

Lattice QCD prediction of pion & kaon form factor at large momentum transfer Q^2

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III Motivation

EPJA 48 (2012) 187 EPJA 52 (2016) 268 arXiv: 2102.09222

- Experiment: JLab, EIC, EicC ...

Gao et al., PRD 96 (2017) 034024

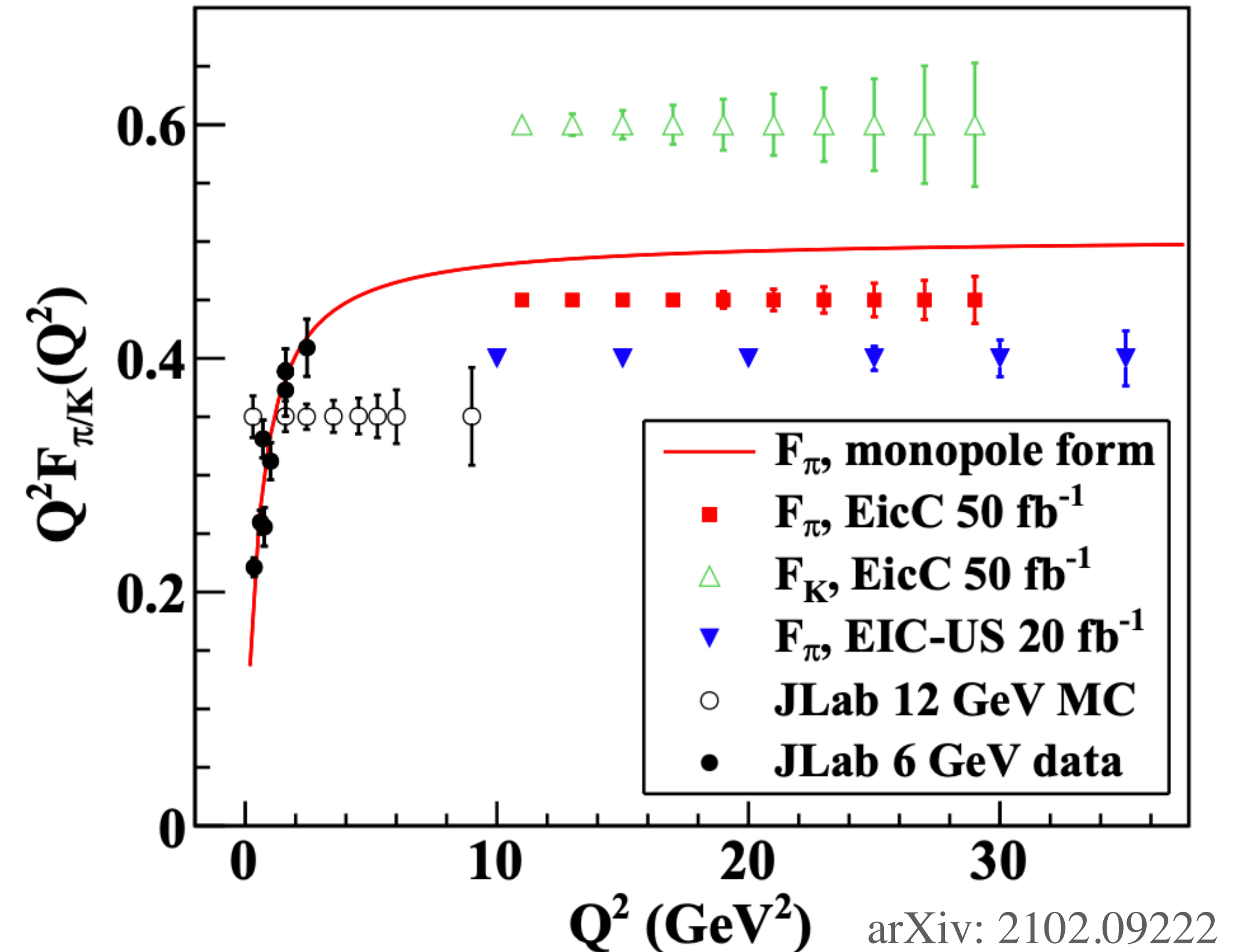
- Effective theory: QCD sum rules, DSE ...

- Lattice QCD

ETMC, PRD 105 (2022) 054502

○ State-of-the-art: $Q^2 \leq 2.5, 3 \text{ GeV}^2$

○ This work: Q^2 up to 10, 28 GeV^2



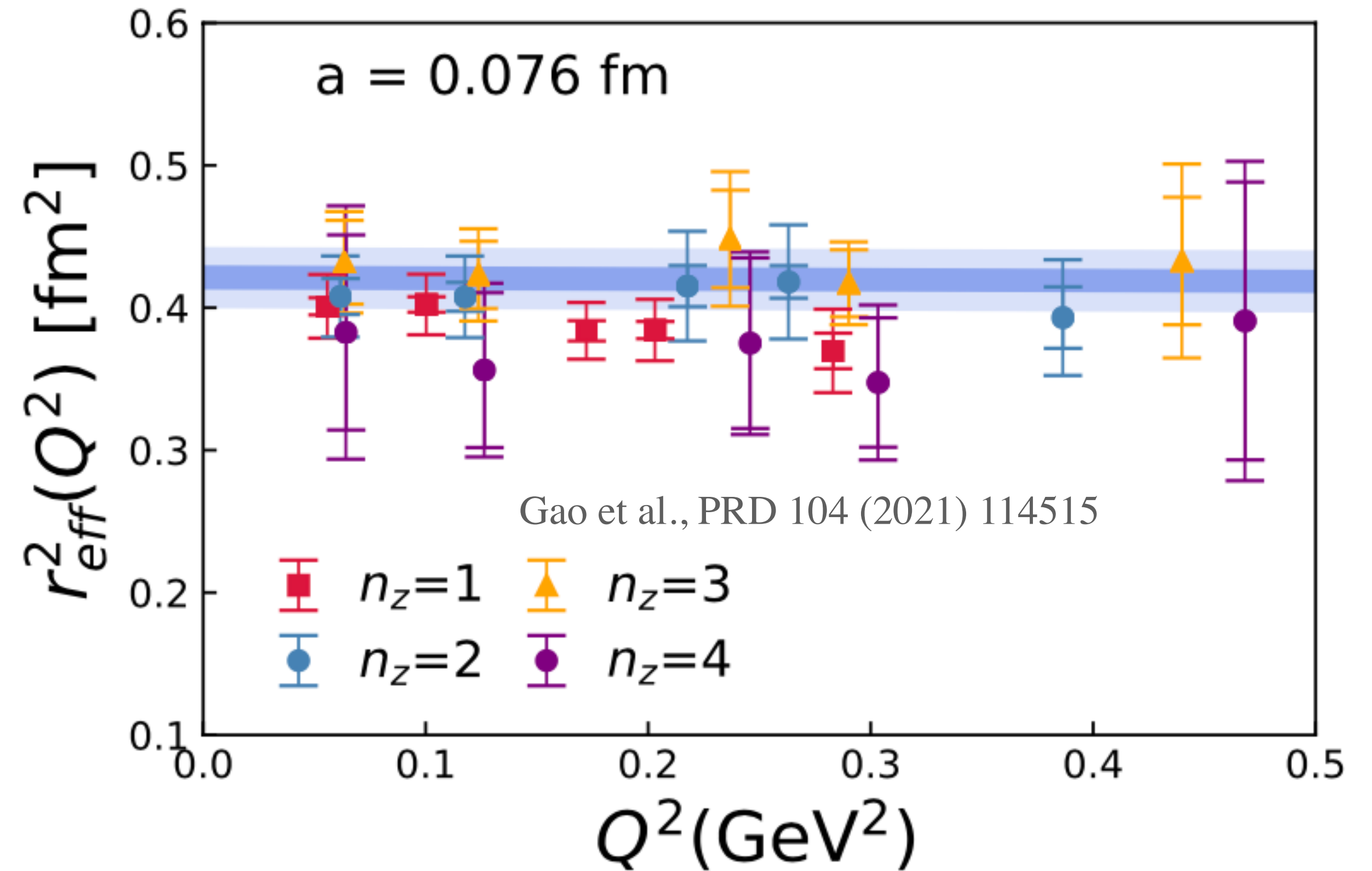
Small Q^2

Hadronic picture

Vector Meson Dominance

$$r_{\text{eff}}^2(Q^2) = \frac{6[1/F_\pi(Q^2) - 1]}{Q^2}$$

$$\langle r_\pi^2 \rangle = 0.42(2) \text{ fm}^2, \langle r_\pi^2 \rangle_{PDG} = 0.434(5) \text{ fm}^2$$



