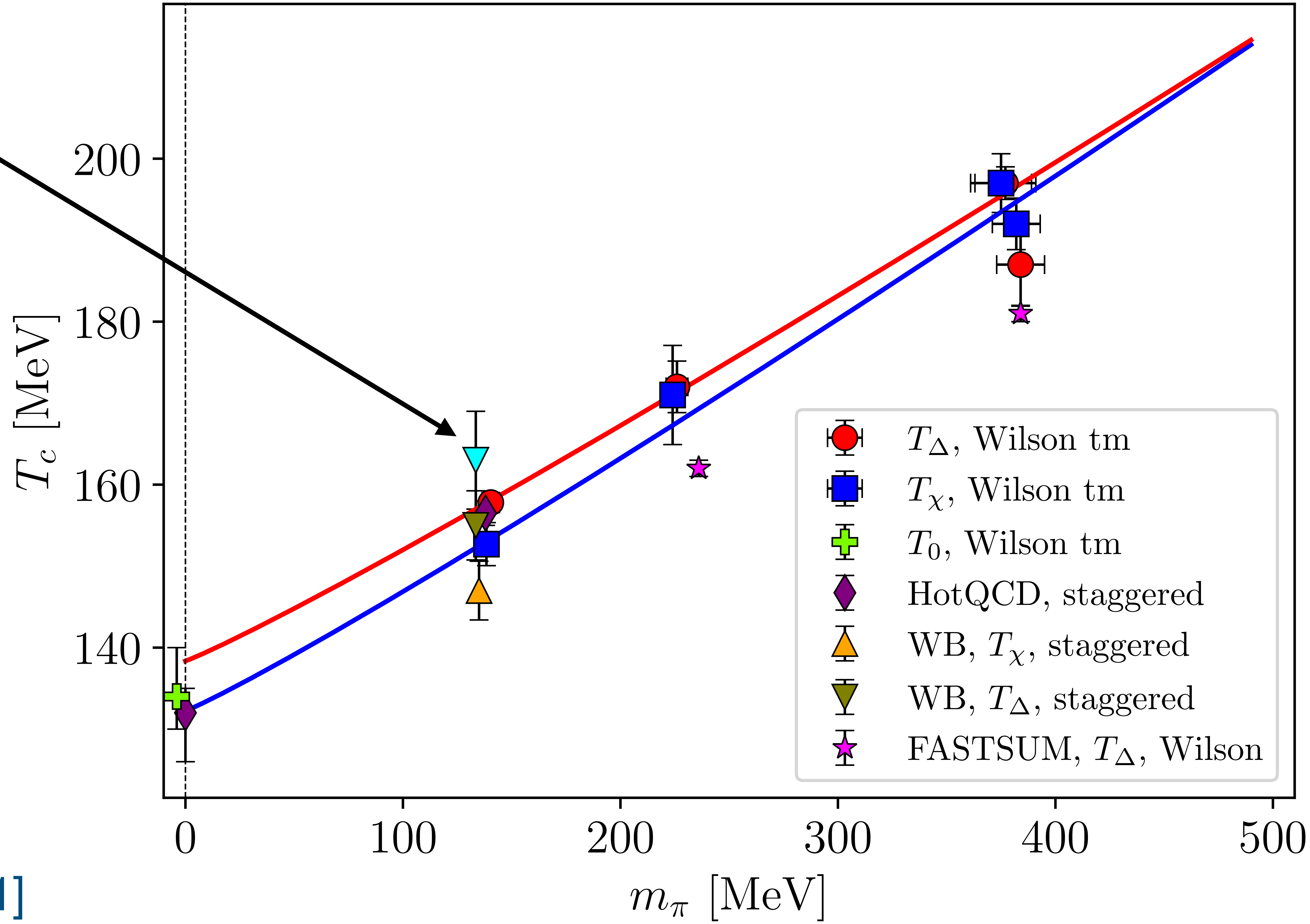
- $\langle\bar\psi\psi\rangle_{\rm ren} = \int_0^{m_s} d\lambda \rho(\lambda)\frac{m_l}{\lambda^2+m_l^2}$ (continuum expression)
- No logarithmic divergence





