A new approach for computing GPDs from asymmetric frames

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Based on: PhysRevD.106.114512 & In Preparation
Generalized Parton Distributions (GPDs)

**Definition:** (See for example Diehl, hep-ph/0307382)

\[
F^{[\Gamma]}(x, \Delta; \lambda, \lambda') = \frac{1}{2} \int \frac{dz}{2\pi} e^{ik \cdot z} \langle p'; \lambda' | \bar{\psi}(\frac{z}{2}) \Gamma W(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) | p; \lambda \rangle \bigg|_{z^+ = 0, \vec{z} = \vec{0}_\perp}
\]
Motivation for GPD studies

Spin sum rule & orbital angular momentum (Ji, 9603249):

\[ J^q = \int_{-1}^{1} dx \ x (H^q + E^q) \bigg|_{t=0} \]

**Imprints of chiral/trace anomalies in GPDs** (SB, Hatta, Vogelsang, 2305.09431):

**Eta-meson mass generation**

\[ \tilde{E}(x) \sim \frac{1}{l^2 - m_{\eta'}^2} \]

**Glueball mass generation**

\[ H(x), E(x) \sim \frac{1}{l^2 - m_G^2} \]

Novel avenue of GPD research

Profound physical implication of anomaly poles:

- Touches questions on mass generations, Chiral symmetry breaking, ...
Motivation for GPD studies

Spin sum rule & orbital angular momentum (ji, 9603249):

\[ J^q = \int_{-1}^{1} dx \, x (H^q + E^q) \bigg|_{t=0} \]

We need GPD measurements from Lattice QCD

Eta-meson mass generation

\[ \tilde{E}(x) \sim \frac{1}{l^2 - m_{\eta'}^2} \]

Glueball mass generation

\[ H(x), E(x) \sim \frac{1}{l^2 - m_G^2} \]

Novel avenue of GPD research

Profound physical implication of anomaly poles:
Touches questions on mass generations, Chiral symmetry breaking, ...
First Lattice QCD results of the x-dependent GPDs

Example:

Excellent progress!!!

\[ E(x) \text{-GPD} \quad H(x) \text{-GPD} \]

Unpolarized

\[ P_3 = 1.25 \text{ GeV} \quad \xi = |1/3| \]

C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)
First Lattice QCD results of the $x$-dependent GPDs

Example:

**Excellent progress!!!**

**But little hiccup...**

Traditionally, GPDs have been calculated from “symmetric frames”

**Practical drawback**

![Diagram showing momentum transfer symmetric between source & sink]

Lattice QCD calculations of GPDs in symmetric frames are expensive
Lattice QCD calculations of GPDs in asymmetric frames

Resolution:

• Perform Lattice QCD calculations of GPDs in asymmetric frames

See Joshua’s talk
Lattice QCD calculations of GPDs in asymmetric frames

Our contribution in a nutshell:

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,1,* Krzysztof Cichy,2 Martha Constantinou,3,† Jack Dodson,3 Xiang Gao,4 Andreas Metz,3 Swagato Mukherjee,1 Aurora Scapellato,3 Fernanda Steffens,5 and Yong Zhao4

In Preparation

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,1,* Krzysztof Cichy,2 Martha Constantinou,3,† Jack Dodson,3 Xiang Gao,4 Andreas Metz,3 Joshua Miller,3,† Swagato Mukherjee,5 Peter Petreczky,5 Aurora Scapellato,3 Fernanda Steffens,5 and Yong Zhao4

Key findings:

• Lorentz covariant formalism for calculating quasi-GPDs in any frame

• Elimination of power corrections potentially allowing faster convergence to light-cone GPDs

This talk
Lattice QCD calculations of GPDs in asymmetric frames

Why is the question of frame-(in)dependence relevant?

