Gauge Fields and Matter with no Fermion Determinant

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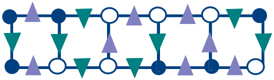
Chicago

arxiv:2305.08917

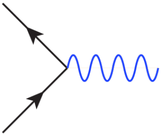
Motivation

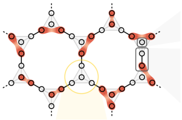


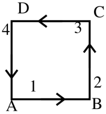
$$H_{\mathbb{Z}_2} = \sum_{\langle xy \rangle} \left[-t \left(\sigma^1_{x,y} c^\dagger_x c_y + \text{h.c.} \right) + \text{interacting terms} \right]$$

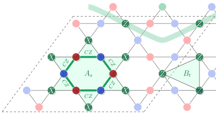
$$H_{U(1)} = \sum_{\langle xy \rangle} \left[-t \left(\sigma^+_{x,y} c^\dagger_x c_y + \text{h.c.} \right) + \text{interacting terms} \right]$$

- Interesting from both particle physics and condensed matter perspectives–for the latter we can see emergent gauge symmetry at low temperature.
- Qubit-friendly models: Models with the desired (even continuous) symmetries but small local Hilbert spaces that are easy to simulate with quantum spins.









Halimeh, Homeier, Hohrdt, Grusdt 2305.06373 (2023)
EH, Garcia Vera, Banerjee, PRD (2022)
Iqbal, Tantivasadakarn, Verresen, Campbell, Dreiling, Figgatt, Gaebler, Johansen, Mills, Moses, Pino, Ransford, Rowe, Siegfried, Stutz, Foss-Feig, Vishwanath, Dreyer 2305.03766 (2023)

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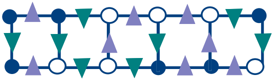

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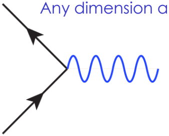
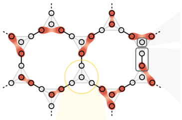
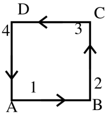
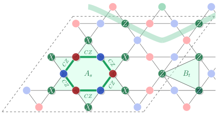
- Interesting from both particle physics and condensed matter perspectives–for the latter we can see emergent gauge symmetry at low temperature.

- Qubit-friendly models: Models with the desired (even continuous) symmetries but small local Hilbert spaces that are easy to simulate with quantum spins.
  Any dimension and any number of flavors simulable.









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Basics of the Meron Cluster Algorithm

- Worldline-based: originally worked out for the t-V model:

$$H = \sum_{\langle xy \rangle} h_{xy}, \qquad h_{xy} = -t\left(c_x^\dagger c_y + c_y^\dagger c_x\right) + 2t\left(n_x - \frac{1}{2}\right)\left(n_y - \frac{1}{2}\right) - \frac{t}{2}.$$

PHYSICAL REVIEW LETTERS

Meron-Cluster Solution of Fermion Sign Problems

Shailesh Chandrasekharan and Uwe-Jens Wiese
Phys. Rev. Lett. **83**, 3116 – Published 18 October 1999

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- No need to work with the fermion determinant–and resulting sampling or stabilization issues.

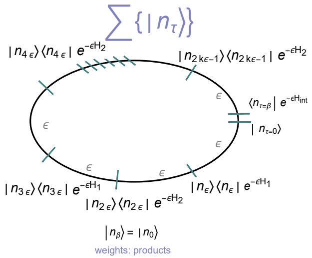
Worldline-based methods: Wiese, Beard, Ceperley, Kawashima, Gubernatis, Evertz, Chandrasekharan, Sandvik, Prokof'ev, Svistunov, Rubtsov, Kaul, Savkin, Lichtenstein, Hohenadler

Partition Function to Path Integral

$$Z = \mathrm{Tr}\left[e^{-\beta H}\right] = \mathrm{Tr}\left[e^{-\epsilon H_2} e^{-\epsilon H_1} ... e^{-\epsilon H_2} e^{-\epsilon H_1}\right]$$

$$= \sum_{[n]} \langle n_0 | ...... | n_{2k\epsilon-1} \rangle \langle n_{2k\epsilon-1} | e^{-\epsilon H_m} ... | n_{2\epsilon} \rangle \langle n_{2\epsilon} | e^{-\epsilon H_2} | n_\epsilon \rangle \langle n_\epsilon | e^{-\epsilon H_1} | n_0 \rangle \, .$$

Worldlines: discrete–time



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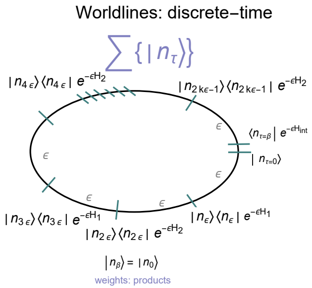


No fermion determinant. Configurations of $|n_\tau\rangle$ are physical–potential uses as synthetic data.

