Hans Böckler Stiftung

Investigations of nucleon-pion states

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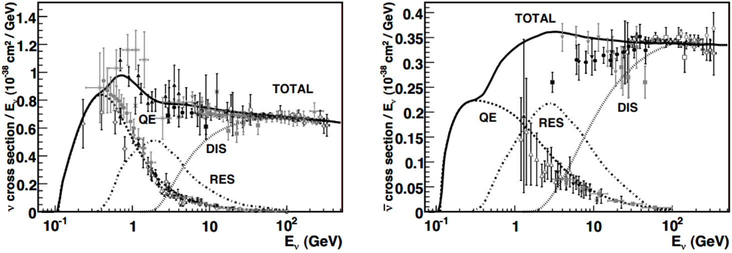
Introduction

Motivation

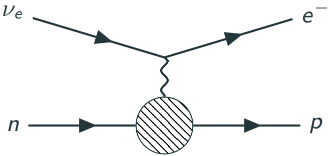
- the axial-vector properties of nucleon states are relevant for neutrino-nucleus scattering events, like e.g. in the DUNE detectors
- one relevant process in the quasi-elastic region ($E_\nu \approx 1$ GeV)

$$\nu_\ell n \to \ell^- p \quad \text{and} \quad \bar{\nu}_\ell p \to \ell^+ n$$



Source: J. A. Formaggio and G. P. Zeller Rev. Mod. Phys. 84, 1307

Axial vector current and nucleon axial charge



- axial-vector current

$$\langle \mathcal{P}(p') | A_\mu^+ | \mathcal{N}(p) \rangle = \bar{u}(p') \left(\gamma_\mu \gamma_5 G_A(Q^2) - i \gamma_5 \frac{Q_\mu}{2M_N} \tilde{G}_P(Q^2) \right) d(p),$$

  with the form factors $G_A(Q^2)$ and $\tilde{G}_P(Q^2)$

- nucleon axial charge

$$g_A = G_A(Q^2 = 0)$$

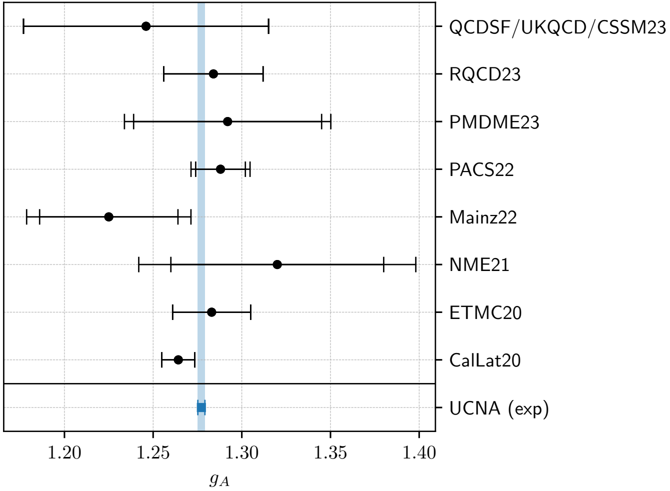
- from Ultracold Neutron experiments: $g_A = 1.2772(20)$ (UCNA)

Axial-Vector current on the Lattice

- axial-vector form factor of nucleon is in general a non-perturbative quantity
- Lattice QCD offers a way for calculations from First Principles
- one particular challenge: on lattice nucleon-pion state and nucleon state can have the same quantum numbers
- Chiral PT predicts a significant nucleon-pion contribution nucleon form factors (O. Bär Phys.Rev.D 101 (2020) 3, 034515)

g_A Lattice-Overview

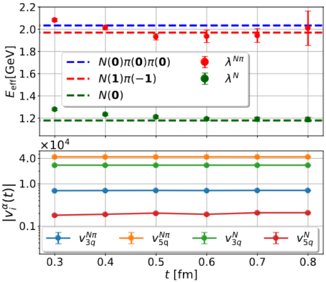
- g_A Lattice computations since 2020:

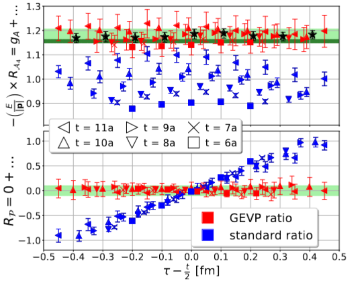


- in this comparison we neglect possible QED corrections for the lattice computations

$N\pi$ interpolating operators

- Idea: use $N\pi$ interpolating operators to increase the variational basis (L. Barca et.al., Phys.Rev.D 107 (2023) 5)

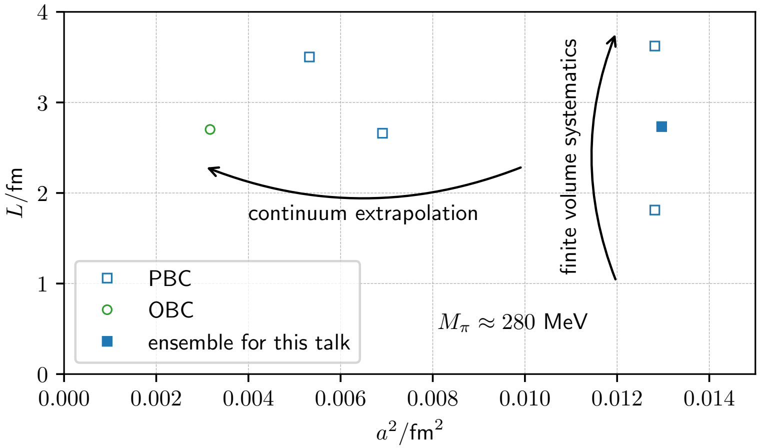




- Goal: study finite-volume nucleon-pion states to scrutinize excited state systematics

Setup

Ensembles



- new RBC/UKQCD ensembles for $g-2$ project, combined with single physical pion mass ensemble (48I)

Ensemble for this talk

- Iwasaki gauge action with $\beta = 2.13$
- 2+1 valence and sea Domain-Wall quarks
- $a^{-1} = 1.74$ GeV
- lattice size: $24^3 \times 48 \times 24$
- $m_\pi = 279$ MeV
- $m_K = 534$ MeV
- $m_\pi L = 3.84$
- $b + c = 2$
- $m_{\text{res}} = 6.2 \times 10^{-4}$

Nucleon Spectrum

Nucleon interpolation operators

- standard proton interpolation operator

$$\mathcal{P}_\mu(x) = \varepsilon_{abc} u^a_\nu(x) u^b_\beta(x) \left(C\gamma_5\right)_{\beta\gamma} d^c_\gamma(x)$$

- we use Coulomb gauge-fixed wall and $\mathbb{Z}_3$-box sources with point sink
- nucleon two-point function

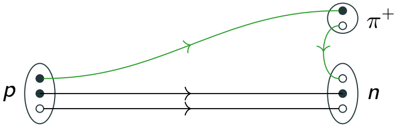
$$C_{2pt}(t) = \sum_{\boldsymbol{x}} \left\langle \mathcal{P}_\nu(\boldsymbol{x}, t) \mathcal{P}^\dagger_\mu(0) \right\rangle P^+_{\mu\nu}$$

- parity projection operator

$$P^+ = \frac{1}{2}\left(\mathbb{1} + \gamma_3\right)$$

- usage of all-mode-averaging (one exact and 48 sloppy time slice)

Nucleon-pion interpolating operator



- at sink replace proton with neutron-pion operator

$$\mathcal{N}\pi^{\boldsymbol{p}}_{\mu\uparrow}(t) = \mathcal{N}_{\mu\downarrow}(\boldsymbol{p}, t)\pi^+(-\boldsymbol{p}, t) - \mathcal{N}_{\mu\downarrow}(-\boldsymbol{p}, t)\pi^+(\boldsymbol{p}, t)$$

