

Rare K decays off and on the lattice

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Lattice 2023-Fermi Lab

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Rare K decays: off & on the lattice; Soni (BNL-HET)

Key Points

1. $K^+ \Rightarrow \pi^+ \nu \nu$ off and on the lattice

Pheno: WIP with E. Lunghi; Impr determination of V_{ub} using
Gounaris+Sakurai;

Lee + Zumino.

→ PALGO

2207.52860 }
2212.01424 }

See also ATLAS et al

→ PRD'67

1910.10544
1806.11520
1701.02855

+ Improvements in ϵ_{ps}' and in K-UT

Expt progress and status

Lattice: RBC-UKQCD several papers

Key points (contd)

- 2. $KL \Rightarrow \pi^0 \nu \nu$; Gold-plated but experimentally exceedingly challenging. Status & Outlook

3. $KL \Rightarrow \pi e e$; Early work; interesting pioneering paper & PhD thesis by German Valencia. *→ Donoghue + Holstein Valencia PRD '87*

Atwood + AS unpublished

Important paper on background issues by Greenlee *→ PRD 1990*

Because of the extremely important point due Greenlee, it does not seem that $KL \Rightarrow \pi e e$ is useable

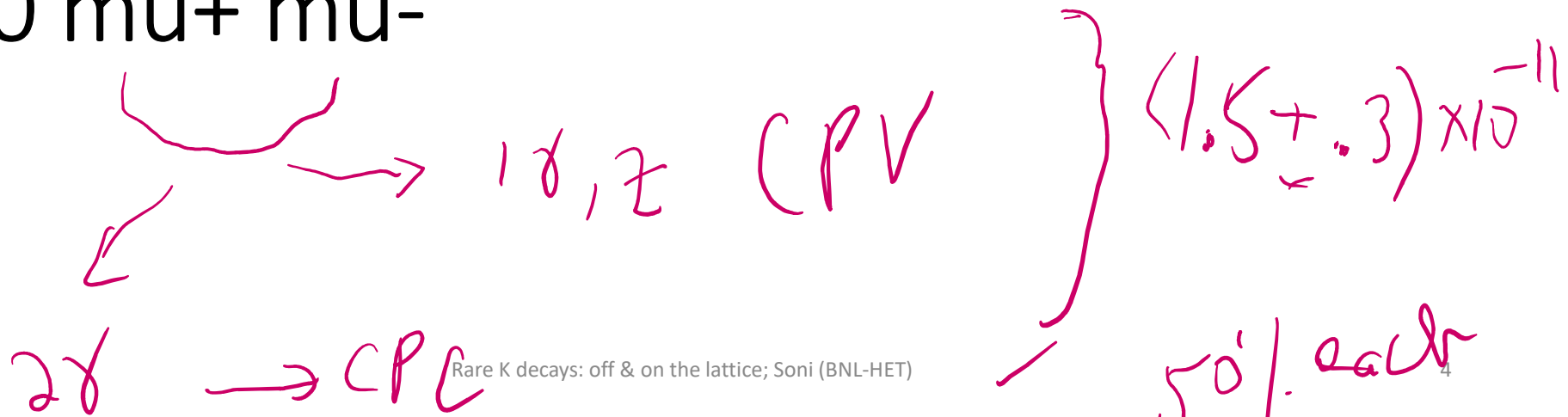
4. $K^0 \Rightarrow \pi^0 \mu \mu$ i.e. KL AND KS extremely useful to study for experiments, pheno AND for the lattice. *$B_{\pi} \sim 10^{-11}$*

WIP with Stefan Schacht

*Existing work: ISIDORE et al
on $K_L \rightarrow \pi^0 \mu \mu$ FPC ~ '04*

Summary

Main purpose: strongly advocate that lattice community should pay high priority to get precise results for K_0 to $\pi^0 \mu^+ \mu^-$