Our contribution in a nutshell:

Key points:

Example: Light-cone GPD H

\[ H(x, \xi, t) \rightarrow \int \frac{dz}{4\pi} e^{ixPz} \langle p' | \bar{q} \gamma^+ q | p \rangle \]

\[ z = (0, z^-, 0^-) \]

GPDs on the light-cone are Lorentz-invariant

\[ H(x, \xi, t) \rightarrow \int \frac{d(P \cdot z)}{4\pi} e^{ixPz} \frac{1}{P \cdot z} \langle p' | \bar{q} \not\! q | p \rangle \]

Arbitrary light-like \( z \)

Key findings:

- Lorentz covariant formalism for calculating quasi-GPDs in any frame
- Elimination of power corrections potentially allowing faster convergence to light-cone GPDs
Lattice QCD calculations of GPDs in asymmetric frames

Why is the question of frame-(in)dependence relevant?

Key point: Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,1, * Krzysztof Cichy,2 Martha Constantinou,3, † Jack Dodson,3 Xiang Gao,4 Andreas Metz,3 Swagato Mukherjee,1 Aurora Scappellato,3 Fernanda Steffens,5 and Yong Zhao4

Key findings: Generalized Parton Distributions

• Lorentz covariant formalism for calculating quasi-GPDs in any frame
• Elimination of power corrections potentially allowing faster convergence to light-cone GPDs

GPDs on the light-cone are Lorentz-invariant

Are quasi-GPDs Lorentz-invariant?
Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs

Definition of quasi-GPDs in symmetric frames: (Historical)

\[
F^0_{\lambda, \lambda'}|_s = \langle p'_s, \lambda' | \bar{q}(-z/2) \gamma^0 q(z/2) | p_s, \lambda \rangle |_{z=0, z'_\perp = 0_\perp}^{x=0, z'_\perp = 0_\perp} = \bar{u}_s(p'_s, \lambda') \left[ \gamma^0 H_{Q(0)}(z, P_s, \Delta_s) |_s + \frac{i\sigma^\mu\Delta_{\mu,s}}{2M} E_{Q(0)}(z, P_s, \Delta_s) |_s \right] u_s(p_s, \lambda)
\]
Definitions of quasi-GPDs

Historic definitions of quasi-GPDs H & E are not manifestly Lorentz invariant

Think about how $\gamma^0$ transforms under Lorentz transformation

"Transverse" Lorentz transformation

Symmetric frame

Asymmetric frame

$F_s^0 = \gamma F_0^a - \gamma / \beta F_1^a$

$\beta = \frac{E_0 - E_f}{E_0 + E_f} < 0$

$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$

"Transverse" with respect to Wilson Line
Definitions of quasi-GPDs

Historic definitions of quasi-GPDs H & E are not manifestly Lorentz invariant

Think about how $\gamma^0$ transforms under Lorentz transformation

Can we come up with a manifestly Lorentz-invariant definition of quasi-GPDs for finite values of momentum?

\[ F_s^0 = \gamma F_0^a - \gamma \beta F_1^a \]

*Transverse* with respect to Wilson Line

\[ \beta = -\frac{E_0 - E_1}{\sqrt{E_0^2 + E_1^2}} < 0 \]

\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]
# Lattice QCD calculations of GPDs in asymmetric frames

**Lorentz covariant formalism**

**Novel parameterization of position-space matrix element:** (Inspired from Meissner, Metz, Schlegel, 2009)

\[
F^\mu(z, P, \Delta) = \bar{u}(p_f, \lambda') \left[ \frac{P^\mu}{m} A_1 + m z^\mu A_2 + \frac{\Delta^\mu}{m} A_3 + i m \sigma^{\mu z} A_4 + \frac{i \sigma^{\mu \Delta}}{m} A_5 + \frac{P^\mu i \sigma^{z \Delta}}{m} A_6 + m z^\mu i \sigma^{z \Delta} A_7 + \frac{\Delta^\mu i \sigma^{z \Delta}}{m} A_8 \right] u(p_i, \lambda)
\]

