A new approach for computing GPDs from asymmetric frames

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Andreas Metz (Temple U.)
Joshua Miller (Temple U.)
Swagato Mukherjee (BNL)
Peter Petreczky (BNL)
Aurora Scapellato (Temple U.)
Fernanda Steffens (Bonn U.)
Yong Zhao (ANL)

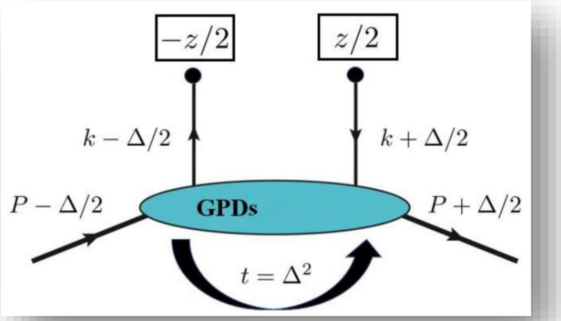
40th International Symposium on Lattice Field Theory

2023 LATTICE

Fermilab

Based on: PhysRevD.106.114512 & In Preparation

Generalized Parton Distributions (GPDs)

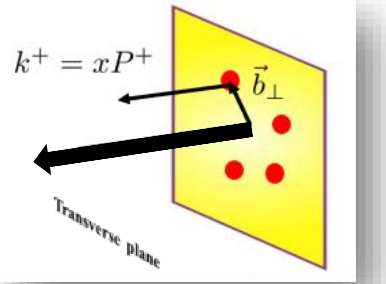


GPD correlator: Graphical representation

Definition: (See for example Diehl, hep-ph/0307382)

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

Motivation for GPD studies

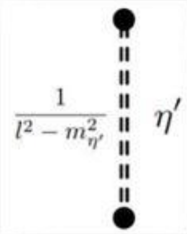


3D imaging (Burkardt, 0005108 ...)

Spin sum rule & orbital angular momentum (Ji, 9603249):

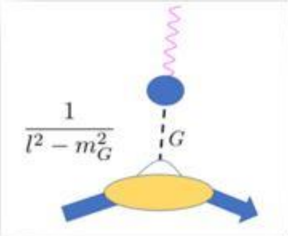
$$J^q = \int_{-1}^{1} dx\, x(H^q + E^q)|_{t=0}$$

Imprints of chiral/trace anomalies in GPDs (SB, Hatta, Vogelsang, 2305.09431):



Eta-meson mass generation

$$\tilde{E}(x) \sim \frac{1}{l^2 - m_{\eta'}^2}$$



Glueball mass generation

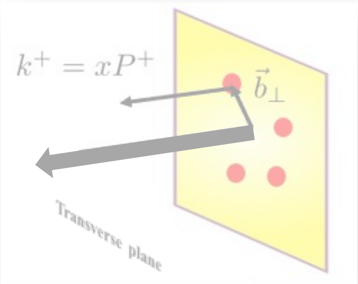
$$H(x), E(x) \sim \frac{1}{l^2 - m_G^2}$$

Novel avenue of GPD research

Profound physical implication of anomaly poles:

Touches questions on mass generations, Chiral symmetry breaking, ...

Motivation for GPD studies

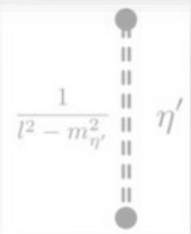


3D imaging (Burkardt, 0005108, ...)

Spin sum rule & orbital angular momentum (Ji, 9603249):

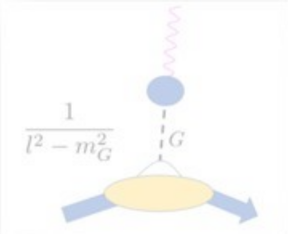
$$J^q = \int_{-1}^{1} dx\, x(H^q + E^q)|_{t=0}$$

We need GPD measurements from Lattice QCD



Eta-meson mass generation

$$\tilde{E}(x) \sim \frac{1}{l^2 - m^2_{\eta'}}$$



Glueball mass generation

$$H(x), E(x) \sim \frac{1}{l^2 - m^2_G}$$

Novel avenue of GPD research

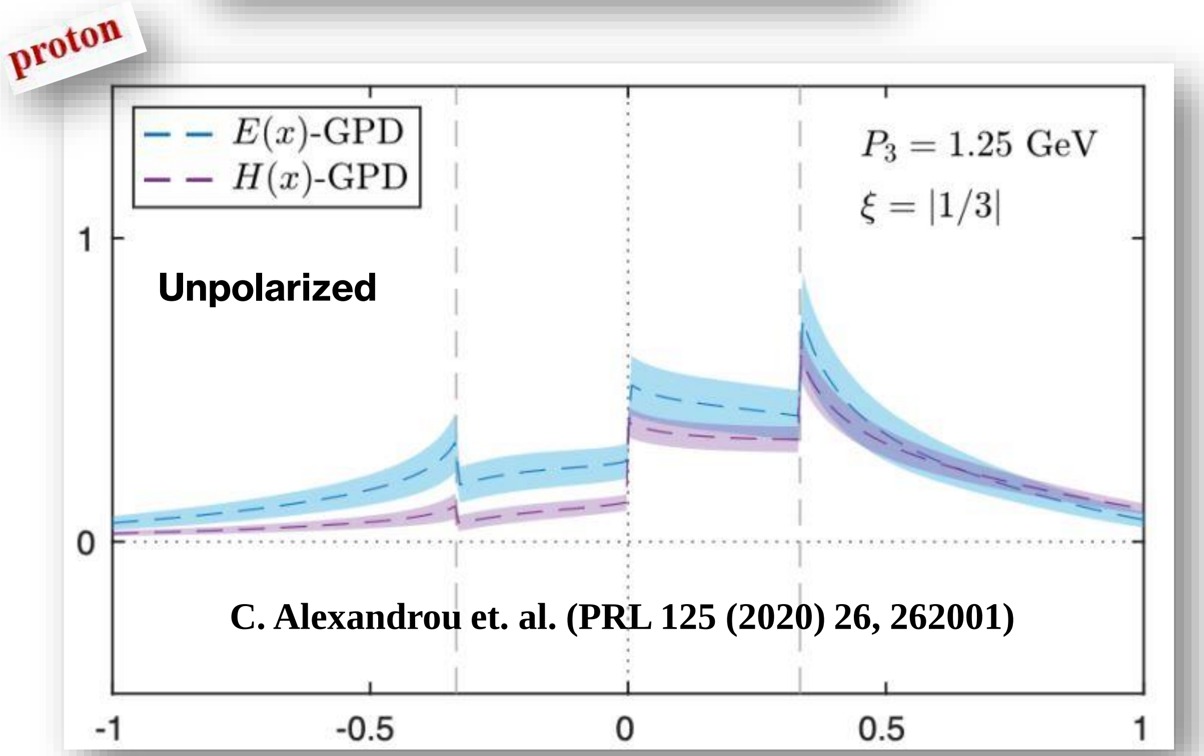
Profound physical implication of anomaly poles:

