Degrees of freedom in various charm subsectors from Lattice QCD

Sipaz Sharma

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Lattice 2023, Fermilab, Batavia, Illinois, USA

Motivation

- Strong interaction matter undergoes a chiral crossover at $\mathrm{T_{pc}} = 156.5 \pm 1.5$ MeV.
  [HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]
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- Questions which heavy-ion experiments aim to answer:
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- Questions which heavy-ion experiments aim to answer:
  - whether the open-charm states can exist above T_{pc} or do melt at T_{pc}.
  - do charm quarks start appearing at T_{pc} or not.
- Charm fluctuations (cumulants) calculated in the framework of Lattice QCD can receive enhanced contributions due the existence of not-yet-discovered open-charm states; it is possible to compare this enhancement to the HRG calculations.

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- For Baryons the argument of $\cosh$ changes to
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- Boltzmann approximation is good in the charm sector not just for mesons and baryons but also for a charm-quark gas.
- $\hat{\mu}_\mathrm{X} = \mu/\mathrm{T}$, $\mathrm{X} \in \{\mathrm{B}, \mathrm{Q}, \mathrm{S}, \mathrm{C}\}$.

Generalized susceptibilities of the conserved charges

$$\mathrm{M_C(T,\overrightarrow{\mu})} = \frac{1}{2\pi^2}\sum_{\mathrm{i}} \mathrm{g_i}\left(\frac{\mathrm{m_i}}{\mathrm{T}}\right)^2 \mathrm{K_2(m_i/T)cosh(Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C)}$$

- $\mathrm{K_2(x)} \sim \sqrt{\pi/2x}\, e^{-x}\, [1 + \mathbb{O}(x^{-1})]$. If $\mathrm{m_i} \gg \mathrm{T}$, then contribution to $\mathrm{P_C}$ will be exponentially suppressed.
- Λ_c^+ mass ~ 2286 MeV, Ξ_{cc}^{++} mass ~ 3621 MeV. At $\mathrm{T_{pc}}$, contribution to $\mathrm{B_C}$ from Ξ_{cc}^{++} will be suppressed by a factor of 10^{-4} in relation to Λ_c^+.

Generalized susceptibilities of the conserved charges

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- Dimensionless generalized susceptibilities of conserved charges:

$$\chi_\mathrm{klmn}^\mathrm{BQSC} = \left.\frac{\partial^{(\mathrm{k+l+m+n})}\;\left[\mathrm{P}\left(\hat{\mu}_\mathrm{B},\hat{\mu}_\mathrm{Q},\hat{\mu}_\mathrm{S},\hat{\mu}_\mathrm{C}\right)/\mathrm{T}^4\right]}{\partial\hat{\mu}_\mathrm{B}^\mathrm{k}\;\partial\hat{\mu}_\mathrm{Q}^\mathrm{l}\;\partial\hat{\mu}_\mathrm{S}^\mathrm{m}\;\partial\hat{\mu}_\mathrm{C}^\mathrm{n}}\right|_{\vec{\mu}=0}$$

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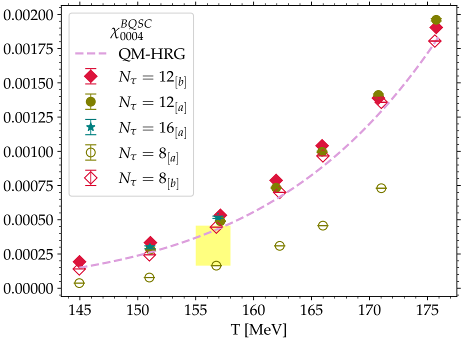
$$\boxed{\mathrm{M_C(T,\vec{\mu}) = \frac{1}{2\pi^2}\sum_i g_i\left(\frac{m_i}{T}\right)^2 K_2(m_i/T)\cosh(Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C)}}$$

