Thermal QCD phase transition with dynamical chiral fermions

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Thermal QCD phase transition

\( N_f = 2 + 1 \) stout smeared staggered fermions, \( m_\pi = m_\pi^{(\text{phys})} \), \( N_t = 4, 6 \)

Chiral fermions on the lattice
Way to avoid Nielsen-Ninomiya «no-go» theorem

- Continuum chiral symmetry: $\gamma_5 D + D\gamma_5 = 0$
- Ginsparg-Wilson relation: $\gamma_5 D + D\gamma_5 = 2aD\gamma_5 D$
- Overlap fermions: $aD_{ov} = \frac{1}{2} \left( 1 + \gamma_5 \text{sign}(\gamma_5 D_w (-m_w)) \right)$
- Very expensive numerically: require multiple tricks

[H.Neuberger, 1998]
Action details

- Symanzik improved gauge action
- Fermion sector: 2 steps of HEX smeared gauge fields
- $N_f = 2 + 1$ flavours of overlap quarks, physical masses
- 2 flavours of Wilson fermions with mass $-m_W$
- Two boson fields with mass $m_B = 0.54$
- Statistics: $O(2000 - 9000)$

\[
\begin{align*}
\text{[H. Fukaya et al., 2006]} & \\
\end{align*}
\]

- $a \to 0$ : irrelevant
- Keep $Q = \text{const} (Q = 0)$
- Make calculations faster
Implementing odd number of flavours

- Monte Carlo: determinant of a hermitian operator \( H^2 = D_{ov}D_{ov}^\dagger \): \( N_f = 2 \)
- To simulate \( N_f = 1 \) (strange quark): need to take the square root

- Chirality projectors: \( P_\pm = \frac{1 \pm \gamma_5}{2} \), \( H_\pm^2 = P_\pm H^2 P_\pm \)
- Fixed topology \( Q = \text{const} \):
  \[
  \det H^2 \sim \det H_+^2 \det H_-^2 \sim (\det H_+^2)^2 \sim (\det H_-^2)^2
  \]
- Take \( \det H_+^2 \) or \( \det H_-^2 \)
Algorithm details

Overlap: \( aD_{ov} = \frac{1}{2} (1 + \gamma_5 \text{sign}(\gamma_5 D_w(-m_w))) \)

- Standard Hybrid Monte Carlo algorithm
- \( \text{sign}X \) implementation:
  - Explicit form on 32-128 low-lying eigenmodes of \( X \)
  - Rest of the \( X \) spectrum: \( \text{sign}X = X/\sqrt{XX^\dagger} \), Chebyshev polynomial approximation for \( \sqrt{XX^\dagger} \)
- Implicitly restarted Lanczos algorithm for Wilson/overlap eigenvalues
- \( D_{ov} \) inversion with FGMRES algorithm and Wilson-Dirac as preconditioner
Lattice details, scale setting

Scale setting from simulations with large $m_\pi$

- Simulations are done along the LCP
- Scale setting: require $T = 0$ simulations
  - $N_f = 3$ staggered simulations, $T = 0$, $w^{(3)}_0 = 0.153(1)$ fm, $m^{(3)}_\pi = 712(5)$ MeV
  - $N_f = 3$ overlap simulations, $T = 0$, at each $\beta$ tune $m^{ov}_s$ to have $m_\pi w_0 \equiv m^{(3)}_\pi w^{(3)}_0$
  - $N_f = 2 + 1$ overlap simulations, $T \neq 0$: $m_s = m^{ov}_s$, $m_{ud} = R m^{ov}_s$, $a = w^{(3)}_0/w^{ov}_0$
- Physical point: $m_{ud} = m^{(phys)}_{ud}$, $m_s = m^{(phys)}_s$

[Sz. Borsanyi et al., 2016]
Chiral condensate

- \( M = 2 \left( m_s \langle \bar{\psi} \psi \rangle_l - m_l \langle \bar{\psi} \psi \rangle_s \right) \)
- Large cutoff effect and FV effects
- \( T_{pc} \approx 163(6) \text{ MeV} \)
Chiral susceptibility

\[ \chi_M = m \partial_m M \]

\[ T_{pc} \sim 160 \text{ MeV} \]
Eigenvalue spectrum

\[ D_{ov}^\dagger(m = 0)D_{ov}(m = 0)|e_i\rangle = \lambda_i^2 |e_i\rangle \]
Eigenvalue spectrum

\[ D_{\text{ov}}^+(m = 0)D_{\text{ov}}(m = 0)|e_i\rangle = \lambda_i^2|e_i\rangle \]

\[ \rho(\lambda \to 0) \text{ disappears at } T \sim 160 - 165 \text{ MeV} \]
Eigenvalue spectrum

\[ D_{ov}^\dagger(m = 0)D_{ov}(m = 0) | e_i \rangle = \lambda_i^2 | e_i \rangle \]

\( \rho(\lambda \to 0) \) disappears at \( T \sim 160 - 165 \, \text{MeV} \)
Eigenvalue spectrum

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Eigenvalue spectrum

$D^\dagger_{ov}(m = 0)D_{ov}(m = 0)|e_i\rangle = \lambda_i^2|e_i\rangle$

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Eigenvalue spectrum

$D^{\dagger}_{ov}(m=0)D_{ov}(m=0)\langle e_i \rangle = \lambda_i^2 \langle e_i \rangle$

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165$ MeV
Eigenvalue spectrum

$D_{ov}^*(m = 0)D_{ov}(m = 0) | e_i \rangle = \lambda_i^2 | e_i \rangle$

$\rho(\lambda \to 0)$ disappears at $T \sim 160 - 165$ MeV
Eigenvalue spectrum

\[ D^\dagger_{\text{ov}}(m = 0)D_{\text{ov}}(m = 0) | e_i \rangle = \lambda_i^2 | e_i \rangle \]

\(\rho(\lambda \rightarrow 0)\) disappears at \(T \sim 160 - 165\) MeV
Renormalised condensate from Dirac spectrum

\[ D_{\text{ov}}^\dagger(m = 0)D_{\text{ov}}(m = 0) | e_i \rangle = \lambda_i^2 | e_i \rangle \]

\[ \langle \bar{\psi}\psi \rangle_{\text{ren}} = \int_0^{m_s} d\lambda \rho(\lambda) \frac{m_l}{\lambda^2 + m_i^2} \]

(continuum expression)

- No logarithmic divergence

\[ \langle \bar{\psi}\psi \rangle_{\text{ren}} = \int m_s \rho(\lambda) \frac{m_l}{\lambda^2 + m_i^2} \]

\( N_c \leftrightarrow \infty, N_f/N_l = 2 \)

\( N_c = 8, N_f/N_l = 2 \)

\( N_c = 8, N_f/N_l = 3 \)

\( N_c = 8, N_f/N_l = 4 \)

\( N_c = 10, N_f/N_l = 2 \)

\( N_c = 10, N_f/N_l = 3 \)

\( N_c = 12, N_f/N_l = 2 \)

\( N_c = 12, N_f/N_l = 3 \)

\( N_c = 12, N_f/N_l = 4 \)

\( \lambda_{\text{DDF}} \in [0, m_i^2] \)
Critical temperature, other fermions

Overlap fermions

[TWEXT, 2021]
Summary

- Thermal QCD phase transition with overlap fermions
- $T_{pc} = 163(6) \text{ MeV}$
- Spectrum of the Dirac operator: consistent with the same picture
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