Touches questions on mass generations, Chiral symmetry breaking, ...

First Lattice QCD results of the x-dependent GPDs

Example:

Excellent progress!!!



First Lattice QCD results of the x-dependent GPDs

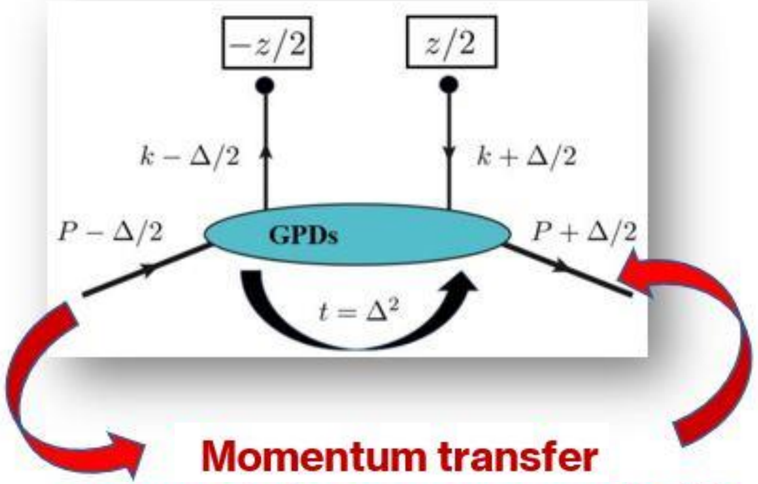
Excellent progress!!!

Example:

But little hiccup ...

Traditionally, GPDs have been calculated from "symmetric frames"

Practical drawback

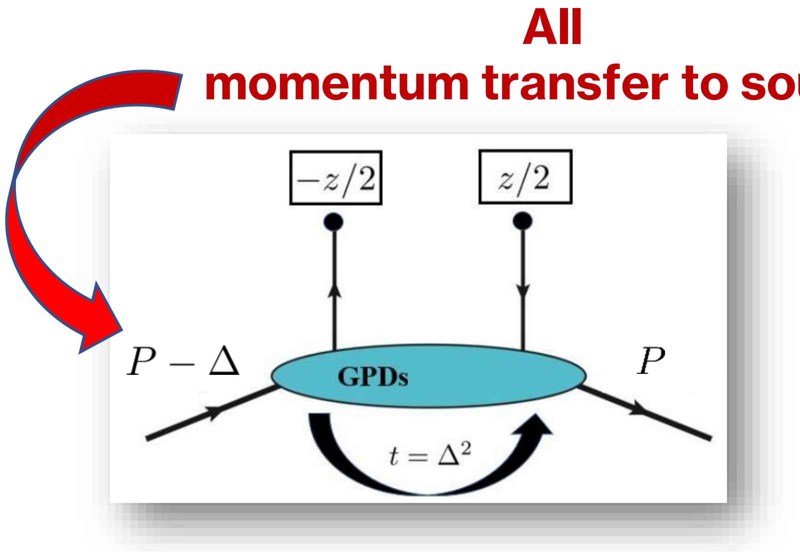


Momentum transfer
symmetric between source & sink

Lattice QCD calculations of GPDs in symmetric frames are expensive

Lattice QCD calculations of GPDs in asymmetric frames

Resolution:



- **Perform Lattice QCD calculations of GPDs in asymmetric frames**

See Joshua's talk

Lattice QCD calculations of GPDs in asymmetric frames

Our contribution in a nutshell:

All ... source

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

In Preparation

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Joshua Miller,[3,‡] Swagato Mukherjee,[5] Peter Petreczky,[5] Aurora Scapellato,[3] Fernanda Steffens,[6] and Yong Zhao[4]

Key findings: ...ce QCD calculations of GPDs in asymmetric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame** — This talk
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

Lattice QCD calculations of GPDs in asymmetric frames

Why is the question of frame-(in)dependence relevant?

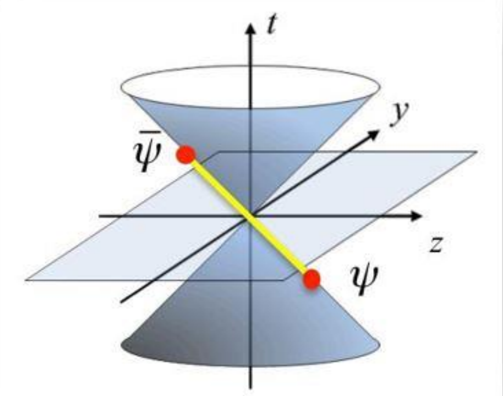
Our contribution in a nutshell:

Key points:

with Asy

Shohini Bhattacharya
Andreas Metz,[3] Swa

Example: Light-cone GPD H



$$H(x,\xi,t) \to \int \frac{dz^-}{4\pi} e^{ixP\cdot z} \langle p'|\bar{q}\gamma^+ q|p\rangle \qquad z = (0, z^-, 0_\perp)$$

$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p'|\bar{q}\not{z}\, q|p\rangle \qquad \text{Arbitrary light-like } z$$

GPDs on the light-cone are Lorentz-invariant

Metz,[3]
Zhao[4]

Key findings: Lattice QCD calculations of GPDs in asymmetric frames

- Lorentz covariant formalism for calculating quasi-GPDs in any frame — This talk
- Elimination of power corrections potentially allowing faster convergence to light-cone GPDs

Lattice QCD calculations of GPDs in asymmetric frames

Why is the question of frame-(in)dependence relevant?