- Dimensionless generalized susceptibilities of conserved charges are given by,

$$\boxed{\chi^{\mathrm{BQSC}}_{\mathrm{klmn}} = \left.\frac{\partial^{(\mathrm{k+l+m+n})}\ \left[\mathrm{P}\ \left(\hat{\mu}_{\mathrm{B}},\hat{\mu}_{\mathrm{Q}},\hat{\mu}_{\mathrm{S}},\hat{\mu}_{\mathrm{C}}\right)/\mathrm{T}^4\right]}{\partial\hat{\mu}^{\mathrm{k}}_{\mathrm{B}}\ \ \partial\hat{\mu}^{\mathrm{l}}_{\mathrm{Q}}\ \ \partial\hat{\mu}^{\mathrm{m}}_{\mathrm{S}}\ \ \partial\hat{\mu}^{\mathrm{n}}_{\mathrm{C}}}\right|_{\vec{\mu}=0}}$$

- $\underbrace{\chi^{BC}_{mn}}_{\chi^{\mathrm{BQSC}}_{\mathrm{m00n}}} = B_{C,1} + 2^n B_{C,2} + 3^n B_{C,3} \simeq B_{C,1}$
- At present, we have gone upto fourth order in calculating various cumulants.

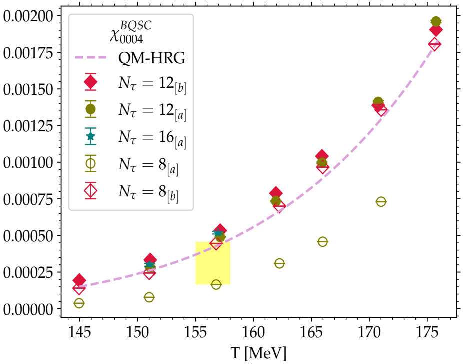
Continuum limit



- Two different LCPs:

  a) charmonium mass, b) $\mathrm{m_c/m_s}$

Continuum limit



- Absolute predictions in the charm sector are particularly sensitive to the precise tuning of the bare input quark masses.

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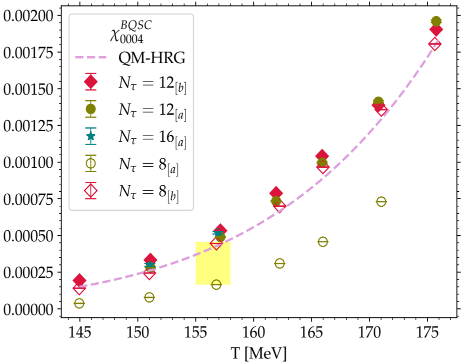
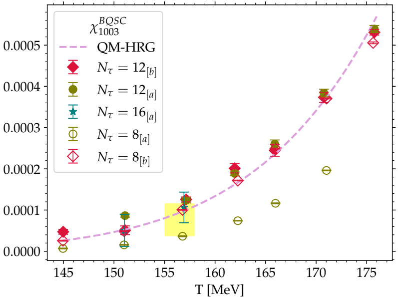
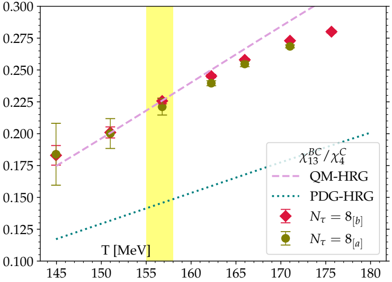


- Absolute predictions in the charm sector are particularly sensitive to the precise tuning of the bare input quark masses.
- Different hadrons have different lattice-cut off effects, therefore, taking the continuum limit is mandatory.

Ratios calculated using different LCPs





- Sensitivity to the choice of LCP cancels to a large extent in the ratios $\implies$ stick to the ratios.

Ratios independent of the hadron spectrum

- Irrespective of the details of the baryon mass spectrum, in the validity range of HRG, $\chi^{\mathrm{BC}}_{\mathrm{mn}}/\chi^{\mathrm{BC}}_{\mathrm{kl}} = 1$, $\forall(\mathrm{m}+\mathrm{n}),(\mathrm{k}+\mathrm{l}) \in$ even.

