The Deconfinement Phase Transition in Sp(2N) Gauge Theories and the Density of States Method

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Overview

Sp(2N) and Beyond the Standard Model Physics

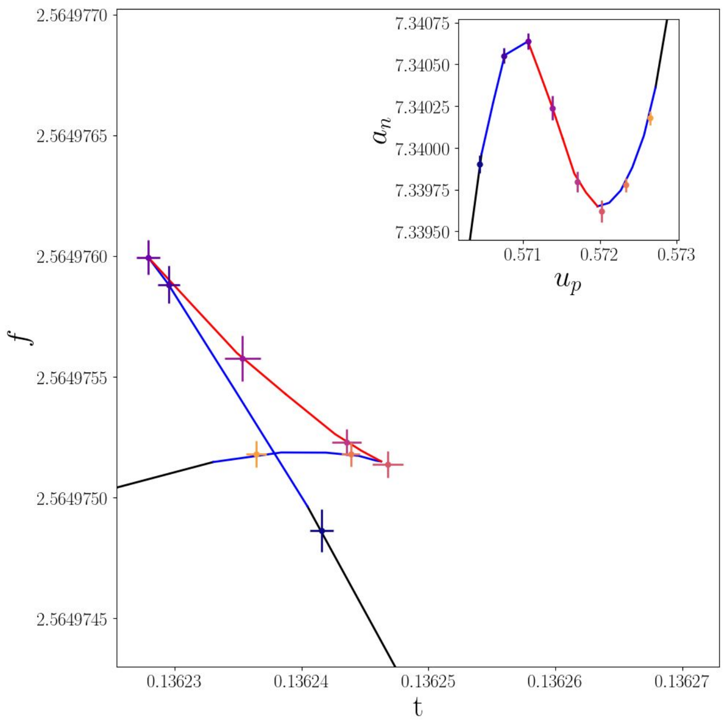
The Deconfinement Phase Transition on the lattice

The Linear Logarithmic Relaxation Method

Results

This work follows of the methodology developed in: arxiv:2305.07463

This talk I will focus on pure Sp(4) with results for a single lattice size 4x20x20x20



Beyond the standard model physics

Extensions to the model to explain:

- Dark matter
- Composite Higgs
- Potential observation of gravitational wave background[1]
- Baryogenesis

Sp(2N) gauge theories have been proposed as a possible solution to each of these problems

Afzal, A., Agazie, G., Anumarlapudi, A., Archibald, A. M., Arzoumanian, Z., Baker, P. T., ... & NANOGrav Collaboration. (2023). The NANOGrav 15 yr Data Set: Search for Signals from New Physics. *The Astrophysical Journal Letters*, *951*(1), L11.

First order phase transitions

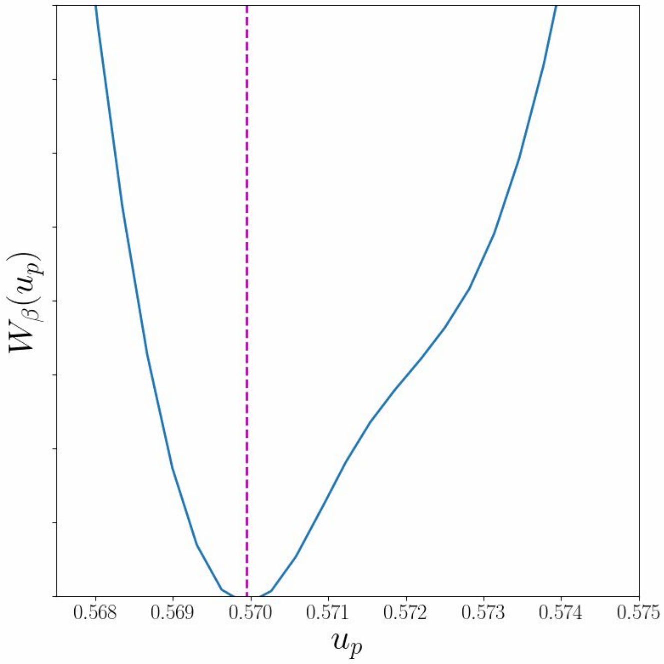
First order phase transitions in the early universe can explain:

- Baryogenesis
- Potential observation of gravitational wave background[1]

Sp(2N) (N>1) pure gauge theories have a first order deconfinement phase transition

Co-existence of solutions at criticality can lead to nucleation of bubbles

Lattice simulations can inform us on the dynamics of the bubbles



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Lattice set-up

$$S[U] = \frac{6\tilde{V}}{a^4}(1 - u_p[U]) \qquad Z(\beta) = \int [DU] e^{-\beta S[U]}$$

Isotropic hypercubic lattice with spacing a and volume $\tilde{V} = a^4 N_t \times N_s^3$

Temperature $T = 1/aN_t$ by changing $\beta(a)$ for $N_t < N_s$

$U_\mu \in Sp(2N) \subset SU(2N)$ satisfying the condition:

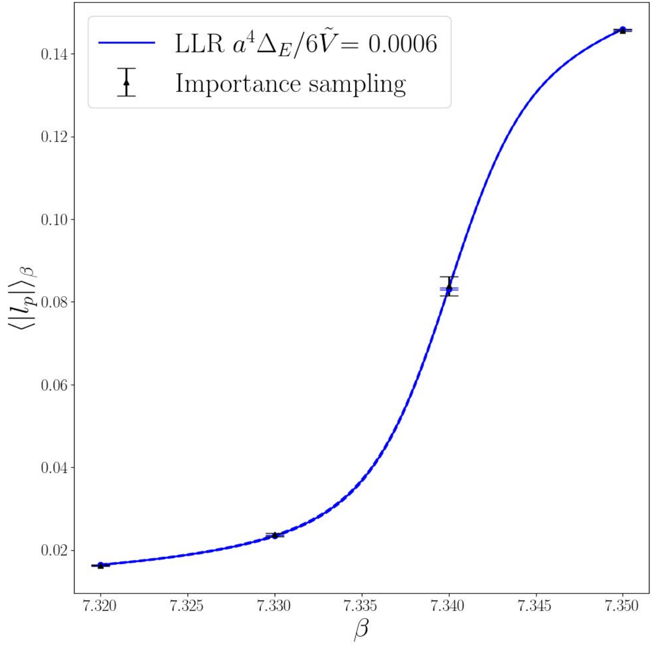
$$U_\mu \Omega (U_\mu)^T = \Omega \qquad \Omega = \begin{pmatrix} 0 & \mathbb{1}_{N\times N} \\ -\mathbb{1}_{N\times N} & 0 \end{pmatrix}$$