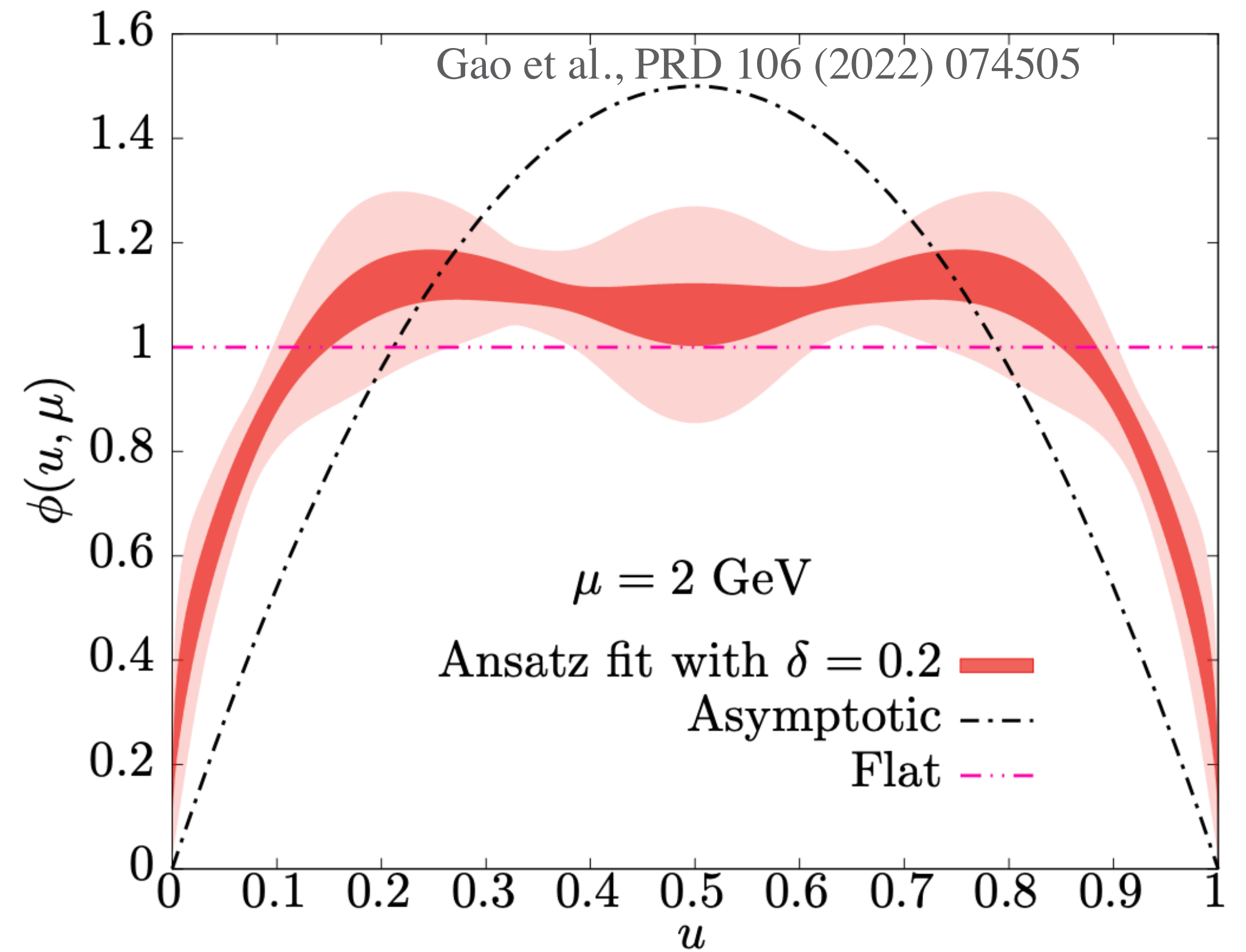
Large Q^2

Partonic picture

Large-Momentum Effective Theory

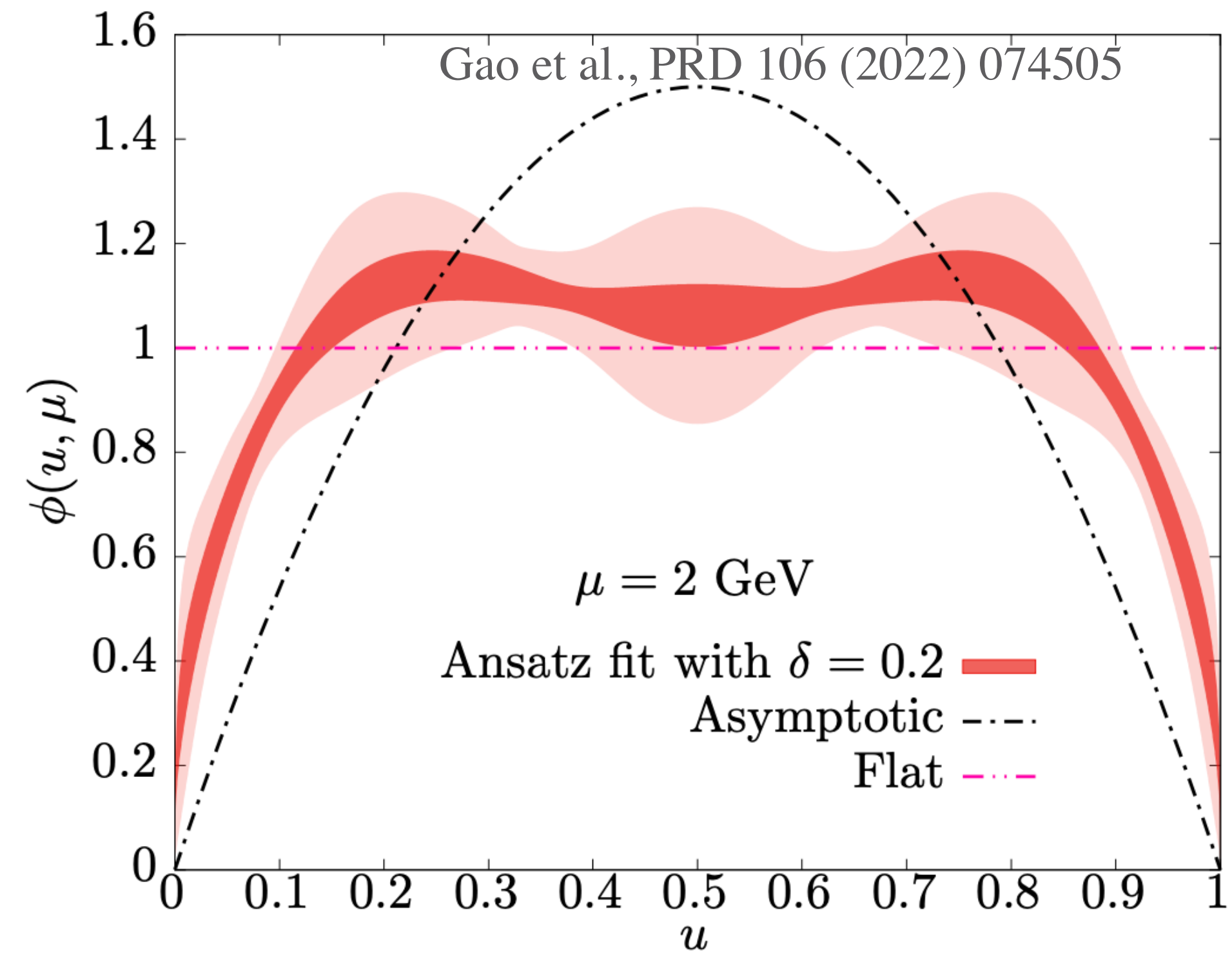
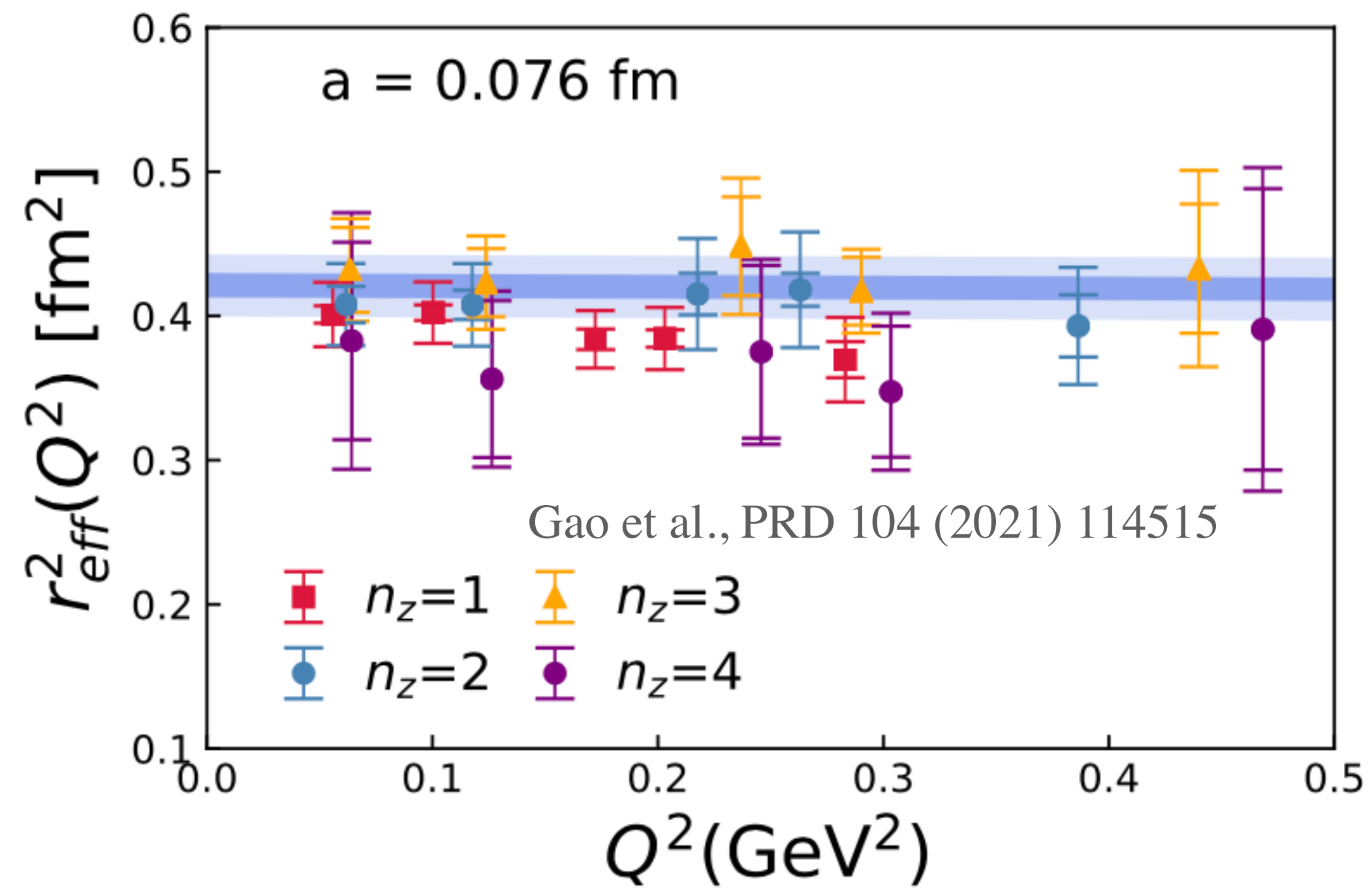
$$F(Q^2) = \int_0^1 \int_0^1 dx dy \phi^*(v, \mu_F^2) T_F(u, v, Q^2, \mu_R^2, \mu_F^2) \phi(u, \mu_F^2)$$