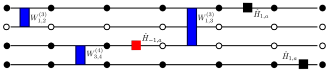
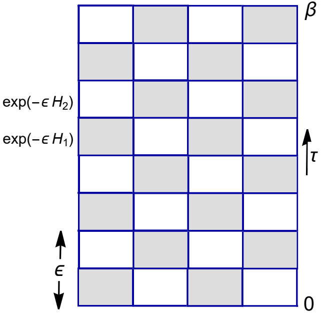


Figure: Merali, De Vlugt, Melko, arXiv:2107.00766

Partition Function Decomposition

$$Z = \mathrm{Tr}\left[e^{-\beta H}\right] = \mathrm{Tr}\left[\left(e^{-\epsilon H}\right)^N\right] = \mathrm{Tr}\left[\left(e^{-\epsilon H_2} e^{-\epsilon H_1}\right)^N\right],$$

- In 1D: $H_1 = \sum_{x \in \mathrm{even}} h_{x,x+1}, \qquad H_2 = \sum_{x \in \mathrm{odd}} h_{x,x+1},$



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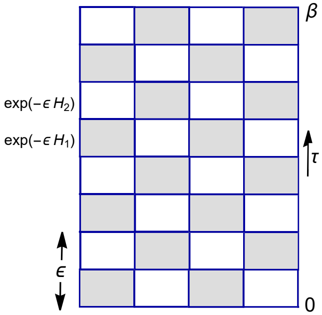

- Can also be done in continuous time.

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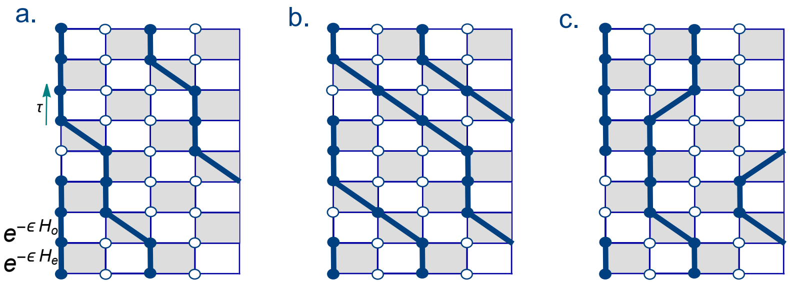

$U(1)$ Gauge Fields and Matter: Meron-Cluster

- Model:

$$H_{U(1)} = \sum_{\langle xy \rangle} \left[-t \left(c_x^\dagger \sigma_{xy}^+ c_y + c_y^\dagger \sigma_{xy}^- c_x \right) \right.$$
$$\left. +2t \left(n_x - \frac{1}{2} \right) \left(n_y - \frac{1}{2} \right) - \frac{t}{2} + h'_{xy} \right],$$

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- $U(1)$ symmetry ($UHU^\dagger = H$):

$$U = \prod_x e^{i\theta_x G_x},$$
$$G_x = \left[n_x + \frac{(-1)^x - 1}{2} - \sum_\alpha \left(\sigma^3_{x,x+\alpha} - \sigma^3_{x-\alpha,x} \right) \right].$$

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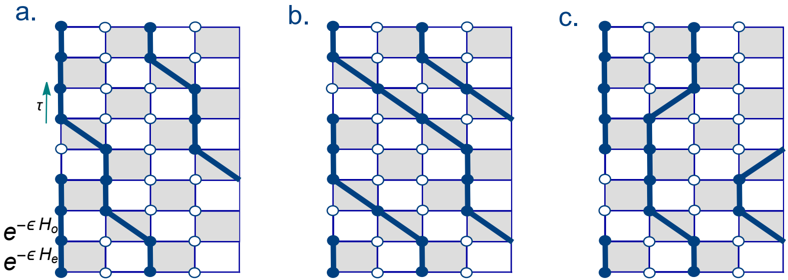
- For simplest algorithm:

$$h'_{xy} = t \Big[(1 - n_x)\, n_y \left(\frac{1}{2} + \sigma^3_{xy} \right)$$

$$+ n_x (1 - n_y) \left(\frac{1}{2} - \sigma^3_{xy} \right) \Big].$$

$U(1)$ model worldline examples

$$H = \sum_x \left[-t \left(\sigma^+_{x,x+1} c^\dagger_x c_{x+1} + \sigma^-_{x,x+1} c^\dagger_{x+1} c_x \right) + \text{diagonal terms} \right]$$