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Generalized Parton Distributions from Lattice QCD
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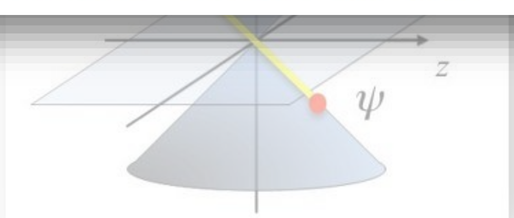
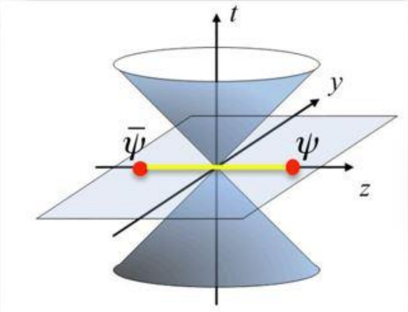


$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p' | \bar{q} \not{z}\, q | p \rangle$$ Arbitrary light-like z

GPDs on the light-cone are Lorentz-invariant

Key findings: ...e QCD c...

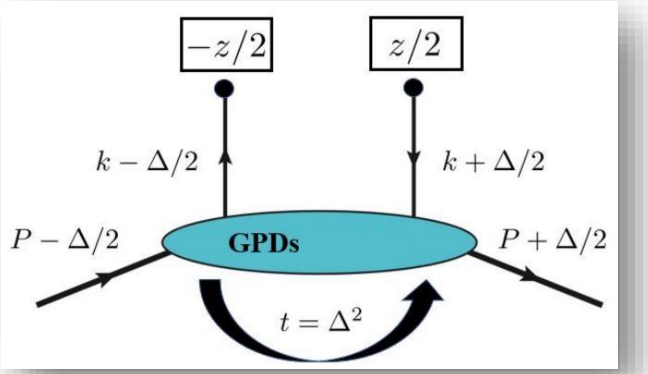
- Lorentz covariant form... ...s talk
- Elimination of power co... ...ight-cone GPDs



Are quasi-GPDs Lorentz-invariant?

Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs



Definition of quasi-GPDs in symmetric frames: (Historical)

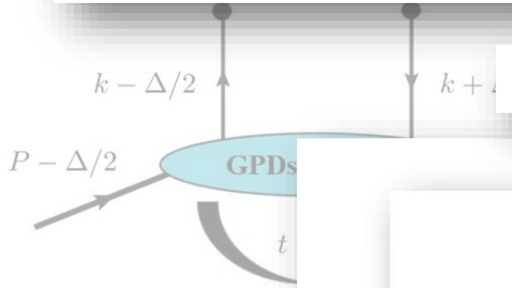
$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s, \lambda' | \bar{q}(-z/2)\gamma^0 q(z/2) | p_s, \lambda\rangle \Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_s(p'_s, \lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(z, P_s, \Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M} E_{\mathrm{Q}(0)}(z, P_s, \Delta_s)\Big|_s\right] u_s(p_s, \lambda)$$

Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs

Historic definitions of quasi-GPDs H & E are not manifestly Lorentz invariant

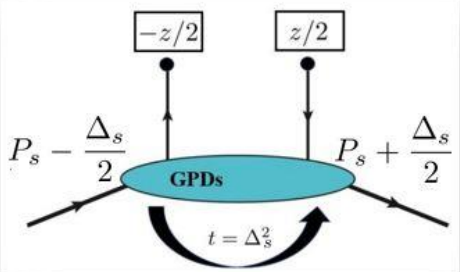


ical)

$z=0,\vec{z}_\perp=\vec{0}_\perp$

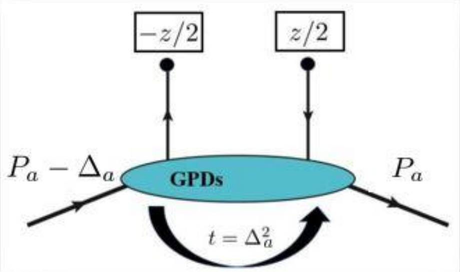
$(z, P_s, \Delta_s)\big|_s \Big] u_s(p_s, \lambda)\big)$

Think about how γ^0 transforms under Lorentz transformation

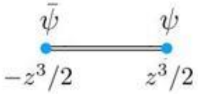


Symmetric frame

"Transverse" Lorentz transformation



Asymmetric frame



"Transverse" with respect to Wilson Line

$$F_s^0 = \gamma F_0^a - \gamma\beta F_\perp^a$$

$$\beta = -\sqrt{\frac{E_i^a - E_f^a}{E_i^a + E_f^a}} < 0$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs

Historic definitions of quasi-GPDs H & E are not manifestly Lorentz invariant

Think about how γ^0 transforms under Lorentz transformation

Can we come up with a manifestly Lorentz-invariant definition of quasi-GPDs for finite values of momentum?

"Transverse" with respect to Wilson Line

$$F^0_s = \gamma F^a_0 - \gamma\beta F^a_\perp$$

$$\beta = -\sqrt{\frac{E^a_i - E^a_f}{E^a_i + E^a_f}} < 0$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Lattice QCD calculations of GPDs in asymmetric frames

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^{\mu}(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}\boldsymbol{A_1} + mz^{\mu}\boldsymbol{A_2} + \frac{\Delta^{\mu}}{m}\boldsymbol{A_3} + im\sigma^{\mu z}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5} + \frac{P^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_6} + mz^{\mu}i\sigma^{z\Delta}\boldsymbol{A_7} + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

Vector operator $F^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_{\perp}=\vec{0}_{\perp}}$

Features:

- **General structure of matrix element based on constraints from Parity**
- **8 linearly-independent Dirac structures**
- **8 Lorentz-invariant amplitudes (or Form Factors)** $\boldsymbol{A_i \equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)}$