Hard-process kernel Distribution amplitude



HF Motivation

How about the intermediate range?



III Lattice Setup

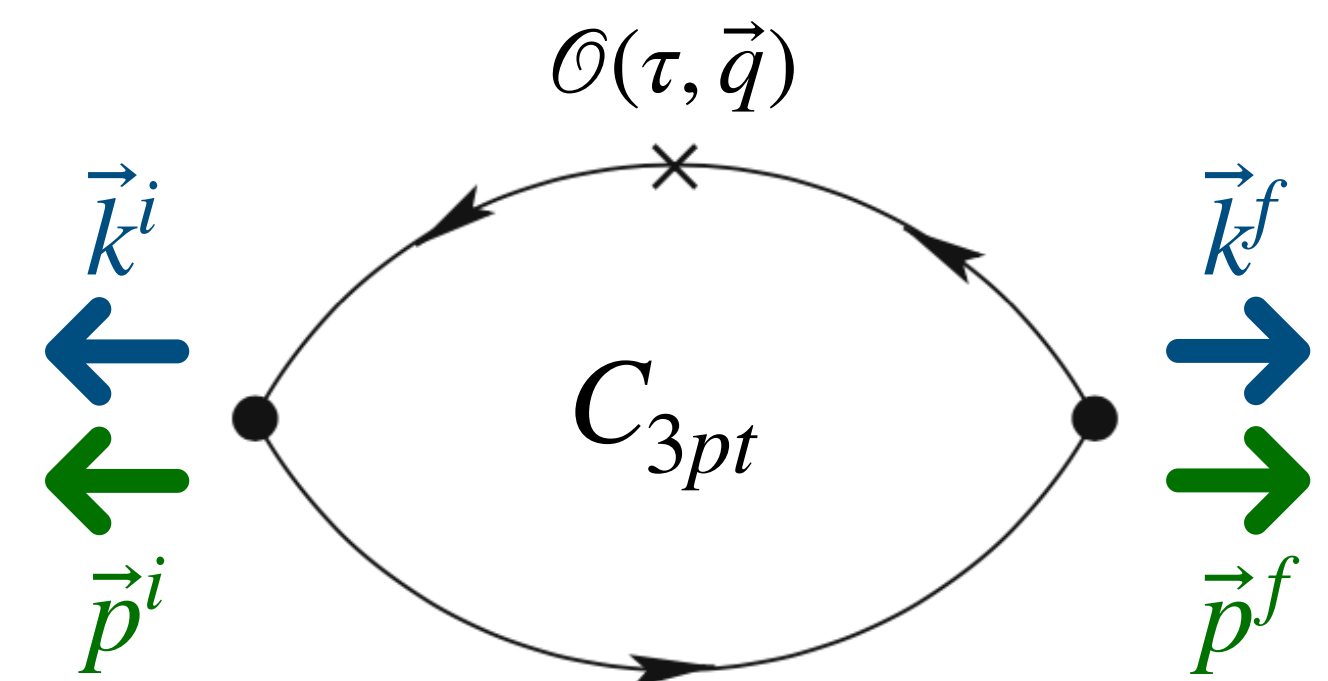
- $N_s^3 \times N_t = 64^3 \times 64$
- Pion / Kaon: $a = 0.076$ fm, $a = 0.04, 0.076$ fm
- HISQ action + Wilson-Clover action,

⇒ **at the physical point**

- Boost smearing back to back:

quark boost momentum \vec{k} & quark momentum \vec{p}

⇒ The largest Q^2 up to **10** (Pion) and **28 GeV²** (Kaon)



