Perturbative renormalization of non-local gluon operators

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Overview

A set of calculations for the renormalization of gauge-invariant non-local gluon operators to one-loop in perturbation theory.

Objectives:

- Obtain the renormalization functions in $\overline{MS}$ scheme using both dimensional and lattice regularizations.
- Compute the conversion factors between the RI$'$ and $\overline{MS}$.
- Find the renormalization mixing matrix, if any.
- Use of the Symanzik improved gluon action.

Impact:

- Derive accurate results for gluon qPDFs through lattice QCD calculations and reduce systematic errors. Disentangle the non-perturbative results of the lattice simulations through the mixing matrix.
- Allow to obtain lattice non-perturbative results in the $\overline{MS}$ scheme using conversion factors (usually extracted in RI’ scheme).

Parton Distribution Functions (PDFs)

- Provide information about the momentum and spin distribution of quarks and gluons inside a hadron, in the infinite momentum frame.
- Defined as expectation values of lightcone correlations in hadronic states → cannot be easily calculated on a Euclidean lattice.

Common approach to determine: Fitting the results of experimental data [1].

Lattice QCD in determining parton observables from first principles:

- Computing their moments, which are matrix elements of local operators. If all moments are known, parton observables can be reconstructed. (Technical difficulties [2])
- Large momentum effective theory (LaMET) [3], quasi-PDFs.

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Quasi Parton Distribution Functions (qPDFs)

LaMET steps to PDFs :

1. Construct a static-operator matrix element (quasi-observable) that approaches the PDF in the infinite momentum limit of the external hadron.
2. Calculate quasi-PDF on the lattice and then renormalize nonperturbatively in an appropriate scheme.
3. The renormalized quasi-observable is matched to the PDF through a factorization formula.

Quasi-observables:

Defined as the matrix elements of gauge-invariant non-local operators that include a finite-length Wilson line, known as Wilson-line operators.

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Lattice action

- Gluons: Symanzik improved action [6], Wilson loops with 4 and 6 links

$$S_G = \frac{2}{g_0^2}\Big[\; c_0 \sum_{\text{plaq.}} \mathrm{Re\,Tr}\,\{1 - U_{\text{plaq.}}\} + c_1 \sum_{\text{rect.}} \mathrm{Re\,Tr}\,\{1 - U_{\text{rect.}}\} + c_2 \sum_{\text{chair}} \mathrm{Re\,Tr}\,\{1 - U_{\text{chair}}\} + c_3 \sum_{\text{paral.}} \mathrm{Re\,Tr}\,\{1 - U_{\text{paral.}}\}\Big]$$

The Symanzik coefficients c_i satisfy the following normalization condition:

$$c_0 + 8c_1 + 16c_2 + 8c_3 = 1$$

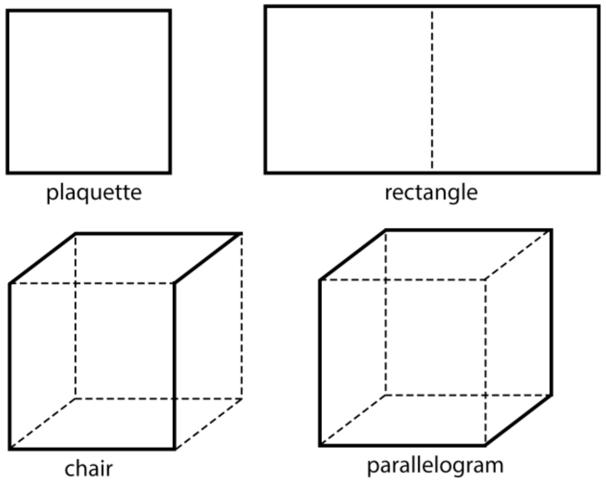

Figure 1. The 4 and 6 link loops contributing to Symanzik improved action.

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Gluon quasi-PDFs

Defined as the matrix elements of gluon non-local correlators in the direction of the lightcone for a given hadron.

For example, the unpolarized gluon distribution can be defined as [7]:

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ixP^+\xi^-} \langle P | G_a^{+i}(\xi^-) \mathcal{W}(\xi^-,0) G_a^{+i}(0) | P \rangle$$

where $x = k^+/P^+$ is the longitudinal momentum fraction carried by the gluon, and μ is the renormalization scale in the $\overline{MS}$ scheme. Here, G_a^{+i} is the Gluon field strength tensor, index i runs over the transverse indices, $P_\mu = (P^0, 0,0, P^z)$ is the hadron momentum, $\xi^\pm = (t \pm z)/\sqrt{2}$ are the lightcone coordinates, and $W(\xi^-, 0)$ is the Wilson line defined as:

$$\mathcal{W}(\xi^-,0) = \exp\left(-ig \int_0^{\xi^-} d\eta^- A^+(\eta^-) \right)$$

Gluon quasi-PDFs

Direct calculation of gluon PDFs is not possible.