Deconfinement

$$\langle l_p \rangle_\beta = \left\langle \frac{1}{2N N_s^3} \sum_{\vec{n_s}} \mathrm{Tr}\left(\prod_{n_t=0}^{N_t-1} U_0(n_t, \vec{n_s}) \right) \right\rangle_\beta \begin{cases} =0 \text{ confined phase} \\ \neq 0 \text{ deconfined phase} \end{cases}$$

The Polyakov loop is the order parameter associated with the breaking of the centre symmetry in the transition

Centre symmetry for Sp(2N) is $\mathbb{Z}_2$

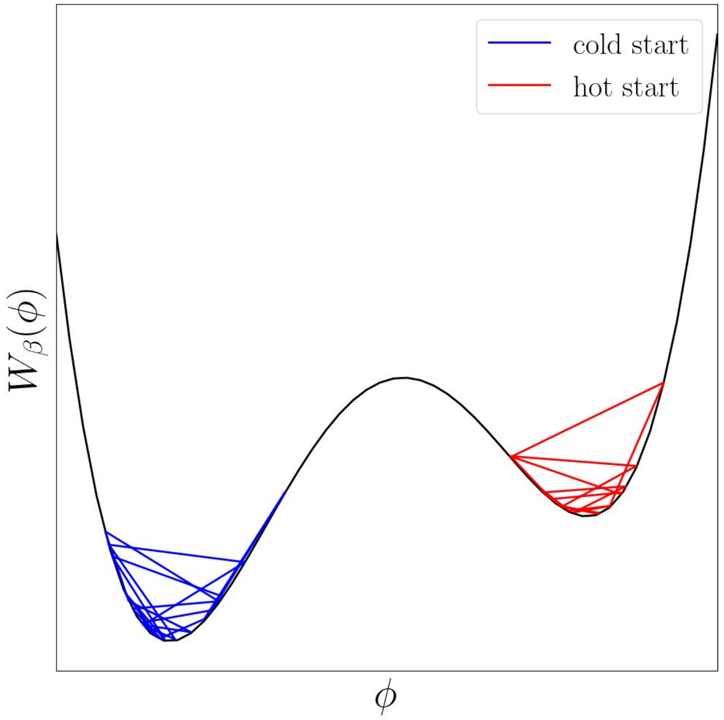


Simulating first order phase transitions

For accurate results configurations must tunnel between phases several times

In the large volume limit the potential barrier grows

More configurations must be generated to ensure the system explores both phases appropriately



Density of States

We want to calculate some observable

$$\langle \mathcal{O} \rangle = \frac{1}{Z_\beta} \int [DU] \mathcal{O}[U] e^{-\beta S[U]} \qquad Z_\beta = \int [DU] e^{-\beta S[U]}$$

Introduce the density of states

$$\rho(E) = \int [DU] \delta(S[U] - E)$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z_\beta} \int dE \rho(E) \mathcal{O}[E] e^{-\beta E} \qquad Z_\beta = \int dE \rho(E) e^{-\beta E}$$

$$P_\beta(E) = \frac{1}{Z_\beta} \rho(E) e^{-\beta E}$$

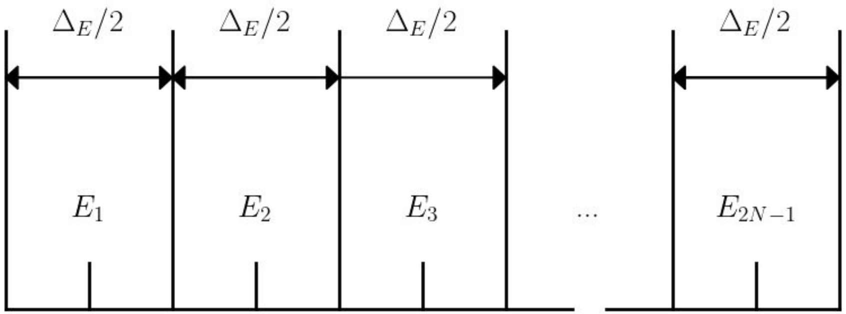
The Linear Logarithmic Relaxation method

Make the ansatz for ϱ(E) for a small region $E_n - \Delta_E/4 \leq E \leq E_n + \Delta_E/4$

$$\ln \tilde{\rho}(E) = a_n(E - E_n) + c_n$$

From continuity of $\tilde{\rho}(E)$

$$c_n = c_1 + \frac{\Delta_E}{4} a_1 + \frac{\Delta_E}{2} \sum_{k=2}^{n-1} a_k + \frac{\Delta_E}{4} a_n$$



Can find a_n by solving:

$$\langle\langle E - E_n \rangle\rangle_n(a_n) = \langle\langle u_p - (u_p)_n \rangle\rangle_n(a_n) = 0$$

$$(u_p)_n = 1 - a^4 E_n / 6\tilde{V}$$

The Linear Logarithmic Relaxation method

Finding a_n :

