Computation of Relativistic Corrections to the Static Potential

from Generalized Wilson Loops at Finite Flow Time

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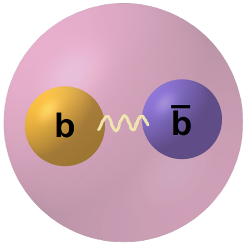
FERMILAB, 03.08.2023

Motivation

$$V(r) = V^{(0)}(r) + \frac{1}{m} V^{(1)}(r) + \frac{1}{m^2}\left(V_{SD}(r) + V_{SI}(r)\right) + \mathcal{O}(1/m^3)$$

[Eichten, Feinberg, 1981], [Barchielli et al. 1988], [Pineda, Vairo, 2001], [Brambilla, 2022]

- Corrections well understood in perturbation theory, but results from lattice QCD incomplete. [Bali, Schilling, Wachter, 1997], [Koma, Koma, 2007]
- **Problems** are UV-noise and renormalization, both can be solved with gradient flow (cf. parallel talk by Julian Mayer-Steudte, Tue 3:10 PM).
- **Analogous** expressions recently derived for hybrid potentials [Brambilla et al. 2020].



Potentials

Potentials are computed using correlators $\langle\langle\ldots\rangle\rangle = \langle\ldots\rangle_W / \langle 1 \rangle_W \Rightarrow V_0$ (in particular $V_{\rm self}(\mu)$ does not appear in resulting spectrum).

$$V_{\mathbf{p}^2} = \frac{1}{2}\left\{\mathbf{p}^2, \left(\mathcal{I}_2(E_z(t,0)E_z(0,0)) + \mathcal{I}_2(E_z(t,r)E_z(0,0))\right)\right\},$$

$$V_{\mathbf{LS}} = \epsilon_{ijz}\frac{c_F(\mu)}{2r}\left(2\mathcal{I}_1(B_i(t,0)E_j(0,0)) + \mathcal{I}_1(B_i(t,r)E_j(0,0))\right)\hat{\mathbf{L}}\hat{\mathbf{S}},$$

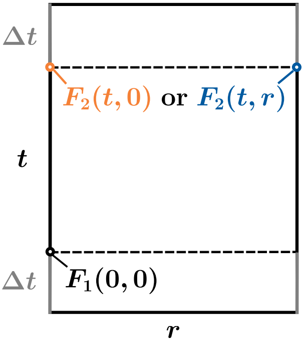
$$V_{\mathbf{S}^2} = \frac{2c_F^2(\mu)}{3}\sum_i \left(\mathcal{I}_0(B_i(t,r)B_i(0,0))\right)(\hat{\mathbf{S}}_1\hat{\mathbf{S}}_2), \ \ldots$$

where

$$\mathcal{I}_n(F_2(t,r_2)F_1(0,r_1)) = \lim_{T\to\infty}\int_0^T dt\ t^n\ \langle\langle g^2 F_2(t,r_2)F_1(0,r_1)\rangle\rangle_{(c)}$$

$\boldsymbol{r} = (0,0,r)$; $c_F(\mu)$: Matching coefficient for B-insertions; $n = 0,1,2$.

Generalized Wilson loop $\langle F_2(t)F_1(0)\rangle_W$:

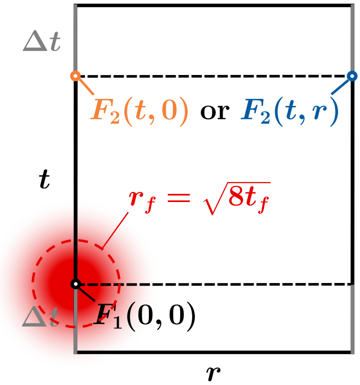


Gradient Flow

- Lattice-$F_{\mu\nu}$ requires renormalization, e.g. clover definition $F_{\mu\nu} = (\Pi_{\mu\nu} - \Pi^\dagger_{\mu\nu})/2$ via $\bar{F}_{\mu\nu} = (\Pi_{\mu\nu} + \Pi^\dagger_{\mu\nu})/2$ (Huntley-Michael).
- Flow equation:

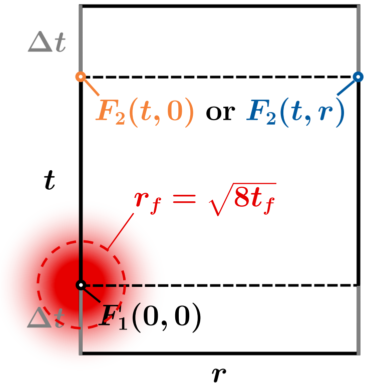
$$\dot{B}_\mu = D_\nu G_{\mu\nu}, \qquad B_\mu|_{t=0} = A_\mu,$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \quad D_\mu = \partial_\mu + [B_\mu, \cdot]$$

- Fields at flow time $t_f > 0$ are smooth and renormalized [Lüscher, 2010].
- Related flow radius is $r_f = \sqrt{8t_f}$.