Using LaMET, a suitable candidate for the unpolarized gluon qPDFs is provided by [5]:

$$\tilde{f}_{g/H}(x,\mu,P^z) = N \int \frac{dz}{2\pi x P^z} e^{izxP^z} \langle P|O_{\mu\nu\rho\sigma}(z,0)|P\rangle$$

where N is a normalization factor, P_μ is the hadron momentum, x is the longitudinal momentum fraction carried by the gluon.

- $O_{\mu\nu\rho\sigma}$ are the non-local gluon operators defined as:

$$O_{\mu\nu\rho\sigma}(z\hat{\tau},0) \equiv 2\ Tr\Big(G_{\mu\nu}(z\hat{\tau}) W(z\hat{\tau},0) G_{\rho\sigma}(0) W^\dagger(z\hat{\tau},0) \Big)$$

where $G_{\mu\nu}$ is the Gluon field strength tensor and $W(z\hat{\tau},0)$is the Wilson line:

$$W(x, x+z\hat{\tau}) = \mathcal{P}\ e^{ig_0 \int_0^z A_\mu(x+\zeta\hat{\tau})d\zeta}$$

Non-local gluon operators

Non-local gluon operators:

$$O_{\mu\nu\rho\sigma}(z\hat{\tau},0) \equiv 2\,Tr\Big(G_{\mu\nu}(z\hat{\tau})W(z\hat{\tau},0)G_{\rho\sigma}(0)W^{\dagger}(z\hat{\tau},0)\Big)$$

- Due to anti-symmetry of $G_{\mu\nu}$, there are 36 non-local operators in total.

Renormalization could lead to mixing with other relevant operators among those defined in the above equation

A recent study using the auxiliary field approach [5]:

- Different components of the operators have nontrivial renormalization patterns, making it challenging to create accurate gluon quasi-PDFs.
- Identifies four gluon operators that are multiplicatively renormalizable and suitable for defining some of the gluon quasi-PDFs.

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Non-local gluon operators

For convenience, we choose to calculate the following Green function :

$$\langle A^{\alpha_1}_{\nu_1}(q_1)\ A^{\alpha_2}_{\nu_2}(q_2)\ O_{\mu\nu\rho\sigma}(z\hat{\tau},0)\rangle$$

where $A^{\alpha_1}_{\nu_1}(q)$ and $A^{\alpha_2}_{\nu_2}(q)$ are two external gluons while $O_{\mu\nu\rho\sigma}(z\hat{\tau},0)$ is the gluon nonlocal operator.

Construct all necessary vertices of the operator and calculate bare Green's functions up to one-loop using both dimensional (DR) and lattice regularization (LR) in the $\overline{MS}$ scheme.

Previous studies of quark non-local operators [4] revealed two important features on the lattice:

- Linear divergences in addition to logarithmic divergences in lattice spacing (eliminate by a non-perturbative method).
- Mixing among certain subsets of the original operators during renormalization.