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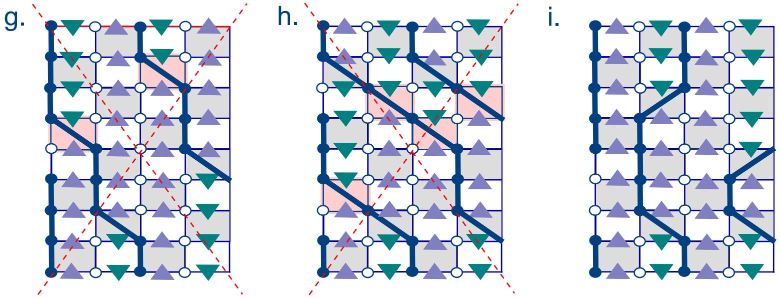

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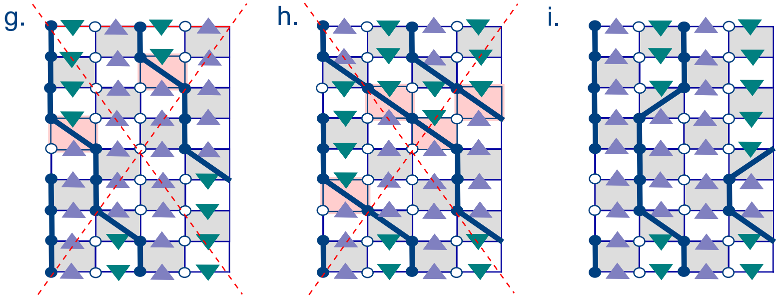
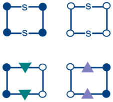
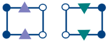


Note: no sign problem in $1+1D$.

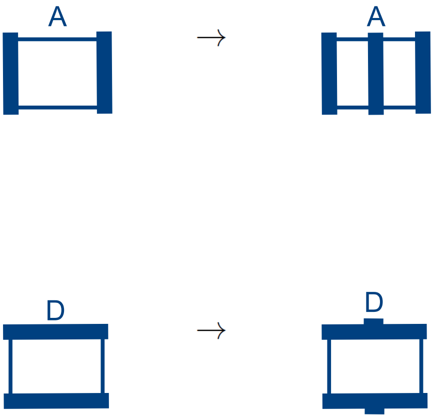
$U(1)$ theory breakups

$$\langle n | \, e^{-\epsilon H_i} \, | n' \rangle$$

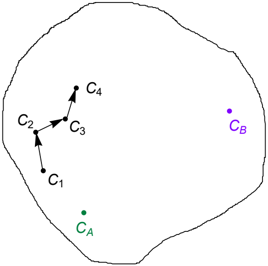
Plaquettes	Weight	Breakups
	1	
	$\exp(\epsilon t)\sinh(\epsilon t)$	
	$\exp(\epsilon t)\cosh(\epsilon t)$	

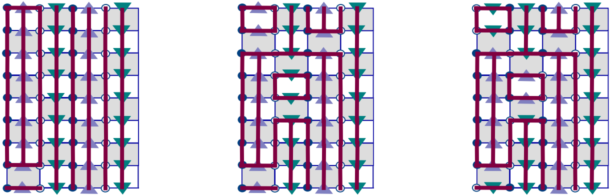
$$\begin{aligned} w_A &= 1 \\ w_D &= \exp\left(\epsilon t\right) \sinh \epsilon t \\ w_A + w_D &= \exp\left(\epsilon t\right) \cosh \epsilon t. \end{aligned}$$

Updated Breakups for Gauge Fields



Configurations

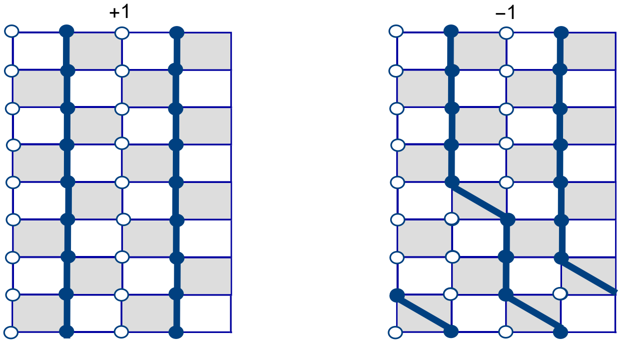




Merons

For fermions, negative signs can come from two places:

- Matrix elements (local).
- Fermion permutations (global).