Differences for gradient flow

- Insertions without gradient flow:
  - − Small signal-to-noise ratio due to UV-fluctuations.
  - − Renormalization is tricky, poor continuum convergence.
  - − B-insertions log-divergent in μ.
- In gradient flow:
  - + UV-fluctuations effectively suppressed.
  - + Additional renormalization of insertions not necessary if flow radius $r_f/a > 1$.
  - + Divergencies regulated by t_f.
  - − Unpredictable behaviour of large r_f, complicated to handle.



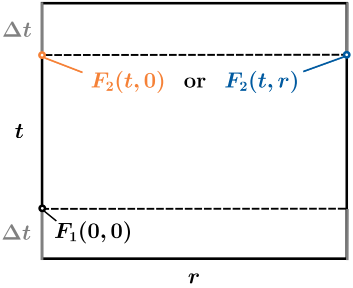
- Find the best compromise: Use flow times *large enough* to make precise measurements, while keeping r_f *small compared to separations*.

Closer look at the Correlator Spectrum

$$\langle\langle F_2(t)F_1(0)\rangle\rangle = \lim_{\Delta t\to\infty} \frac{\langle F_2(t)F_1(0)\rangle_W}{\langle 1\rangle_W} = \sum_{\Lambda_\eta^\epsilon} \langle \Sigma_g^+ | \hat{F}_1 | \Lambda_\eta^\epsilon\rangle \langle \Lambda_\eta^\epsilon | \hat{F}_2 | \Sigma_g^+\rangle \, e^{-\Delta E_{\Lambda_\eta^\epsilon} t},$$

with $\Delta E_{\Lambda_\eta^\epsilon} = E_{\Lambda_\eta^\epsilon} - E_{\Sigma_g^+}$, where Λ_η^ϵ can be:

Λ_η^ϵ	$\langle\langle F_2(t)F_1(0)\rangle\rangle^a$
$\Sigma_{g,u}^+$	$\langle\langle E_z(t)E_z(0)\rangle\rangle$
$\Sigma_{g,u}^-$	$\langle\langle B_z(t)B_z(0)\rangle\rangle$
$\Pi_{g,u}$	$\langle\langle E_{x,y}(t)E_{x,y}(0)\rangle\rangle$, $\langle\langle B_{x,y}(t)E_{y,x}(0)\rangle\rangle$, $\langle\langle B_{x,y}(t)B_{x,y}(0)\rangle\rangle$



[a] $\boldsymbol{r} = (0, 0, r)$

⇒ **Group** correlators for combined fits - Integrate fits analytically.

Lattice ensembles

- Simulation: $SU(3)$ heatbath with CL2QCD.
- Wilson flow with adaptive step size.
- Whole Wilson loop, including insertions (clover definition), flowed.
- APE-smearing for ground state enhancement used.
- Statistical errors propagated via pyerrors.

Table: Lattice ensembles used for the computation (black), or planned (grey) - Results will be shown for $\beta = 6.451$.

β	T/a	L/a	a [fm]	# confs.
6.091	36	18	0.08	20000
6.284	48	24	0.06	10000
6.451	60	30	0.048	2000
6.594	72	36	0.04	800
6.816	96	48	0.03	–

Static potential

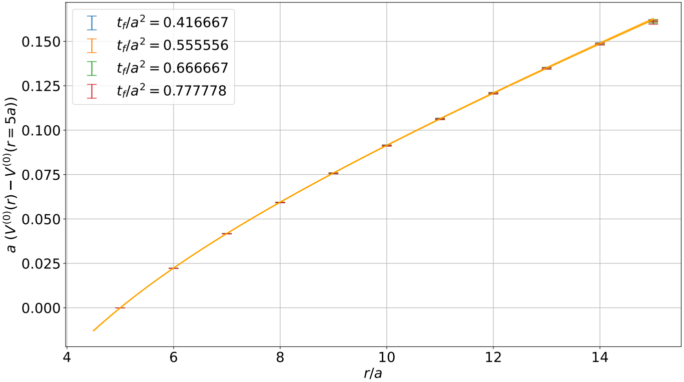


Figure: Consistency check: Static potential for $a = 0.048$ fm, normalized with $V_0(r/a = 5)$, at different flow times.