**Vector operator** \[ F^\mu_{\lambda, \lambda'} = \langle p', \lambda' | q(-z/2) \gamma^\mu q(z/2) | p, \lambda \rangle \bigg|_{z=0, z_\perp=0} \]

**Features:**

- General structure of matrix element based on constraints from Parity
- 8 linearly-independent Dirac structures
- 8 Lorentz-invariant amplitudes (or Form Factors) \[ A_i \equiv A_i(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) \]
Lattice QCD calculations of GPDs in asymmetric frames

**Novel parameterization of position-space matrix element:** (Inspired from Meissner, Metz, Schlegel, 2009)

\[ F^\mu(z, P, \Delta) = \bar{u}(p_f, \lambda') \left[ \frac{P^\mu}{m} A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m} A_3 + i\sigma^\mu z A_4 + i\sigma^\mu \Delta A_5 + \frac{P^\mu i\sigma^z \Delta}{m} A_6 + mz^\mu i\sigma^z A_7 + \frac{\Delta^\mu i\sigma^z \Delta}{m} A_8 \right] u(p_i, \lambda) \]

**Main point:**
Calculate quasi-GPD in symmetric frame through matrix elements of asymmetric frame

\[ F^8 \leftrightarrow F^{\alpha} \]

**Niilo’s talk:**
Unveil GPDs through the amplitude formalism in the pseudo-distribution approach
<table>
<thead>
<tr>
<th>Lattice QCD calculations of GPDs in asymmetric frames</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Re-exploring historical definitions of quasi-GPDs</strong></td>
</tr>
<tr>
<td>Mapping amplitudes to the historical definitions of quasi-GPDs: (Sample results)</td>
</tr>
</tbody>
</table>
Lattice QCD calculations of GPDs in asymmetric frames

Re-exploring historical definitions of quasi-GPDs

Frame-dependent expressions: Explicit non-invariance from kinematics factors

**Symmetric frame:**

\[
H_{Q(0)}(z, P_s, \Delta_s)|_{s} = A_1 + \frac{\Delta_0}{P_0^{s}} A_3 \left( \frac{\Delta_0^{0}}{2P_0^{s}P_3^{s}} - \frac{\Delta_0^{0}z^3}{2M^2P_3^{s}} + \frac{\Delta_0^{0}\Delta_3^{3}P_0^{s}}{2M^2(P_3^{s})^2} - \frac{z^3\Delta_1^{3}}{2M^2P_3^{s}} \right) A_6
\]

\[
+ \left( \frac{\Delta_0^{3}z_{3}^{3}}{2M^2P_0^{s}P_3^{s}} - \frac{\Delta_0^{0}z^{3}\Delta_3^{3}}{2M^2(P_3^{s})^2} - \frac{\Delta_0^{3}z_{3}^{3}\Delta_1^{3}}{2M^2P_0^{s}P_3^{s}} \right) A_8
\]

**Asymmetric frame:**

\[
H_{Q(0)}|_{a}(z, P_a, \Delta_a) = A_1 + \frac{\Delta_0}{P_0^{a}} A_3 \left( \frac{\Delta_0^{0}}{2P_0^{a}P_3^{a}} - \frac{1}{(1 + \frac{\Delta_3^{a}}{2P_0^{a}})} + \frac{\Delta_0^{0}\Delta_3^{a}z_{3}^{3}}{4P_0^{a}(P_3^{a})^2} \right) A_4
\]

\[
+ \left( \frac{\Delta_0^{3}z_{3}^{3}}{2M^2P_0^{a}P_3^{a}} - \frac{\Delta_0^{0}z^{3}\Delta_3^{a}z_{3}^{3}}{4M^2(P_3^{a})^2} - \frac{1}{(1 + \frac{\Delta_3^{a}}{2P_0^{a}})} \right) A_6
\]

\[
+ \left( \frac{\Delta_0^{0}z^{3}\Delta_3^{a}z_{3}^{3}}{4M^2P_0^{a}(P_3^{a})^2} - \frac{1}{(1 + \frac{\Delta_3^{a}}{2P_0^{a}})} \right) A_8
\]
Relation between light-cone GPD H & amplitudes:

\[ H(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg, s/a} \cdot z} A_3 \]