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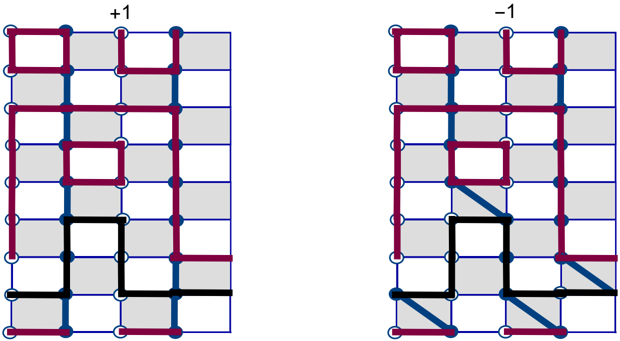

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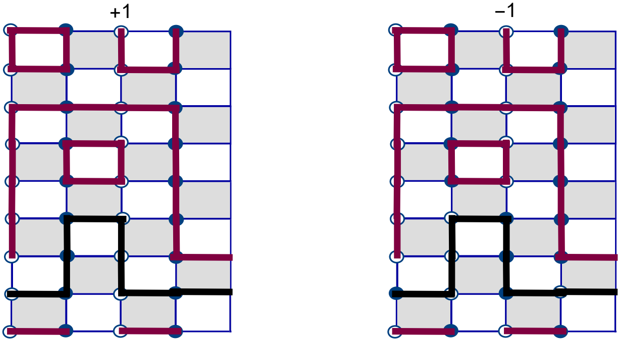
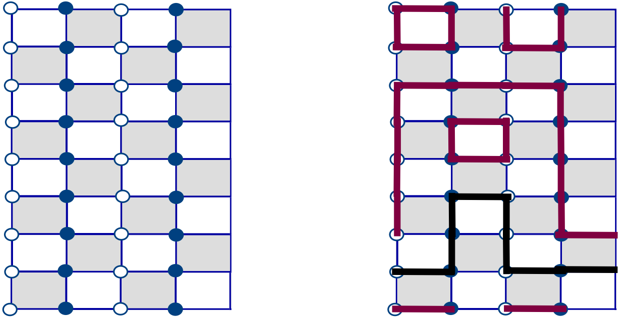


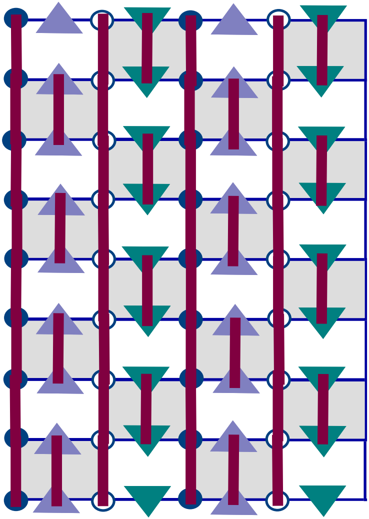
$n_w + n_h/2$ even is a meron.

Reference Configuration

Ensures ergodicity, clarifies the configuration signs.

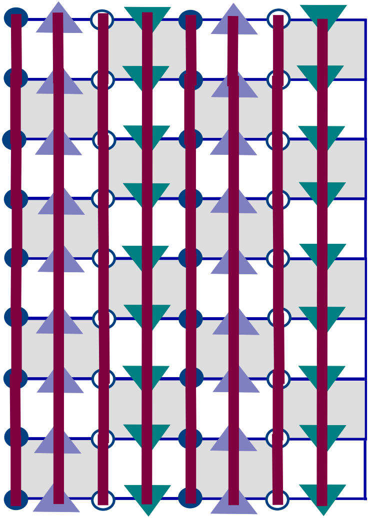


First Configuration

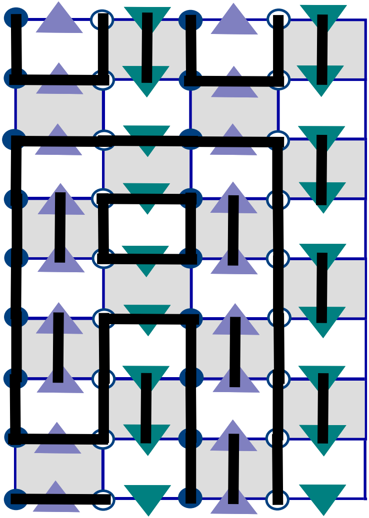


Note: this is a reference configuration.