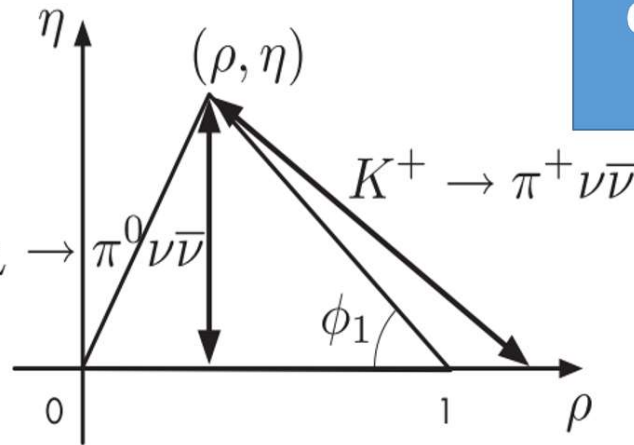
K-UT: A dream for some

Blucher, Winstein and Yamanaka '09; see also Buras

- Click to add t

Construction of a Kaon UT

GOLD
LITTENBERG PAD '89



Lehner+Lunghi+AS PLB '2016




Fig. 14. Unitarity triangle.

Instead of [or in addition to] $K_L \rightarrow \pi^0 \nu \bar{\nu}$ can now plan on using ϵ'/ϵ

C. H. Kim
et al

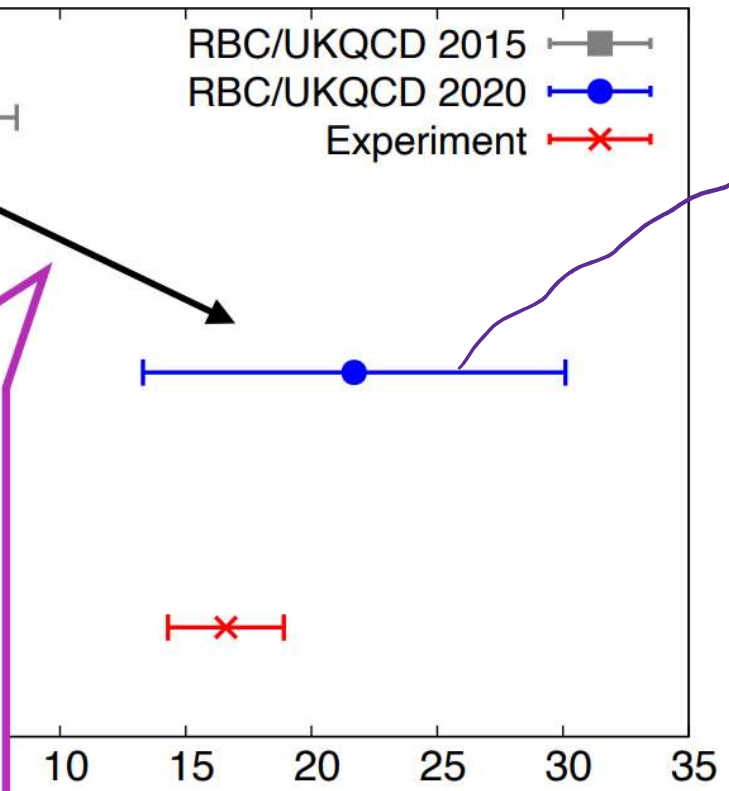
PRL '15

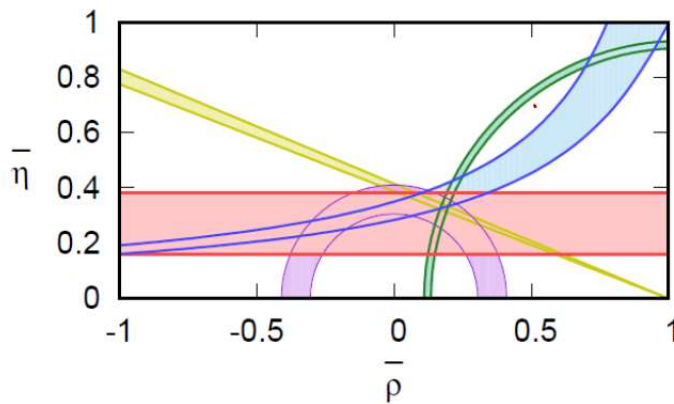
$\text{Re}(\epsilon'/\epsilon) (\times 10^4)$

RBC/UKQCD 2015 
RBC/UKQCD 2020 
Experiment 

PRL
2020

- 3+ times more confs
- Better control of excited-state contaminations
- Step scaling in NPR





