

Doubly charmed tetraquark T_{cc}^+

in (2+1)-flavor QCD near physical point

Sinya Aoki

Center for Gravitational Physics and Quantum Information,
Yukawa Institute for Theoretical Physics, Kyoto University



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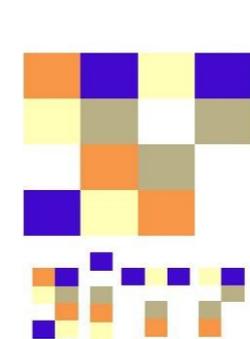
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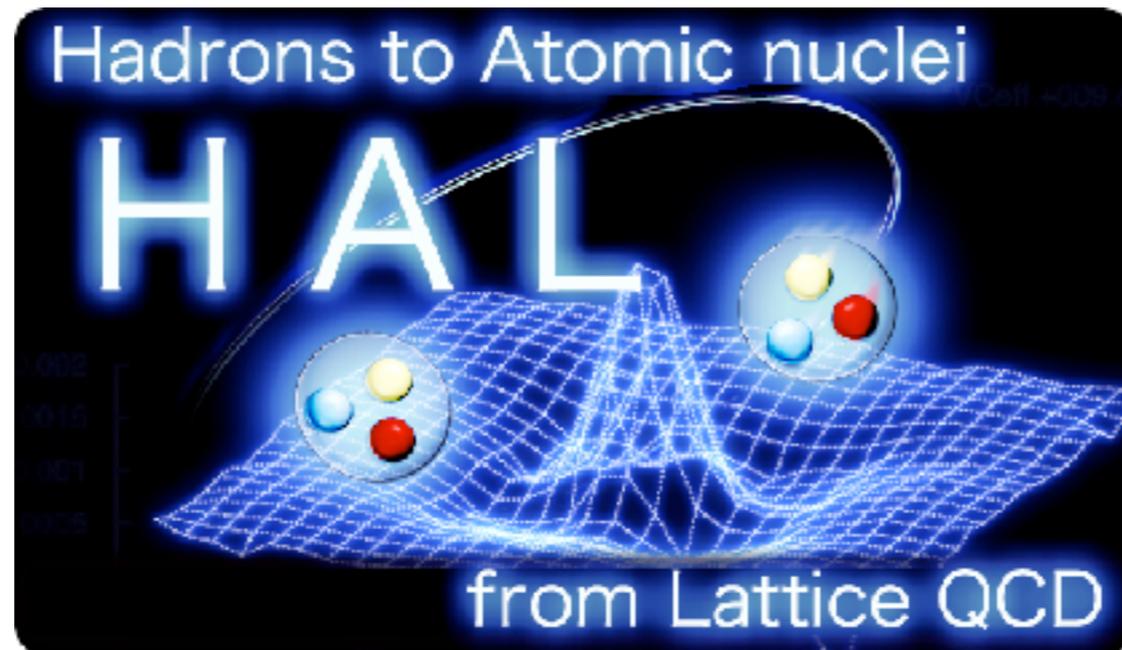
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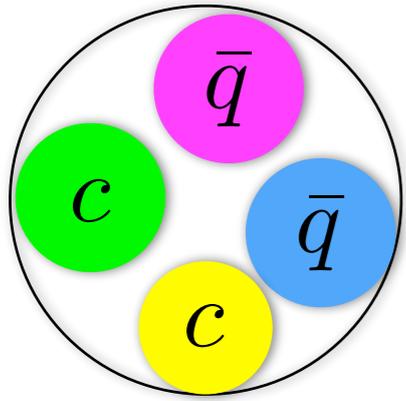
References

Yan Lyu, Sinya Aoki, Takumi Doi, Tetsuo Hatsuda, Yoichi Ikeda and Jie Meng,
“Doubly Charmed Tetraquark T_{cc}^+ from Lattice QCD near Physical Point”, arXiv:2302.04505.