Lattice QCD calculations of GPDs in asymmetric frames

Lorentz covariant formalism

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$$F^{\mu}(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}A_1 + mz^{\mu}A_2 + \frac{\Delta^{\mu}}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6 + mz^{\mu}i\sigma^{z\Delta}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$

Main point:

Calculate quasi-GPD in symmetric frame through matrix elements of asymmetric frame

$$F^s \longleftrightarrow F^a$$

Niilo's talk:

Unveil GPDs through the amplitude formalism in the pseudo-distribution approach

Lattice QCD calculations of GPDs in asymmetric frames

Re-exploring historical definitions of quasi-GPDs

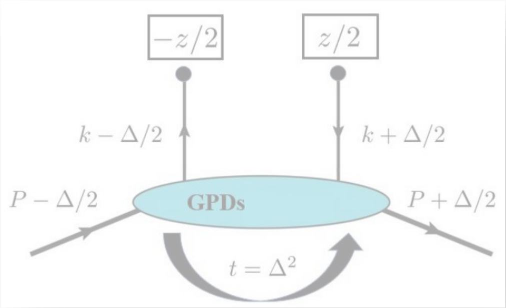
Mapping amplitudes to the historical definitions of quasi-GPDs: (Sample results)

Lattice QCD calculations of GPDs in asymmetric frames

Re-exploring historical definitions of quasi-GPDs

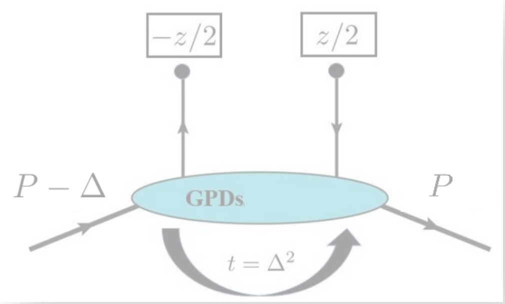
Frame-dependent expressions: Explicit non-invariance from kinematics factors

Symmetric frame:



$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = \boldsymbol{A_1} + \frac{\Delta_s^0}{P_s^0}\boldsymbol{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}\boldsymbol{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)\boldsymbol{A_6} + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)\boldsymbol{A_8}$$

Asymmetric frame:



$$H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) = \boldsymbol{A_1} + \frac{\Delta_a^0}{P^0_{avg,a}}\boldsymbol{A_3} - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)\boldsymbol{A_4}$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\right)\boldsymbol{A_6}$$
$$+ \left(\frac{(\Delta_a^0)^3 z^3}{2M^2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\right)\boldsymbol{A_8}$$

Lattice QCD calculations of GPDs in asymmetric frames

Relation between light-cone GPD H & amplitudes:

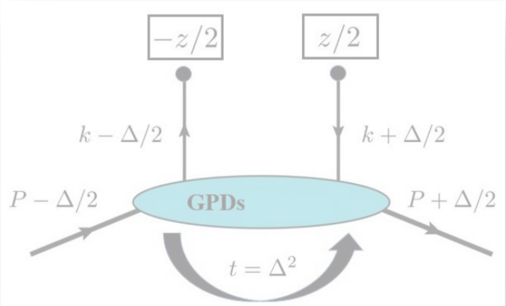
$$H(z\cdot P, z\cdot \Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

Lorentz-invariant expression

Re-exploring historical definitions

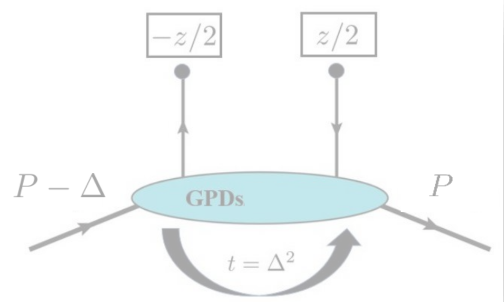
Frame-dependent expressions: Explicit non-in

Symmetric frame:



$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2 \Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

Asymmetric frame:



$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0}A_3 - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0 \Delta_a^3 z^3}{4P_{avg,a}^0 (P_{avg,a}^3)^2}\right)A_4$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0 \Delta_a^0 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_{avg,a}^3}\right)A_6$$
$$+ \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3 \Delta_a^3 z^3}{4M^2 P_{avg,a}^0 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2 \Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right)A_8$$

Lattice QCD calculations of GPDs in asymmetric frames

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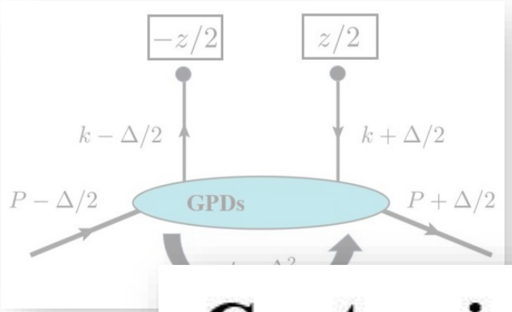
Novel definition of quasi-GI

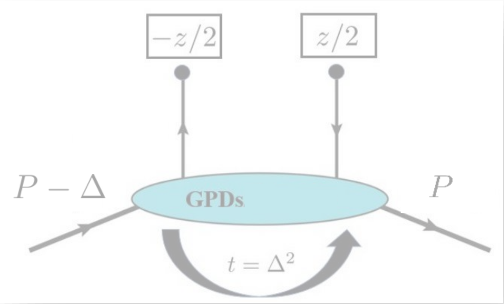
Mapping amplitudes to the historical definitions of quasi-GP

Symmetric frame:



$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3z^3P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$

Contamination from additional amplitudes or power corrections



$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P^0_{avg,a}}A_3 - \left(\frac{\Delta_a^0z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{\Delta_a^0\Delta_a^3z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)A_4$$
$$+\left(\frac{(\Delta_a^0)^2z^3}{2M^2P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\right)A_6$$
$$+\left(\frac{(\Delta_a^0)^3z^3}{2M^2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^3\Delta_a^3z^3}{4M^2P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\right)A_8$$

