Running Dark Energy in Lattice Quantum Gravity

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Aug 1st, 2023

Asymtotic Safety with EDT

It was pointed out by Weinberg that if gravity is asymptotically safe, it would be renormalizable non-perturbatively.[1]

Euclidean dynamical triangulations (EDT) is an approach to lattice quantum gravity. Geometry is constructed by gluing 4-simplices together. In [J. Laiho, S. Bassler,2016], it was shown that when a non-trivial measure term is added and associated coupling is fine-tuned, EDT gives the correct Hausdorff dimension and spectral dimension.

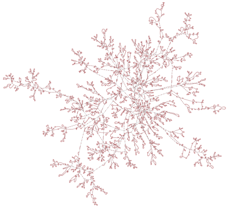

Figure 1: Visualization of one configuration. Dots represent 4-simplices and the lines show connection between the nearest neighbors.

The path integral of 4-d Euclidean Einstein gravity in the **continuum** is:

$$Z_E = \int \mathcal{D}[g] e^{-S_{EH}[g]}, \tag{1}$$

For **lattice** QG, the path integral becomes:

$$Z_E = \sum_T \frac{1}{C_T} \left[\prod_{j=1}^{N_2} \mathcal{O}(t_j)^{\beta}\right] e^{-S_{ER}} \tag{2}$$

where C_T divides out equivalant ways of labeling the vertices in a given geometry, N_2 is the total number of triangles in our geometry, β is a free parameter, and S_{ER} is the Einstein-Regge action:[J. Ambjorn,J. Jurkiewicz,1992]

$$S_{ER} = -\kappa_2 N_2 + \kappa_4 N_4, \tag{3}$$

The volume of a d-simplex can be calculated:

$$V_d = \frac{\sqrt{d+1}}{d! \sqrt{2^d}} a^d. \tag{4}$$

Phase Diagram

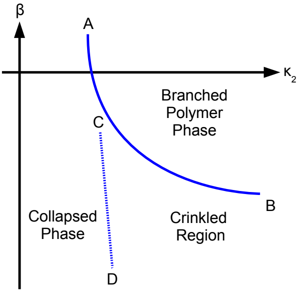

Figure 2: Schematic of the phase diagram as a function of κ_2 and β.[D. Coumbe, J. Laiho,2014]

Lattice spacings I

Two units of lattice spacing:

- Link units a: Distance between two vertices. Edge/link length.
- Simplex units ℓ: Distance between two 4-simplexes.

Quantities can be measured in both units and on different lattice spacings. Therefore, to put quantities in a common unit, we need the value of $\frac{a}{\ell}$ and ℓ_{rel}, which is the relative dual lattice spacing:

$$\ell_{\text{rel}} = \frac{\ell}{\ell_{\text{fiducial}}}$$

Determination of $\frac{a}{\ell}$ I

The shelling function of a particular configuration can be found by choosing a source simplex and find its neighbors and neighbors of neighbors.

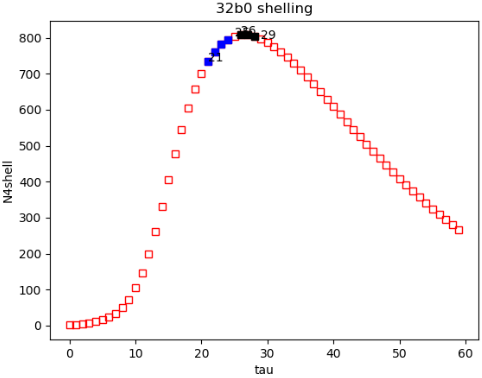

Figure 3: Shelling of one configuration. x axis shows the geodesic distance (Euclidean time) and y axis shows the number of 4-simplices in that slice (3-volumes).

Determination of $\frac{a}{\ell}$ II

We fit our shelling to the function:

$$f(\tau) = A\cos^3(B\tau + C) \tag{5}$$

The scale factor in Euclidean space should have the form of:

$$A(\tau) = \sqrt{\frac{3}{\Lambda}}\cos(\sqrt{\frac{\Lambda}{3}}\tau) \tag{6}$$

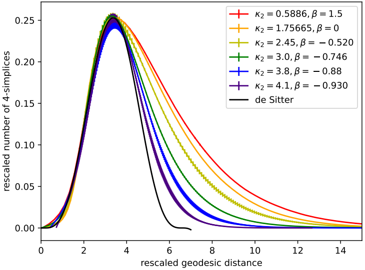

Figure 4: Finer lattices are closer to the de Sitter solution.

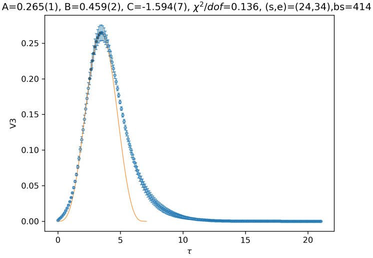

Figure 5: The fit of shelling to $f(\tau)$ on 8k fine.