- neutron and pion operator are each point-like
- we use a sequential propagator to include the pion
- internal momentum $\boldsymbol{p} \neq 0$ necessary for positive parity
- in the following, $\boldsymbol{p} = \frac{2\pi}{L}\hat{e}_x$
- use Coulomb gauge-fixed wall source with standard proton operator
- proton at source is isospin and G_1 projected

Effective masses and constrained fits

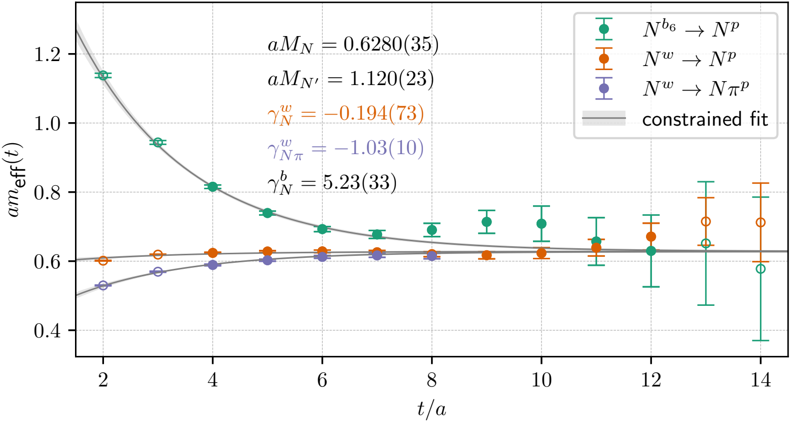
- result of a lot of experimentation (GEVP, Matrix Prony, different ways to compute the effective mass curve)
- computation of the effective mass curve using

$$am_{\text{eff}}(t) = -\frac{1}{n} \log \left(\frac{C(t+an)}{C(t)} \right)$$

- constrained fit: each effective mass curve is simultaneously fitted against

$$am_{\text{eff}}(t) = aM_N + \gamma_i \frac{1 - e^{-an\Delta E}}{n} e^{-a\Delta E t}$$

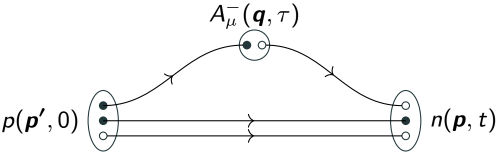
- in the following, we use $n = 3$



p: point-like   *w*: gauge-fixed wall b_k: gauge-fixed $\mathbb{Z}_3$-Box of size *k*

- $\chi^2/\text{dof} = 1.23$ and $p = 0.22$
- difference between γ_N^w and $\gamma_{N\pi}^w$ indicates the effect of the nucleon-pion operator

Nucleon Axial Charge (Preliminary)



- we need additionally a 3-point function with inserted vector-axial current

$$\mathcal{C}^{j,\uparrow/\downarrow}_{\mu,3pt}(t, \boldsymbol{p}, \tau, \boldsymbol{q}) = \left\langle \mathcal{N}_\alpha(\boldsymbol{p}, t) A^-_\mu(\boldsymbol{q}, \tau) \mathcal{P}^\dagger_\beta(\boldsymbol{p}', 0) \right\rangle \left(P^+ P^{\uparrow/\downarrow}_j \right)_{\beta\alpha}$$

- parity projection P^+ and spin projection

$$P^{\uparrow/\downarrow}_j = \frac{1}{2} \left(\mathbb{1} \pm i \gamma_5 \gamma_j \right)$$

- axial-vector insertion $A^-_\mu(x) = \bar{d}(x) \gamma_\mu \gamma_5 u(x)$ for $\boldsymbol{q} = \boldsymbol{0}$