$\Delta M_s / \Delta M_d$ (green)
 $\epsilon_K + |V_{cb}|$ (blue)
 $\sin 2\beta$ (yellow)
 $|V_{ub}/V_{cb}|$ (purple)
 ϵ' (red)

→ current systematic ~35%
 Aim to reduce this
 in N3 to ~15%

FIG. 12: The horizontal-band constraint on the CKM matrix unitarity triangle in the $\bar{\rho} - \bar{\eta}$ plane obtained from our calculation of ϵ' , along with constraints obtained from other inputs [6, 70, 71]. The error bands represent the statistical and systematic errors combined in quadrature. Note that the band labeled ϵ' is historically (e.g. in Ref. [72]) labeled as ϵ'/ϵ , where ϵ is taken from experiment.

Precision performance

Error % (statistical)

	32 ³ G-parity BC (previous work)	24 ³ Periodic BC	32 ³ Periodic BC (w/o AMA correction)
# of configurations	741	258	107
$\Delta I = 1/2$ ME via Q_2^{lat}	10%	14%	14%
$\Delta I = 1/2$ ME via Q_6^{lat}	6.5%	8.9%	11%
Re A_0	11%	13%	14%

Preliminary

PBC
effort
Masaaki
et al

- Good precision performance of PBC (ME with excited-state $\pi\pi$) compared to G-parity BC calculation (ME with ground-state $\pi\pi$)

Quantity	This work	Experiment
$\text{Re}(A_2)$	$1.74(15)(48) \times 10^{-8} \text{ GeV}$	$1.479(4) \times 10^{-8} \text{ GeV}$
$\text{Im}(A_2)$	$-5.91(13)(1.75) \times 10^{-13} \text{ GeV}$...
$\text{Re}(A_0)$	$3.13(69)(95) \times 10^{-7} \text{ GeV}$	$3.3201(18) \times 10^{-7} \text{ GeV}$
$\text{Im}(A_0)$	$-9.3(1.5)(2.8) \times 10^{-11} \text{ GeV}$...
$\text{Re}(A_0)/\text{Re}(A_2)$	$18.0(4.4)(7.4)$	$22.45(6)$
$\omega = \text{Re}(A_2)/\text{Re}(A_0)$	$0.056(14)(23)$	$0.04454(12)$
$\text{Re}(\varepsilon'/\varepsilon)$	$31.8(6.3)(11.8)(5.0) \times 10^{-4}$	$16.6(2.3) \times 10^{-4}$

masinaki T
ct d
2306 = 06781

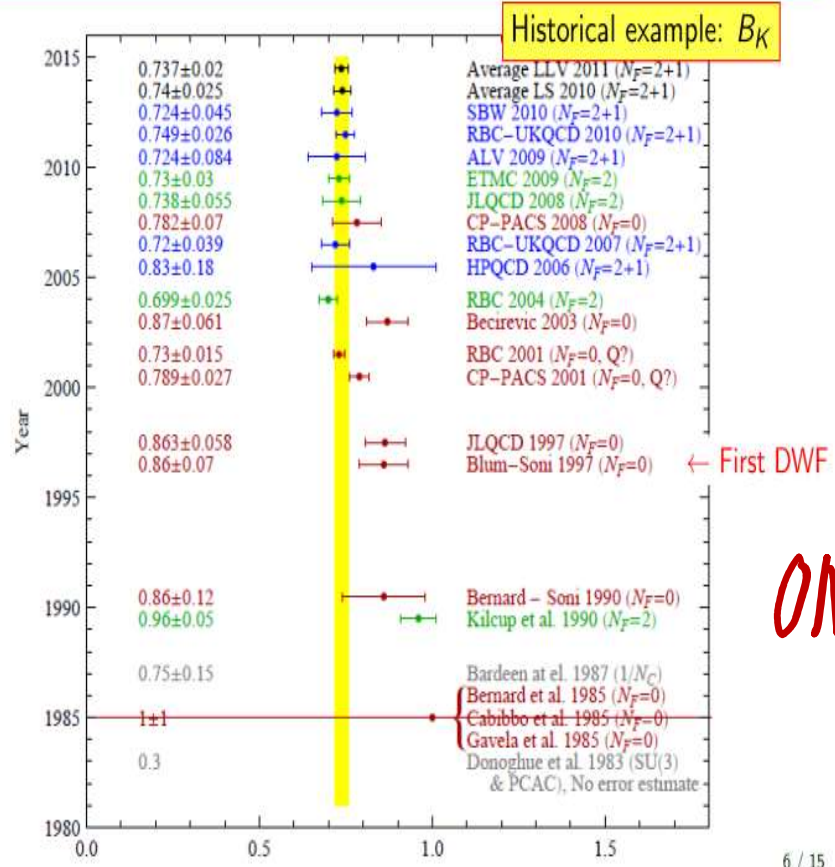
TABLE I. A summary of the primary results of this work shown in the middle column. The values in parentheses give the statistical and systematic errors, respectively. For the last entry the systematic error associated with electromagnetic and isospin breaking effects is listed separately as the third error, which we inherit from the estimation in Ref. [2] based on the large- N_c expansion of QCD and ChPT [49]. The corresponding experimental values are shown in the right column if applicable.

