Running Dark Energy in Lattice Quantum Gravity

Mingwei Dai (speaker)
Jack Laiho, Judah Unmuth-Yockey, Marc Schiffer

Syracuse University
mdai07@syr.edu

Aug 1st, 2023
It was pointed out by Weinberg that if gravity is asymptotically safe, it would be renormalizable non-perturbatively.[1]

Euclidean dynamical triangulations (EDT) is an approach to lattice quantum gravity. Geometry is constructed by gluing 4-simplices together. In [J. Laiho, S. Bassler, 2016], it was shown that when a non-trivial measure term is added and associated coupling is fine-tuned, EDT gives the correct Hausdorff dimension and spectral dimension.

Figure 1: Visualization of one configuration. Dots represent 4-simplices and the lines show connection between the nearest neighbors.
The path integral of 4-d Euclidean Einstein gravity in the **continuum** is:

\[ Z_E = \int \mathcal{D}[g] e^{-S_{EH}[g]}, \]  

(1)

For **lattice** QG, the path integral becomes:

\[ Z_E = \sum_T \frac{1}{C_T} \left[ \prod_{j=1}^{N_2} \mathcal{O}(t_j)^{\beta} \right] e^{-S_{ER}} \]  

(2)

where \( C_T \) divides out equivalent ways of labeling the vertices in a given geometry, \( N_2 \) is the total number of triangles in our geometry, \( \beta \) is a free parameter, and \( S_{ER} \) is the Einstein-Regge action:[J. Ambjorn, J. Jurkiewicz, 1992]

\[ S_{ER} = -\kappa_2 N_2 + \kappa_4 N_4, \]  

(3)

The volume of a d-simplex can be calculated:

\[ V_d = \frac{\sqrt{d+1}}{d! \sqrt{2}^d} a^d. \]  

(4)
Figure 2: Schematic of the phase diagram as a function of $\kappa_2$ and $\beta$. [D. Coumbe, J. Laiho, 2014]
Two units of lattice spacing:

- Link units $a$: Distance between two vertices. Edge/link length.
- Simplex units $\ell$: Distance between two 4-simplexes.

Quantities can be measured in both units and on different lattice spacings. Therefore, to put quantities in a common unit, we need the value of $\frac{a}{\ell}$ and $\ell_{\text{rel}}$, which is the relative dual lattice spacing:

$$\ell_{\text{rel}} = \frac{\ell}{\ell_{\text{fiducial}}}$$
The shelling function of a particular configuration can be found by choosing a source simplex and find its neighbors and neighbors of neighbors.

**Figure 3:** Shelling of one configuration. $x$ axis shows the geodesic distance (Euclidean time) and $y$ axis shows the number of 4-simplices in that slice (3-volumes).
Determination of $\frac{a}{\ell}$ II

We fit our shelling to the function:

$$f(\tau) = A \cos^3 (B\tau + C)$$  \hspace{1cm} (5)

The scale factor in Euclidean space should have the form of:

$$A(\tau) = \sqrt{\frac{3}{\Lambda}} \cos(\sqrt{\frac{\Lambda}{3}} \tau)$$  \hspace{1cm} (6)

**Figure 4:** Finer lattices are closer to the de Sitter solution.

**Figure 5:** The fit of shelling to $f(\tau)$ on 8k fine.
$A^{1/3}$ should just be $1/B$ if the space and time units are consistent.

$$\frac{1}{A^{1/3}B} = \frac{\ell_V}{\ell} \quad (7)$$

which is the ratio between the unit length in the 3-volume and the Euclidean time.

And $\frac{a}{\ell}$ can be obtained by:

$$\frac{a}{\ell} = \left( \frac{2\pi^2}{\sqrt{5}} \left( \frac{\ell_V}{\ell} \right)^3 \right)^{1/4} \quad (8)$$

**Figure 6:** Extrapolation of $\frac{a}{\ell}$ on for ensembles with $\kappa_2 = 3.4$. $\frac{a}{\ell} = 16.586(680)$ with $\chi^2$/d.o.f. = 0.698.
Determination of $\ell_{\text{rel}}$ and Absolute Lattice Spacing

Relative lattice spacing

$$\ell_{\text{rel}} = \frac{m_k}{m_{\text{fid}}} \quad (9)$$

where $m_k$ is the renormalized mass at which it turns for a particular lattice spacing and $m_{\text{fid}}$ is the mass of the fiducial lattice spacing.

Figure 7: Knee of three $\kappa_2 = 3.0$ ensembles.