- non-renormalized nucleon axial charge $\tilde{g}_A^N$ can be obtained from the ratios (analog to CalLat: *Phys. Rev. C 105, 065203 (2022)*)

$$\mathcal{R}_j^{A_j}(t,\tau) = -i\frac{\mathcal{C}^j_{j,3pt}(t,\boldsymbol{p}=0,\tau,\boldsymbol{q}=0)}{\mathcal{C}_{2pt}(t)}$$

$$= \overbrace{\tilde{g}_A^N}^{\text{ground state}} + \overbrace{(\tilde{g}_A^{N'} - \tilde{g}_A^N) r_p r_w^\star e^{-\Delta E t}}^{\text{excited state}}$$

$$+ \underbrace{\tilde{g}_A^{N\to N'}\left(r_w^\star e^{-\Delta E \tau} + r_p e^{-\Delta E (t-\tau)}\right)}_{\text{exchange}} + \ldots,$$

with

- $\tilde{g}^{N'}$, $\tilde{g}^{N\to N'}$ further matrix elements of axial-vector current
- r_p and $r_w^\star$ the normalized overlap with the point-like nucleon at sink and wall proton at source

- with the axial-vector renormalization constant Z_A we can renormalize the axial charge

$$g_A = Z_A \tilde{g}_A$$

Constrained fit for g_A^N determination

- we make a constrained fit of
  - the ratio data $\mathcal{R}_0^{A_0}(t,\tau)$ for $\tau/a = 3,4,5,6$ and $t/a = 8,9,10$ against

$$\mathcal{R}_j^{A_j}(t,\tau) = g_A^N + (g_A^{N'} - g_A^N) r_p r_w^\star e^{-\Delta E t} + g_A^{N\to N'} \left(r_w^\star e^{-\Delta E \tau} + r_p e^{-\Delta E (t-\tau)} \right)$$

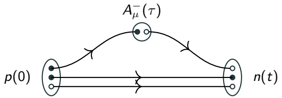
  - the effective mass curve of the 2-point correlation function $am_{\text{eff},3} = -\frac{1}{3} \log \frac{C_{2pt}(t+3a)}{C(t)}$ against

$$am_{\text{eff}}(t) = aM_N + r_p r_w^\star \frac{1 - e^{-3a\Delta E}}{3} e^{-\Delta E t}$$

- we use $\Delta E = M_{N'} - M_N$
- in total 7 independent real fit parameter: $g_A^N, g_A^{N'}, g_A^{N\to N'}, r_p, r_w^\star, M_N, M_{N'}$

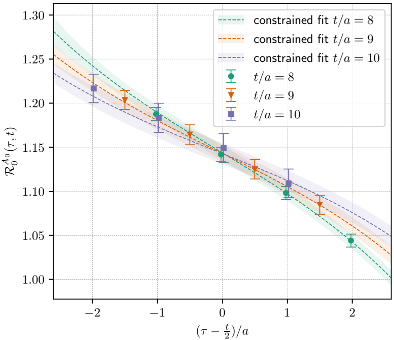
Preliminary results for the axial charge

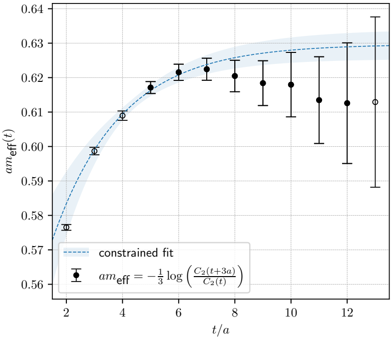
preliminary



constrained fit results:

- $g_A^N = 1.143(20) \quad (1.71\%)$
- $aM_N = 0.6297(43) \quad (0.69\%)$
- $aM_{N'} = 1.042(77) \quad (7.41\%)$



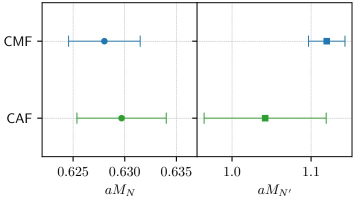


Comparison: Constrained mass and axial charge fits

preliminary

- Compare the constrained axial charge fit (CAF) and constrained mass fit (CMF)

