Degrees of freedom in various charm subsectors from Lattice QCD

Sipaz Sharma

F.Karsch, P.Petreczky, C.Schmidt

Lattice 2023, Fermilab, Batavia, Illinois, USA
Motivation

- Strong interaction matter undergoes a chiral crossover at $T_{pc} = 156.5 \pm 1.5$ MeV.
  
  [HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]

- In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below $T_{pc}$.
Motivation

- Strong interaction matter undergoes a chiral crossover at $T_{pc} = 156.5 \pm 1.5$ MeV.
  [HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]

- In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below $T_{pc}$.

- Questions which heavy-ion experiments aim to answer:
  - whether the open-charm states can exist above $T_{pc}$ or do melt at $T_{pc}$.
  - do charm quarks start appearing at $T_{pc}$ or not.
Motivation

- Strong interaction matter undergoes a chiral crossover at $T_{pc} = 156.5 \pm 1.5$ MeV.
  [HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]

- In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below $T_{pc}$.

- Questions which heavy-ion experiments aim to answer:
  - whether the open-charm states can exist above $T_{pc}$ or do melt at $T_{pc}$.
  - do charm quarks start appearing at $T_{pc}$ or not.

- Charm fluctuations (cumulants) calculated in the framework of Lattice QCD can receive enhanced contributions due the existence of not-yet-discovered open-charm states; it is possible to compare this enhancement to the HRG calculations.
Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$.

- Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure:

$$P_{C}(T, \mu) = M_{C}(T, \mu) + B_{C}(T, \mu)$$

[C. R. Allton et al., 2005]

- For Baryons the argument of $\cosh$ changes to $B_{i}^{\mu} + Q_{i}^{\mu} + S_{i}^{\mu} + C_{i}^{\mu}$.

- Boltzmann approximation is good in the charm sector not just for mesons and baryons but also for a charm-quark gas.

- $\hat{\mu}_X = \mu/T, X \in \{B, Q, S, C\}$. 

Sipaz Sharma  
Bielefeld University  
August 1, 2023  
3 / 22
Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$.
- Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure: $P_C(T, \mu)/T^4 = M_C(T, \mu) + B_C(T, \mu)$. [C. R. Allton et al., 2005]
Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$.

- Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure: $P_C(T, \mu)/T^4 = M_C(T, \mu) + B_C(T, \mu)$. [C. R. Allton et al., 2005]

\[
M_C(T, \mu) = \frac{1}{2\pi^2} \sum_i g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \cosh(Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C)
\]

[A. Bazavov et al., 2014]
Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$.
- Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure:

  $$P_C(T, \mu)/T^4 = M_C(T, \mu) + B_C(T, \mu).$$

  \[C. R. Allton et al., 2005\]

\[
M_C(T, \mu) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C)
\]

\[A. Bazavov et al., 2014\]

- For Baryons the argument of $\cosh$ changes to

  $$B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C$$

- Boltzmann approximation is good in the charm sector not just for mesons and baryons but also for a charm-quark gas.
Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$.
- Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure: $P_C(T, \mu)/T^4 = M_C(T, \mu) + B_C(T, \mu)$. [C. R. Allton et al., 2005]

\[
M_C(T, \mu) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C)
\]

[A. Bazavov et al., 2014]

- For Baryons the argument of $\cosh$ changes to $B_i \hat{\mu}_B + Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C$
- Boltzmann approximation is good in the charm sector not just for mesons and baryons but also for a charm-quark gas.
- $\hat{\mu}_X = \mu/T, \ X \in \{B, Q, S, C\}$.
Generalized susceptibilities of the conserved charges

\[ M_C(T, \overrightarrow{\mu}) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C) \]

- \( K_2(x) \sim \sqrt{\pi/2x} \ e^{-x} \ [1 + O(x^{-1})] \). If \( m_i \gg T \), then contribution to \( P_C \) will be exponentially suppressed.

- \( \Lambda_c^+ \) mass \( \sim 2286 \) MeV, \( \Xi_{cc}^{++} \) mass \( \sim 3621 \) MeV. At \( T_{pc} \), contribution to \( B_C \) from \( \Xi_{cc}^{++} \) will be suppressed by a factor of \( 10^{-4} \) in relation to \( \Lambda_c^+ \).
Generalized susceptibilities of the conserved charges

\[ M_C(T, \vec{\mu}) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C) \]

- \[ K_2(x) \sim \sqrt{\pi/2x} \ e^{-x} \left[ 1 + O(x^{-1}) \right] \]. If \( m_i \gg T \), then contribution to \( P_C \) will be exponentially suppressed.