Frame-dependent expressions: Explicit non-invariant representations

Symmetric frame:

\[ H_{Q(0)}(z, P_s, \Delta_s) = A_1 + \frac{\Delta_0}{P_0} A_3 - \frac{\Delta_0 \cdot z}{2 P_0 P_s} A_4 + \left( \frac{\Delta_0 \cdot z^3}{2 M^2 P_s^3} - \frac{\Delta_0 \cdot \Delta_3 \cdot z \cdot P_0}{2 M^2 (P_s^3)^2} - \frac{z^3 \Delta_1}{2 M^2 P_s^3} \right) A_6 \]

\[ + \left( \frac{(\Delta_0^0)^2 \cdot z^3}{2 M^2 P_s^3} - \frac{(\Delta_0^0) \cdot 2 \Delta_3 \cdot z \cdot P_0}{2 M^2 (P_s^3)^2} - \frac{z^3 \Delta_1}{2 M^2 P_s^3} \right) A_8 \]

Asymmetric frame:

\[ H_{Q(0)}|_{a}(z, P_a, \Delta_a) = A_1 + \frac{\Delta_0}{P_0} A_3 - \frac{\Delta_0 \cdot z}{2 P_0 P_a} A_4 + \left( \frac{\Delta_0 \cdot z^3}{2 M^2 P_a^3} - \frac{\Delta_0 \cdot \Delta_3 \cdot z \cdot P_0}{2 M^2 (P_a^3)^2} - \frac{z^3 \Delta_1}{2 M^2 P_a^3} \right) A_6 \]

\[ + \left( \frac{(\Delta_0^0)^2 \cdot z^3}{2 M^2 P_a^3} - \frac{(\Delta_0^0) \cdot 2 \Delta_3 \cdot z \cdot P_0}{2 M^2 (P_a^3)^2} - \frac{z^3 \Delta_1}{2 M^2 P_a^3} \right) A_8 \]

Lattice QCD calculations of GPDs in asymmetric frames: Re-exploring historical definitions of quasi-GPDs
Novel definition of quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs

Symmetric frame:

\[
H_{Q}(z, P_s, \Delta_s) |_{s} = A_1 + \frac{\Delta^0}{P^0_s} A_3 - \frac{(\Delta^0)_{s}^3}{2 P^0_s P^3} A_4 + \left( \frac{1}{2 M^2 P^3} \right)^2 A_6
\]

\[
+ \left( \frac{(\Delta^0)_{s}^3 z^3}{2 M^2 P^0 P^3} - \frac{(\Delta^0)_{s}^2 z^3}{2 M^2 (P^3)^2} - \frac{\Delta^0 z^3 \Delta^3}{2 M^2 P^3} \right) A_8
\]

Contamination from additional amplitudes or power corrections

\[
H_{Q}(z, P_a, \Delta_a) |_{a} = A_1 + \frac{\Delta^0}{P^0_{avg.a}} A_3 - \frac{(\Delta^0)_{a}^3}{2 P^0_{avg.a} P^3_{avg.a}} A_4
\]

\[
+ \left( \frac{1}{(1 + \frac{\Delta^3}{2 P^3_{avg.a}}) 4 P^0_{avg.a} (P^3_{avg.a})^2} \right)^2 A_6
\]

\[
+ \left( \frac{(\Delta^0)_{a}^3 z^3}{2 M^2 P^0_{avg.a} P^3_{avg.a}} - \frac{(\Delta^0)_{a}^2 z^3}{2 M^2 (P^3_{avg.a})^2} - \frac{\Delta^0 z^3 \Delta^3}{2 M^2 P^3_{avg.a}} \right) A_8
\]
Novel definition of quasi-GPDs

Contrary to quasi-PDFs, $\gamma^0$ operator for quasi-GPDs is contaminated with additional amplitudes or power corrections.