Choose initial guess $a_n^{(0)}$

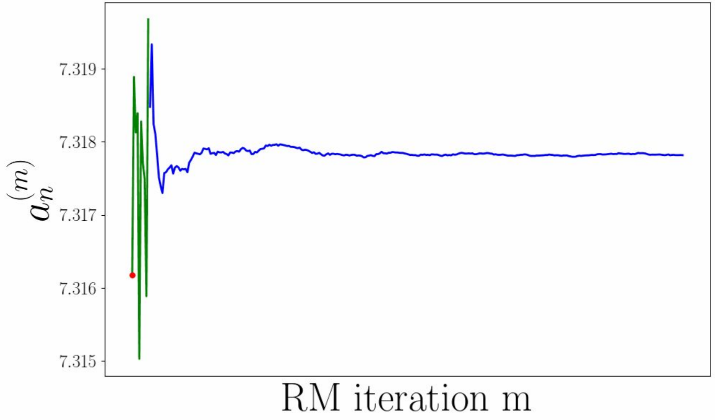
Improve guess iteratively using

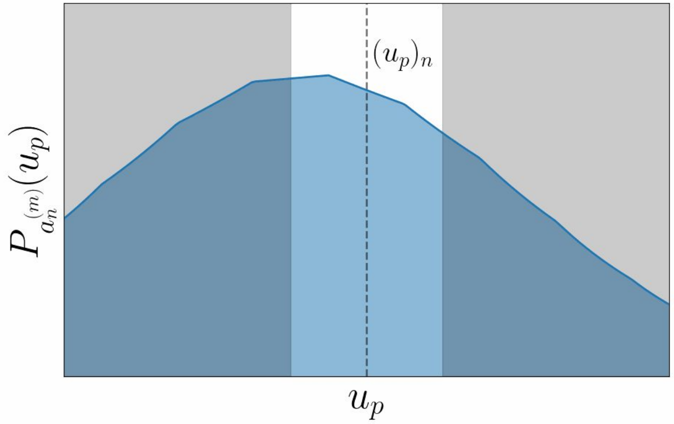
Newton-Raphson Method:

$$a_n^{(m+1)} \simeq a_n^{(m)} - \left(\frac{12}{\Delta_E^2}\right) \langle\langle E - E_n \rangle\rangle_n (a_n^{(m)})$$

Then, Robbins-Monro method:

$$a_n^{(m+1)} \simeq a_n^{(m)} - \frac{1}{m+1} \left(\frac{12}{\Delta_E^2}\right) \langle\langle E - E_n \rangle\rangle_n (a_n^{(m)})$$



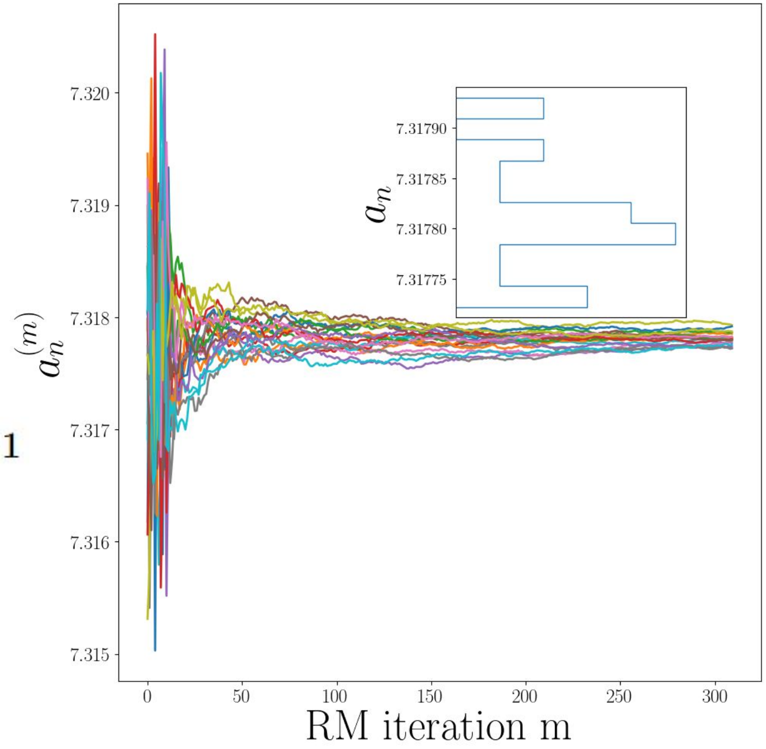


Errors on a_n

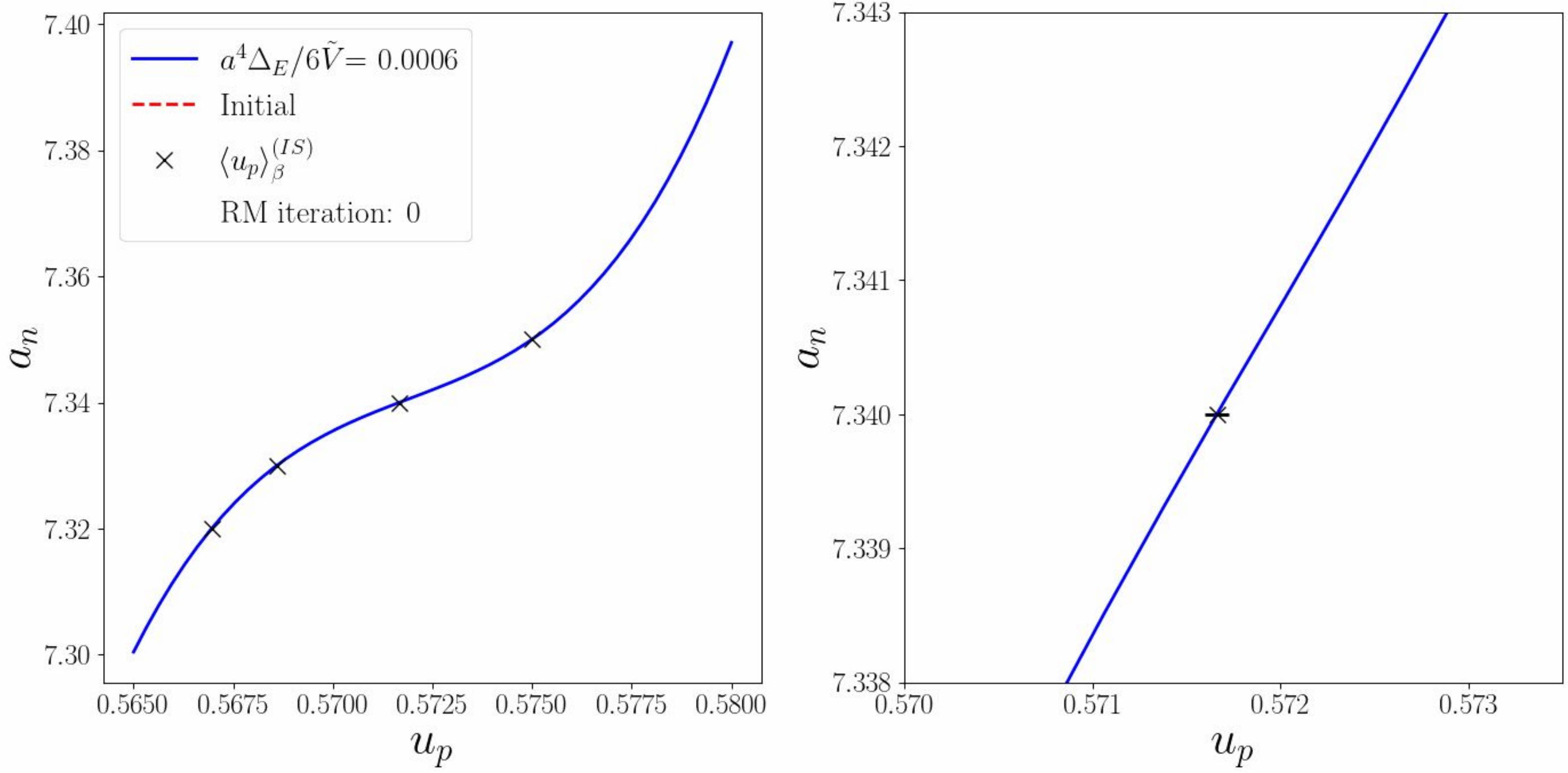
Truncation of RM iterations leads to error