for
HAL QCD collaboration

Heavy tetra-quark states T_{cc}



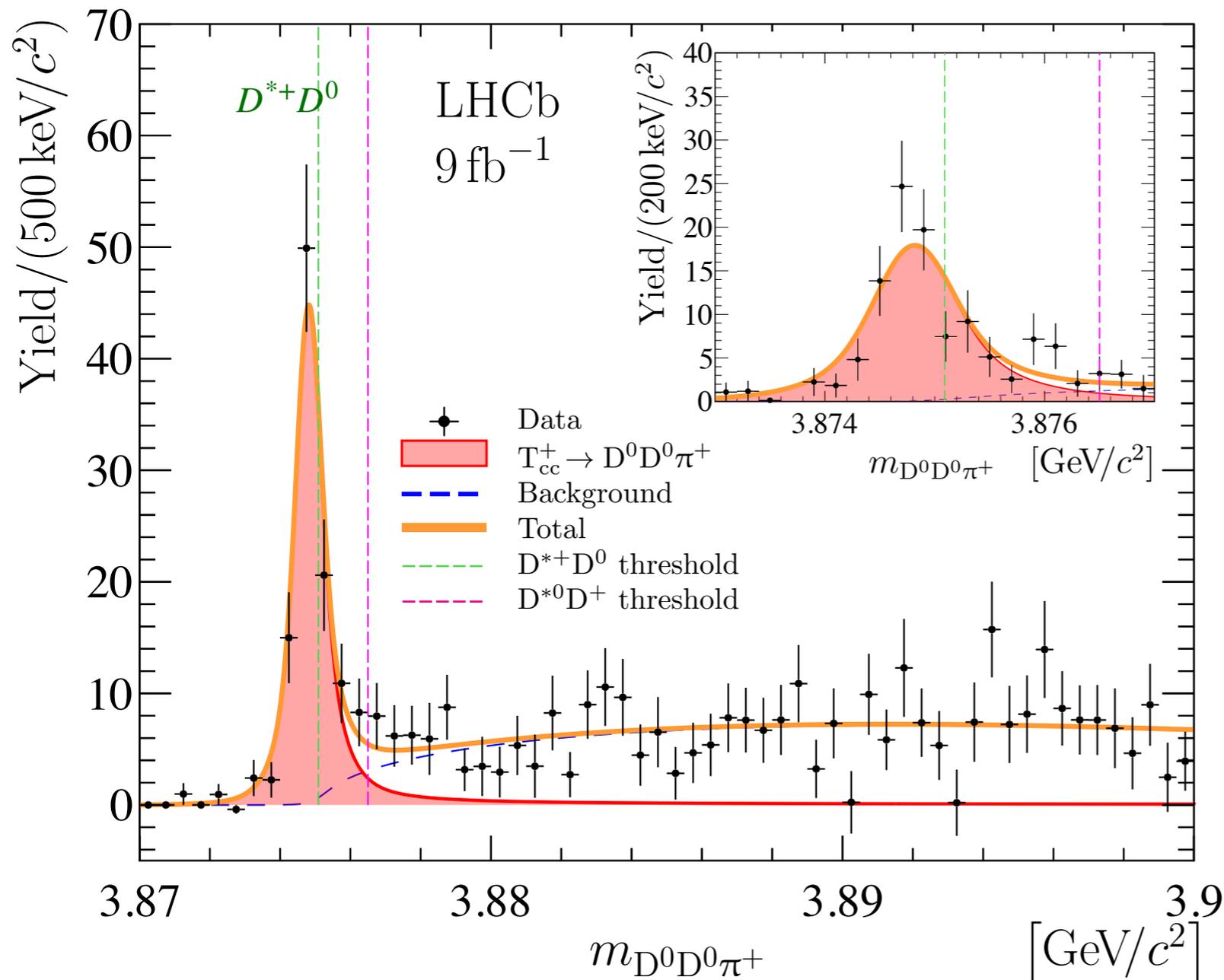
\bar{q} : light anti-quark

genuine tetra-quark states

$T_{cc}(cc\bar{u}\bar{d})$

observation by LHCb.

Aaij et al. (LHCb Collaboration),
Nature Phys. (2022)