In spirit of what’s done for PDFs:

Asymmetric frame:
Lattice QCD calculations of GPDs in asymmetric frames

Contrary to quasi-PDFs, $\gamma^0$ operator for quasi-GPDs is contaminated with additional amplitudes or power corrections.

**Main finding:**

Schematic structure: $H_Q \rightarrow c_0 \langle \bar{\psi} \gamma^0 \psi \rangle + c_1 \langle \bar{\psi} \gamma^1 \psi \rangle + c_2 \langle \bar{\psi} \gamma^2 \psi \rangle$

Note: Here c’s are frame-dependent kinematic factors that cancel additional amplitudes.

**Lorentz-invariant definition of quasi-GPDs:**

**In spirit of what’s done for PDFs:**

**Relation between light-cone GPD H & amplitudes:**

$H(\mathbf{z} \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} A_3$

**Novel definition of quasi-GPDs**

Mapping amplitudes to the historical definitions of quasi-GPDs: (Sample results)

Lattice QCD calculations of GPDs in asymmetric frames
**Main finding:**

Schematic structure: \( H_Q \rightarrow c_0 \langle \bar{\psi} \gamma^0 \psi \rangle + c_1 \langle \bar{\psi} \gamma^1 \psi \rangle + c_2 \langle \bar{\psi} \gamma^2 \psi \rangle \)

Note: Here \( c \)'s are frame-dependent kinematic factors that cancel additional amplitudes to project quasi-GPD potentially faster (vs historic def.) onto light-cone GPD
QCD calculations of GPDs in asymmetric frames

\[ H(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} A_3 \]

**Key points:**

1) Lorentz-invariant generalization of LC definition to \( z^2 \neq 0 \) might converge faster

2) Lorentz-invariant definition \( \rightarrow \) differences suppressed by frame-independent power corrections

Schematic structure:

\[ H_Q \rightarrow c_0 \langle \bar{\psi} \gamma^0 \psi \rangle + c_1 \langle \bar{\psi} \gamma^1 \psi \rangle + c_2 \langle \bar{\psi} \gamma^2 \psi \rangle \]

Note: Here \( c \)'s are frame-dependent kinematic factors that cancel additional amplitudes to project quasi-GPD potentially faster (vs historic def.) onto light-cone GPD
QCD calculations of GPDs in asymmetric frames

Relation between light-cone GPD H & amplitudes:

\[ H(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} A_3 \]

Lorentz-invariant generalization of LC definition to \( z^2 \neq 0 \):

\[ H_Q(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} A_3 \]

Key points:

1) Lorentz-invariant generalization of LC definition to \( z^2 \neq 0 \) might converge faster

Caveat:

amplitudes themselves also contain implicit power corrections. Moreover, it is worth noting that the presence of additional amplitudes in the first place could potentially serve to mitigate the implicit power corrections inherent in the amplitudes. Ultimately, the actual convergence of the different quasi-GPD definitions is determined by the underlying non-perturbative dynamics. Therefore, it is important to perform numerical comparisons.
Lattice QCD calculations of GPDs in asymmetric frames

Why is the question of frame-(in)dependence relevant?