$A^{1/3}$ should just be $1/B$ if the space and time units are consistent.

$$\frac{1}{A^{1/3}B} = \frac{\ell_V}{\ell} \tag{7}$$

which is the ratio between the unit length in the 3-volume and the Euclidean time. And $\frac{a}{\ell}$ can be obtained by:

$$\frac{a}{\ell} = \left(\frac{2\pi^2}{\frac{\sqrt{5}}{96}} \left(\frac{\ell_V}{\ell} \right)^3 \right)^{\frac{1}{4}} \tag{8}$$

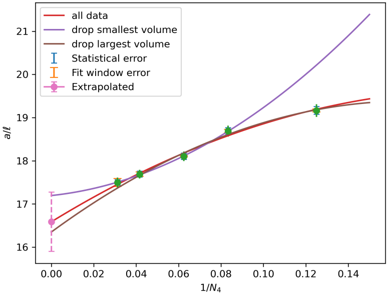

Figure 6: Extrapolation of $\frac{a}{\ell}$ on for ensembles with $\kappa_2 = 3.4$. $\frac{a}{\ell} = 16.586(680)$ with $\chi^2/\text{d.o.f.} = 0.698$.

Relative lattice spacing

$$\ell_{\text{rel}} = \frac{m_k}{m_{\text{fid}}} \tag{9}$$

where m_k is the renormalized mass at which it turns for a particular lattice spacing and m_{fid} is the mass of the fiducial lattice spacing.

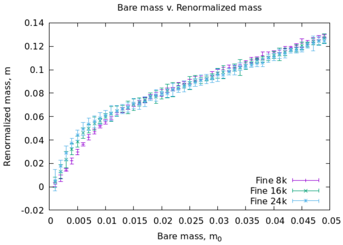

Figure 7: Knee of three $\kappa_2 = 3.0$ ensembles.

Determination of absolute lattice spacing can be done by calculating G on the lattice.[S. Bassler,M. Schiffer,J. Laiho,2021][M. Dai,M. Schiffer,J. Laiho,2021][Marc Schiffer, today at 2:30 WH3NW]

We can extract the scale factor $A(\tau)$ from the 3-volume using the formula:

$$A(\tau) = \left(\frac{1}{2\pi^2} \frac{\sqrt{5}}{96} V_3(\tau) \left(\frac{a}{\ell} \right)^4 \right)^{\frac{1}{3}} \tag{10}$$

From the first Friedmann equation in the Euclidean space, we have

$$\Lambda = \frac{3}{A^2} \left[- \left(\frac{\mathrm{d}A}{\mathrm{d}\tau} \right)^2 + 1 \right] \tag{11}$$

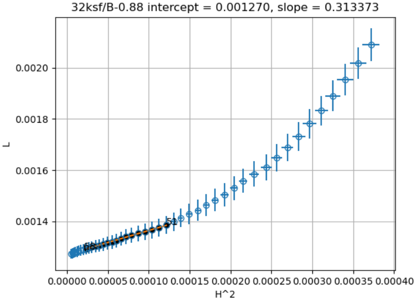

Figure 8: The fit of Λ as a function of H^2 for 32k $\kappa_2 = 3.8$

We fit our Λ to a running dark energy model introduced by Joan Solà:[arXiv:1501.03832]

$$\Lambda = \Lambda_0 + 3\nu H^2 + 3\tilde{\nu}\dot{H} + \mathcal{O}\left(H^4\right) \tag{12}$$

where H is the Hubble parameter. ν and $\tilde{\nu}$ are dimensionless parameters.

Running of ν and $\tilde{\nu}$ I

If we define $s = \frac{\ddot{A}}{A}$ so that $\dot{H} = s - H^2$ then we can rewrite:

$$\Lambda = \Lambda_0 + 3\tilde{\nu}s + 3\nu' H^2 \tag{13}$$

where $\nu' = \nu - \tilde{\nu}$. And we treat s as a constant.