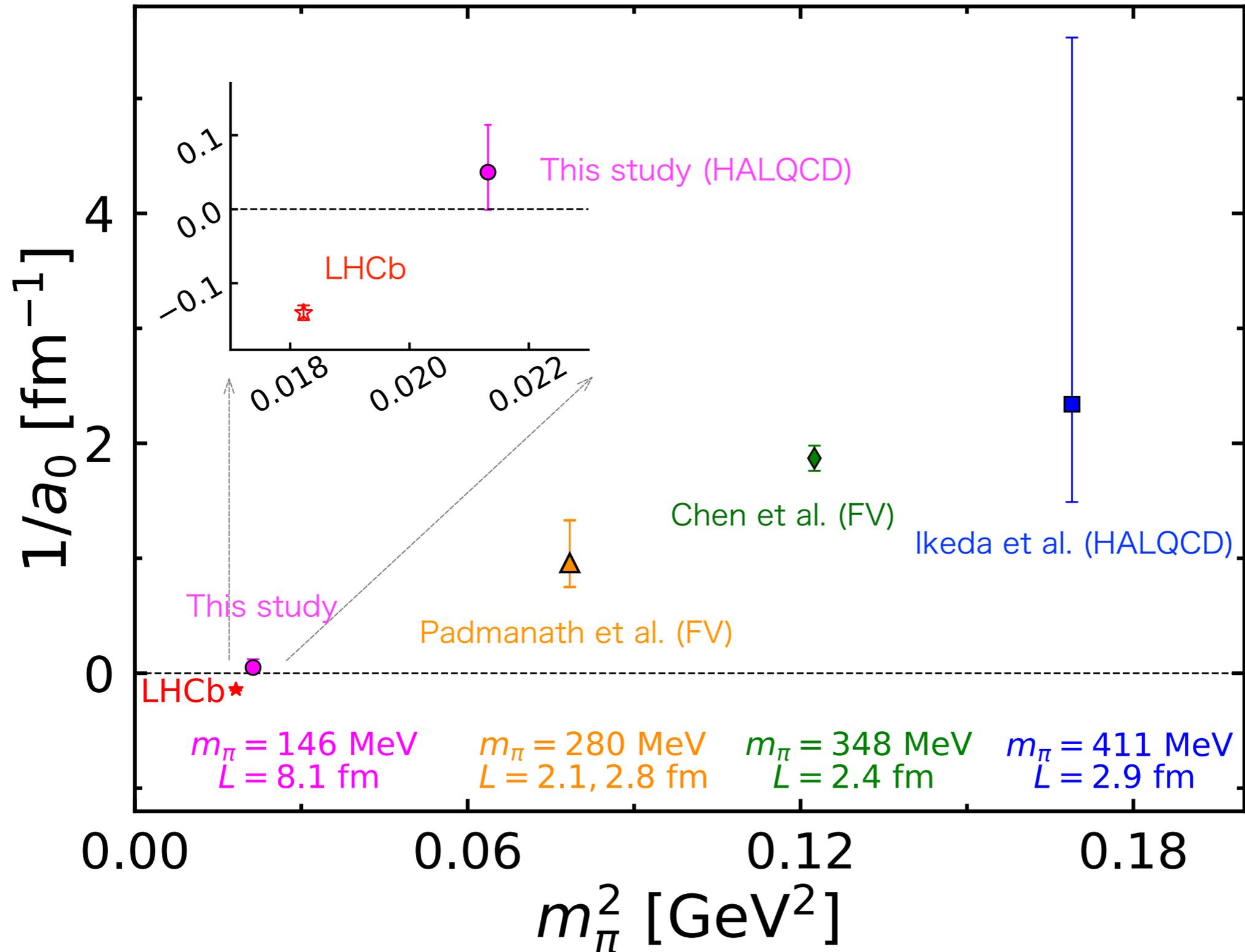
360 keV below $D^{*+} D^0$ threshold

$$(I, J^P) = (0, 1^+)$$

(Latest) Lattice QCD results

inverse scattering length

significant pion mass dependence



HAL QCD method

N. Ishii, S. Aoki, T. Hatsuda, PRL99(2007) 022001.

Nambu-Bethe-Salpeter (NBS) wave function

$$\psi_W^{H_1+H_2}(\mathbf{r})e^{-Wt} := \frac{1}{\sqrt{Z_{H_1}}} \frac{1}{\sqrt{Z_{H_2}}} \sum_{\mathbf{x}} \langle 0 | H_1(\mathbf{x} + \mathbf{r}, t) H_2(\mathbf{x}, t) | (H_1 + H_2); W \rangle$$

Non-local potential

$$\left(\frac{\nabla^2}{2\mu} + \frac{p_W^2}{2\mu} \right) \psi_W(\mathbf{r}) = \int d^3\mathbf{r}' U(\mathbf{r}, \mathbf{r}') \psi_W(\mathbf{r}'),$$

derivative expansion

$$U(\mathbf{r}, \mathbf{r}') = V(\mathbf{r}, \nabla) \delta(\mathbf{r} - \mathbf{r}') = \sum_{k=0}^{\infty} V^{(k)}(\mathbf{r}) \nabla^k \delta(\mathbf{r} - \mathbf{r}')$$

Leading order (LO) potential

$$V^{(0)}(\mathbf{r}; W) = \frac{1}{\psi_W(\mathbf{r})} \left(\frac{\nabla^2}{2\mu} + \frac{p_W^2}{2\mu} \right) \psi_W(\mathbf{r})$$

+ Some improvements

time-dependent HAL QCD method

Ishii *et al.* (HAL QCD), PLB712(2012)437.

partial wave decomposition

Miyamoto *et al.* (HAL QCD), PRD101(2020)074514.

higher order terms in the derivative expansion

Iritani *et al.* (HAL QCD), PRD99(2019)014514.

Lattice setup

2+1 flavor gauge configuration on 96^4 lattice

with Iwasaki gauge + NP $O(a)$ improved clover quark

$a \simeq 0.0846$ fm, $m_\pi \simeq 146$ MeV, $m_K \simeq 525$ MeV (near **physical point**)

$La \simeq 8.1$ fm

Ishikawa *et al.* (PACS), PoS Lattice2015(2016) 075.

relativistic heavy quark action (Tsukuba-type) for quenched charm quark

S. Aoki, Y. Kuramashi, S-i. Tominaga, PTEP 109(2003) 383.

charm quark mass from a spin-averaged mass of charmonium

	$(m_{\eta_c} + 3m_{J/\Psi})/4$ [MeV]	$m_{\Omega_{ccc}}$ [MeV]
set 1	3096.6(0.3)	4837.3(0.7)
set 2	3051.4(0.3)	4770.2(0.7)
Interpolation	3068.5(0.3)	4795.6(0.7)
Exp.	3068.5(0.1)	-

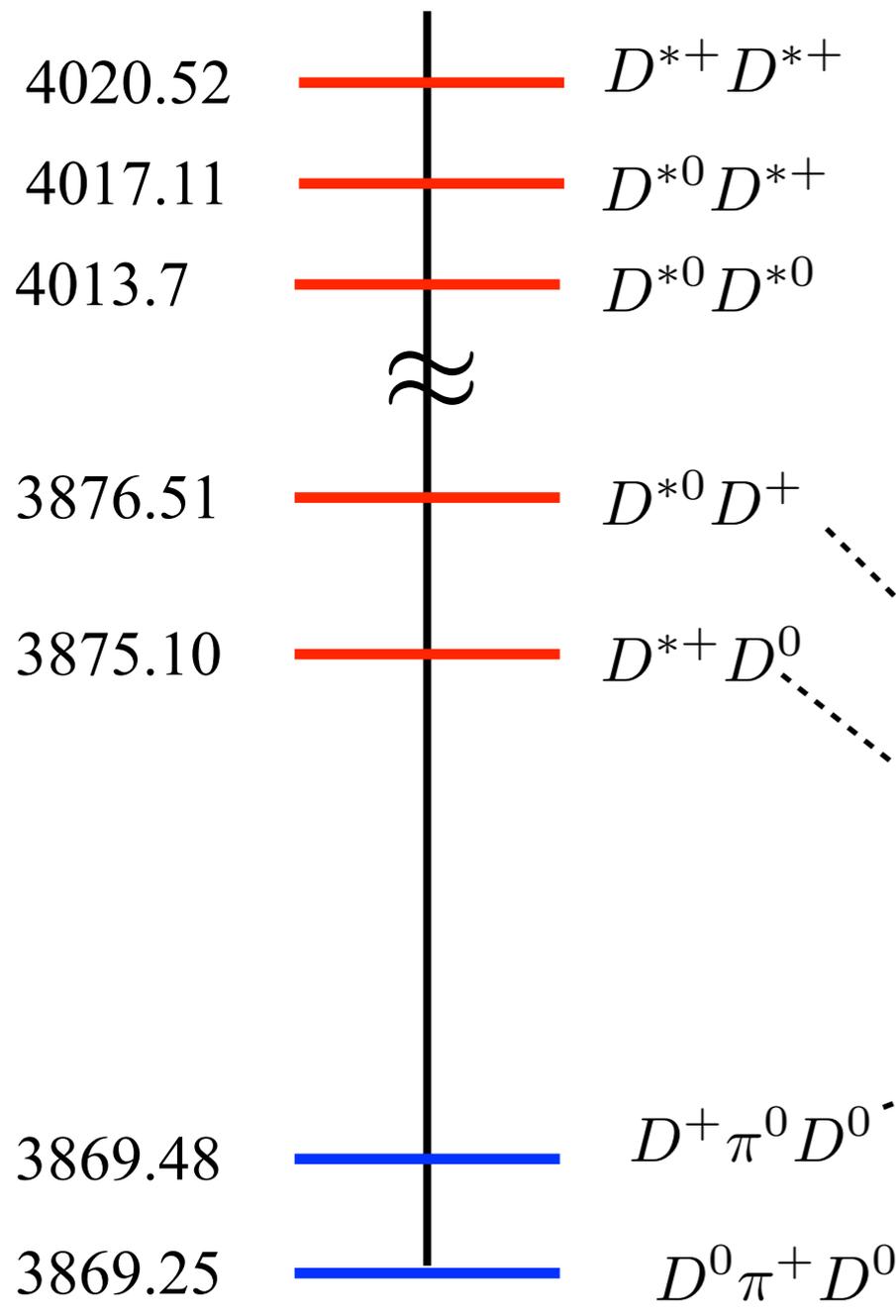
c.f. Y. Namekawa,(PACS), PoS Lattice2016(2017) 125.