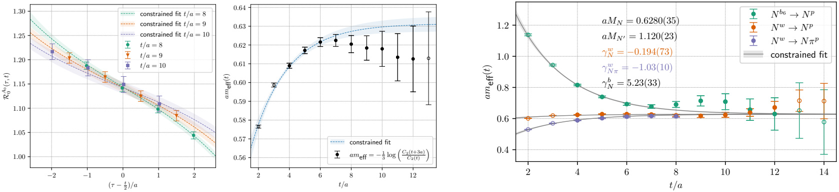
	CMF	CAF
aM_N	0.6280(35)	0.6297(43)
aM'_N	1.120(23)	1.042(77)
g_A^N		1.143(20)



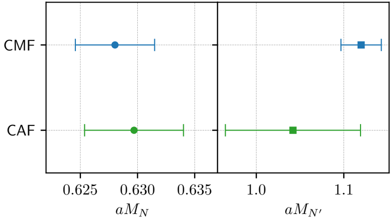
- energies obtained from CMF and CAF are consistent

Summary and Outlook

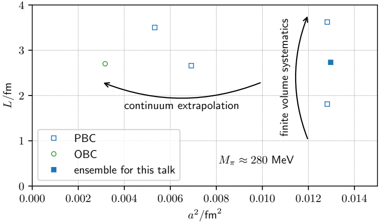
Summary



- nucleon-pion operator enhances coupling to excited state
- constrained fits of 2-point and 3-point functions give stable and coherent estimates
- we obtain consistent energies for constrained axial charge and mass fits



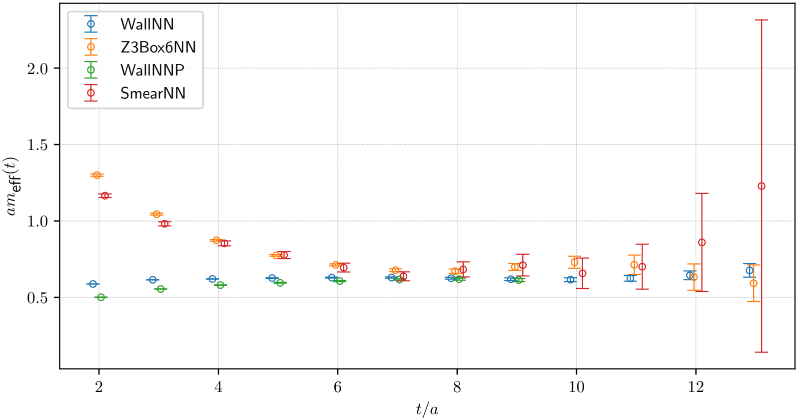
Outlook



- further analysis of the nucleon axial-vector current on the ensemble of this talk:
  - include nucleon-pion operator in 3-point function
  - more statistic
- expand the analysis to the other ensembles
- **Final goal**: Determination of nucleon axial charge and other quantities in the continuum and for physical pion mass with full error budget