Repeat the determination of $\{a_n\}_{n=1}^{2N-1}$ use statistical error on repeats to estimate truncation error

Calculate observables using each set $\{a_n\}_{n=1}^{2N-1}$ and bootstrap to find errors



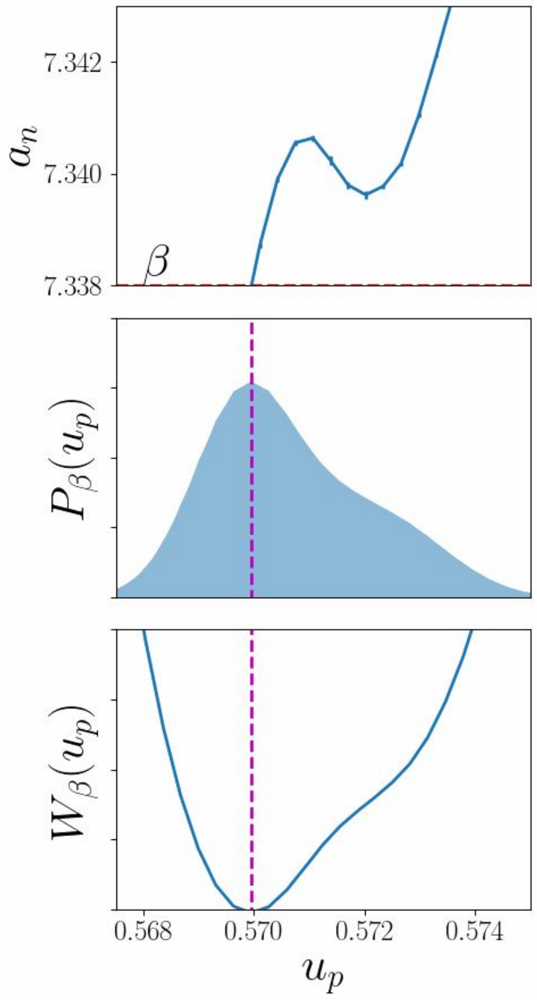
Behaviour of density of states



Probability distribution