calculate D^*D potential

isospin-symmetric, single channel

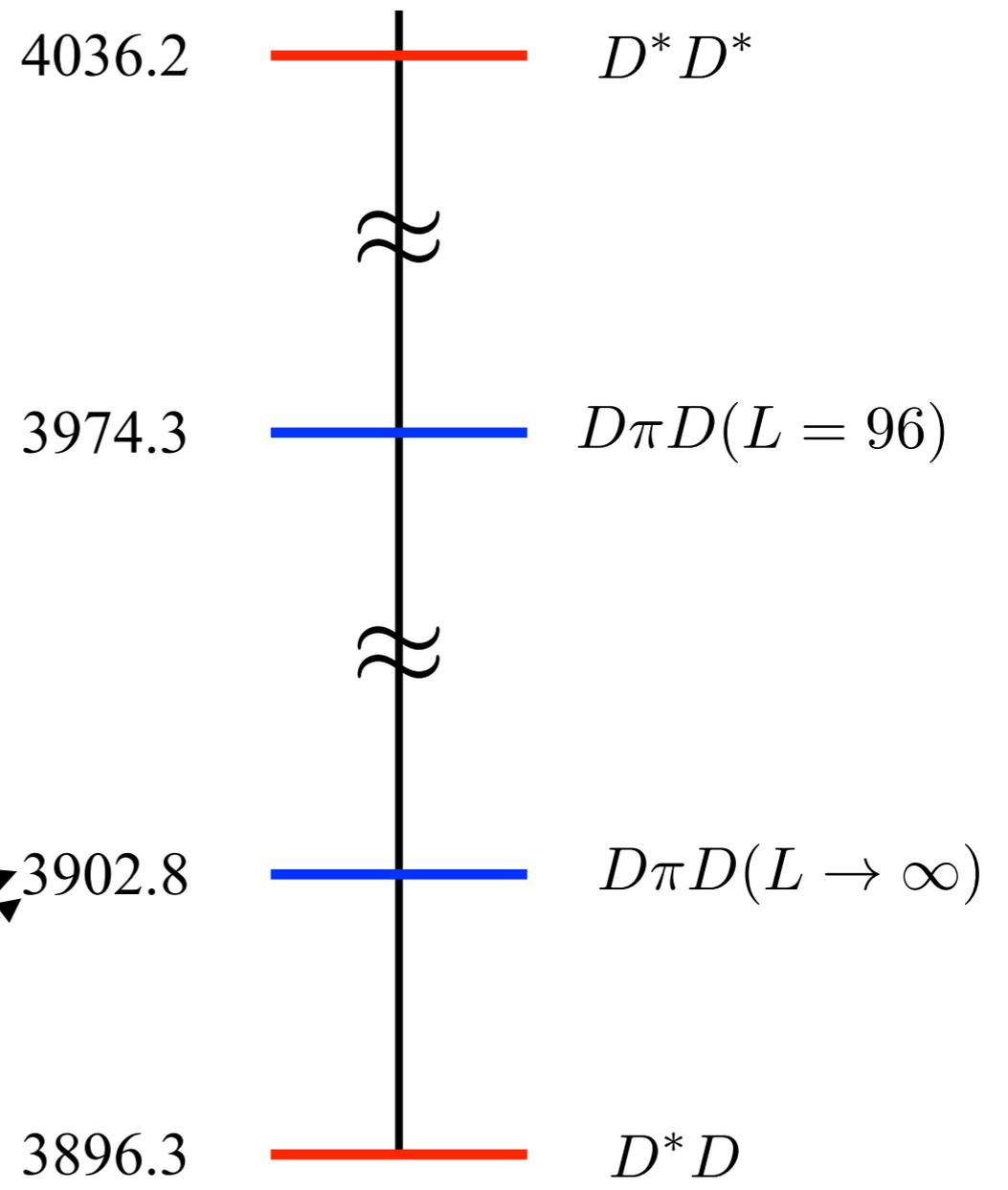
Nature

$\pi^0(134.98)$ $\pi^+(139.57)$
 $D^0(1864.84)$ $D^+(1869.66)$
 $D^{*0}(2006.85)$ $D^{*+}(2010.26)$