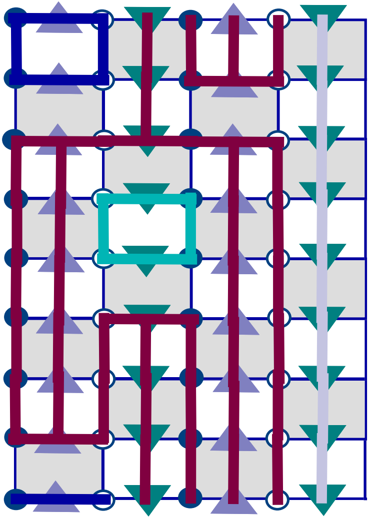
First Configuration



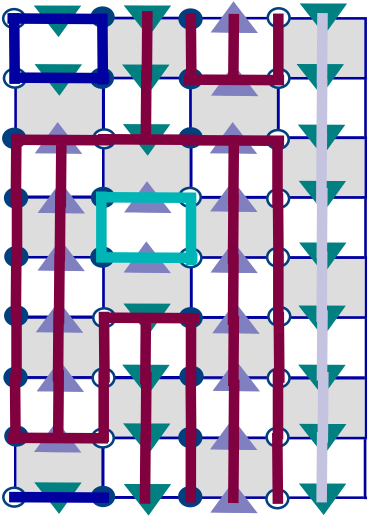
Assigning Breakups



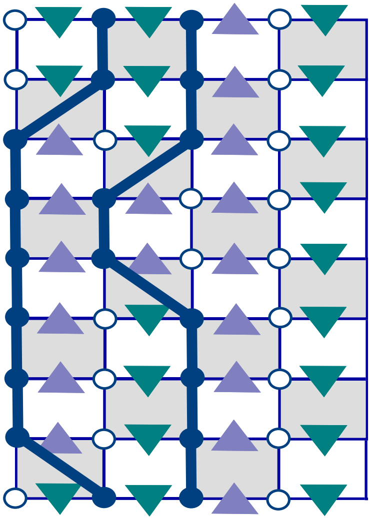
Identifying Clusters



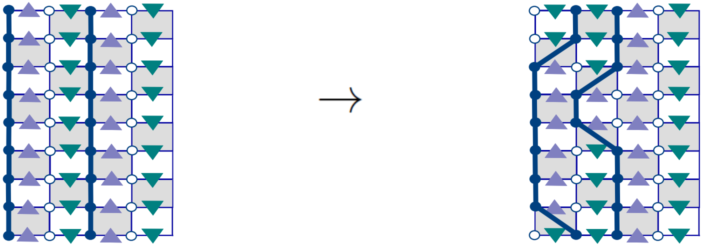
Flipping Clusters



New Configuration: Worldlines



Update Worldline Summary



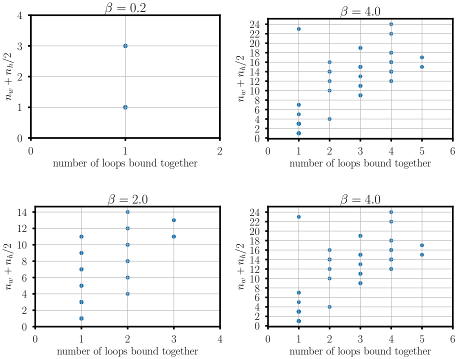
No Merons

$$H_{U(1)} = \sum_{\langle xy \rangle} \left[-t \left(c_x^\dagger \sigma_{xy}^+ c_y + c_y^\dagger \sigma_{xy}^- c_x \right) + 2t \left(n_x - \frac{1}{2} \right) \left(n_y - \frac{1}{2} \right) - \frac{t}{2} + {h'}_{xy} \right],$$

For one loop, we must have $n_w + n_h/2$ odd to not be a meron.