$$P_\beta(E) = \frac{1}{Z_\beta}\rho(E)e^{-\beta E}$$

$$W_\beta(E) = -\ln P_\beta(E)$$



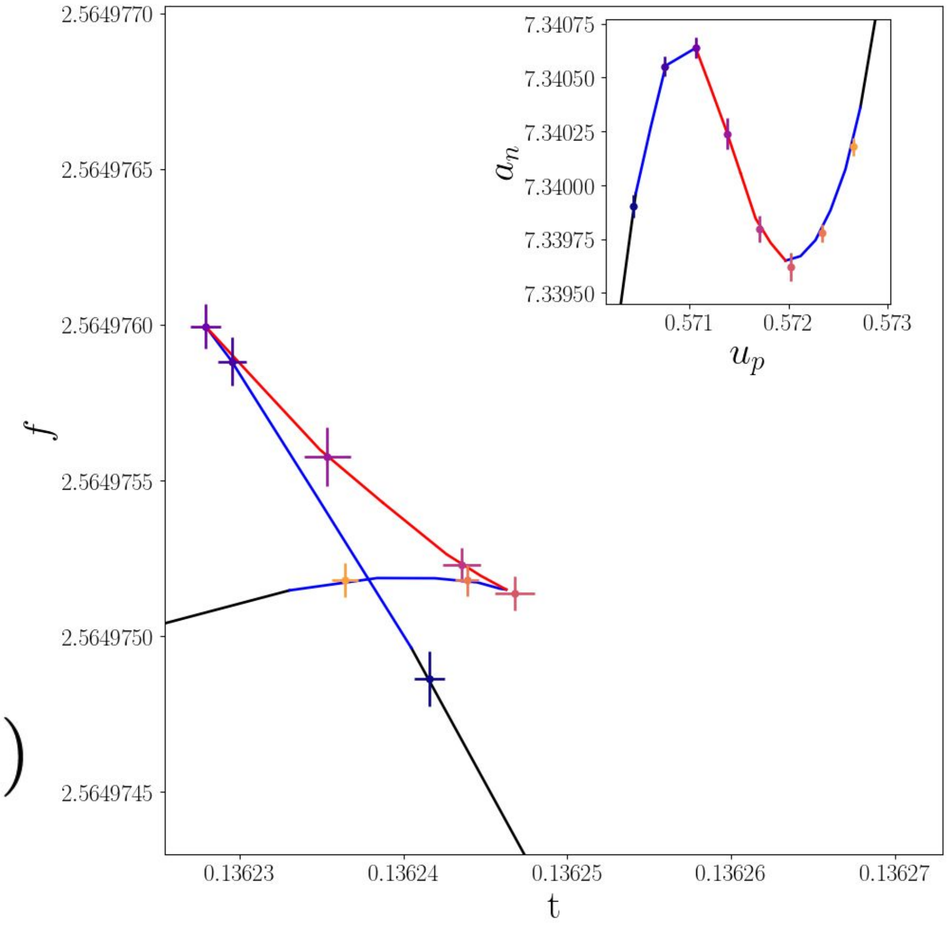
Free energy

$$F(t) = E - ts$$

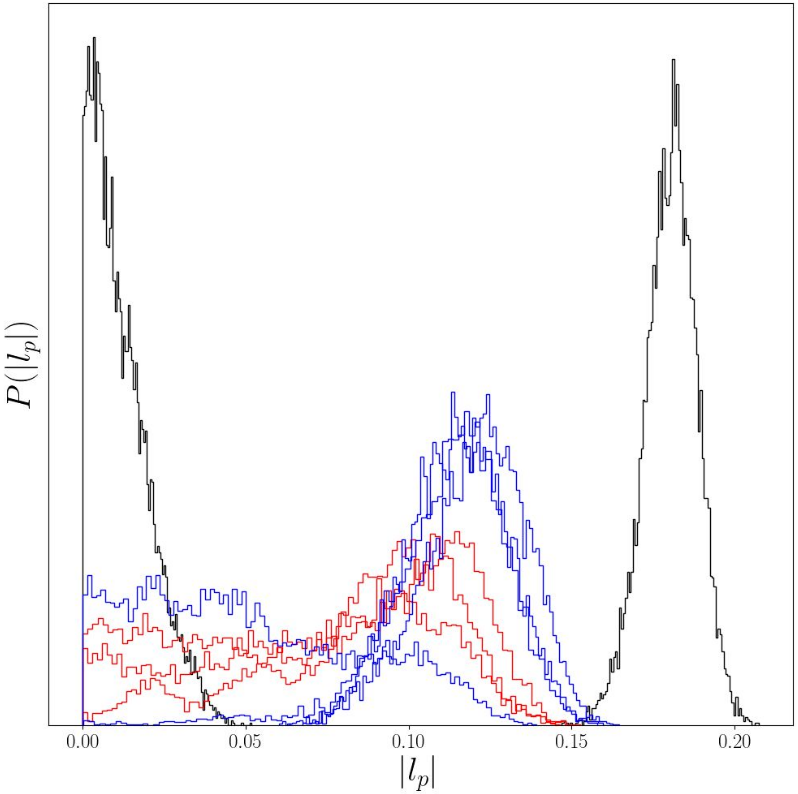
$$t = 1/a_n$$

$$s = \ln \rho$$

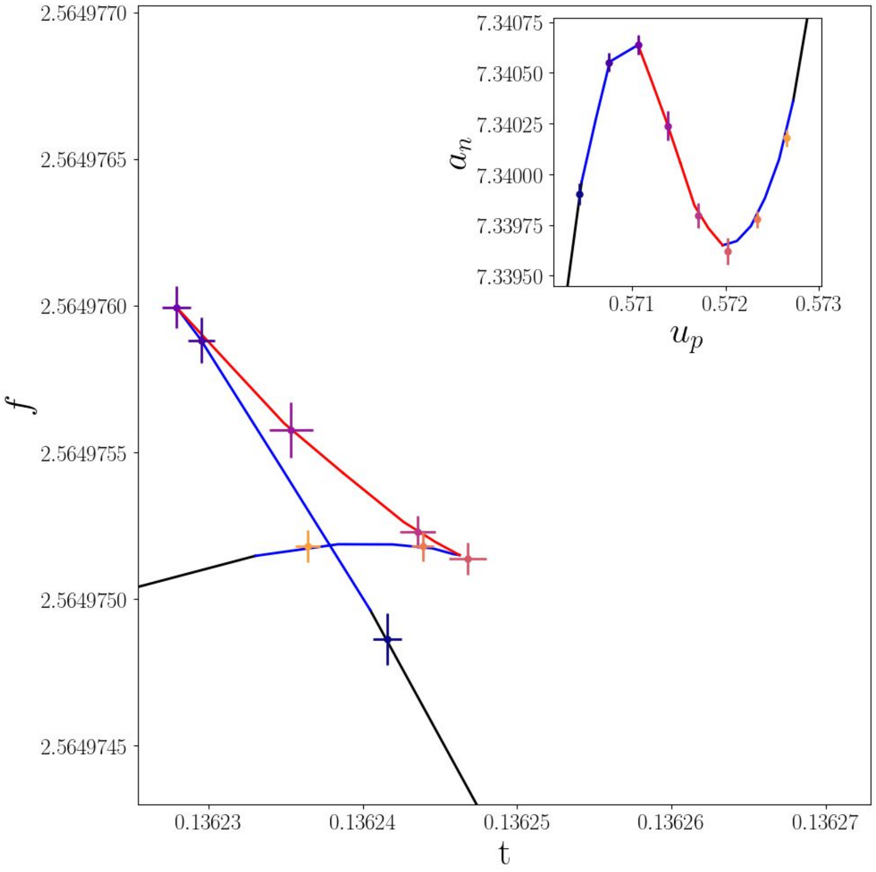
$$f(t) = a^4(F(t) + \Sigma t)/(\tilde{V})$$