Key points (so far) on our PBC effort

- **Demonstrated that with GEVP matrix elements of ground and 1st two excited states can be extracted quite well**
- **Good quality of signals with PBC obtained rather efficiently**
- **On our way to get results from 2 lattice spacings**
- **Optimistic that we can get epsilon' in the continuum limit (for the iso-symmetric) case in**
about a year...That should appreciably reduce one of the major
source of systematic errors.



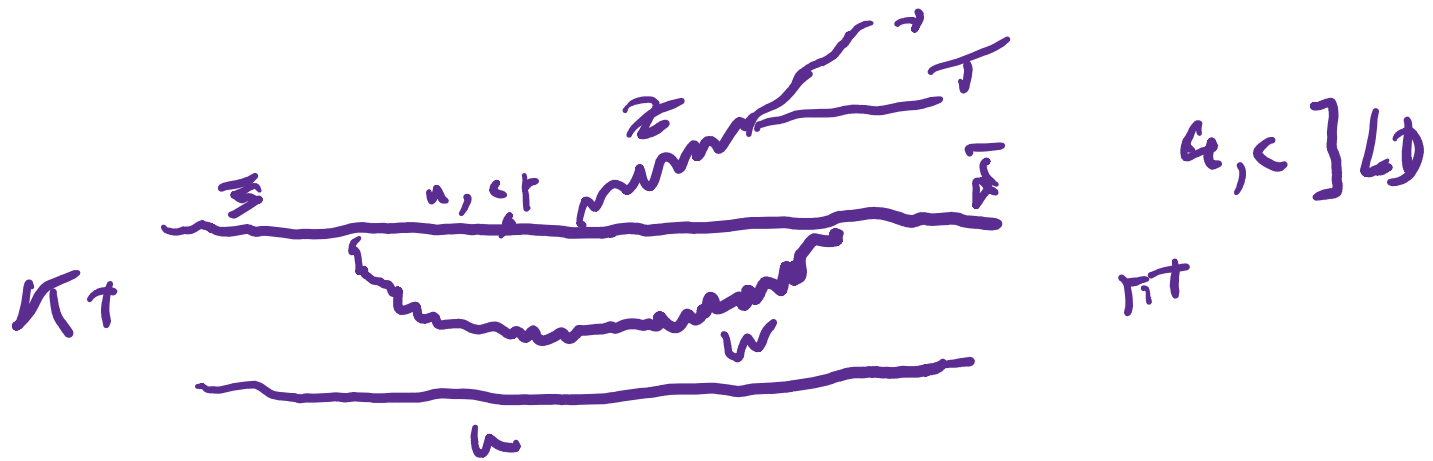
Power of the lattice: Only method to systematically reduce the NP error!