Lattice

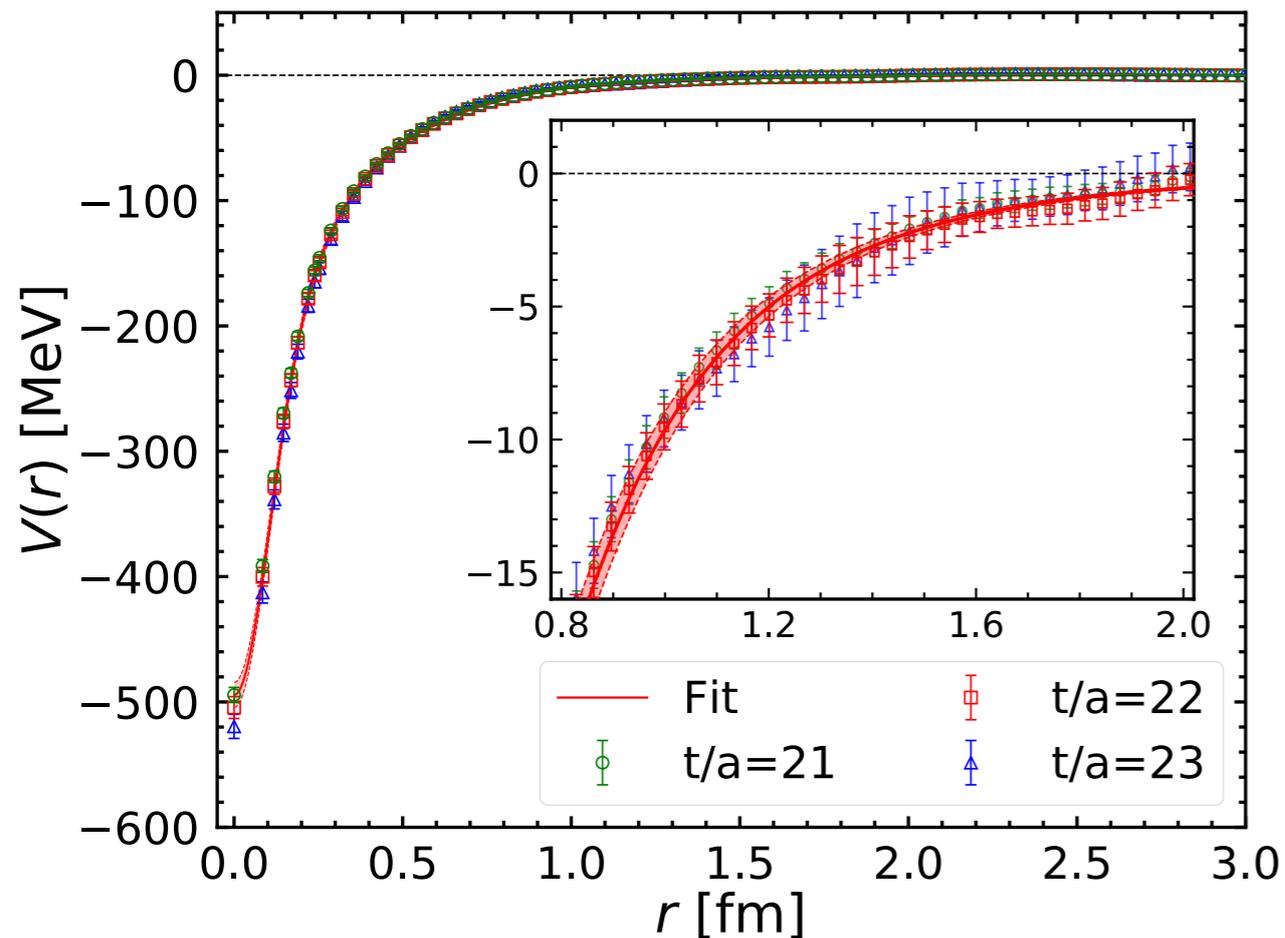
$\pi(146.4)$
 $D(1878.2)$
 $D^*(2018.1)$