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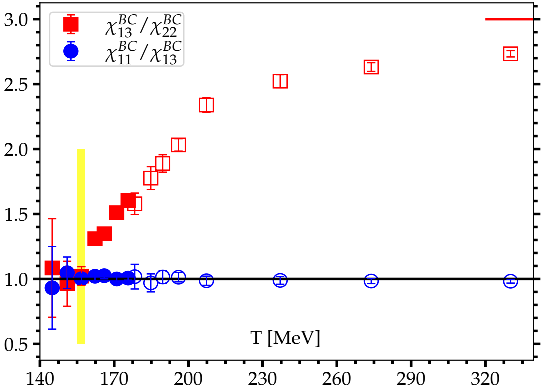
- $\chi^{\rm BC}_{\rm 1n}/\chi^{\rm BC}_{\rm 1l} = 1$, $\forall {\rm n}, {\rm l} \in$ odd, for the entire temperature range.

Change in the charm degrees of freedom



- States with fractional B start appearing near $\mathrm{T_{pc}}$. Is it possible to determine this fractional B?

Baryonic and mesonic contributions to $\mathrm{P_C}$

In the low temperature range, where HRG works,

- χ_{13}^{BC} is the partial pressure from the charmed-baryonic subsector.
- $\chi_{4}^{\mathrm{C}} - \chi_{13}^{\mathrm{BC}}$ can be interpreted as the partial pressure from the charmed-mesonic subsector.

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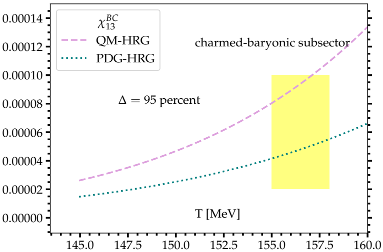
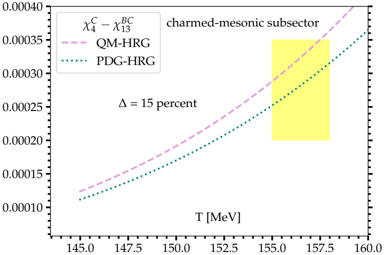
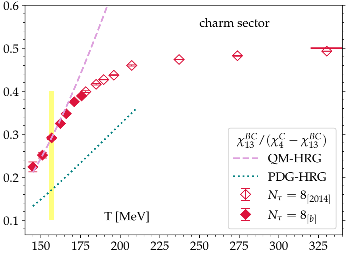
- Unlike the previous quantities shown, ratios such as $\chi_{13}^{\mathrm{BC}}/(\chi_{4}^{\mathrm{C}} - \chi_{13}^{\mathrm{BC}})$ will partially depend upon the hadron spectrum.

Ratios of baryonic and mesonic contributions to $\mathrm{P_C}$

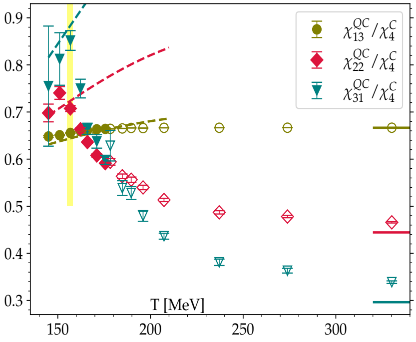


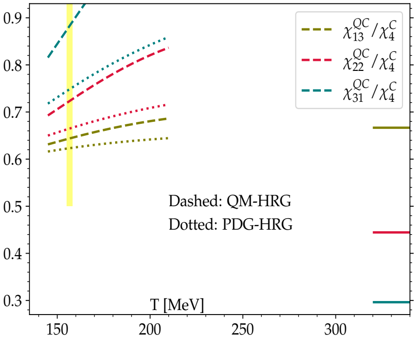




- Missing charmed-baryonic states below $\mathrm{T_{pc}}$.
- $\Delta = (|1 - \text{QM-HRG}/\text{PDG-HRG}|)|_{\mathrm{T_{pc}}}$