Free energy

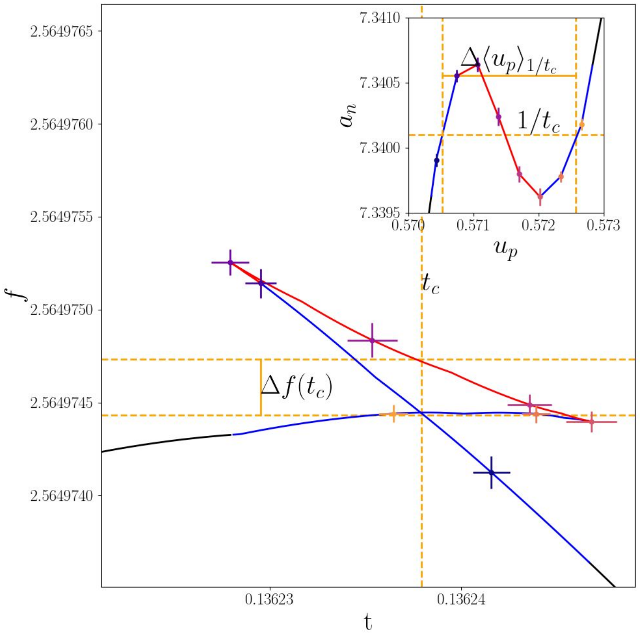


$P(|l_p|)$
0.00
0.05
0.10
0.15
0.20
$|l_p|$

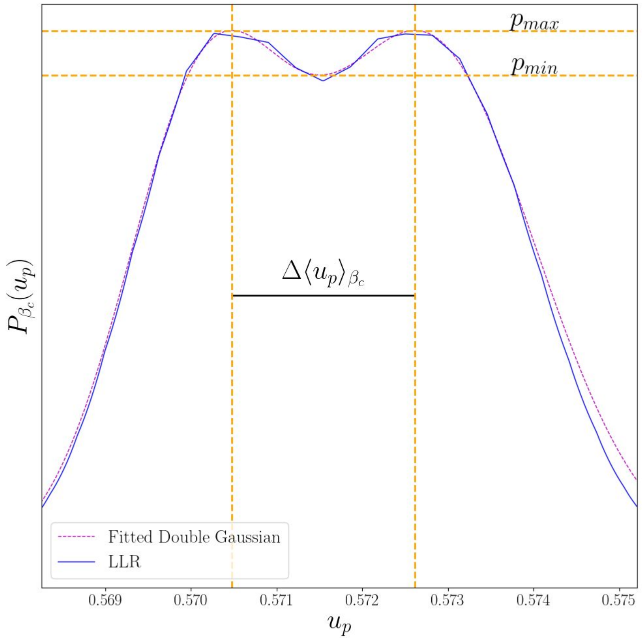


2.5649770
2.5649765
2.5649760
2.5649755
2.5649750
2.5649745
f
0.13623
0.13624
0.13625
0.13626
0.13627
t
7.34075
7.34050
7.34025
7.34000
7.33975
7.33950
a_η
0.571
0.572
0.573
u_p

Latent heat and surface tension

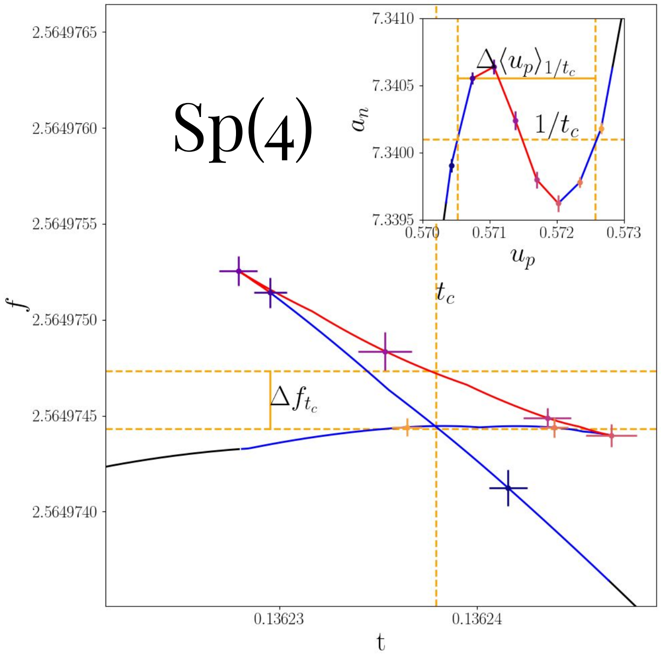


$$\Delta\langle u_p\rangle_{\beta_c} = 0.00205(3)$$
$$\beta_c = 1/t_c = 7.34010(3)$$
$$\frac{\tilde{V}\Delta f(t_c)}{a^4 t_c} = -\ln\left(\frac{pmin}{pmax}\right) = 0.0708(48)$$



$$\Delta\langle u_p\rangle_{\beta_c} = 0.00203(2)$$
$$\beta_c = 1/t_c = 7.34009(3)$$
$$\frac{\tilde{V}\Delta f(t_c)}{a^4 t_c} = -\ln\left(\frac{pmin}{pmax}\right) = 0.0714(43)$$

Comparison with SU(3)



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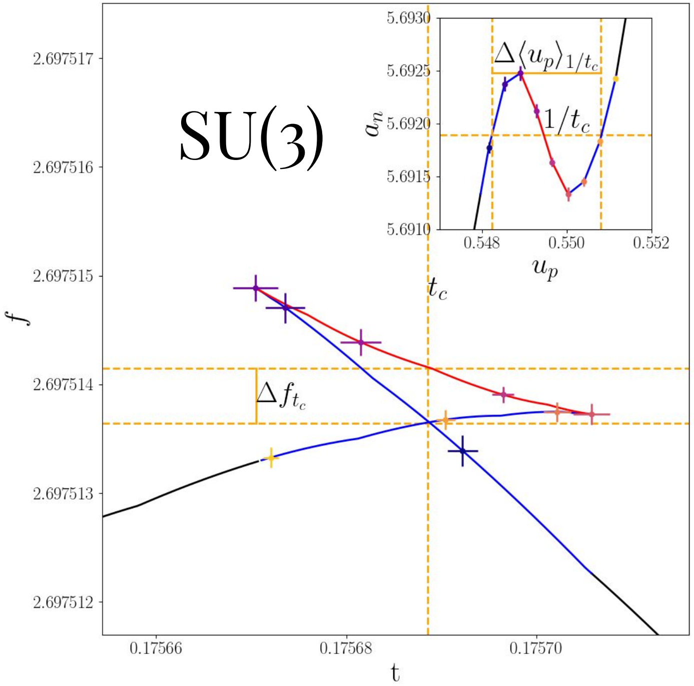


$$\Delta\langle u_p\rangle_{\beta_c} = 0.00257(3)$$
$$\beta_c = 1/t_c = 5.69189(4)$$
$$\frac{\tilde{V}\Delta f(t_c)}{a^4 t_c} = -\ln(\frac{pmin}{pmax}) = 0.0919(83)$$

Conclusion

First order phase transitions from Beyond the Standard physics can lead to observable signatures

The dynamics of the phase transition can be probed through lattice methods

The linear logarithmic relaxation method can be used to solve some of the metastability problems when using Monte Carlo methods around first order phase transitions