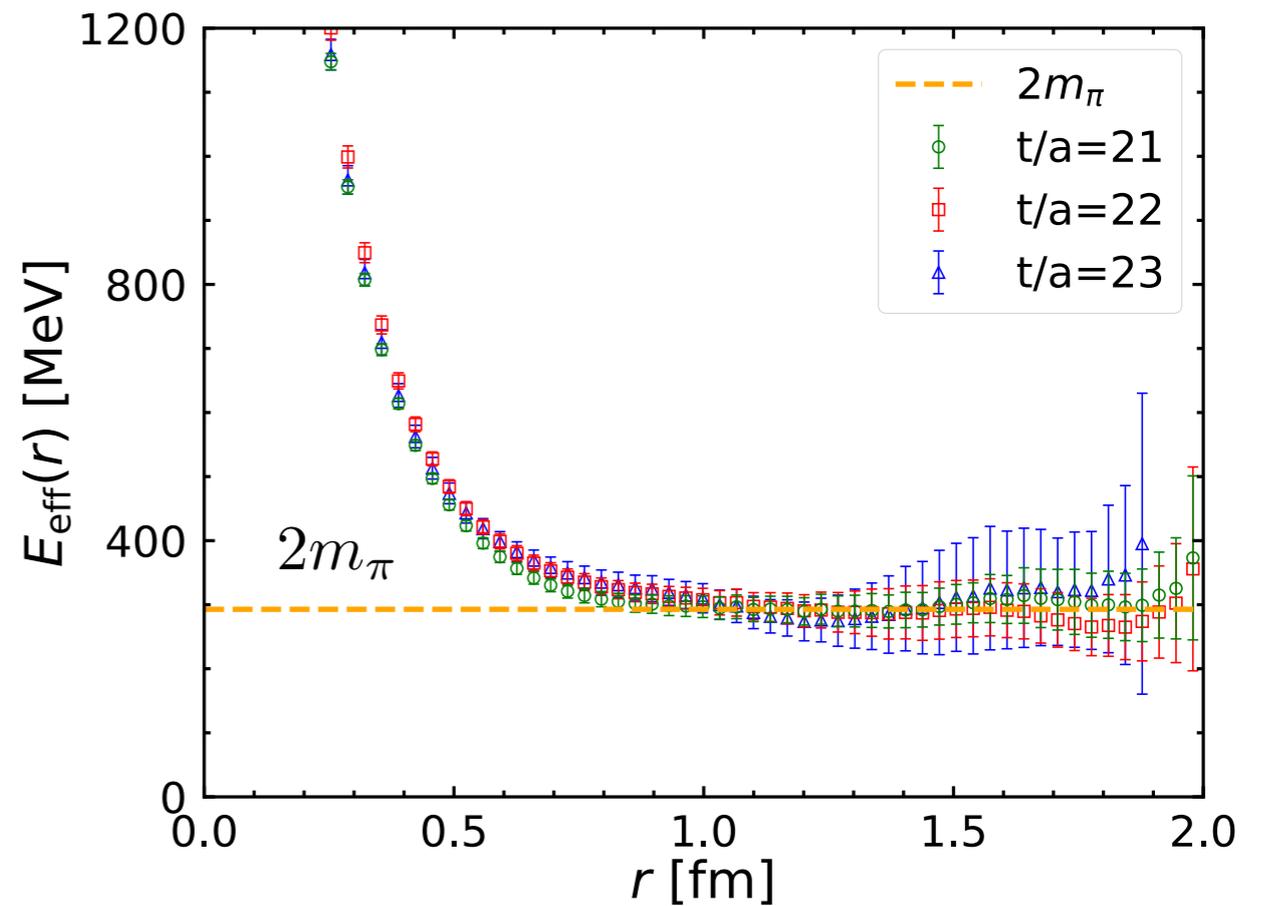
D^*D Potentials

$m_\pi \simeq 146$ MeV

D^*D potential



Effective energy in space



attractive at all distances

$$E_{\text{eff}}(r) = -\frac{\ln[V(r)r^2/a_3]}{r}$$

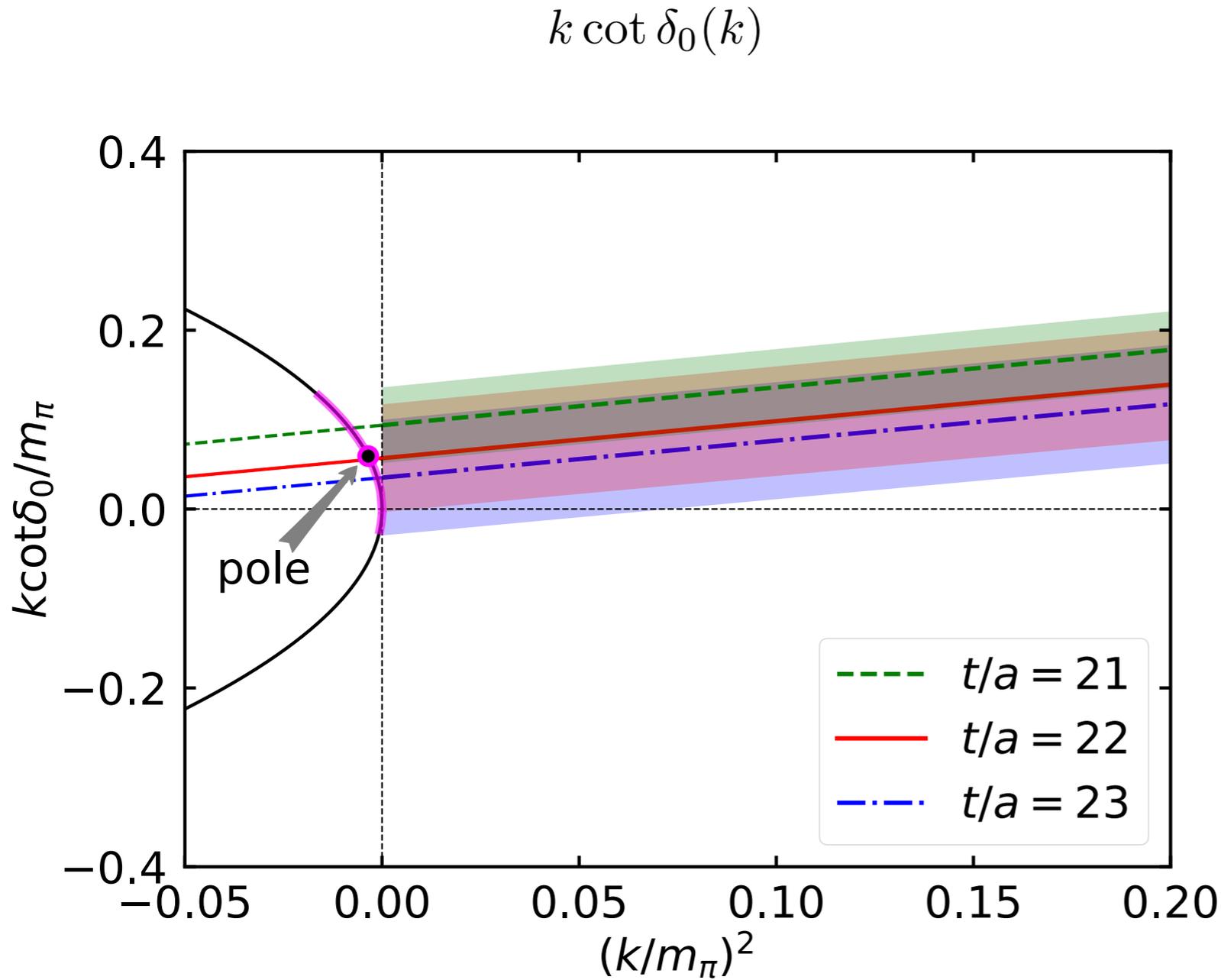
2-Gauss + Yukawa²

consistent with Yukawa² at large r

$$V_{\text{fit}}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r} \right)^2$$