For odd numbers of loops fused together, the number must be odd,
for even numbers of loops fused together, the number must be even.

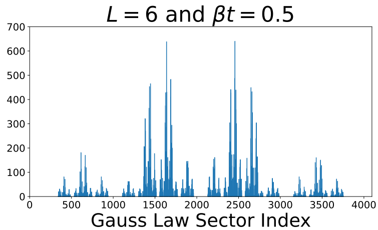
$1 + 1d$ Data:

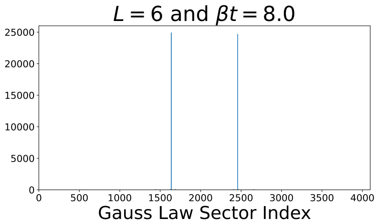


Emergence of the Gauss Law, All Sectors, $L = 6$

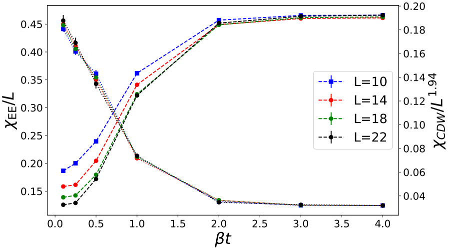
The meron cluster method allows us to easily separate data by Gauss law sector.

$$G_x = \left[n_x + \frac{(-1)^x - 1}{2} - \left(\sigma^3_{x,x+1} - \sigma^3_{x-1,x} \right) \right]$$



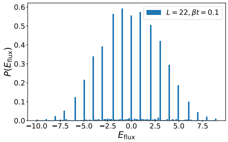


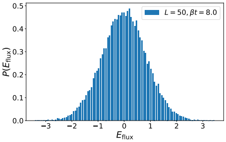
Ordering at Low Temperature



$$\chi_{\mathrm{CDW}} = \frac{1}{N_t} \sum_{x,y,t} \left\langle (-1)^x \left(n_x(t) - \frac{1}{2} \right) (-1)^y \left(n_y(t) - \frac{1}{2} \right) \right\rangle$$

$$\chi_{\mathcal{E}\mathcal{E}} = \frac{1}{N_t} \sum_{x,y,t} \left\langle S^z_x(t) S^z_y(t) \right\rangle$$





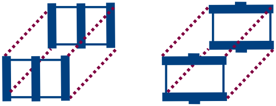
Flavor Extensions

- "Products" of the previous models.

$$H_{\mathbb{Z}_2} = -\prod_{\lambda}\sum_{\langle xy\rangle}\left[t\sigma^1_{xy,\lambda}\left(c^\dagger_{x,\lambda}c_{y,\lambda} + c^\dagger_{y,\lambda}c_{x,\lambda}\right) - 2t\left(n_{x,\lambda} - \frac{1}{2}\right)\left(n_{y,\lambda} - \frac{1}{2}\right) + \frac{t}{2}\right]$$

$$H_{U(1)} = -\prod_{\lambda}\sum_{\langle xy\rangle}\left[t\left(\sigma^+_{xy,\lambda}c^\dagger_{x,\lambda}c_{y,\lambda} + \sigma^-_{xy,\lambda}c^\dagger_{y,\lambda}c_{x,\lambda}\right) - 2t\left(n_{x,\lambda} - \frac{1}{2}\right)\left(n_{y,\lambda} - \frac{1}{2}\right)\right.$$
$$\left. + \frac{t}{2} + h'_{xy}\right]$$

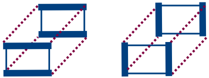
- Sign-problem-free for any number of flavors and any dimension. (For $U(1)$ theory, auxiliary fields only work for compact theories with even numbers of flavors.)



Hubbard model and spin-polarization extensions

$$H_U = \sum_x (n_{x\uparrow} - 1/2)(n_{x\downarrow} - 1/2)$$

- Quantum phase transition for two flavors of fermions in 1D (massive VBS phase to conformal $SU(2)_1$ WZW theory phase).