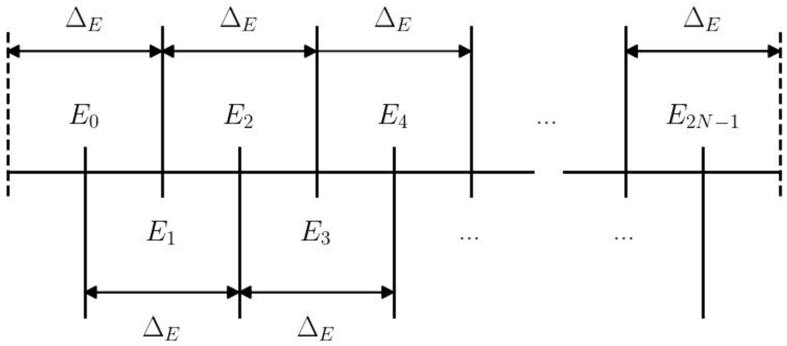
Through the linear logarithmic relaxation method we get access to micro-canonical information about the system

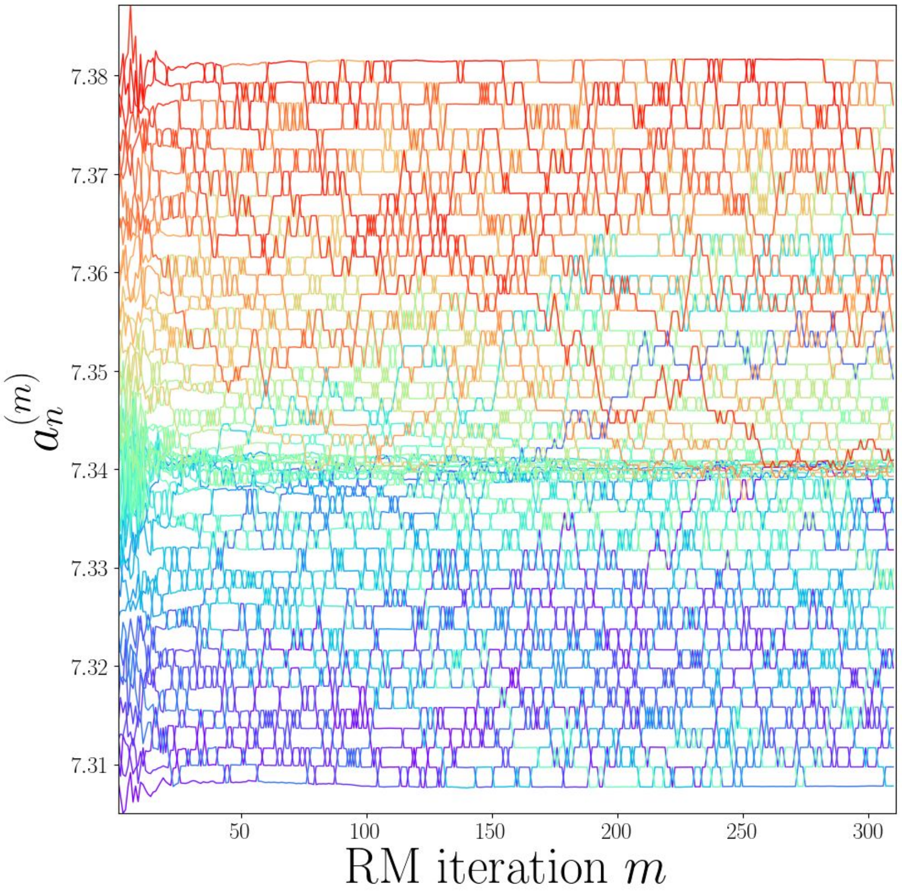
First results reported for Sp(4) Yang-Mills

Backup: Ergodicity

To solve ergodicity problem we use replica exchange
After each RM iteration if replicas are in crossover region, consider swap with probability:

$$P_{\text{swap}} = \min\left(1, e^{(a_n^{(m)} - a_{n-1}^{(m)})(E_n^{(m)} - E_{n-1}^{(m)})}\right)$$