Scattering phase shift

$$m_\pi \simeq 146 \text{ MeV}$$



ERE (effective Range Expansion)

$$k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + O(k^4)$$

$$\frac{1}{a_0} [\text{fm}^{-1}] = 0.05(5) \left(\begin{matrix} +2 \\ -2 \end{matrix} \right)$$

one shallow “virtual” state

$$\kappa_{\text{pole}} = -8(8) \left(\begin{matrix} +3 \\ -5 \end{matrix} \right) \text{ MeV}$$

$$E_{\text{pole}} = -59 \left(\begin{matrix} +53 \\ -99 \end{matrix} \right) \left(\begin{matrix} +2 \\ -67 \end{matrix} \right) \text{ keV}$$

“chiral” correction to the potential

2-Gauss + Yukawa² $V_{\text{fit}}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r} \right)^2$

$m_\pi : 146.4 \text{ MeV} \rightarrow 135 \text{ MeV}$

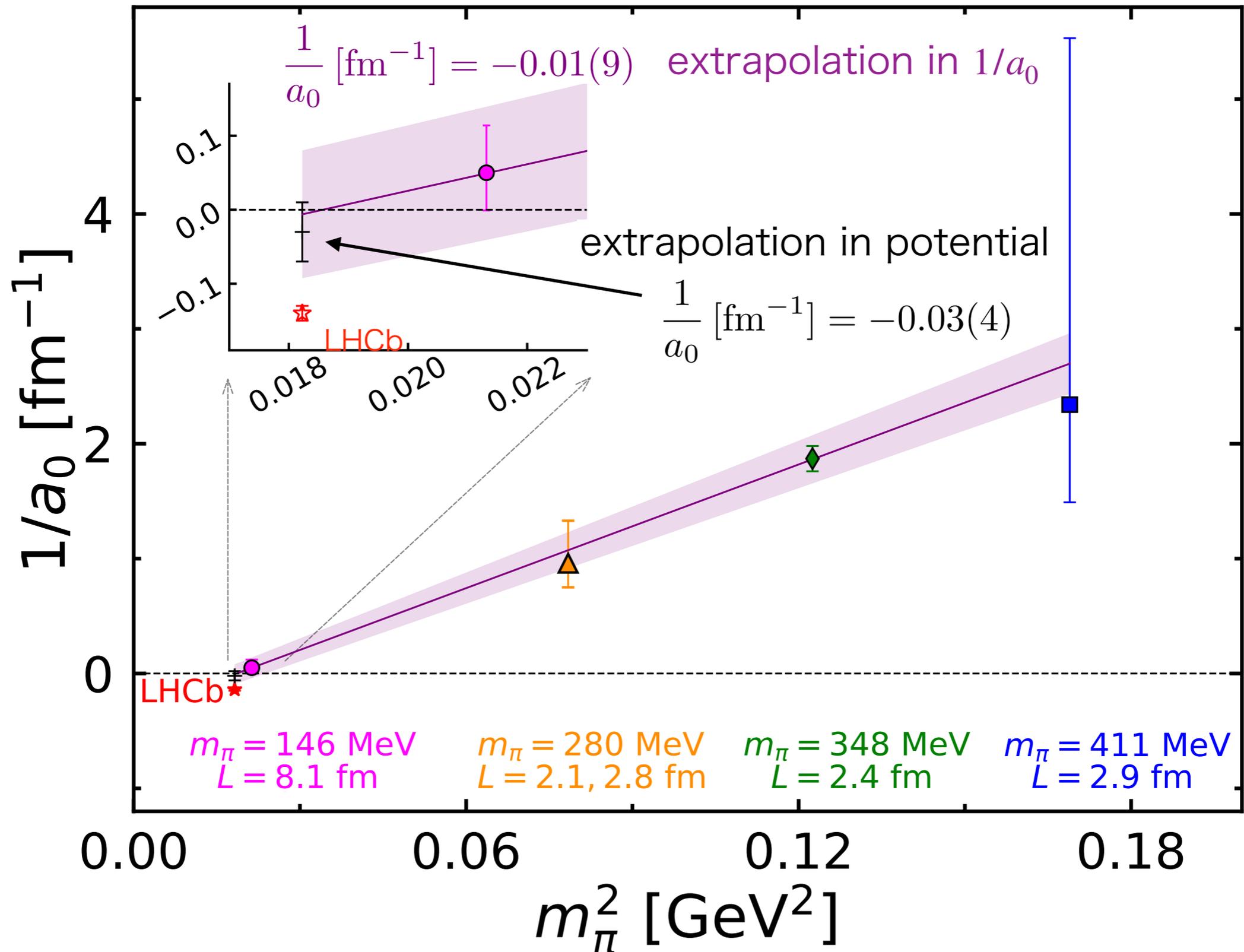
ERE $k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + O(k^4)$

m_π [MeV]	146.4	135.0
$1/a_0$ [fm ⁻¹]	0.05(5) $\left(\begin{smallmatrix} +2 \\ -2 \end{smallmatrix} \right)$	-0.03(4)
r_{eff} [fm]	1.12(3) $\left(\begin{smallmatrix} +3 \\ -8 \end{smallmatrix} \right)$	1.12(3)
κ_{pole} [MeV]	-8(8) $\left(\begin{smallmatrix} +3 \\ -5 \end{smallmatrix} \right)$	+5(8)
E_{pole} [keV]	-59 $\left(\begin{smallmatrix} +53 \\ -99 \end{smallmatrix} \right) \left(\begin{smallmatrix} +2 \\ -67 \end{smallmatrix} \right)$	-45 $\left(\begin{smallmatrix} +41 \\ -78 \end{smallmatrix} \right)$