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One loop Feynman diagrams

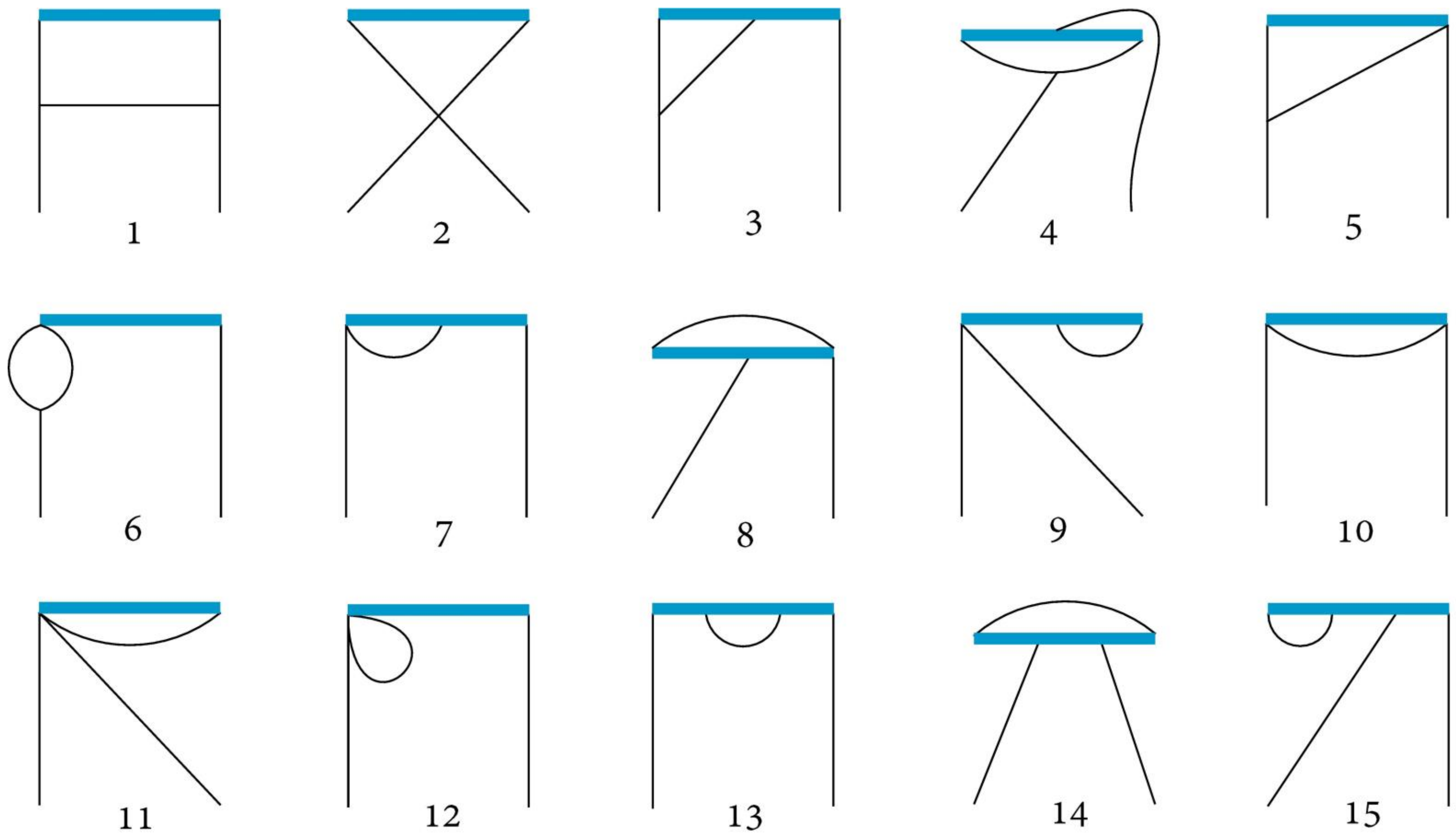

Figure 2. Feynman diagrams contributing to the one-loop calculation of the Green's functions of the non-local operators. Solid lines represent gluons. The operator insertion is denoted by a solid box.