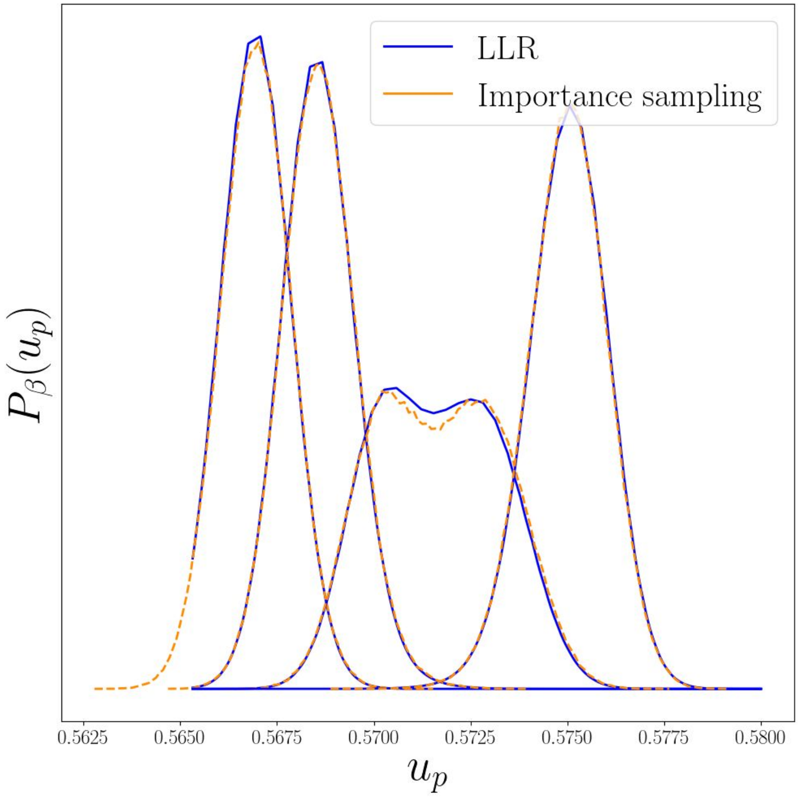


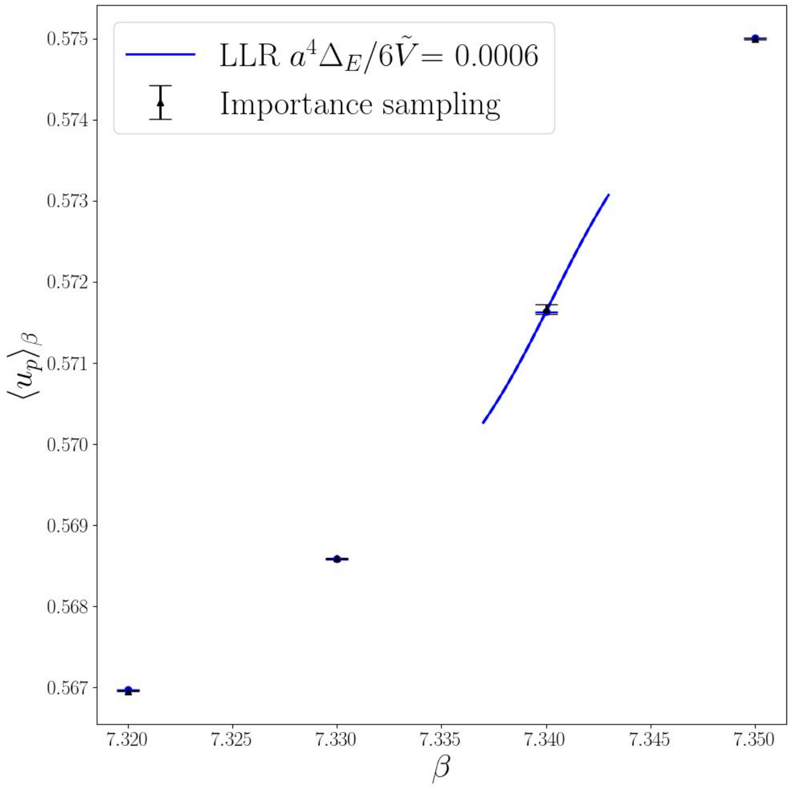
Backup: Bulk phase transition

Fortunately, in contrast to $SU(N)$ with $N \geq 4$, both in $Sp(2)$ and in $Sp(3)$ no bulk phase transition (which could obscure the finite temperature transition) has been found.

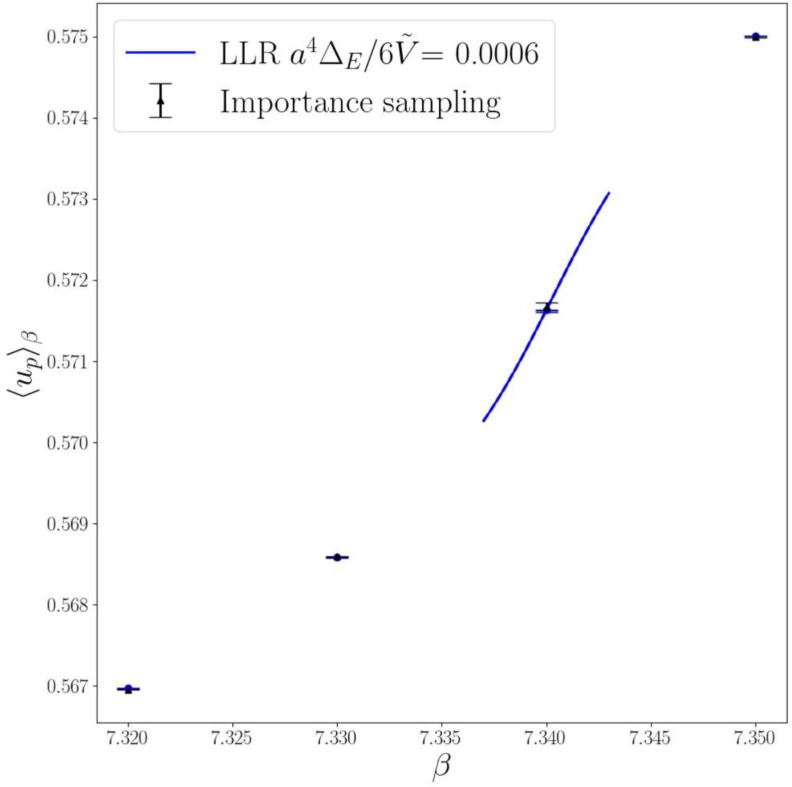
Holland, K., M. Pepe, and U-J. Wiese. "The deconfinement phase transition of Sp (2) and Sp (3) Yang–Mills theories in 2+ 1 and 3+ 1 dimensions." *Nuclear physics B* 694.1-2 (2004): 35-58.

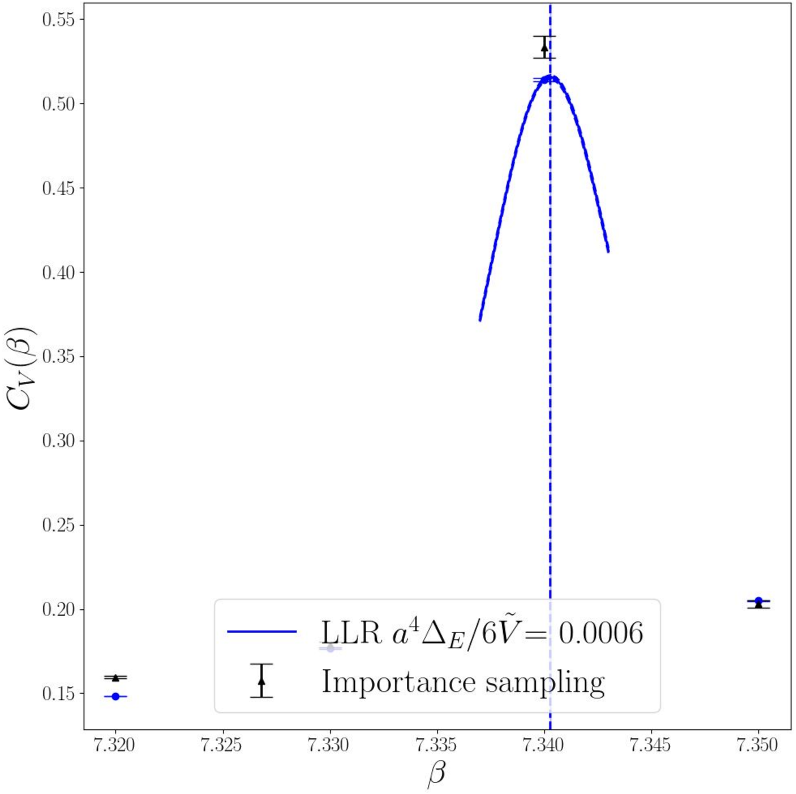
Backup: Comparison to importance sampling





Backup: Comparison to importance sampling





Backup: Comparison to importance sampling