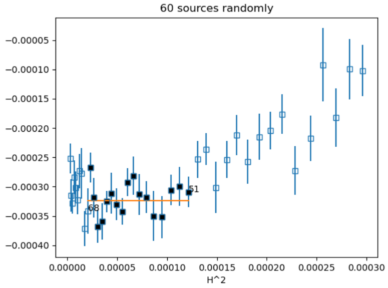

Figure 9: s as a function of H^2 for 32k $\kappa_2 = 3.8$

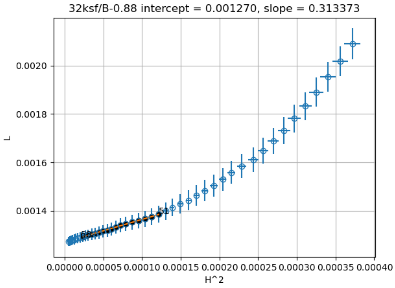

Figure 10: The fit of Λ as a function of H^2 for 32k $\kappa_2 = 3.8$

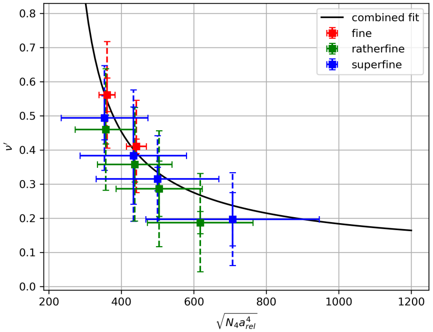

Figure 11: $\nu' = \nu - \tilde{\nu}$ for different lattice spacings and volumes with a logarithmic fit. We get $[A, B] = [0.284(109), 0.005(1)]$ with $\chi^2/\text{d.o.f.} = 0.153$.

ν is dimensionless.

$$\nu(\mu_f^2) = \frac{\nu(\mu_i^2)}{1 + b\log(\frac{\mu_f^2}{\mu_i^2})} \tag{14}$$

$$\mu^2 \to H^2 \propto R \propto 1/\sqrt{V_4}$$

$$\nu' = \frac{A}{\log(B\sqrt{V_4})} \tag{15}$$

Running of ν and $\tilde{\nu}$ III

We did separate fits for 3 lattice spacings.

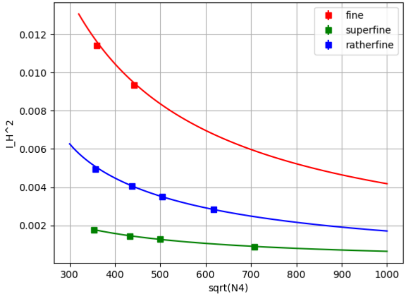

Figure 12: $\tilde{\nu}$ for different lattice spacings.

$$I_{H^2} = \Lambda_0 + 3\tilde{\nu}s$$

$$I_{H^2} = \frac{C}{\sqrt{V_4}} + \frac{3\tilde{A}s}{\log\left(B\sqrt{V_4}\right)}$$

The fit result of $\tilde{\nu}$ is roughly consistent with 0. $\kappa_2 = 3.8$ ensembles give $\tilde{\nu} = -0.038(51)$ with $\chi^2/\text{d.o.f.} = 2.370$. $\kappa_2 = 3.4$ ensembles give $\tilde{\nu} = -0.050(45)$ with $\chi^2/\text{d.o.f.} = 0.026$.

ν'' I

- $\nu' = \nu - \tilde{\nu}$ is from 1st Friedmann equation
- Classically, $s = \frac{\ddot{A}}{A} = -\frac{\Lambda}{3}$. It receives a correction when Λ starts running.
- $\nu'' = \nu - 2\tilde{\nu}$ from the 2nd Friedmann equation and continuity equation

$$I_{H^2} + 3s \approx -3\nu'' s \tag{16}$$

[Jack Laiho, today 4:40, WH3NE]

When $\tilde{\nu}$ is 0, both ν' and ν'' reduce to ν.

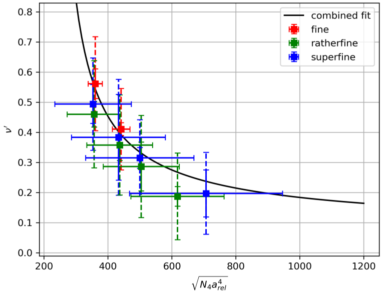

Figure 13: $\nu' = \nu - \tilde{\nu}$ for different lattice spacings and volumes with a logarithmic fit. We get $[A, B] = [0.284(109), 0.005(1)]$ with $\chi^2/\text{d.o.f.} = 0.153$.

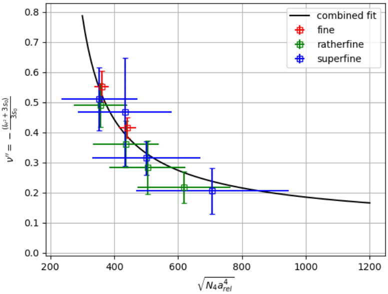

Figure 14: $\nu'' = \nu - 2\tilde{\nu}$ for different lattice spacings and volumes with a logarithmic fit. We get $[A, B] = [0.291(111), 0.005(1)]$ with $\chi^2/\text{d.o.f.} = 0.099$.

We can see that ν' and ν'' are pretty consistent, which means that $\tilde{\nu}$ is 0. This is in agreement with the calculation of renormalized ρ_Λ with QFT on FLRW spacetime[C. Moreno-Pulido,2023].

Conclusion

- Our results show that the dark energy runs naturally in lattice quantum gravity and can be well described by the Solà model.
- The running of Λ only depends on H^2.
- What particles does dark energy decay to?
- How does adding matter field affect the result?

References

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Thank you!