How to get form factor

$$C_{2pt}(t, \vec{p}) = \langle H(t_s, \vec{p}) H^\dagger(0, \vec{p}) \rangle$$

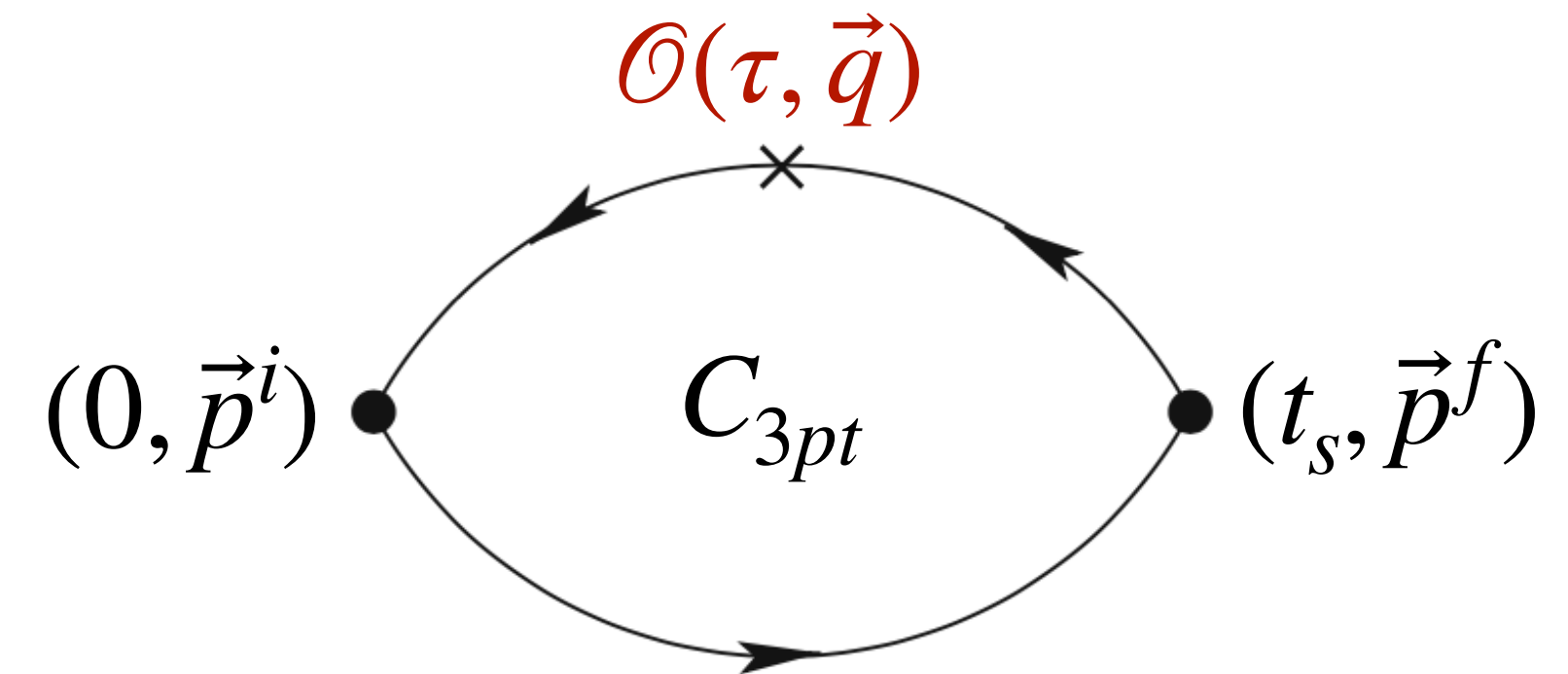
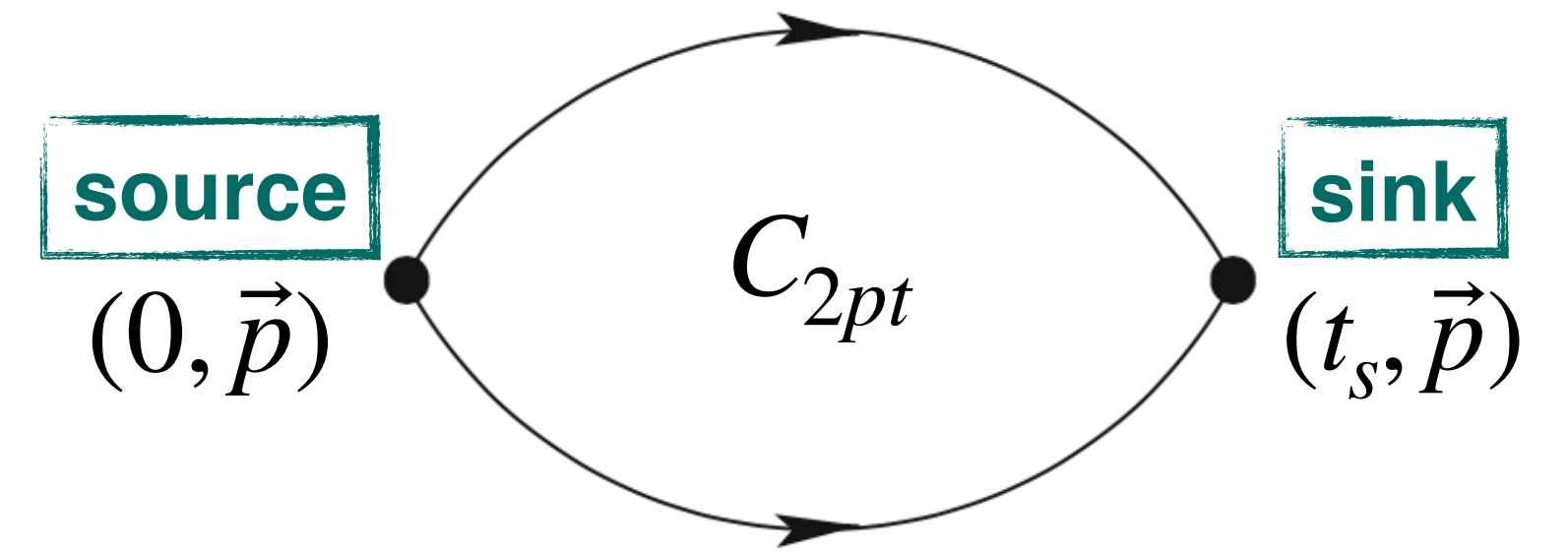
Insert $\mathcal{O}(\tau, \vec{q})$



$$\vec{p}^f = \vec{p}^i + \vec{q}$$

$$C_{3pt}(\tau, t_s; \vec{p}^i, \vec{p}^f) = \langle H(t_s, \vec{p}^f) \hat{\mathcal{O}}_\Gamma(\tau, \vec{q}) H^\dagger(0, \vec{p}^i) \rangle$$

$\Gamma : \hat{1}, \gamma^\mu, \sigma^{\mu\nu}$



$$F^B = \langle E_0, \vec{p}^f | \hat{\mathcal{O}}_{\gamma^\mu}(\tau, \vec{q}) | E_0, \vec{p}^i \rangle$$