Determination of absolute lattice spacing can be done by calculating $G$ on the lattice.[S. Bassler, M. Schiffer, J. Laiho, 2021][M. Dai, M. Schiffer, J. Laiho, 2021][Marc Schiffer, today at 2:30 WH3NW]
Running dark energy

We can extract the scale factor $A(\tau)$ from the 3-volume using the formula:

$$A(\tau) = \left( \frac{1}{2\pi^2} \frac{\sqrt{5}}{96} V_3(\tau) \left( \frac{a}{\ell} \right)^4 \right)^{\frac{1}{3}} \tag{10}$$

From the first Friedmann equation in the Euclidean space, we have

$$\Lambda = \frac{3}{A^2} \left[ - \left( \frac{dA}{d\tau} \right)^2 + 1 \right] \tag{11}$$

Figure 8: The fit of $\Lambda$ as a function of $H^2$ for 32k $\kappa_2 = 3.8$

We fit our $\Lambda$ to a running dark energy model introduced by Joan Solà:[arXiv:1501.03832]

$$\Lambda = \Lambda_0 + 3\nu H^2 + 3\tilde{\nu} \dot{H} + O \left( H^4 \right) \tag{12}$$

where $H$ is the Hubble parameter. $\nu$ and $\tilde{\nu}$ are dimensionless parameters.
Running of $\nu$ and $\tilde{\nu}$ I

If we define $s = \frac{\ddot{A}}{A}$ so that $\dot{H} = s - H^2$ then we can rewrite:

$$\Lambda = \Lambda_0 + 3\tilde{\nu} s + 3\nu' H^2$$

(13)

where $\nu' = \nu - \tilde{\nu}$. And we treat $s$ as a constant.

Figure 9: $s$ as a function of $H^2$ for $32k \kappa_2 = 3.8$

Figure 10: The fit of $\Lambda$ as a function of $H^2$ for $32k \kappa_2 = 3.8$
Running of $\nu$ and $\tilde{\nu}$ II

$\nu' = \nu - \tilde{\nu}$ for different lattice spacings and volumes with a logarithmic fit. We get $[A, B] = [0.284(109), 0.005(1)]$ with $\chi^2$/d.o.f. $= 0.153$.

$\nu$ is dimensionless.

$$\nu(\mu_f^2) = \frac{\nu(\mu_i^2)}{1 + b \log(\frac{\mu_f^2}{\mu_i^2})}$$  \hspace{1cm} (14)$$

$$\mu^2 \rightarrow H^2 \propto R \propto 1/\sqrt{V_4}$$

$$\nu' = \frac{A}{\log(B\sqrt{V_4})}$$  \hspace{1cm} (15)$$
Running of $\nu$ and $\tilde{\nu}$ III

We did separate fits for 3 lattice spacings.

![Graph showing running of $\nu$ for different lattice spacings.](image)

**Figure 12:** $\tilde{\nu}$ for different lattice spacings.

The fit result of $\tilde{\nu}$ is roughly consistent with 0. $\kappa_2 = 3.8$ ensembles give $\tilde{\nu} = -0.038(51)$ with $\chi^2$/d.o.f. = 2.370. $\kappa_2 = 3.4$ ensembles give $\tilde{\nu} = -0.050(45)$ with $\chi^2$/d.o.f. = 0.026.

$$I_{H2} = \Lambda_0 + 3\tilde{\nu}s$$

$$I_{H2} = \frac{C}{\sqrt{V_4}} + \frac{3\tilde{A}s}{\log(B\sqrt{V_4})}$$
\( \nu' = \nu - \tilde{\nu} \) is from 1st Friedmann equation

- Classically, \( s = \frac{\ddot{A}}{A} = -\frac{\Lambda}{3} \). It receives a correction when \( \Lambda \) starts running.
- \( \nu'' = \nu - 2\tilde{\nu} \) from the 2nd Friedmann equation and continuity equation

\[ I_{H^2} + 3s \approx -3\nu''s \]  

[Jack Laiho, today 4:40, WH3NE]
When $\tilde{\nu}$ is 0, both $\nu'$ and $\nu''$ reduce to $\nu$.

**Figure 13:** $\nu' = \nu - \tilde{\nu}$ for different lattice spacings and volumes with a logarithmic fit. We get $[A, B] = [0.284(109), 0.005(1)]$ with $\chi^2$/d.o.f. = 0.153.

**Figure 14:** $\nu'' = \nu - 2\tilde{\nu}$ for different lattice spacings and volumes with a logarithmic fit. We get $[A, B] = [0.291(111), 0.005(1)]$ with $\chi^2$/d.o.f. = 0.099.

We can see that $\nu'$ and $\nu''$ are pretty consistent, which means that $\tilde{\nu}$ is 0. This is in agreement with the calculation of renormalized $\rho_\Lambda$ with QFT on FLRW spacetime[C. Moreno-Pulido, 2023].
Our results show that the dark energy runs naturally in lattice quantum gravity and can be well described by the Solà model.

- The running of $\Lambda$ only depends on $H^2$.
- What particles does dark energy decay to?
- How does adding matter field affect the result?
References


Thank you!