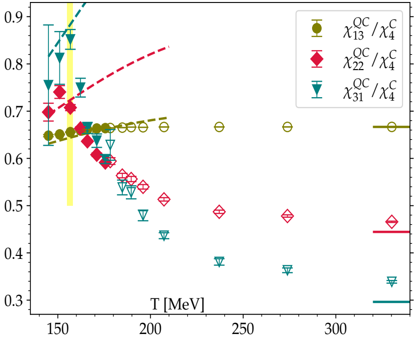
Electrically-charged-charm subsector

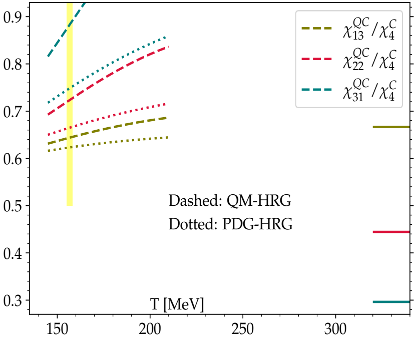




- With now available statistics, possibility of distinguishing $|\mathrm{Q}| = 0, 1, 2$ charm subsectors in the hadronic phase.

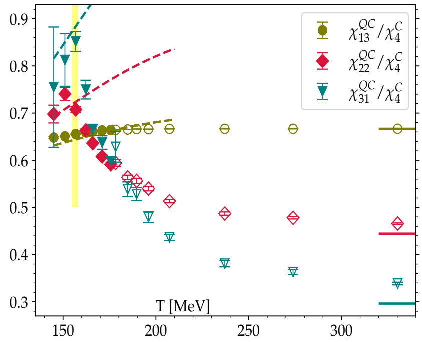
Electrically-charged-charm subsector

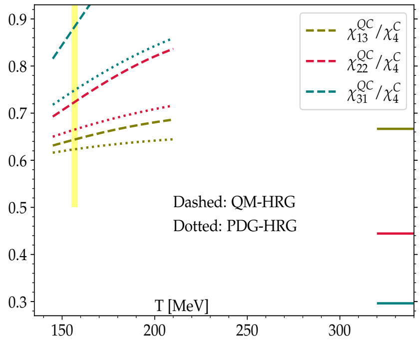




- Ratio of QM-HRG/PDG-HRG increases with increasing Q-moments $\implies$ $|\mathrm{Q}| = 2$ sector more sensitive to 'missing resonances'.
- χ^{QC}_{22} and χ^{QC}_{31} give evidence for 'missing resonances'.

Electrically-charged-charm subsector



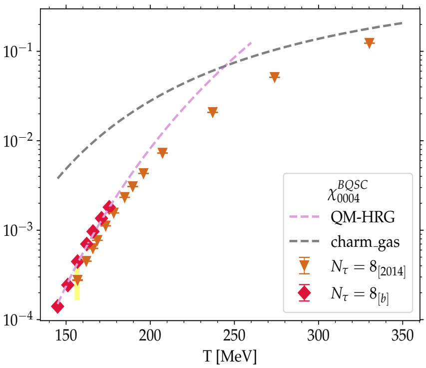


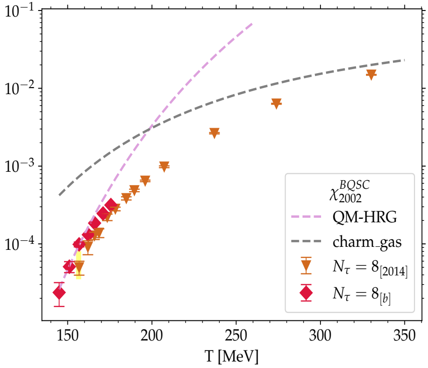
- Ratio of QM-HRG/PDG-HRG increases with increasing Q-moments.
- Close to freeze-out, an enhancement over the PDG expectation in the fractional contribution of the $|\mathrm{Q}| = 2$ (Σ_c^{++}) charm subsector to the total charm partial pressure.

Approach to free charm-quark gas limit

$$\mathrm{P_c(T,\vec{\mu})} = \frac{3}{\pi^2}\left(\frac{\mathrm{m_c}}{\mathrm{T}}\right)^2 \mathrm{K_2(m_c/T)cosh}\left(\frac{2}{3}\hat{\mu}_\mathrm{Q} + \frac{1}{3}\hat{\mu}_\mathrm{B} + \hat{\mu}_\mathrm{C}\right)$$

$\mathrm{m_c} = 1.27$ GeV.