from $\sim C_{3pt} / C_{2pt}$

Renormalization

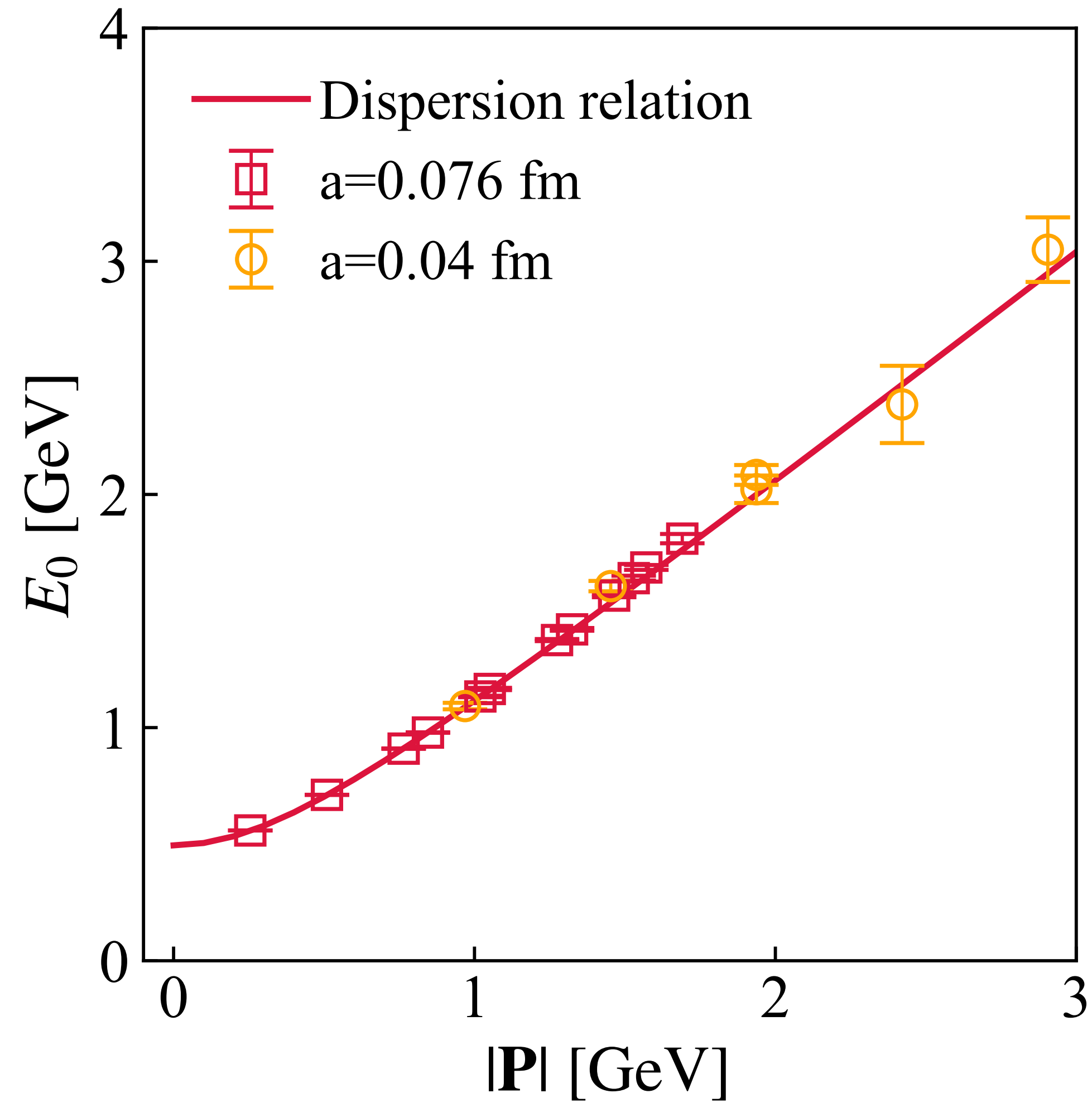


$$F(Q^2) = F^B \times Z_V^{-1}$$

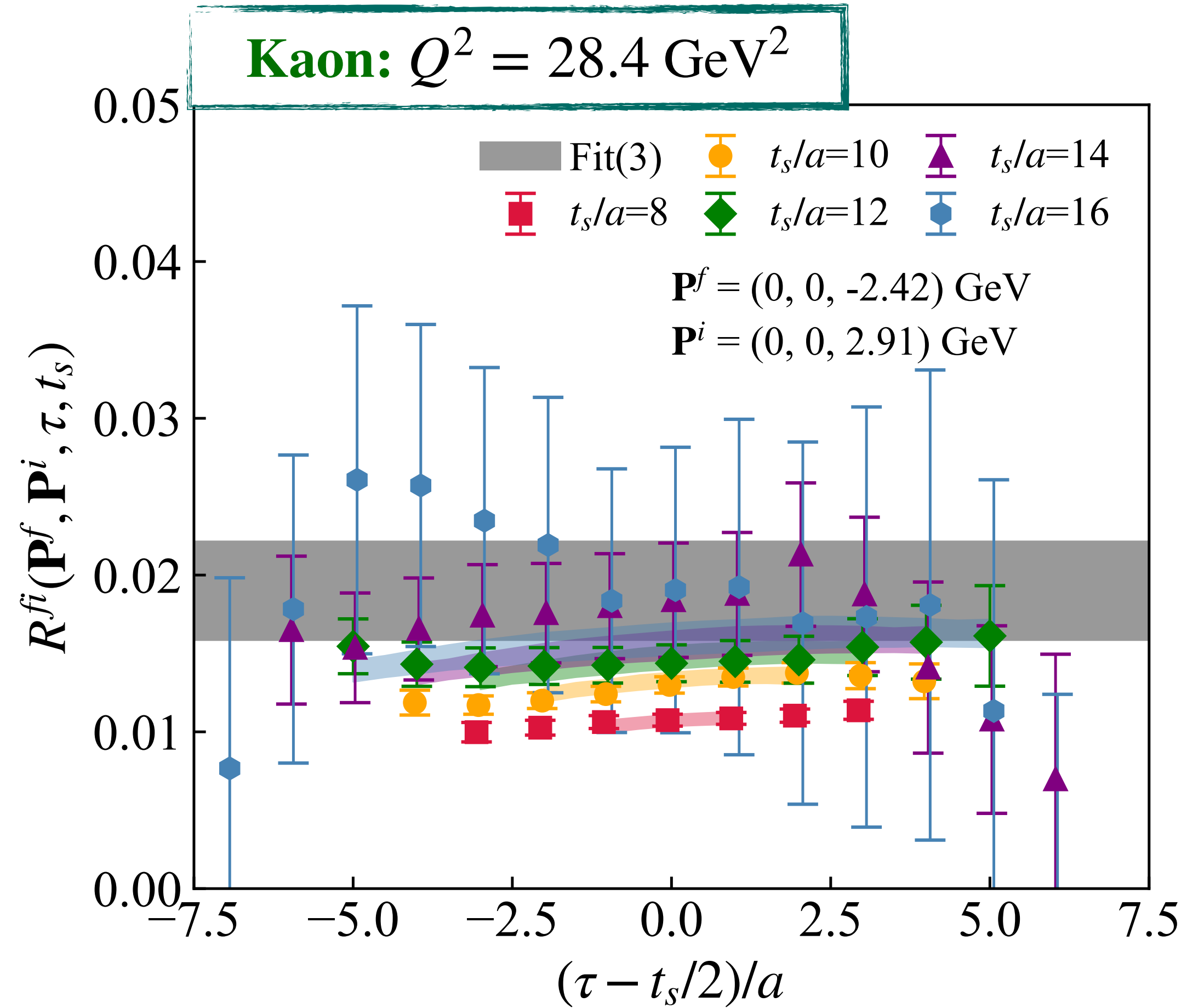
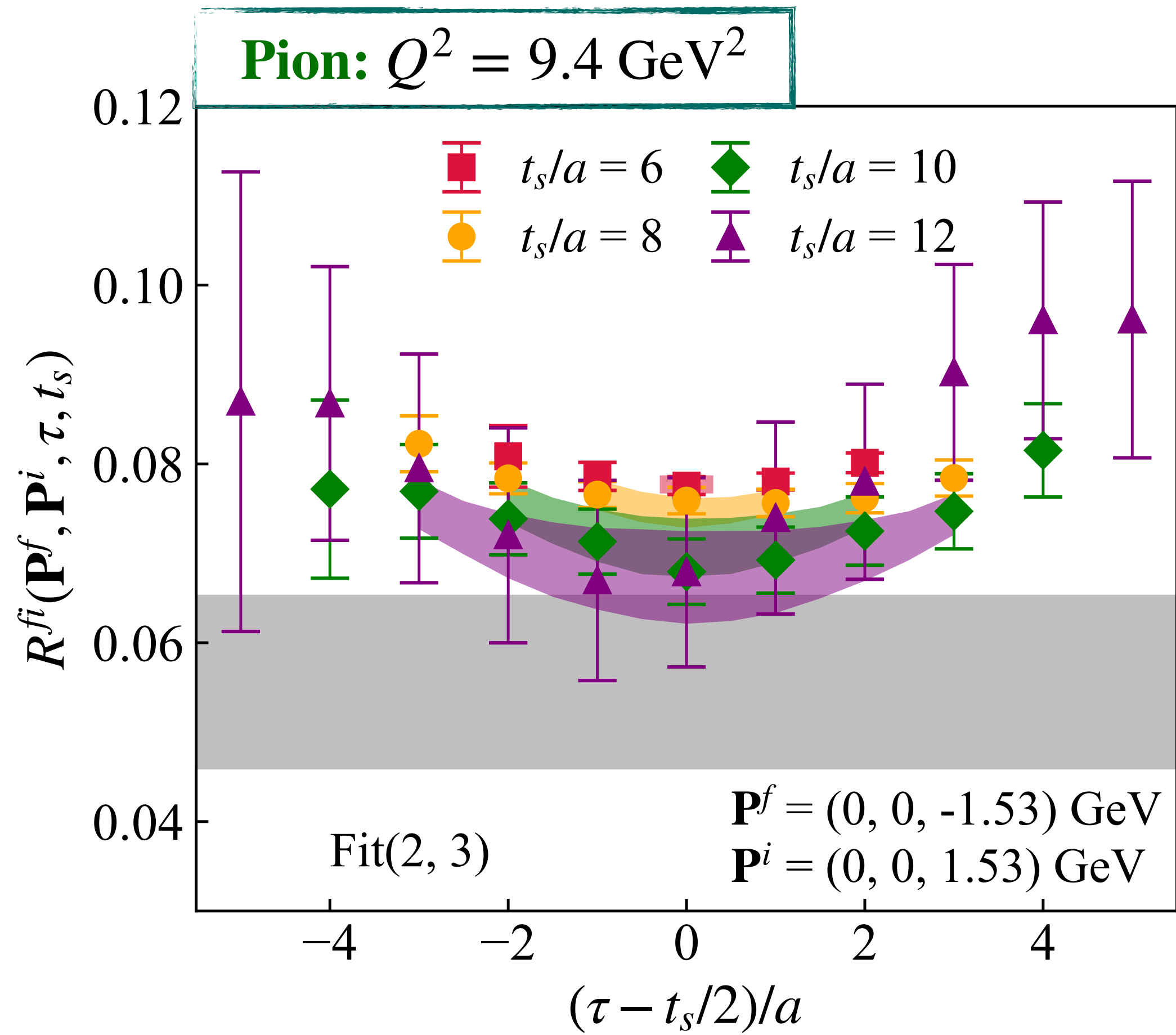
$$Q^2 = -q^2$$