Noise reduction

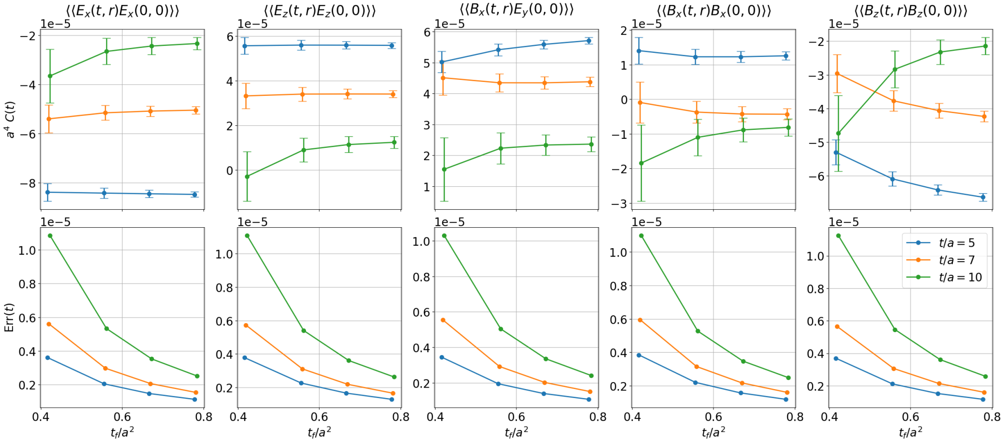
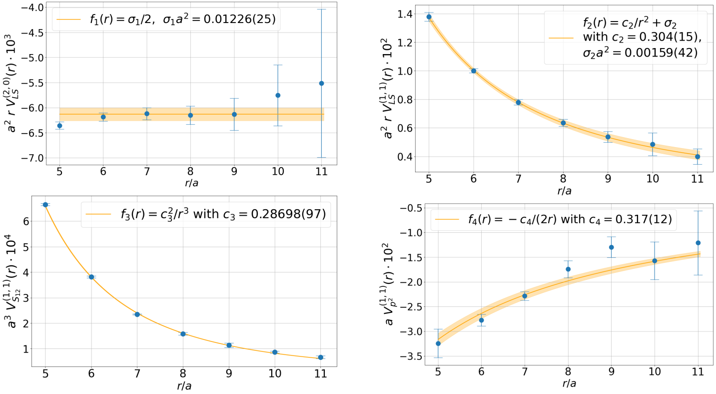


Figure: Mean values and errors for different $\langle\langle F_2(t)F_1(0)\rangle\rangle$ as function of t_f/a^2 for $a = 0.048$ fm, $r/a = 8$ and various t/a.

Spin-Potentials and $V_{\boldsymbol{p}^2}^{(1,1)}$

Preliminary

- $a = 0.048\ \mathrm{fm},\ t_f/a^2 = 0.778$.
- Fit ansätze from models, to be replaced with matching to pNRQCD (continuum limit).
- $c_2 \approx c_3 \approx c_4 \approx c =$ 0.3002(33) and $\sigma_1 \approx \sigma = 0.012227(86)$ (Cornell parameters), good agreement with models.
- $\sigma_2 \neq 0$ surprising, possibly overestimation of integrals at large r.



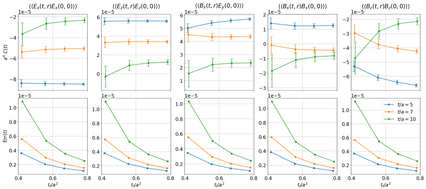
Conclusions

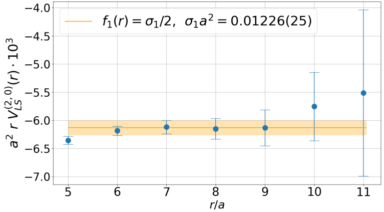
Summary

- $\mathcal{O}(1/m^2)$-corrections to the static potential have been computed at several lattice spacings and finite t_f for distances up to $\sim$ 0.5 fm.
- *Significant reduction* of noise of correlators.
- Potential results are *very close* to expectations, possibly due to renormalization.
- *Difficult to get integral values from fitting*.

Outlook

- Try out alternative to fitting, e.g. determine hybrid energies and directly compute respective amplitudes.
- Find continuum and zero-flow-time limit.
- Access smaller min. separations $<$ 0.2 fm and match with perturbation theory.





Thank you for your attention!

Correlators

Preliminary

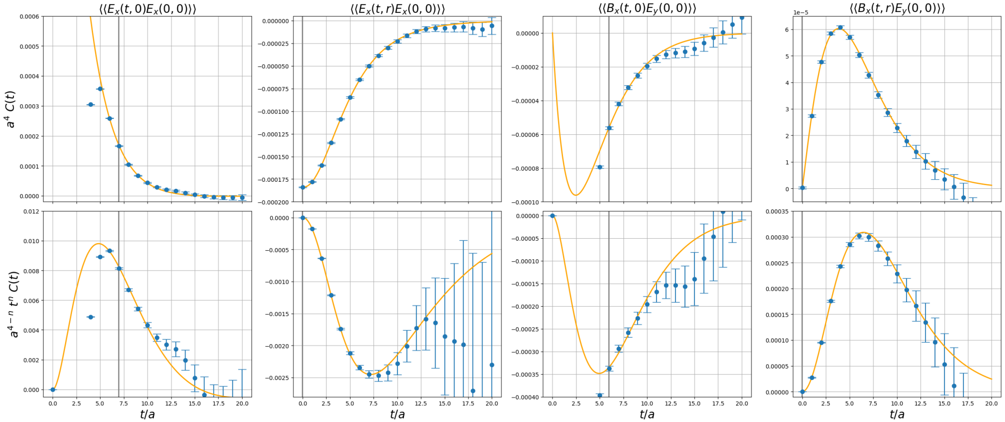


Figure: Correlators where integral is weighted with t or t^2 for $a = 0.048\ \mathrm{fm}$ and $t_f/a^2 = 0.778$. Grey lines indicate where separation surpasses $2r_f$.