Key points:

Case 2: Transverse boost in the x-direction
Operator distance remains spatial (same)
Results:
Symmetric & asymmetric frames Related via Lorentz transformation? What kind?
Lattice QCD calculations of GPDs in asymmetric frames

In Preparation

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,1, * Krzysztof Cichy,2 Martha Constantinou,3, † Jack Dodson,3 Xiang Gao,4 Andreas Metz,3 Swagato Mukherjee,1 Aurora Scapellato,3 Fernanda Steffens,5 and Yong Zhao4
Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Definition: (Historic)

\[
\tilde{F}^3(z, P^{s/a}, \Delta^{s/a}) = \langle p_f; \lambda' | \bar{\psi}(-\frac{z}{2}) \gamma^3 \gamma_5 \mathcal{W}(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) | p_i; \lambda \rangle
\]

\[
= \bar{u}^{s/a}(p^s_f, \lambda') \left[ \gamma^3 \gamma_5 \tilde{H}^{s/a}_3(z, P^{s/a}, \Delta^{s/a}) + \frac{\Delta^3 \gamma_5}{2m} \tilde{E}^{s/a}_3(z, P^{s/a}, \Delta^{s/a}) \right] u^{s/a}(p^s_i, \lambda)
\]
Lattice QCD calculations of GPDs in asymmetric frames

**Helicity quasi-GPDs**

**Definition: (Historic)**

\[
\tilde{F}^3(z, P^{s/a}, \Delta^{s/a}) = \langle p_f; \lambda' | \bar{\psi}(-\frac{z}{2}) \gamma^3 \gamma_5 \mathcal{W}(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) | p_i; \lambda \rangle \\
= \bar{u}^{s/a}(p_f^{s/a}, \lambda') \left[ \gamma^3 \gamma_5 \tilde{H}^{s/a}_3(z, P^{s/a}, \Delta^{s/a}) - \frac{\Delta^{s/a} \gamma_5}{2m} \tilde{E}^{s/a}_3(z, P^{s/a}, \Delta^{s/a}) \right] u^{s/a}(p_i^{s/a}, \lambda)
\]

**GPD** $\tilde{E}$ **can not be accessed at zero skewness because it simply does not contribute to the matrix element at this point**

See Martha’s talk:

Glimpse into GPD $\tilde{E}$ through twist 3 at zero skewness
Lattice QCD calculations of GPDs in asymmetric frames

Novel parameterization of position-space matrix element:

$$\tilde{F}^\mu = \bar{u}(p_f, \lambda') \left[ \frac{i\epsilon^{\mu \nu \rho \sigma}}{m} \tilde{A}_1 + \gamma^\mu \gamma_5 \tilde{A}_2 \right. + \gamma_5 \left( \frac{P^\mu}{m} \tilde{A}_3 + mz^\mu \tilde{A}_4 + \frac{\Delta^\mu}{m} \tilde{A}_5 \right) + m \gamma_5 \left( \frac{P^\mu}{m} \tilde{A}_6 + mz^\mu \tilde{A}_7 + \frac{\Delta^\mu}{m} \tilde{A}_8 \right) \bigg] u(p_i, \lambda)$$

Axial-vector operator $$\tilde{F}^\mu_{\lambda,\lambda'} = \langle p', \lambda' | \bar{q}(z/2) \gamma^\mu \gamma_5 q(z/2) | p, \lambda \rangle \bigg|_{z=0, \tilde{z}_\perp = 0}$$

Features:

- General structure of matrix element based on constraints from Parity
- 8 linearly-independent Dirac structures (similar to vector case)
Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:

\[ \tilde{H}_3(z, P^{s/a}, \Delta^{s/a}) = \tilde{A}_2 - z^3 P^{3,s/a} \tilde{A}_6 - m^2 (z^3)^2 \tilde{A}_7 - z^3 \Delta^{3,s/a} \tilde{A}_8 \]

Features:

- Same functional form in both symmetric & asymmetric frames

Frame-independence of \( \gamma^3 \gamma_5 \) understood by considering "transverse boosts" that preserve the 3-component
Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:

\[
\hat{H}_3(z, P^{s/a}, \Delta^{s/a}) = \tilde{A}_2 - z^3 P^{3,s/a} \tilde{A}_6 - m^2(z^3)^2 \tilde{A}_7 - z^3 \Delta^{3,s/a} \tilde{A}_8
\]

\[
= \tilde{A}_2 + (P^{s/a} \cdot z) \tilde{A}_6 + m^2 z^2 \tilde{A}_7 + (\Delta^{s/a} \cdot z) \tilde{A}_8
\]