III Extract Energy and Amplitude

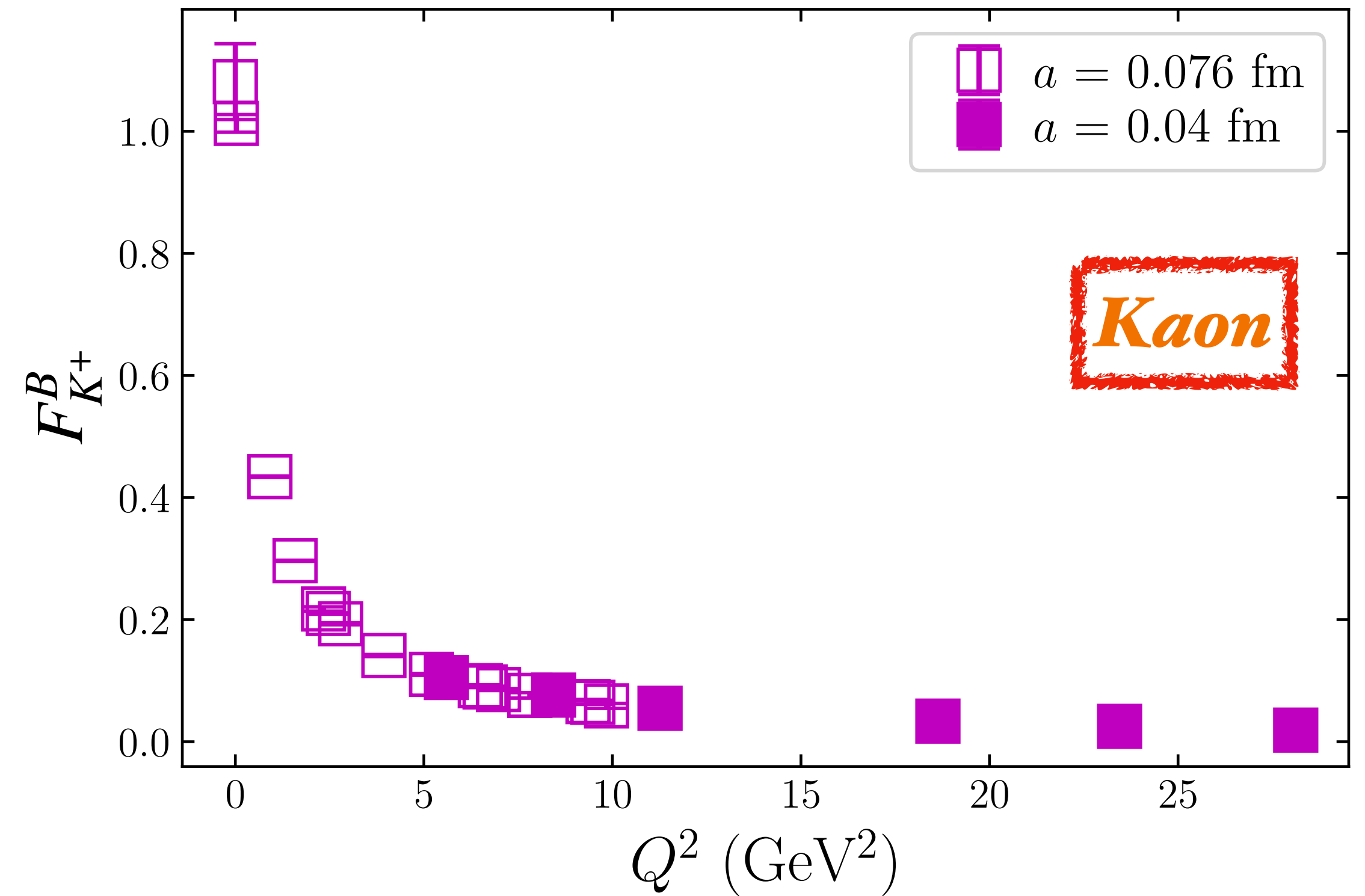
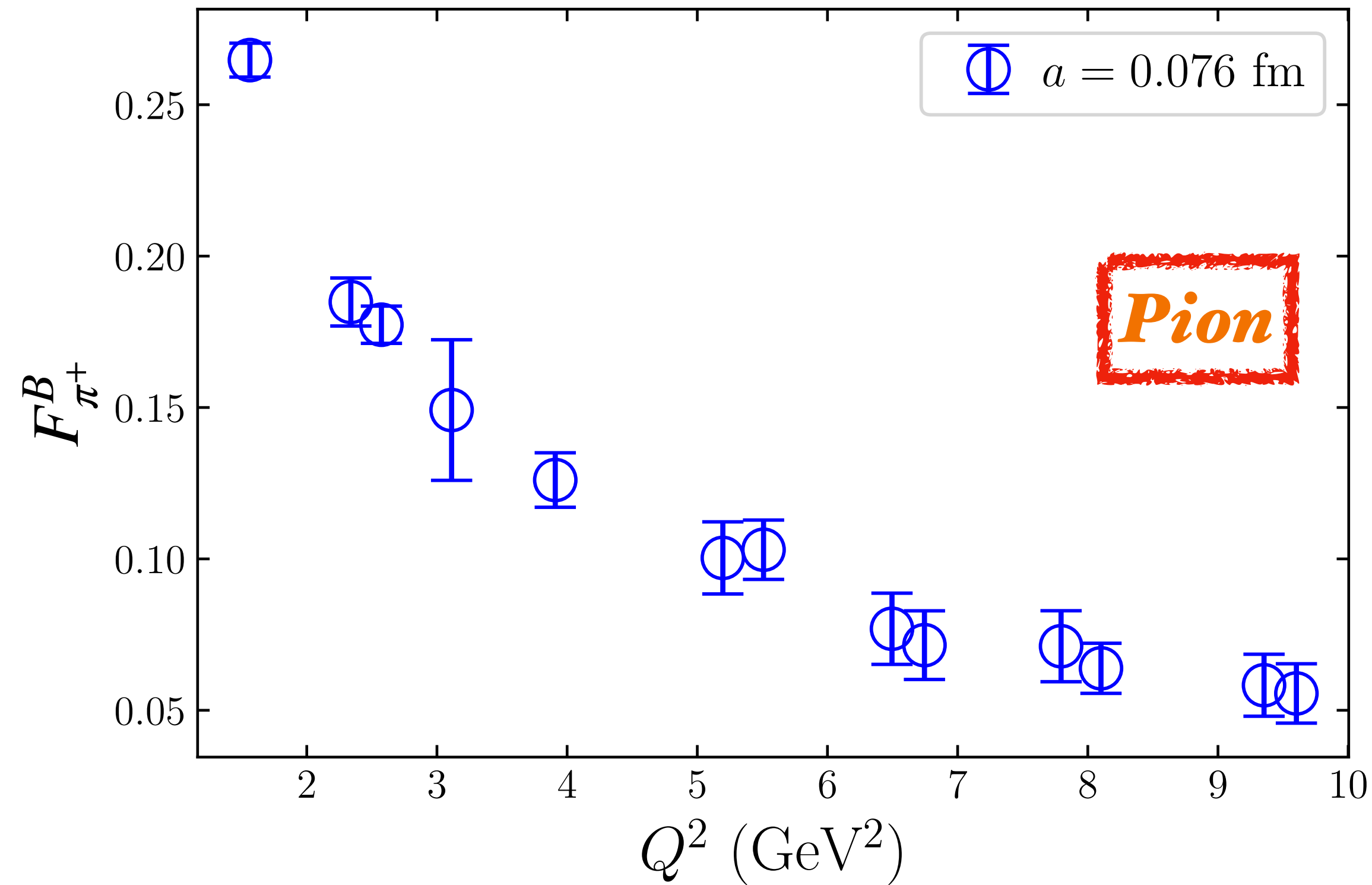
Take kaon data as an example