- \( \Lambda_c^+ \) mass \( \sim 2286 \) MeV, \( \Xi_{cc}^{++} \) mass \( \sim 3621 \) MeV. At \( T_{pc} \), contribution to \( B_C \) from \( \Xi_{cc}^{++} \) will be suppressed by a factor of \( 10^{-4} \) in relation to \( \Lambda_c^+ \).

- Dimensionless generalized susceptibilities of conserved charges:

\[ \chi_{BQSC}^{klmn} = \left. \frac{\partial^{(k+l+m+n)} \left[ P (\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C) / T^4 \right]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \partial \hat{\mu}_S^m \partial \hat{\mu}_C^n} \right|_{\vec{\mu} = 0} \]
Generalized susceptibilities of the conserved charges

\[ M_C(T, \vec{\mu}) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C) \]

- Dimensionless generalized susceptibilities of conserved charges are given by,

\[ \chi_{BQSC}^{klmn} = \frac{\partial^{(k+1+m+n)} \left[ P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C) / T^4 \right]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \partial \hat{\mu}_S^m \partial \hat{\mu}_C^n} \bigg|_{\mu=0} \]

- \[ \chi_{mn}^{BC} = B_{C,1} + 2^n B_{C,2} + 3^n B_{C,3} \approx B_{C,1} \]

- At present, we have gone up to fourth order in calculating various cumulants.
Two different LCPs:

a) charmonium mass, b) $m_c/m_s$
Absolute predictions in the charm sector are particularly sensitive to the precise tuning of the bare input quark masses.
Absolute predictions in the charm sector are particularly sensitive to the precise tuning of the bare input quark masses.

Different hadrons have different lattice-cut off effects, therefore, taking the continuum limit is mandatory.
Ratios calculated using different LCPs

▶ Sensitivity to the choice of LCP cancels to a large extent in the ratios → stick to the ratios.
Ratios independent of the hadron spectrum

Irrespective of the details of the baryon mass spectrum, in the validity range of HRG, $\chi_{mn}^{BC}/\chi_{kl}^{BC} = 1$, $\forall (m + n), (k + l) \in$ even.
Ratios independent of the hadron spectrum

- Irrespective of the details of the baryon mass spectrum, in the validity range of HRG, \( \frac{\chi_{mn}^{BC}}{\chi_{kl}^{BC}} = 1, \forall (m + n), (k + l) \in \text{even}. \)

- \( \frac{\chi_{1n}^{BC}}{\chi_{1l}^{BC}} = 1, \forall n, l \in \text{odd}, \) for the entire temperature range.
States with fractional $B$ start appearing near $T_{pc}$. Is it possible to determine this fractional $B$?
Baryonic and mesonic contributions to $P_C$

In the low temperature range, where HRG works,

- $\chi_{13}^{BC}$ is the partial pressure from the charmed-baryonic subsector.
- $\chi_4^C - \chi_{13}^{BC}$ can be interpreted as the partial pressure from the charmed-mesonic subsector.
Baryonic and mesonic contributions to $P_C$

In the low temperature range, where HRG works,

- $\chi_{13}^{BC}$ is the partial pressure from the charmed-baryonic subsector.
- $\chi_4^C - \chi_{13}^{BC}$ can be interpreted as the partial pressure from the charmed-mesonic subsector.
- Unlike the previous quantities shown, ratios such as $\chi_{13}^{BC}/(\chi_4^C - \chi_{13}^{BC})$ will partially depend upon the hadron spectrum.
Ratios of baryonic and mesonic contributions to $P_C$

- **Charmed-baryonic subsector**
  - $\Delta = 95\%$

- **Charmed-mesonic subsector**
  - $\Delta = 15\%$

- **Charm sector**
  - $\chi_{BC}^{13}/(\chi_C^4 - \chi_{BC}^{13})$

- **QM-HRG**
- **PDG-HRG**

- **Missing charmed-baryonic states below $T_{pc}$**.