Backup slides

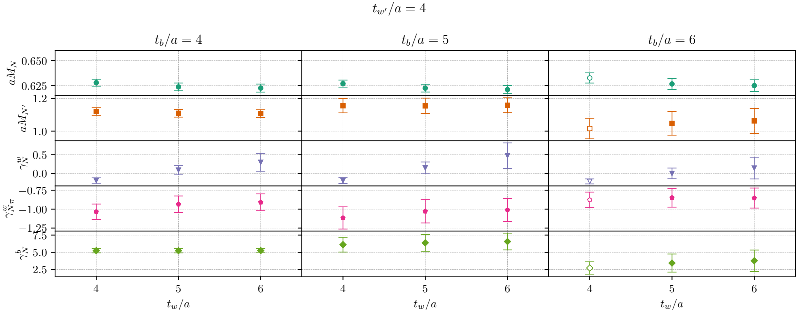
Comparison with smeared source



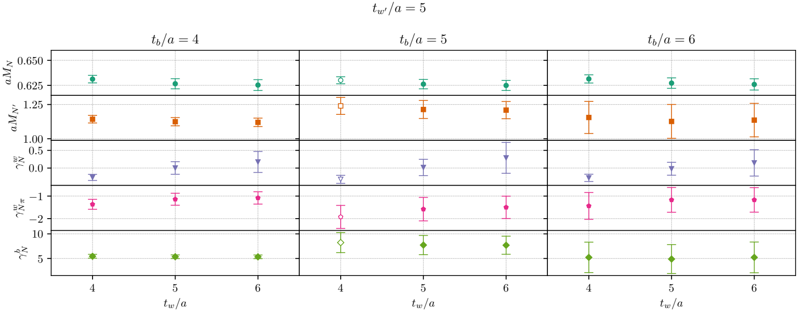
smearing parameter:

- Gauss smearing
- Wilson flowed gauge fields, $N_{wf} = 100$, $\varepsilon_{wf} = 0.01$
- $\sigma = 2.0$, $N = 5$

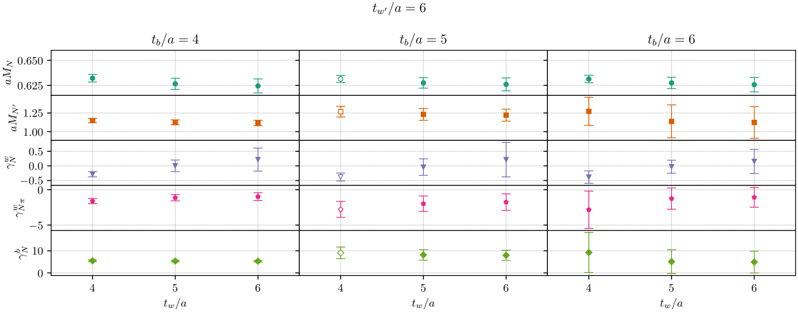
Spectrum constrained fit (fit range analysis)



Spectrum constrained fit (fit range analysis)



Spectrum constrained fit (fit range analysis)

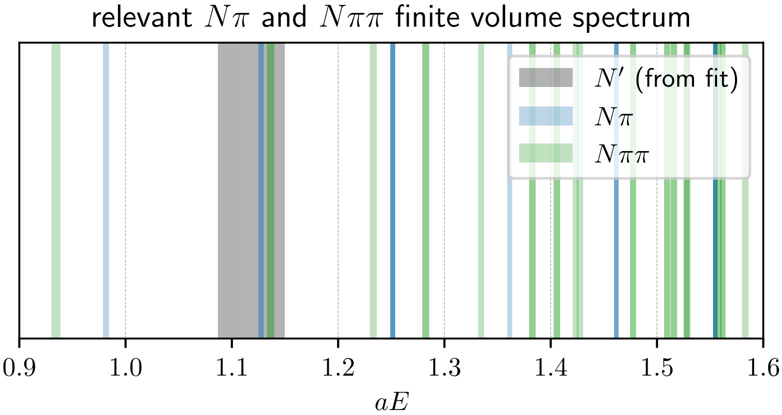


Excited states

- on the lattice there are multiple types of states with the same quantum number as the nucleon state:
    1. excited states of the nucleon
    2. nucleon-pion states with different momentum configurations
    3. nucleon-pion-pion states with different momentum configurations
- we identify the multi-particle states with the free particle approximation, e.g., the nucleon-pion states with

$$M_{N\pi} = \sqrt{M_N^2 + \boldsymbol{p}_N^2} + \sqrt{M_\pi^2 + \boldsymbol{p}_\pi^2}$$

Excited states



- $L = 2.73$ fm, $m_\pi = 279$ MeV and $a^{-1} = 1.74$ GeV
- excited state fit is combination of multiple excited states
- the spectrum gets more dense in the infinite volume limit

Fit range analysis for g_A-Fit