Extract Bare Form Factor



Renormalization

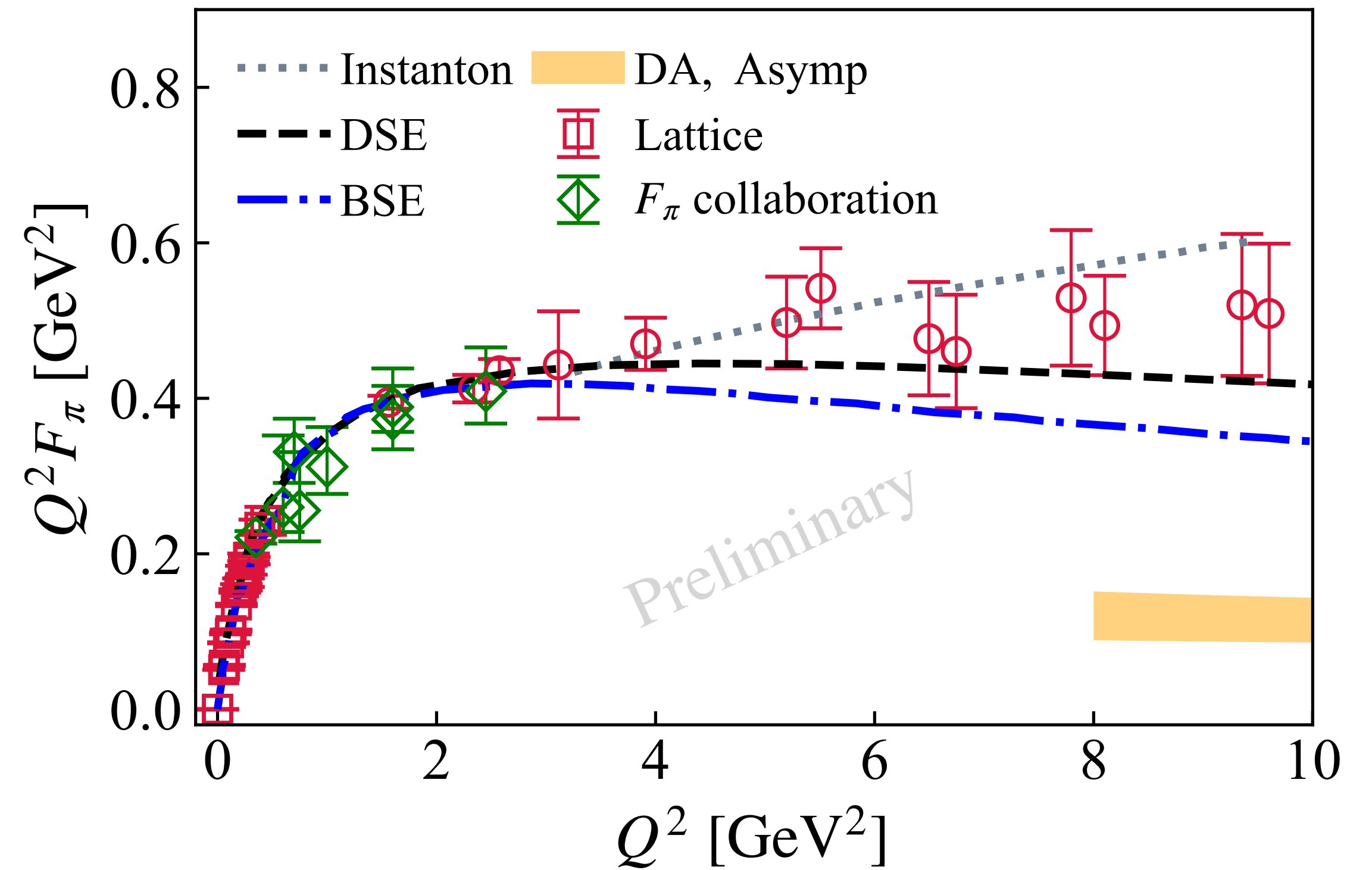
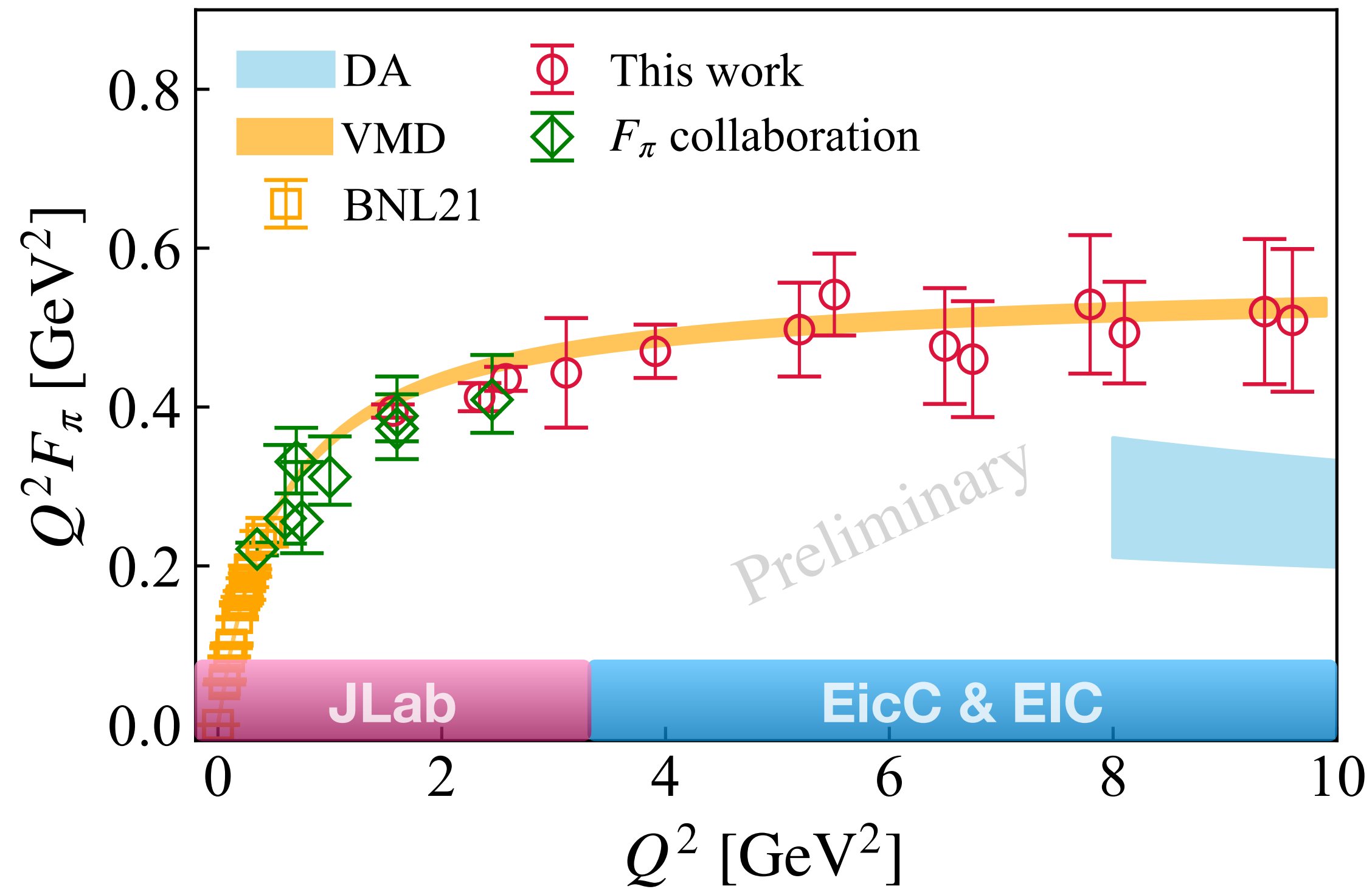


- Renormalization: $F = F^B \times Z_V^{-1}$

$$Z_V = \langle 0; \vec{p} | \hat{\mathcal{O}} | \vec{p}; 0 \rangle = 1.048, 1.024 \text{ for } a = 0.076, 0.04 \text{ fm}$$