Features:

- Same functional form in both symmetric & asymmetric frames
- Kinematical prefactor of amplitudes can be uniquely promoted to a Lorentz-invariant status

The historic definition involving \( \gamma^3\gamma_5 \) is a contender for a Lorentz invariant definition
Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:

\[
\tilde{H}_3(z, P^{s/a}, \Delta^{s/a}) = \tilde{A}_2 - z^3 P^{3,s/a} \tilde{A}_6 - m^2 (z^3)^2 \tilde{A}_7 - z^3 \Delta^{3,s/a} \tilde{A}_8
\]

\[
= \tilde{A}_2 + (P^{s/a} \cdot z) \tilde{A}_6 + m^2 z^2 \tilde{A}_7 + (\Delta^{s/a} \cdot z) \tilde{A}_8
\]

Features:

- Non-uniqueness of LI definitions for quasi-GPDs

Formulation in terms of a new operator:

\[(\gamma^0, \gamma^1, \gamma^2)\gamma_5\]

Lorentz-invariant definition of LC definition to \(z^2 \neq 0\):

\[
\tilde{H} = \tilde{A}_2 + (P^{s/a} \cdot z) \tilde{A}_6 + (\Delta^{s/a} \cdot z) \tilde{A}_8, \quad A_i = A_i(z^2 \neq 0)
\]

Same functional form as LC GPD
Lattice QCD calculations of GPDs in asymmetric frames

Helicity quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs:

$$\tilde{\mathcal{E}}_3(z, P^{s/a}, \Delta^{s/a}) = 2 \frac{P^{3,s/a}}{\Delta^{3,s/a}} \tilde{A}_3 + 2m^2 \frac{z^3}{\Delta^{3,s/a}} \tilde{A}_4 + 2 \tilde{A}_5$$

Features:

- $\tilde{\mathcal{E}}$ expression for $\xi \neq 0$

Based on symmetry arguments we expect $\tilde{A}_{3/4}$ to exhibit at least linear scaling with respect to $\xi$.

Hence appearance of $1/\xi$ in above expression is innocuous.
Lattice QCD calculations of GPDs in asymmetric frames

**Helicity quasi-GPDs**

Mapping amplitudes to the historical definitions of quasi-GPDs:

\[
\mathcal{E}_3(z, P^{s/a}, \Delta^{s/a}) = 2 \frac{P^{3,s/a}}{\Delta^{3,s/a}} \mathcal{A}_3 + 2m^2 \frac{z^3}{\Delta^{3,s/a}} \mathcal{A}_4 + 2 \mathcal{A}_5
\]

Features:

- \( \mathcal{E} \) expression for \( \xi \neq 0 \)

- To calculate \( \mathcal{E} \) at \( \xi = 0 \) using above expression, one needs to determine the zero-skewness limit of \( \mathcal{A}_3/\xi, \mathcal{A}_4/\xi \) (well-defined limit)
Lattice QCD calculations of GPDs in asymmetric frames

**Helicity quasi-GPDs**

Mapping amplitudes to the historical definitions of quasi-GPDs:

\[
\tilde{E}_3\left(z, P^{s/a}, \Delta^{s/a}\right) = 2 \frac{P^{s/a}}{\Delta^{s/a}} \tilde{A}_3 + 2m^2 \frac{z^3}{\Delta^{s/a}} \tilde{A}_4 + 2\tilde{A}_5
\]

See Joshua’s talk:

Validation of formalism & Lattice QCD results

- To calculate \( \tilde{E} \) at \( \xi = 0 \) using above expression, one needs to determine the zero-skewness limit of \( \tilde{A}_3/\xi, \tilde{A}_4/\xi \) (well-defined limit)
Goal: Connecting dots: Ending with what I started with

Perform Lattice QCD calculations of GPDs in asymmetric frames

All momentum transfer to source
Connecting dots: Ending with what I started with

**Summary**

**Why is the question of frame-(in)dependence relevant?**

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,1,* Krzysztof Cichy,2 Martha Constantinou,3,† Jack Dodson,3 Xiang Gao,4 Andreas Metz,4 Swagato Mukherjee,1 Aurora Scopellito,3 Fernanda Steffen,3 and Yong Zhao4

1) **Historic definitions of quasi-GPDs H & E are not manifestly Lorentz invariant**

Key findings:

Contamination from additional amplitudes or power corrections
Why is the question of frame-(in)dependence relevant?

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Shohini Bhattacharya,1, * Krzysztof Cichy,2 Martha Constantinou,3,1 Jack Dodson,3 Xiang Gao,3 Andreas Metz,3 Swagato Mukherjee,1 Aurora Scapellato,3 Fernanda Steffens,3 and Yong Zhao4

All momentum transfer to source

2) Novel parameterization of position-space matrix element: (Vector case)

\[ F^\mu(z, P, \Delta) = \bar{u}(p_f, \lambda) \left[ \frac{P^\mu}{m} A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m} A_3 + i m \sigma^{\mu z} A_4 + \frac{i \sigma^{\mu \Delta}}{m} A_5 + \frac{P^\mu i \sigma^{z \Delta}}{m} A_6 + mz^\mu i \sigma^{z \Delta} A_7 + \frac{\Delta^\mu i \sigma^{z \Delta}}{m} A_8 \right] u(p_i, \lambda) \]

Key findings:

- Lorentz covariant formalism for calculating quasi-GPDs in any frame
Summary

Why is the question of frame-(in)dependence relevant?

3) Lorentz-invariant definition of quasi-GPDs:

\[
H_Q(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_3}{P_{avg, \Delta} \cdot z} A_3
\]

Key findings:

- Lorentz covariant formalism for calculating quasi-GPDs in any frame
- Elimination of power corrections potentially allowing faster convergence to light-cone GPDs
Summary

Why is the question of frame-(in)dependence relevant?

**Key findings:**

- Lorentz covariant formalism for calculating quasi-GPDs in any frame
Summary

Why is the question of frame-(in)dependence relevant?

Key findings:
- Lorentz covariant formalism for calculating quasi-GPDs
- Demonstrated non-uniqueness of LI definitions of quasi-GPDs
- Lattice QCD calculations of GPDs in asymmetric frames

In Preparation: Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Axial-vector case

Shobini Bhattacharya,1, * Krzysztof Cichy,2 Martha Constantinou,3, 1 Jack Dodson,3 Xiang Gao,4 Andreas Metsa,5 Joshua Miller,3, 1 Swagato Mukherjee,6 Peter Petrovsky,7 Aurora Scapellato,8 Fernanda Steffen,9 and Yong Zhao1

2) Contender 1: Historic definition $\gamma^3 \gamma_5$

$\widetilde{\mathcal{H}}(x, P^{s/a}, \Delta^{s/a}) = \widetilde{A}_2 + (P^{s/a} \cdot z) \widetilde{A}_6 + m^2 z^2 \widetilde{A}_7 + (\Delta^{s/a} \cdot z) \widetilde{A}_8$

Contender 2: LI generalization of light-cone definition

$\tilde{H} = \tilde{A}_2 + (P^{s/a} \cdot z) \tilde{A}_6 + (\Delta^{s/a} \cdot z) \tilde{A}_8$

Formulation in terms of a new operator:
$(\gamma^n, \gamma^1, \gamma^2)_{\gamma_5}$

Same functional form as LC GPD