CD calculations of GPDs in asymmetric frames

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

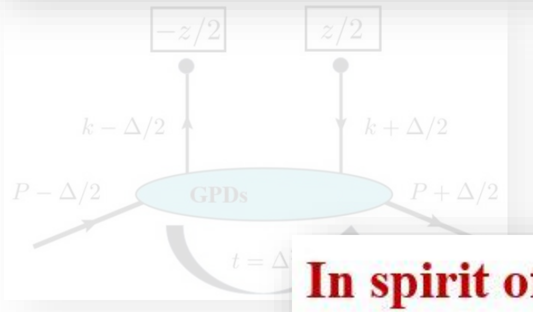
A. V. Radyushkin
Old Dominion University, Norfolk, VA 23529, USA and
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

Novel definition of quasi-GI

Contrary to quasi-PDFs, γ^0 operator for quasi-GPDs is contaminated with additional amplitudes or power corrections



$$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0} A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3} A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right) A_6$$

$+ \Big((\Delta_s^0 \ldots$

In spirit of what's done for PDFs:

You can think of eliminating additional amplitudes by the addition of other operators:

$$(\gamma^1, \gamma^2)$$

Asymmetric frame:

$$H_{Q(0)}\big|_a(z, P_a, \Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0} A_3 - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0 \Delta_a^3 z^3}{4P_{avg,a}^0 (P_{avg,a}^3)^2}\right) A_4$$

$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0 \Delta_a^0 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_{avg,a}^3}\right) A_6$$

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Lattice QCD calculations of GPDs in asymmetric frames

Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

Novel definition of quasi-GPDs

Contrary to quasi-PDFs, γ^0 operator for quasi-GPDs is contaminated with additional amplitudes or power corrections

$$\ldots_{(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6$$

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

In spirit of what's done for PDFs:

You can think of eliminating additional amplitudes by the addition of other operators:

$$(\gamma^1, \gamma^2)$$

Asymmetric frame:

Lorentz-invariant definition of quasi-GPDs:

Main finding:

Schematic structure: $H_Q \to c_0\langle\bar\psi\gamma^0\psi\rangle + c_1\langle\bar\psi\gamma^1\psi\rangle + c_2\langle\bar\psi\gamma^2\psi\rangle$

Note: Here c's are frame-dependent kinematic factors that cancel additional amplitudes

QCD calculations of GPDs in asymmetric frames

Same functional forms

Relation between light-cone GPD H & amplitudes:

$$H(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} A_3$$

$$A_i \equiv A_i(z^2 = 0)$$

Lorentz-invariant generalization of LC definition to $z^2 \neq 0$:

$$H_Q(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} A_3$$

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Same functional forms

QCD calculations of GPDs in asymmetric frames

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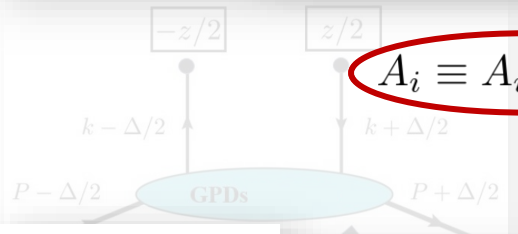
Con... contam... quasi-GPDs is



$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0} A_3$

You can think of eliminating additional amplitudes by the addition of other operators:

Key points:

In spirit of what's done for PDFs:

1) **Lorentz-invariant generalization of LC definition to $z^2 \neq 0$ might converge faster**

Lorentz-inva...

2) **Lorentz-invariant definition** ⟶ **differences suppressed by frame-independent power corrections**

Schematic structure: $H_Q \rightarrow c_0\langle\bar\psi\gamma^0\psi\rangle + c_1\langle\bar\psi\gamma^1\psi\rangle + c_2\langle\bar\psi\gamma^2\psi\rangle$

Note: Here c's are frame-dependent kinematic factors that cancel additional amplitudes to project quasi-GPD potentially faster (vs historic def.) onto light-cone GPD

QCD calculations of GPDs in asymmetric frames

Same functional forms

Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

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Lorentz-invariant generalization of LC definition to $z^2 \neq 0$:

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$A_i \equiv A_i(z^2 \neq 0)$

Key points:

1) Lorentz-invariant generalization of LC definition to $z^2 \neq 0$ might converge faster

Caveat:

However, it is essential to acknowledge that the amplitudes themselves also contain implicit power corrections. Moreover, it is worth noting that the presence of additional amplitudes in the first place could potentially serve to mitigate the implicit power corrections inherent in the amplitudes. Ultimately, the actual convergence of the different quasi-GPD definitions is determined by the underlying non-perturbative dynamics. Therefore, it is important to perform numerical comparisons

Lattice QCD calculations of GPDs in asymmetric frames

Why is the question of frame-(in)dependence relevant?

Key points:

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4]
Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

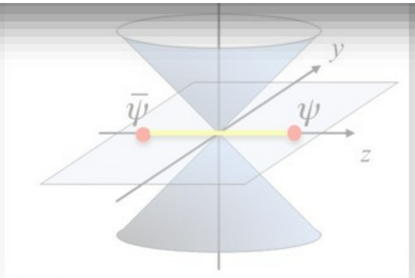
Case 2: Tra

In Preparation

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3]
Joshua Miller,[3,‡] Swagato Mukherjee,[5] Peter Petreczky,[5] Aurora Scapellato,[3] Fernanda Steffens,[6] and Yong Zhao[4]

Transverse boost: This Lor ... si-GPDs in symmetric frame



Are quasi-GPDs Lorentz-invariant?