Gao et al., PRD 104 (2021) 114515

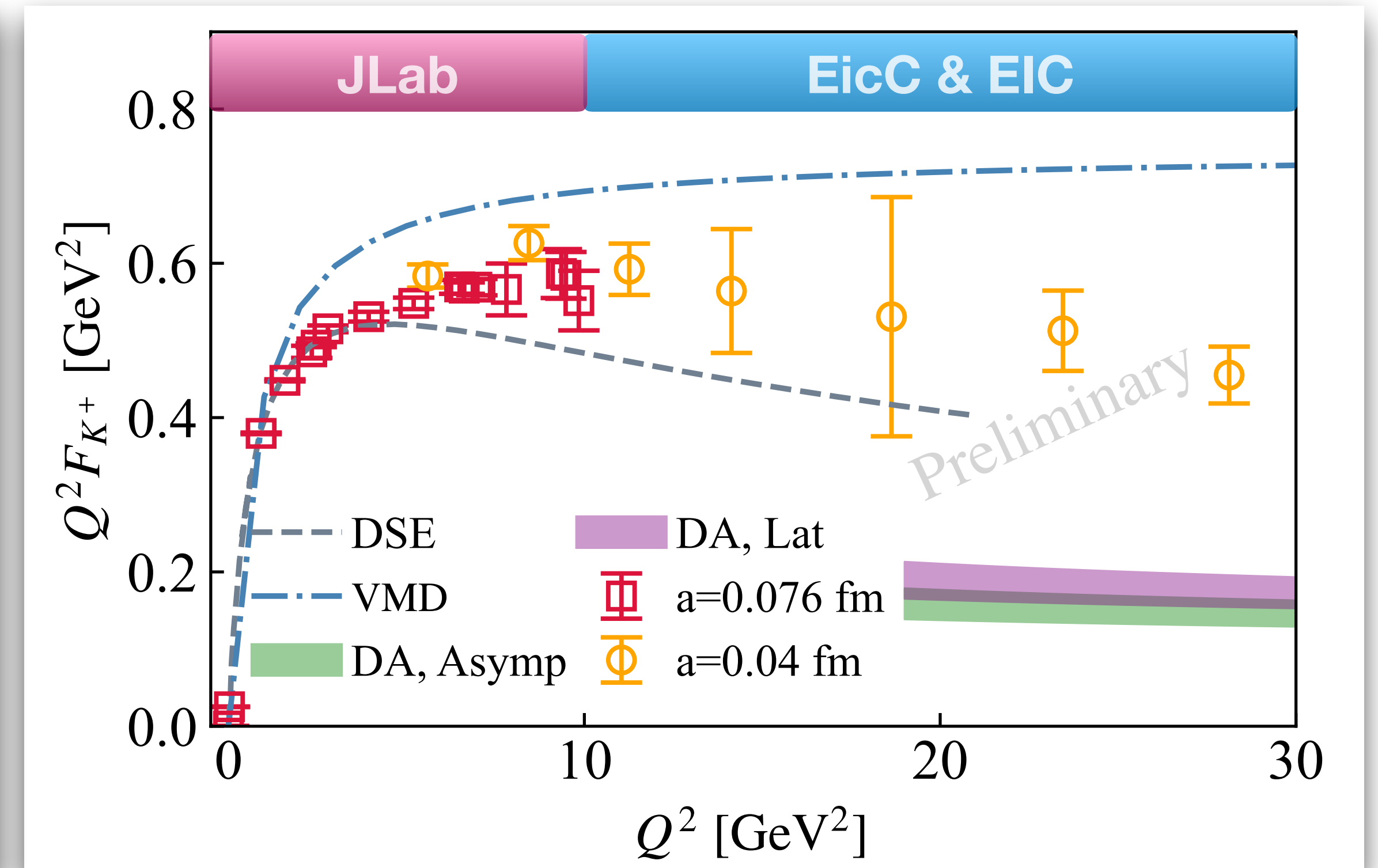
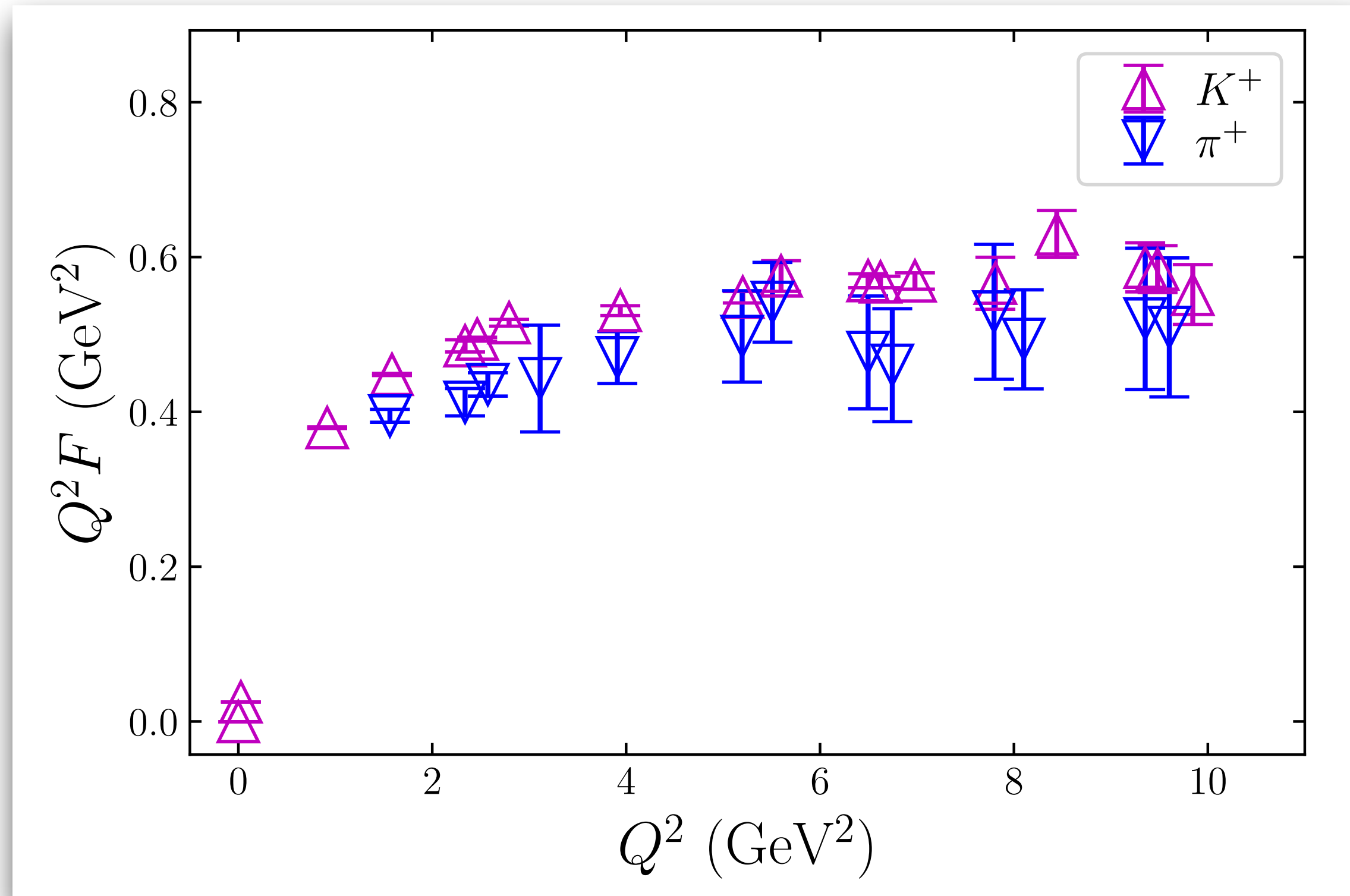
Results of Pion



DA: Gao et al., arXiv:2206.04084
 VMD: $Q^2/(1 + Q^2\langle r_\pi^2 \rangle/6)$
 BNL21: Gao et al., PRD 104 (2021) 114515
 F_π collaboration: Huber et al., PRC 78 (2008) 045203

Instanton: Shuryak et al., PRD 103 (2021) 054028
 DSE: Gao et al., PRD 96 (2017) 034024
 BSE: Ydrefors et al., PLB 820 (2021) 136494
 DA, Asymp: $\phi(x) = 6x(1 - x)$

Results of Kaon



DSE: Gao et al., PRD 96 (2017) 034024
 DA, Lat: Bali et al., JHEP 08 (2019) 065
 VMD: $Q^2/(1 + Q^2\langle r_K^2 \rangle/6)$
 DA, Asymp: $\phi(x) = 6x(1 - x)$

Summary

- Pion and kaon electromagnetic form factor at the physical point.
- Up to 10 and 28 GeV^2 for pion and kaon.
- Up to $Q^2 \sim 10 \text{ GeV}^2$, no significant flavor dependence

Even up to $Q^2 \sim 30 \text{ GeV}^2$, haven't reached the partonic picture

Thanks for your attention!