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One loop Feynman diagrams

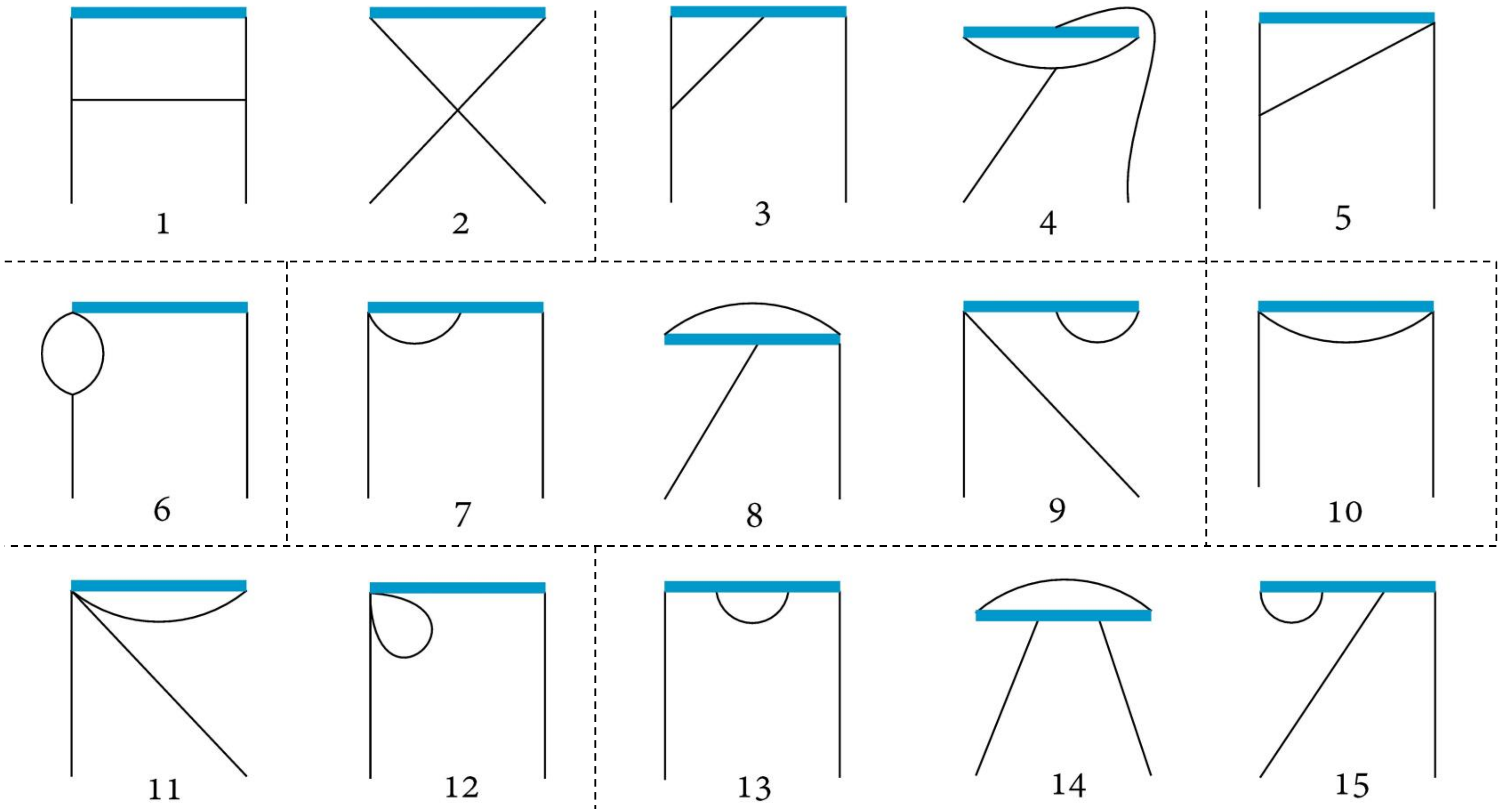

Figure 2. Feynman diagrams contributing to the one-loop calculation of the Green’s functions of the non-local operators. Solid lines represent gluons. The operator insertion is denoted by a solid box.

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Tree level Green's functions

The tree level of equation of the chosen Green's functions is quite trivial. We find the following result in both dimensional and lattice regularization:

$$
\begin{aligned}
\langle A^{\alpha_1}_{\nu_1}(q_1)\ A^{\alpha_2}_{\nu_2}(q_2)\ O_{\mu\nu\rho\sigma}\rangle^{\text{tree}} = 2\ \delta^{\alpha_1\alpha_2}\Big(& + q_{1\mu}q_{2\rho}\ \delta_{\nu_1\nu}\delta_{\nu_2\sigma}\ e^{-izq_1\hat{\tau}} + q_{1\mu}q_{2\rho}\ \delta_{\nu_1\sigma}\delta_{\nu_2\nu}\ e^{izq_1\hat{\tau}} \\
& - q_{1\nu}q_{2\rho}\ \delta_{\nu_1\mu}\delta_{\nu_2\sigma}\ e^{-izq_1\hat{\tau}} - q_{1\nu}q_{2\rho}\ \delta_{\nu_1\sigma}\delta_{\nu_2\mu}\ e^{izq_1\hat{\tau}} \\
& - q_{1\mu}q_{2\sigma}\ \delta_{\nu_1\nu}\delta_{\nu_2\rho}\ e^{-izq_1\hat{\tau}} - q_{1\mu}q_{2\sigma}\ \delta_{\nu_1\rho}\delta_{\nu_2\nu}\ e^{izq_1\hat{\tau}} \\
& + q_{1\nu}q_{2\sigma}\ \delta_{\nu_1\mu}\delta_{\nu_2\rho}\ e^{-izq_1\hat{\tau}} + q_{1\nu}q_{2\sigma}\ \delta_{\nu_1\rho}\delta_{\nu_2\mu}\ e^{izq_1\hat{\tau}}\Big)
\end{aligned}
$$

where z and $\hat{\tau}$ is the length and direction of the Wilson line.