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Definition: (Historic)

$$\widetilde{F}^3(z, P^{s/a}, \Delta^{s/a}) = \langle p_f; \lambda' | \bar{\psi}(-\tfrac{z}{2})\, \gamma^3 \gamma_5\, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \psi(\tfrac{z}{2}) | p_i; \lambda \rangle$$

$$= \bar{u}^{s/a}(p_f^{s/a}, \lambda') \left[\gamma^3 \gamma_5\, \widetilde{\mathcal{H}}_3^{s/a}(z, P^{s/a}, \Delta^{s/a}) + \frac{\Delta^3 \gamma_5}{2m}\, \widetilde{\mathcal{E}}_3^{s/a}(z, P^{s/a}, \Delta^{s/a}) \right] u^{s/a}(p_i^{s/a}, \lambda)$$

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

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GPD $\widetilde{E}$ can not be accessed at zero skewness because it simply does not contribute to the matrix element at this point

See Martha's talk :

Glimpse into GPD $\widetilde{E}$ through twist 3 at zero skewness

Lattice QCD calculations of GPDs in asymmetric frames

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

$$\widetilde{F}^{\mu} = \bar{u}(p_f,\lambda')\left[\frac{i\epsilon^{\mu P z \Delta}}{m}\widetilde{A}_1 + \gamma^{\mu}\gamma_5\widetilde{A}_2 + \gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A}_3 + m z^{\mu}\widetilde{A}_4 + \frac{\Delta^{\mu}}{m}\widetilde{A}_5\right) + m\not{z}\gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A}_6 + m z^{\mu}\widetilde{A}_7 + \frac{\Delta^{\mu}}{m}\widetilde{A}_8\right)\right]u(p_i,\lambda)$$

Axial-vector operator $\widetilde{F}^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}\gamma_5 q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$

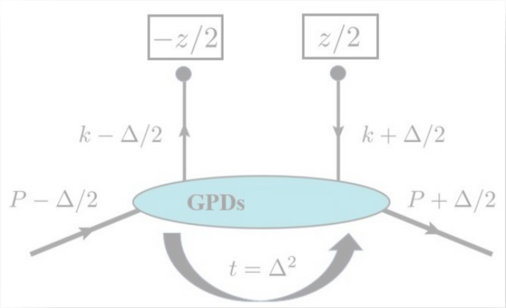
Features:

- **General structure of matrix element based on constraints from Parity**
- **8 linearly-independent Dirac structures (similar to vector case)**

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

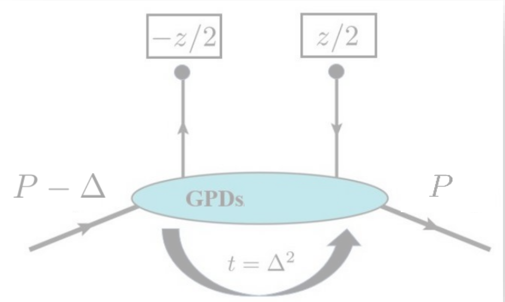
Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\widetilde{\mathcal{H}}_3(z, P^{s/a}, \Delta^{s/a}) = \widetilde{A}_2 - z^3 P^{3,s/a} \widetilde{A}_6 - m^2 (z^3)^2 \widetilde{A}_7 - z^3 \Delta^{3,s/a} \widetilde{A}_8$$

Features:

- **Same functional form in both symmetric & asymmetric frames**

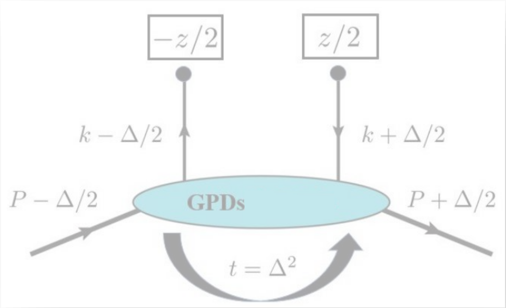


Frame-independence of $\gamma^3\gamma_5$ understood by considering "transverse boosts" that preserve the 3-component

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:

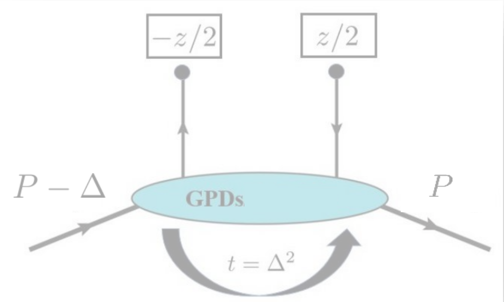


$$\widetilde{\mathcal{H}}_3(z, P^{s/a}, \Delta^{s/a}) = \widetilde{A}_2 - z^3 P^{3,s/a} \widetilde{A}_6 - m^2 (z^3)^2 \widetilde{A}_7 - z^3 \Delta^{3,s/a} \widetilde{A}_8$$

$$= \widetilde{A}_2 + (P^{s/a} \cdot z) \widetilde{A}_6 + m^2 z^2 \widetilde{A}_7 + (\Delta^{s/a} \cdot z) \widetilde{A}_8$$

Features:

- **Same functional form in both symmetric & asymmetric frames**
- **Kinematical prefactor of amplitudes can be uniquely promoted to a Lorentz-invariant status**



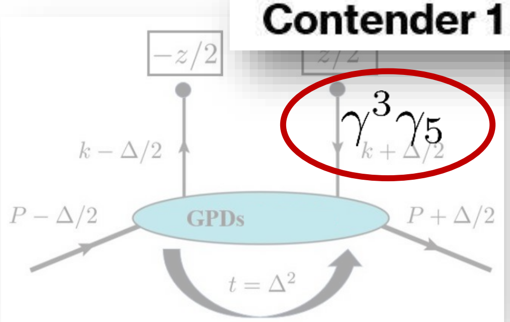
The historic definition involving $\gamma^3\gamma_5$ is a contender for a Lorentz invariant definition

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:

Contender 1

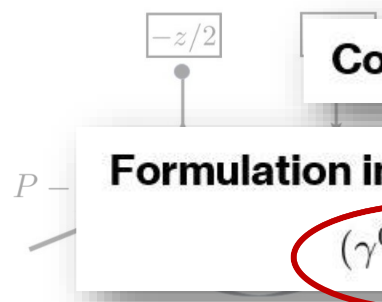


$$\widetilde{\mathcal{H}}_3(z, P^{s/a}, \Delta^{s/a}) = \widetilde{A}_2 - z^3 P^{3,s/a}\widetilde{A}_6 - m^2(z^3)^2\widetilde{A}_7 - z^3\Delta^{3,s/a}\widetilde{A}_8$$
$$= \widetilde{A}_2 + (P^{s/a}\cdot z)\widetilde{A}_6 + m^2 z^2\widetilde{A}_7 + (\Delta^{s/a}\cdot z)\widetilde{A}_8$$

Features:

- <u>**Non-uniqueness of LI definitions for quasi-GPDs**</u>

Contender 2



Formulation in terms of a new operator:

$(\gamma^0, \gamma^1, \gamma^2)\gamma_5$

Lorentz-invariant definition of LC definition to $z^2 \neq 0$:

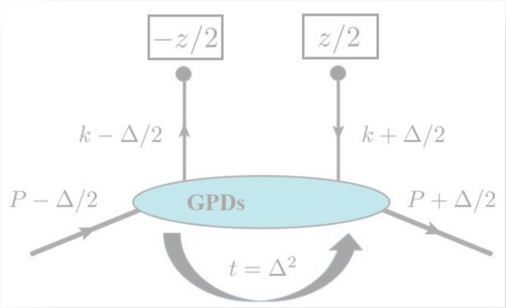
$$\widetilde{\mathcal{H}} = \widetilde{A}_2 + (P^{s/a}\cdot z)\widetilde{A}_6 + (\Delta^{s/a}\cdot z)\widetilde{A}_8 \qquad A_i \equiv A_i(z^2 \neq 0)$$

Same functional form as LC GPD

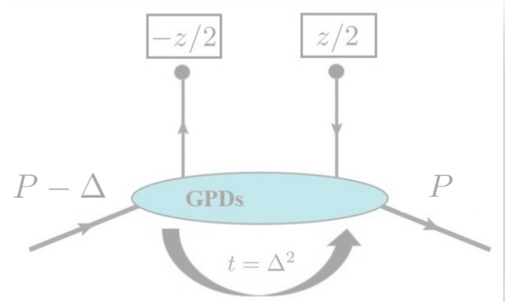
Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\widetilde{\mathcal{E}}_3(z, P^{s/a}, \Delta^{s/a}) = 2\frac{P^{3,s/a}}{\Delta^{3,s/a}}\widetilde{A}_3 + 2m^2\frac{z^3}{\Delta^{3,s/a}}\widetilde{A}_4 + 2\widetilde{A}_5$$



Features:

- $\widetilde{\mathcal{E}}$ **expression for** $\xi \neq 0$

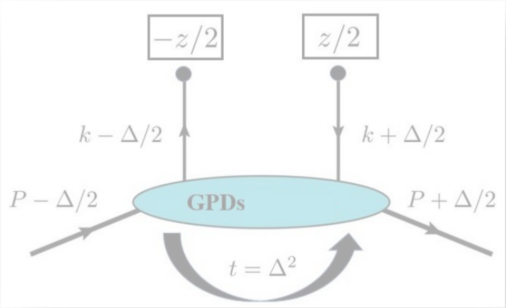
Based on symmetry arguments we expect $\widetilde{A}_{3/4}$ to exhibit at least linear scaling with respect to ξ

Hence appearance of $1/\xi$ in above expression is innocuous

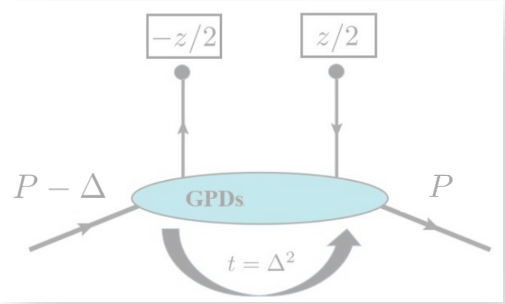
Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\widetilde{\mathcal{E}}_3(z, P^{s/a}, \Delta^{s/a}) = 2\,\frac{P^{3,s/a}}{\Delta^{3,s/a}}\,\widetilde{\boldsymbol{A}}_{\boldsymbol{3}} + 2m^2\,\frac{z^3}{\Delta^{3,s/a}}\,\widetilde{\boldsymbol{A}}_{\boldsymbol{4}} + 2\widetilde{\boldsymbol{A}}_{\boldsymbol{5}}$$



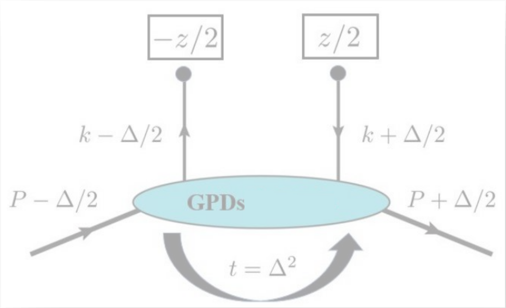
Features:

- $\widetilde{\mathcal{E}}$ **expression for** $\xi \neq 0$
- **To calculate** $\widetilde{\mathcal{E}}$ **at** $\xi = 0$ **using above expression, one needs to determine the zero-skewness limit of** $\widetilde{A}_3/\xi,\ \widetilde{A}_4/\xi$ **(well-defined limit)**

Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

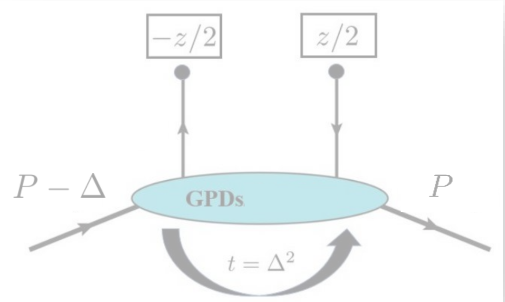
Mapping amplitudes to the historical definitions of quasi-GPDs:



$$\widetilde{\mathcal{E}}_3(z, P^{s/a}, \Delta^{s/a}) = 2\frac{P^{3,s/a}}{\Delta^{3,s/a}}\widetilde{A}_3 + 2m^2\frac{z^3}{\Delta^{3,s/a}}\widetilde{A}_4 + 2\widetilde{A}_5$$

See Joshua's talk:

Validation of formalism & Lattice QCD results



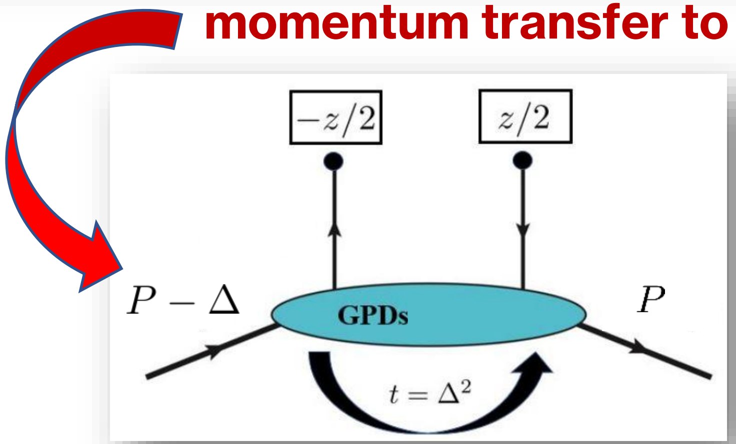
- To calculate $\widetilde{\mathcal{E}}$ at $\xi=0$ using above expression, one needs to determine the zero-skewness limit of $\widetilde{A}_3/\xi$, $\widetilde{A}_4/\xi$ (well-defined limit)

Summary

Goal: **Connecting dots: Ending with what I started with**

Perform Lattice QCD calculations of GPDs in asymmetric frames

All momentum transfer to source



Summary

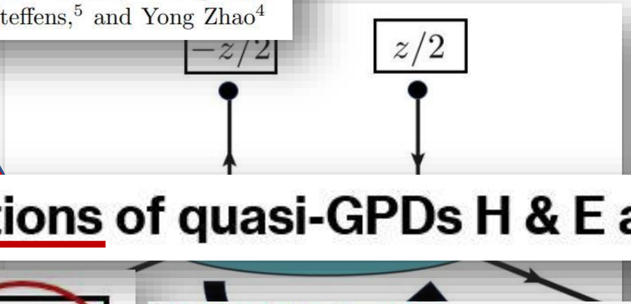
at I started with

Why is the question of frame-(in)dependence relevant?

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

All
um transfer to source



1) Historic definitions of quasi-GPDs H & E are not manifestly Lorentz invariant

$$H_Q \to c\langle\bar{\psi}\gamma^0\psi\rangle$$

Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2 \Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

Contamination from additional amplitudes or power corrections

Key findings:

e QCD calculations of

Summary

at I started with

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Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4]
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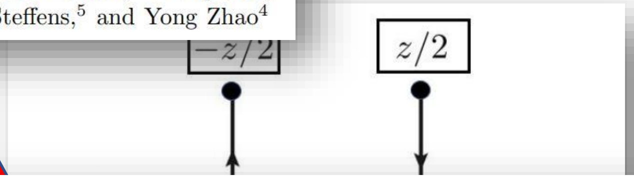

2) **Novel parameterization of position-space matrix element: (Vector case)**

$$F^{\mu}(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}A_1 + mz^{\mu}A_2 + \frac{\Delta^{\mu}}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6 + mz^{\mu}i\sigma^{z\Delta}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$

Key findings:

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**

Summary

at I started with

Why is the question of frame-(in)dependence relevant?

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

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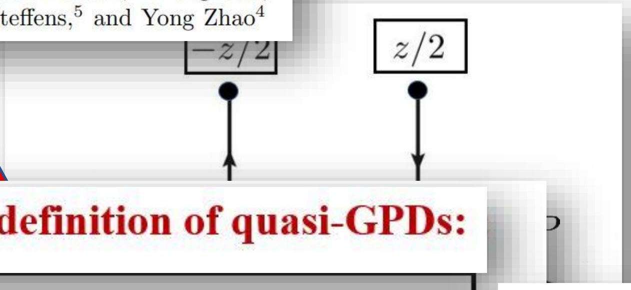

3) **Lorentz-invariant definition of quasi-GPDs:**

$$H_{\mathrm{Q}}(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

$$H_{\mathrm{Q}} \to c_0 \langle \bar{\psi}\gamma^0\psi \rangle + c_1 \langle \bar{\psi}\gamma^1\psi \rangle + c_2 \langle \bar{\psi}\gamma^2\psi \rangle$$

Same functional form as LC GPD

Key findings:

ce QCD cal... metric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

Summary

at I started with

Why is the question of frame-(in)dependence relevant?

All

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In Preparation

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Joshua Miller,[3,‡] Swagato Mukherjee,[5] Peter Petreczky,[5] Aurora Scapellato,[3] Fernanda Steffens,[6] and Yong Zhao[4]

1) **Novel parameterization of position-space matrix element: (Axial-vector case)**

$$\widetilde{F}^{\mu} = \bar{u}(p_f,\lambda')\left[\frac{i\epsilon^{\mu P z \Delta}}{m}\widetilde{A}_1 + \gamma^{\mu}\gamma_5\widetilde{A}_2 + \gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A}_3 + mz^{\mu}\widetilde{A}_4 + \frac{\Delta^{\mu}}{m}\widetilde{A}_5\right) + m\not{z}\gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A}_6 + mz^{\mu}\widetilde{A}_7 + \frac{\Delta^{\mu}}{m}\widetilde{A}_8\right)\right]u(p_i,\lambda)$$

Axial-vector operator $\widetilde{F}^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}\gamma_5 q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$

Key findings:

ce QCD

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**

Summary

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Why is the question of frame-(in)dependence relevant?

In Preparation

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Axial-vector case

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2) Contender 1: Historic definition $\gamma^3\gamma_5$

$$\widetilde{\mathcal{H}}_3(z, P^{s/a}, \Delta^{s/a}) = \widetilde{A}_2 + (P^{s/a}\cdot z)\widetilde{A}_6 + m^2 z^2 \widetilde{A}_7 + (\Delta^{s/a}\cdot z)\widetilde{A}_8$$

Contender 2: LI generalization of light-cone definition

$$\widetilde{\mathcal{H}} = \widetilde{A}_2 + (P^{s/a}\cdot z)\widetilde{A}_6 + (\Delta^{s/a}\cdot z)\widetilde{A}_8$$

Formulation in terms of a new operator: $(\gamma^0, \gamma^1, \gamma^2)\gamma_5$

Same functional form as LC GPD

Key findings:

e QCD calculations of GPDs in as

- **Lorentz covariant formalism for calculating qua**
- **Demonstrated non-uniqueness of LI definitions of quasi-GPDs**