- $\Delta = (|1 - \text{QM-HRG/PDG-HRG}|)T_{pc}$
With now available statistics, possibility of distinguishing $|Q| = 0, 1, 2$ charm subsectors in the hadronic phase.
Ratio of QM-HRG/PDG-HRG increases with increasing $|Q|$-moments.

$\implies |Q| = 2$ sector more sensitive to ‘missing resonances’.

$\chi_{22}^{QC}$ and $\chi_{31}^{QC}$ give evidence for ‘missing resonances’.

Sipaz Sharma
Bielefeld University
August 1, 2023 14 / 22
Ratio of QM-HRG/PDG-HRG increases with increasing $Q$-moments.

Close to freeze-out, an enhancement over the PDG expectation in the fractional contribution of the $|Q| = 2 \, (\Sigma_c^{++})$ charm subsector to the total charm partial pressure.
Approach to free charm-quark gas limit

\[ P_c(T, \mu) = \frac{3}{\pi^2} \left( \frac{m_c}{T} \right)^2 K_2 \left( \frac{m_c}{T} \right) \cosh \left( \frac{2}{3} \hat{\mu}_Q + \frac{1}{3} \hat{\mu}_B + \hat{\mu}_C \right) \]

\[ m_c = 1.27 \text{ GeV}. \]
Charm degrees of freedom in the intermediate $T$ range

Quasi-particle model:

$$\frac{P^C(T, \mu_C, \mu_B)}{T^4} = \frac{P^C_M(T)}{T^4} \cosh(\mu_C) + \frac{P^C_B(T)}{T^4} \cosh(\mu_C + \mu_B)$$
$$+ P^C_q(T) \cosh(\mu_C + \mu_B/3)$$

$$P^C_q = 9(\chi^{BC}_{13} - \chi^{BC}_{22})/2$$
$$P^C_B = (3\chi^{BC}_{22} - \chi^{BC}_{13})/2$$
$$P^C_M = \chi^C_4 + 3\chi^{BC}_{22} - 4\chi^{BC}_{13}$$

Constraint on cumulants in a simple quasi-particle model:

$$c = \chi^{BC}_{13} + 3\chi^{BC}_{31} - 4\chi^{BC}_{22} = 0$$

[S. Mukherjee et al., 2016]
The constraint holds true $\implies$ quasi-particle states with $B = 0, 1$ or $1/3$ exist in the intermediate temperature range.
Charm-quark-like excitations in QGP

Right after $T_{pc}$, $P_q$ starts contributing to $P_C$, which is compensated by a reduction (and deviation from HRG) in the fractional contribution of the hadron-like states to $P_C$. 
Charm-quark-like excitations in QGP

Quantum numbers of the charm-quark like excitations in QGP?

\[
P^{Q=2/3}_C = \frac{1}{8} [54\chi^{QC}_{13} - 81\chi^{QC}_{22} + 27\chi^{QC}_{31}] \tag{1}
\]

\[
P^{B=1/3, Q=2/3}_C = \frac{27}{4} [\chi^{BQC}_{112} - \chi^{BQC}_{211}] \tag{2}
\]
Charm-quark-like excitations in QGP

Clear agreement between three independent observables which correspond to the partial pressures of
i) $B = 1/3$, ii) $Q = 2/3$, and iii) $B = 1/3$ and $Q = 2/3$ charm subsectors.
Conclusions & Outlook

▶ Deviations from HRG in the open-charm sector near $T_{pc} = 156.5 \pm 1.5$ MeV.
▶ Analysis shows that there are missing states in the PDG record.
Conclusions & Outlook

- Deviations from HRG in the open-charm sector near $T_{pc} = 156.5 \pm 1.5$ MeV.
- Analysis shows that there are missing states in the PDG record.
- Quasi-particle model can describe the lattice results in the intermediate temperature range.
  - Presence of charm-quark-like excitations in QGP.
  - No evidence for the existence of charmed diquarks above $T_{pc}$. 
Conclusions & Outlook

- Deviations from HRG in the open-charm sector near $T_{pc} = 156.5 \pm 1.5$ MeV.
- Analysis shows that there are missing states in the PDG record.
- Quasi-particle model can describe the lattice results in the intermediate temperature range.
  - Presence of charm-quark-like excitations in QGP.
  - No evidence for the existence of charmed diquarks above $T_{pc}$.
- Continuum limit with two different LCPs is in progress; it will enable us to make a statement based on the absolute cumulants.