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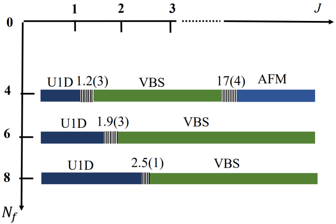
Phys. Rev. D **103**, 054033 – Published 25 March 2021



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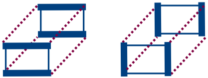

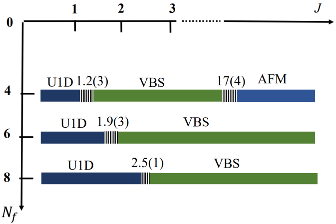

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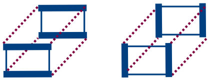

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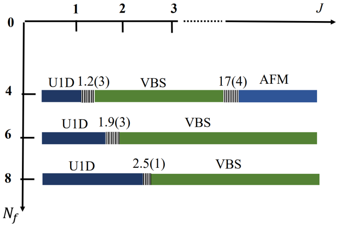

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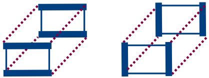
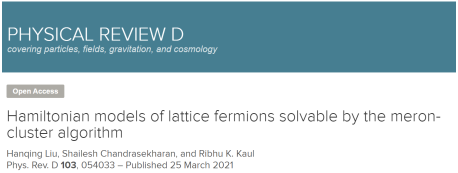
- Interplay between σ^1 or σ^+/σ^- and σ^3 terms, fermionic transitions too? Quantum phase transitions in similar models for Assaad, Grover PRX 2016, Xu, Qi, Zhang, Assaad, Xu, Meng, PRX 2019, Janssen, Wang, Scherer, Meng, Xu, PRB 2020.





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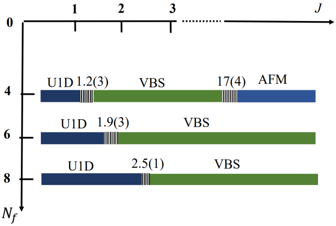

Conclusions

arxiv:2305.08917

- Models involving gauge fields interacting with fermions can be studied using meron clusters–results in limitations of fermionic worldlines. Any dimension and any number of flavors possible for the $\mathbb{Z}_2$ and $U(1)$ symmetry classes.

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- More potential opportunities to study quantum critical behavior for gauge theories, in a way more amenable to qubits as well.

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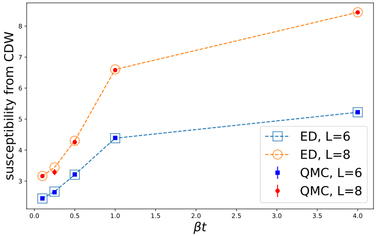
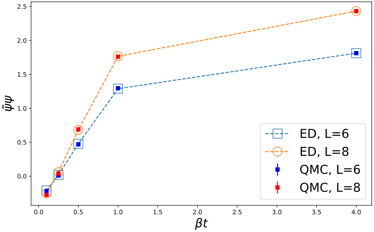
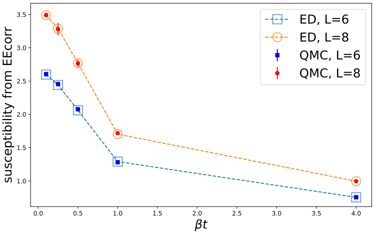
Results: Compare with ED







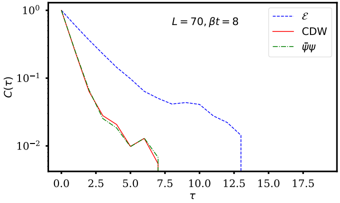
Algorithm Data

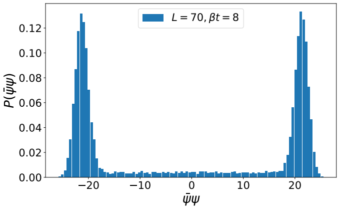
- Observables

$$\bar{\psi}\psi = \frac{1}{N_t}\sum_{x,t}\langle(-1)^x n_x(t)\rangle$$

$$\mathrm{CDW} = \frac{1}{N_t}\sum_{x,y,t}\left\langle(-1)^x\left(n_x(t)-\frac{1}{2}\right)(-1)^y\left(n_y(t)-\frac{1}{2}\right)\right\rangle$$

$$\mathcal{E} = \frac{1}{N_t}\sum_{x,t}\langle S_x^z(t)\rangle$$





$$C_{\mathcal{O}}(\tau) = \frac{\left\langle(\mathcal{O}(i)-\overline{\mathcal{O}})(\mathcal{O}(i+\tau)-\overline{\mathcal{O}})\right\rangle}{\left\langle(\mathcal{O}(i)-\overline{\mathcal{O}})^2\right\rangle}$$