AB-initio Calculation

$$B_K = \frac{\langle K | S^{\text{eff}} | K \rangle}{8/3 g^2 m_K^2}$$

ONE ILLUSTRATION



WITH ENRICO LUNGI
 TRY Reduce LD uncertainty
 WIP

=

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.39 \pm 0.30) \times 10^{-11} \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}} \right]^{2.8} \left[\frac{\gamma}{73.2^\circ} \right]^{0.74}$$

NLO

In the above formula, the explicit numerical uncertainty is the theoretical one originating from QCD and electroweak uncertainties, which amounts to 3.6%. Taking the latest values (28) for $|V_{cb}|_{\text{avg}} = (41.0 \pm 1.4) \times 10^{-3}$ and $\gamma = (72.1_{-4.5}^{+4.1})^\circ$, one finds the following:

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (8.5 \pm 1.0) \times 10^{-11} \quad \leftarrow$$

The predictions are currently dominated by the parametric uncertainty that will plausibly be reduced by new measurements of $|V_{cb}|$ and γ by LHCb and Belle II.

cannot be detected. A long series of decay-at-rest searches for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ have culminated with the final results of the BNL E787/E949 experiments, which found the following (50):

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{E787/E949}} = (17.3_{-10.5}^{+11.5}) \times 10^{-11}$$

$\leftarrow \approx 26\%$

From these analyses, the best upper limit, at 90% confidence level (CL), has been obtained:

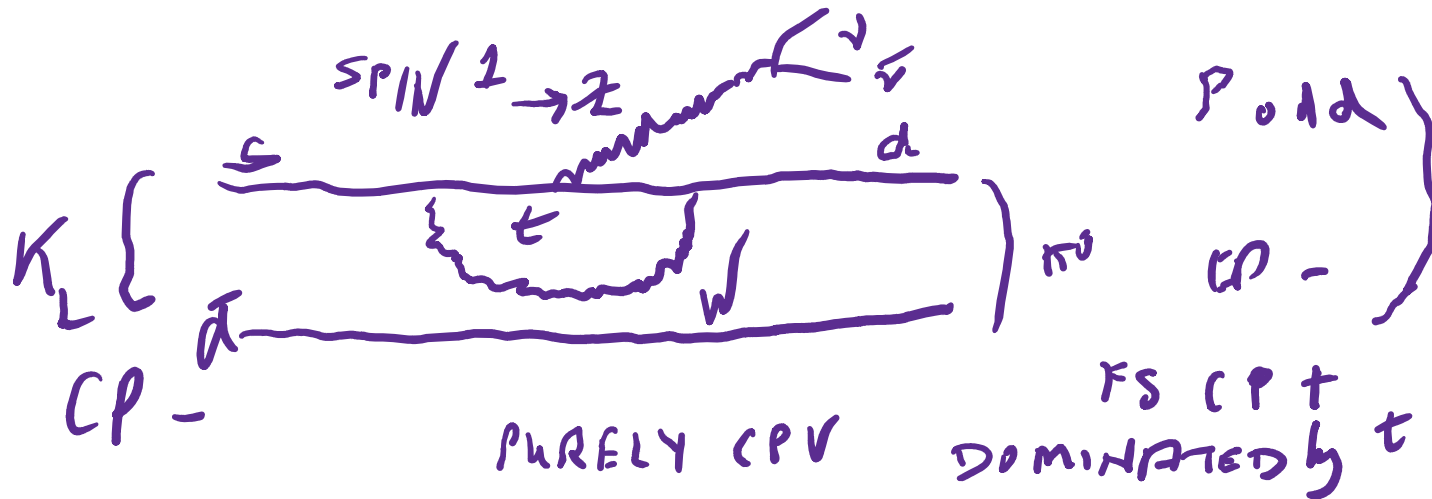
$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{NA62(2016-2017)}} \leq 17.8 \times 10^{-11}$$

The 2016–2017 data also allow one to set a 68% CL mean value for the branching ratio:

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{NA62(2016-2017)}} = (4.8_{-4.8}^{+7.2}) \times 10^{-11}$$