Charm degrees of freedom in the intermediate T range

Quasi-particle model:

$$\mathrm{P^C(T,\hat{\mu}_C,\hat{\mu}_B)/T^4} = \mathrm{P^C_M(T)\cosh(\hat{\mu}_C) + P^C_B(T)\cosh(\hat{\mu}_C+\hat{\mu}_B)} \\ \mathrm{+P^C_q(T)\cosh(\hat{\mu}_C+\hat{\mu}_B/3)}$$

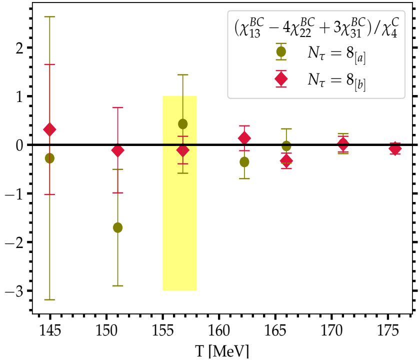
$$\mathrm{P^C_q} = 9(\chi^{BC}_{13} - \chi^{BC}_{22})/2$$
$$\mathrm{P^C_B} = (3\chi^{BC}_{22} - \chi^{BC}_{13})/2$$
$$\mathrm{P^C_M} = \chi^{C}_{4} + 3\chi^{BC}_{22} - 4\chi^{BC}_{13}$$

Constraint on cumulants in a simple quasi-particle model:

$$\boxed{\mathrm{c} = \chi^{BC}_{13} + 3\chi^{BC}_{31} - 4\chi^{BC}_{22} = 0}$$

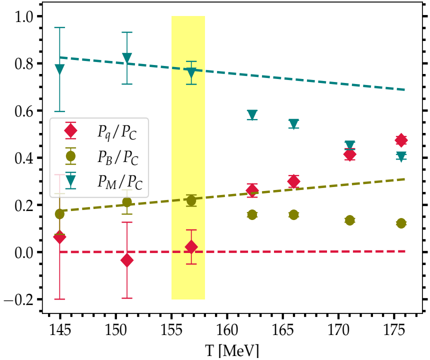
[S. Mukherjee et al., 2016]

Quasi-particle model



The constraint holds true $\implies$ quasi-particle states with $\mathrm{B} = 0, 1$ or $1/3$ exist in the intermediate temperature range.

Charm-quark-like excitations in QGP



Right after $\mathrm{T_{pc}}$, $\mathrm{P_q}$ starts contributing to $\mathrm{P_C}$, which is compensated by a reduction (and deviation from HRG) in the fractional contribution of the hadron-like states to $\mathrm{P_C}$.

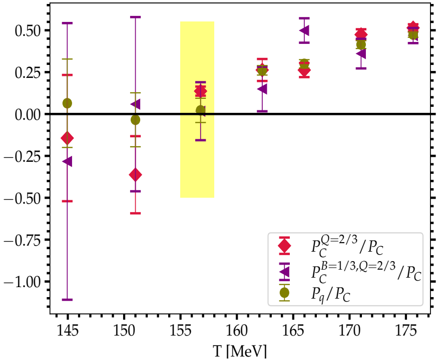
Charm-quark-like excitations in QGP

Quantum numbers of the charm-quark like excitations in QGP?

$$\mathrm{P_C^{Q=2/3}} = \frac{1}{8}\left[54\chi_{13}^{\mathrm{QC}} - 81\chi_{22}^{\mathrm{QC}} + 27\chi_{31}^{\mathrm{QC}}\right] \tag{1}$$

$$\mathrm{P_C^{B=1/3,Q=2/3}} = \frac{27}{4}\left[\chi_{112}^{\mathrm{BQC}} - \chi_{211}^{\mathrm{BQC}}\right] \tag{2}$$

Charm-quark-like excitations in QGP



Clear agreement between three independent observables which correspond to the partial pressures of
i) $\mathrm{B} = 1/3$, ii) $\mathrm{Q} = 2/3$, and iii) $\mathrm{B} = 1/3$ and $\mathrm{Q} = 2/3$ charm subsectors.

Conclusions & Outlook

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  - No evidence for the existence of charmed diquarks above $\mathrm{T_{pc}}$.
- Continuum limit with two different LCPs is in progress; it will enable us to make a statement based on the absolute cumulants.