Critical temperature, other fermions

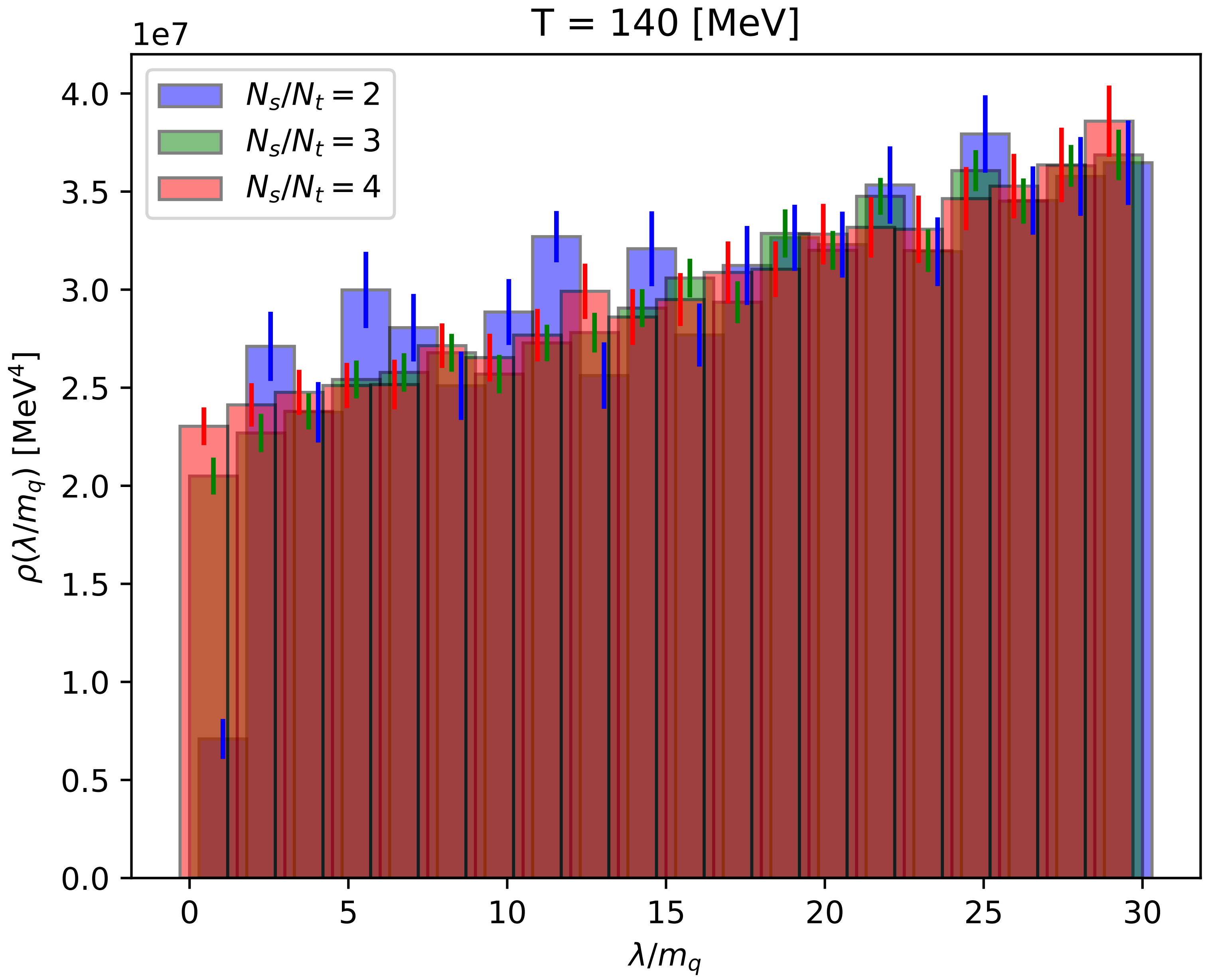
Overlap fermions

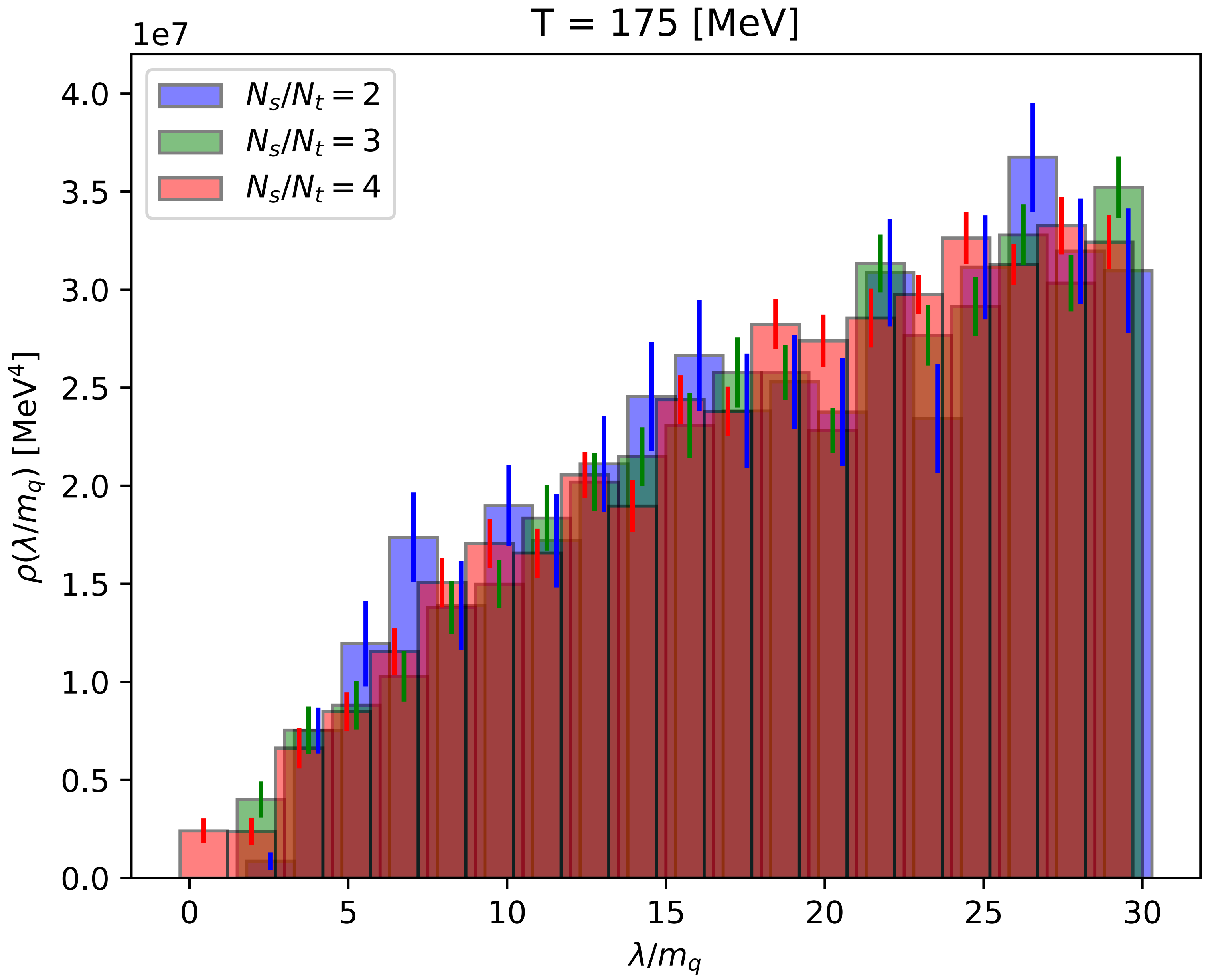


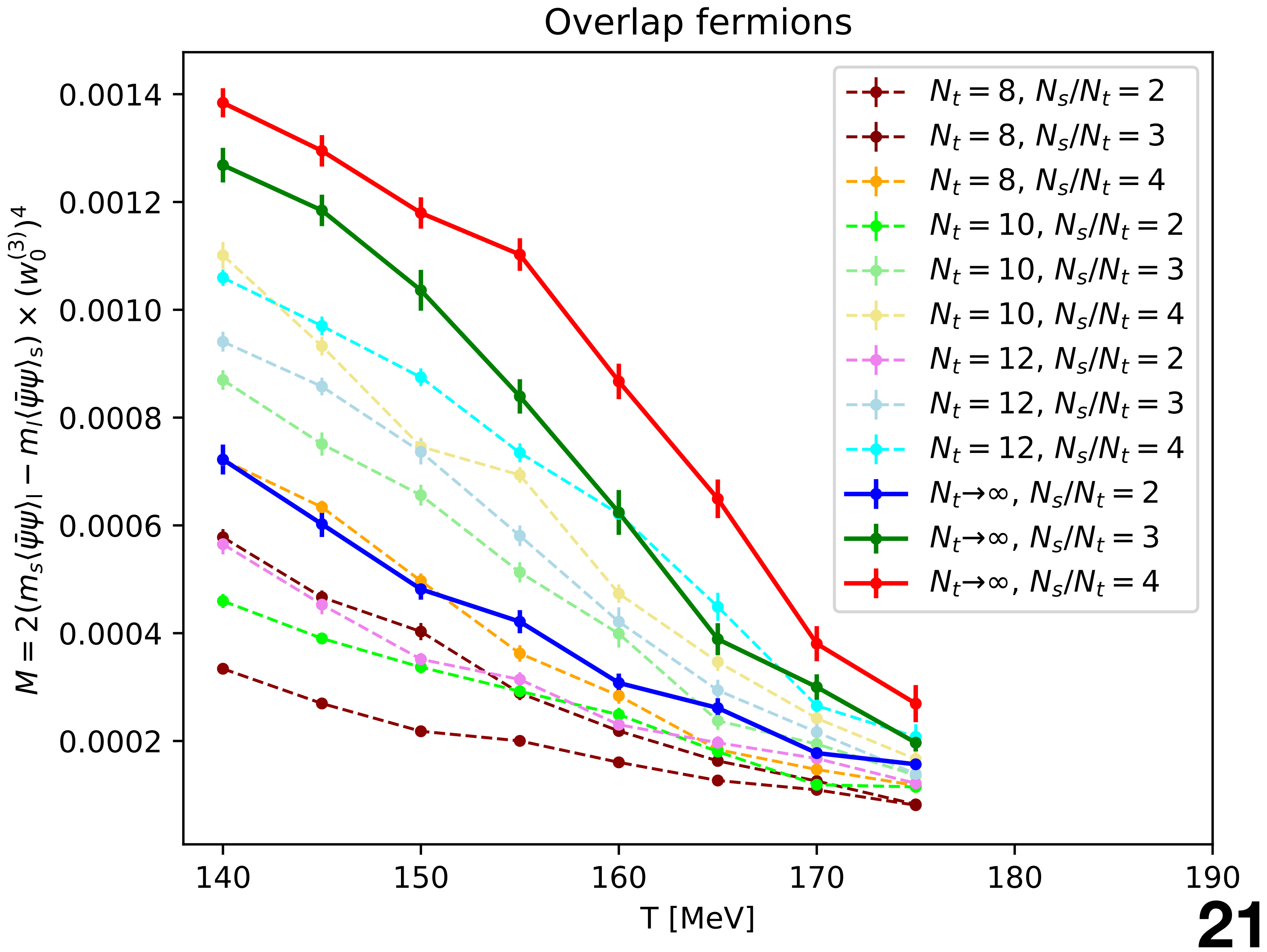
[TWEXT, 2021]

Summary

- Thermal QCD phase transition with overlap fermions
- $T_{pc} = 163(6)\ \text{MeV}$
- Spectrum of the Dirac operator: consistent with the same picture







Summary

Thank you for your attention!

- Thermal QCD phase transition with overlap fermions

- $T_{pc} = 163(6)\ \mathrm{MeV}$

- Spectrum of the Dirac operator: consistent with the same picture