$\leftarrow \approx 15\%$

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LITTENBERG PRD 1989

GOLD PLATED

KOTO_pJPARC

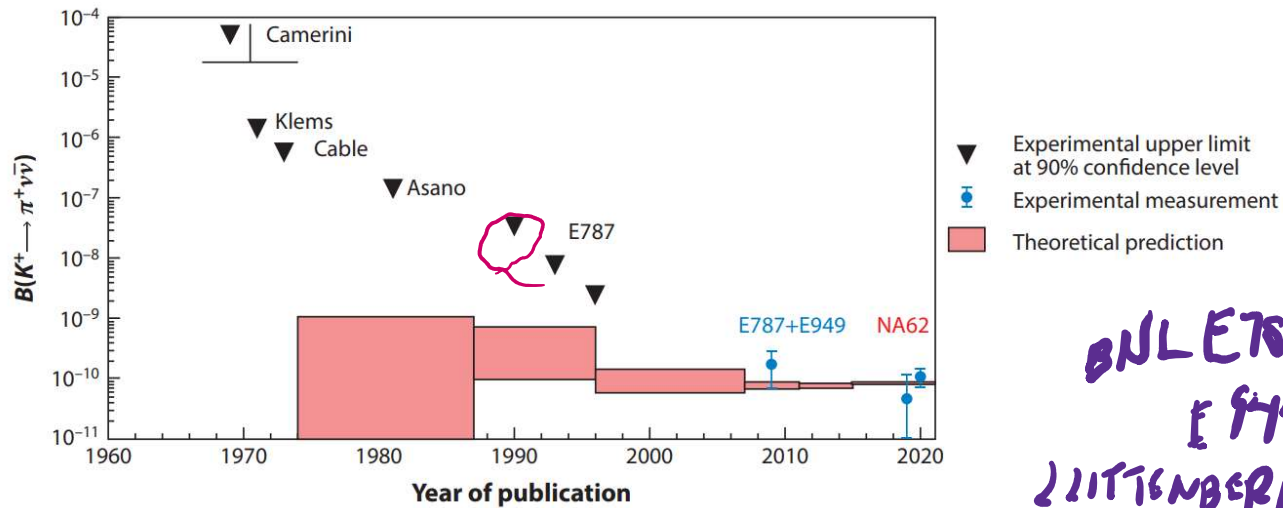
$$B(K_L^0 \rightarrow \pi^0 \nu \bar{\nu}) = (3.36 \pm 0.05) \times 10^{-11} \left[\frac{|V_{ub}|}{3.88 \times 10^{-3}} \right]^2 \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}} \right]^2 \left[\frac{\sin \gamma}{\sin 73.2^\circ} \right]^2,$$

which, taking the latest values (28) for $|V_{cb}|_{\text{avg}} = (41.0 \pm 1.4) \times 10^{-3}$, $|V_{ub}|_{\text{avg}} = (3.82 \pm 0.24) \times 10^{-3}$, and $\gamma = (72.1_{-4.5}^{+4.1})^\circ$, leads to the following numerical prediction:

SM $B(K_L^0 \rightarrow \pi^0 \nu \bar{\nu}) = (3.2 \pm 0.6) \times 10^{-11}$. ←

While the experimental situation for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ shows that we have two independent experimental techniques that can reach SM sensitivities, with the NA62 experiment on the way to making a precise measurement, the situation for the neutral mode is more complex. Progress has been hampered by the lack of a clean experimental signature because no redundancy is available once the π^0 mass is used as a constraint to reconstruct the decay vertex. The KOTO experiment at J-PARC builds on the experience of the predecessor experiment E391a (67), which was performed at KEK. It is based on the technique of letting a well-collimated “pencil” beam enter the decay region surrounded by high-performance photon vetoes. By vetoing extra photons and applying a transverse momentum cut (150 MeV/c) to eliminate residual $\Lambda \rightarrow n\pi^0$ decays, KOTO is expected to reach SM sensitivities by the mid-2020s. The KOTO experiment has published the best upper limit (68):

~ about 2 orders of magnitude to go $B(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})_{\text{KOTO}} < 3.0 \times 10^{-9}$ (90% CL). ←