one shallow “bound” state appears at “physical” pion mass

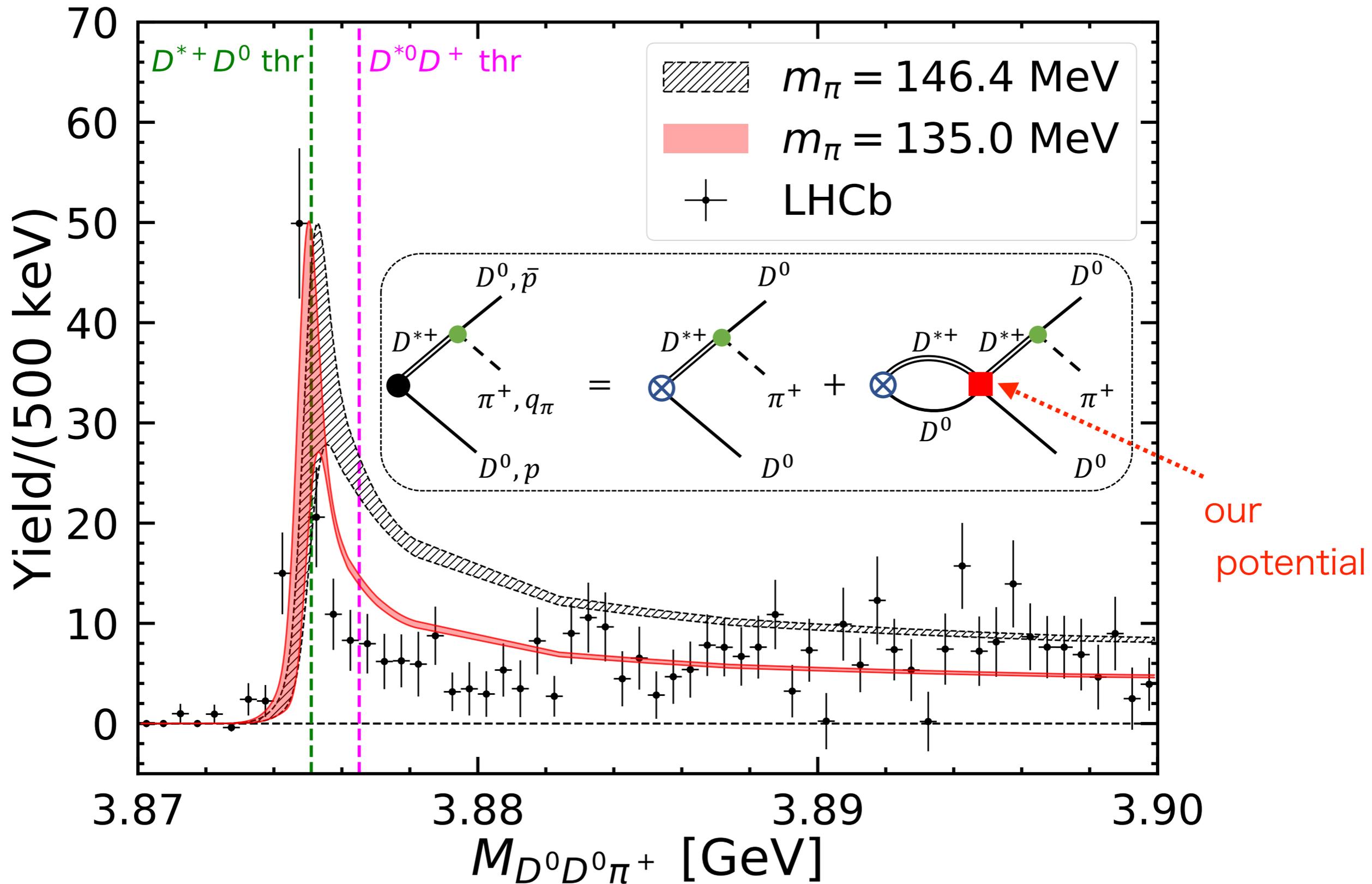
$$E_{\text{pole}} = -45 \left(\begin{smallmatrix} +41 \\ -78 \end{smallmatrix} \right) \text{ keV}$$

chiral extrapolation of $1/a_0$ linear in m_π^2



Two chiral “extrapolations” are consistent.

D⁰D⁰π⁺ mass spectrum



Our potential at “physical” pion mass explains LHCb data better.

Summary & Outlook

- We employ the HAL QCD method to investigate a doubly charmed tetraquark T_{cc}^+ at almost physical pion $m_\pi = 146$ MeV .
 - D^*D potential is attractive at all distances.
 - The system appears very close to unitarity, so that a small change in pion mass from 146 MeV to 135 MeV leads to significant changes in physical observables.
 - from a virtual state to a bound state
 - better agreement in the mass spectrum with LHCb
 - A more reliable chiral extrapolation is required.
 - This may be a challenge for the finite volume method, since energy shift is very small.
 - configurations at a “physical” pion mass ($m_\pi \simeq 135$ MeV) are generated on Fugaku. Stay tuned.
- [E. Itou \(HALQCD\), poster.](#) [T.M. Doi \(HALQCD\), Thu 03/08 17:00@Hadron and Nuclear Spectrum and interactions](#)
- (Challenge) Inclusion of iso-spin breaking effect (quark mass and QED) might be required. Then a coupled channel analysis is mandatory.

Thank you !