Antisymmetric in {μ,ν} and {ρ,σ} as expected.

1-loop Green’s functions in DR

In contrast to 2-point Green’s functions involving local operators, integration results become significantly more complicated due to :

- the presence of both the external momentum and the length of the Wilson line in the integrands.
- a nontrivial dependence on the preferred direction of the Wilson line

New techniques for assessing one-loop tensor integrals with an exponential factor in D dimensions similar to [8]

Examples of integrals:

$$\int \frac{d^D p}{(2\pi)^D} \frac{e^{-izp_{\hat{\tau}}}\ p_\nu\, p_\rho\, p_\sigma}{p^2\,(p+q)^2} \qquad \int_0^z d\zeta \int \frac{d^D p}{(2\pi)^D} \frac{e^{-i\zeta p_{\hat{\tau}}}\ p_\nu}{(p^2)^2\ (p+q)^2}$$

$$\int_0^z d\zeta_1 \int_{\zeta_1}^z d\zeta_2 \int \frac{d^D p}{(2\pi)^D} \frac{e^{i(\zeta_1-\zeta_2)p_{\hat{\tau}}}}{p^2}$$

1-loop Green's functions in DR

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Below we present the $\mathcal{O}(1/\varepsilon)$ contributions of these calculations, including all combinatorial factors:

$$\langle A^{\alpha_1}_{\nu_1}(q_1)\ A^{\alpha_2}_{\nu_2}(q_2)\ O_{\mu\nu\rho\sigma}\rangle^{\text{1-loop}}\Big|_{\mathcal{O}(1/\epsilon)} = \frac{g^2 N_c}{16\epsilon\pi^2}\left(\delta_{\mu\hat{\tau}} + \delta_{\nu\hat{\tau}} + \delta_{\rho\hat{\tau}} + \delta_{\sigma\hat{\tau}} - \frac{7}{2}\beta\right)\langle A^{\alpha_1}_{\nu_1}(q_1)\ A^{\alpha_2}_{\nu_2}(q_2)\ O_{\mu\nu\rho\sigma}\rangle^{\text{tree}}$$

where β is the gauge fixing parameter and $\hat{\tau}$ is the direction of the Wilson line.

It should be noted that, at the one-loop level in dimensional regularization, the pole terms are proportional to the tree-level values:

→indicating no mixing with operators of equal or lower dimension.

Also, as expected from gauge invariance, the β dependence disappears in the renormalization function of the operators, upon taking into account the renormalization function for the external gluon fields.

1-loop Green's functions in LR

We are currently evaluating the corresponding lattice renormalization factors; this is a complicated calculation:

- There is an additional factor of $1/a^2$ (a is the lattice spacing) in the expression due to the presence of the external gluons.
- We have encountered some well-known lattice integrals with IR divergences [9]:

$$\int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} \frac{\overset{\circ}{p}_\mu \overset{\circ}{p}_\nu}{(\hat{p}^2)^2 \, \widehat{p+aq}^{\,2}} \qquad \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} \frac{\hat{p}_\mu^4}{(\hat{p}^2)^2}$$

- We encountered the following integrals (use of Schwinger parametrization)

$$\int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} \cos(p_\tau/2) \frac{(e^{inp_\tau} - 1)}{\sin(p_\tau/2)} \frac{\overset{\circ}{p}_\mu \overset{\circ}{p}_\nu}{(\hat{p}^2)^2}$$

where $\hat{p}^2 \equiv 4 \sum_\mu sin^2(p_\mu/2)$, while $\overset{\circ}{p}_\mu \equiv \sin(p_\mu)$, index τ is the direction of the Wilson line, indices μ and ν is one of the four directions, and $n \equiv z/\alpha$

The calculations are in progress.

Summary and Upcoming calculations

Summary:

Obtain the renormalization functions of non-local gluon operators in $\overline{MS}$ scheme using both dimensional and lattice regularizations.

We expect that lattice calculations will reveal a finite mixing among these operators in pairs as happens with quark non-local operators.

These results can help the non-perturbative studies of quasi-PDFs.

Upcoming calculations:

- Compute the renormalization factors of the non-local gluon operators using the pole terms of the one-loop result in DR.
- Utilising the one-loop result in DR, we can find the conversion factors between the RI$'$ and $\overline{MS}$ schemes (only obtainable within perturbation theory) , to convert the corresponding lattice non-perturbative results to the $\overline{MS}$ scheme.
- Complete the evaluation of the corresponding lattice renormalization factors

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Thank you!