BNL E787 +
E949
LUTTENBERG
PANOFSKY
PRIZE

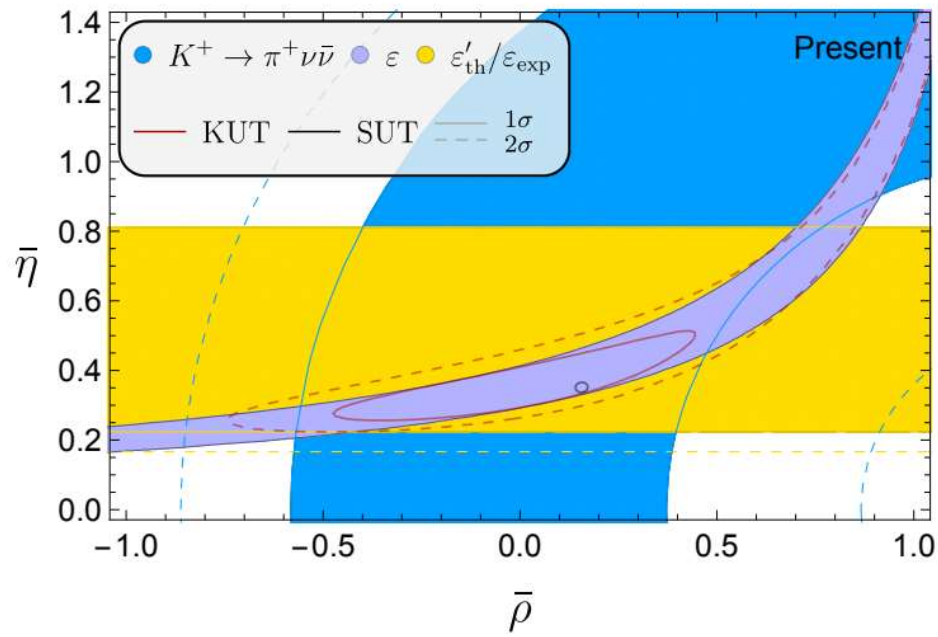
Figure 4

Timeline of theoretical predictions and experimental results for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (10, 51, 57–64). Figure adapted with permission from Reference 58; copyright 2020 CERN for the benefit of the NA62 Collaboration.

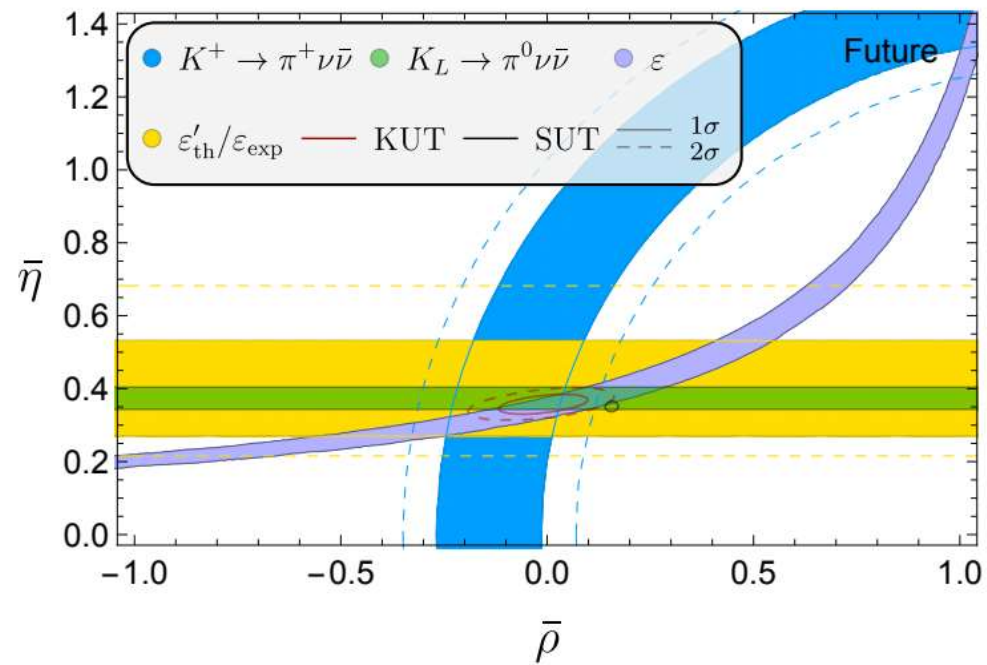
the NA62 Collaboration reported the following:

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{NA62(2016-2018)}} = (11.0_{-3.5}^{+4.0} \text{stat} \pm 0.3_{\text{sys}}) \times 10^{-11},$$

$10^{-8} \rightarrow 10^{-10}$
 $\sim 20 \text{ years}$



Lehner +
 Luyhi +
 AS
 PLB'16



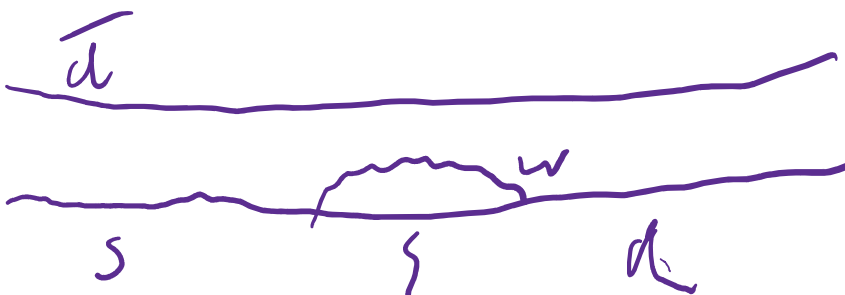
Remaining points

- Given that $KL \Rightarrow \pi^0 \nu \nu$ is exptally so challenging why don't (can't) we try replace with $KL \Rightarrow \pi^0 e^+ e^-$?
- 1st very nice attempt in this direction was made in a beautiful PhD thesis work of German Valencia; see also PRD'87 paper by DHV
- Also Atwood + AS (unpublished)
- (insurmountable) difficulty is the "Greenlee" background
- (see PRD Greenlee '90)
- Why not consider $KL \Rightarrow \pi^0 \mu^+ \mu^-$?
- Existing work on $KL \Rightarrow \pi^0 \mu^+ \mu^-$ by Isidori et al; WIP by Schacht+AS

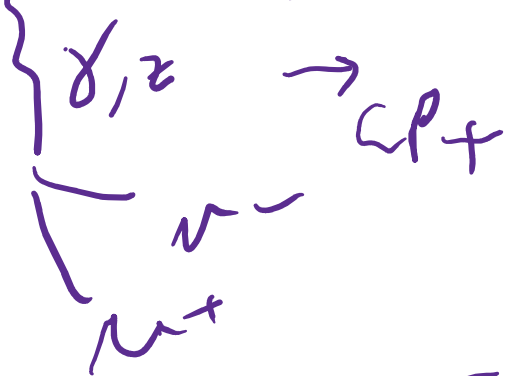
KOTO

\odot CP- K_L
JPARC

$K_S \rightarrow$ CPC
LHCb

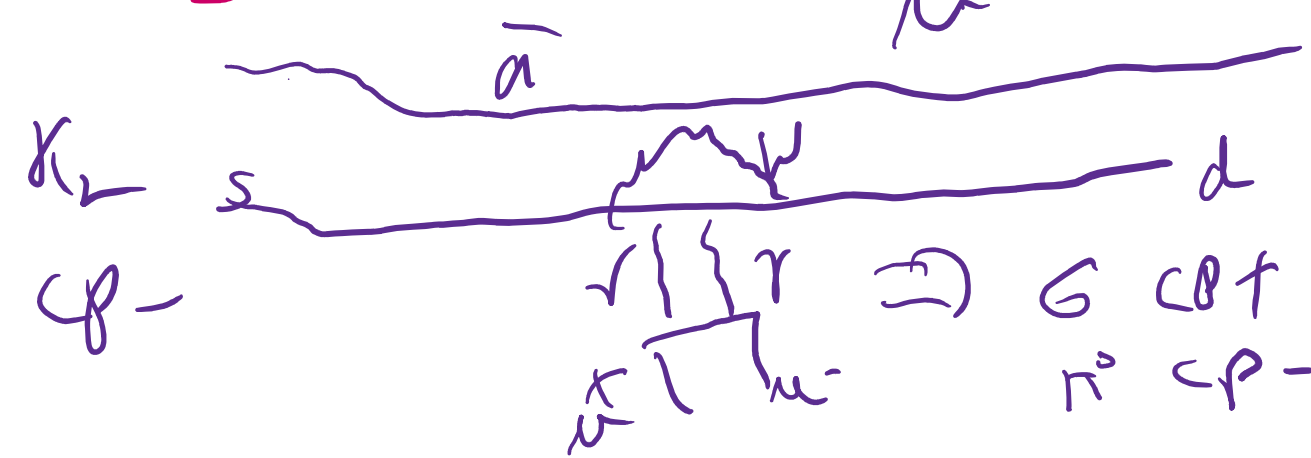


\rightarrow CP-
 $P-$



CP+
CPV

\rightarrow CP-



CPC
+
CPV

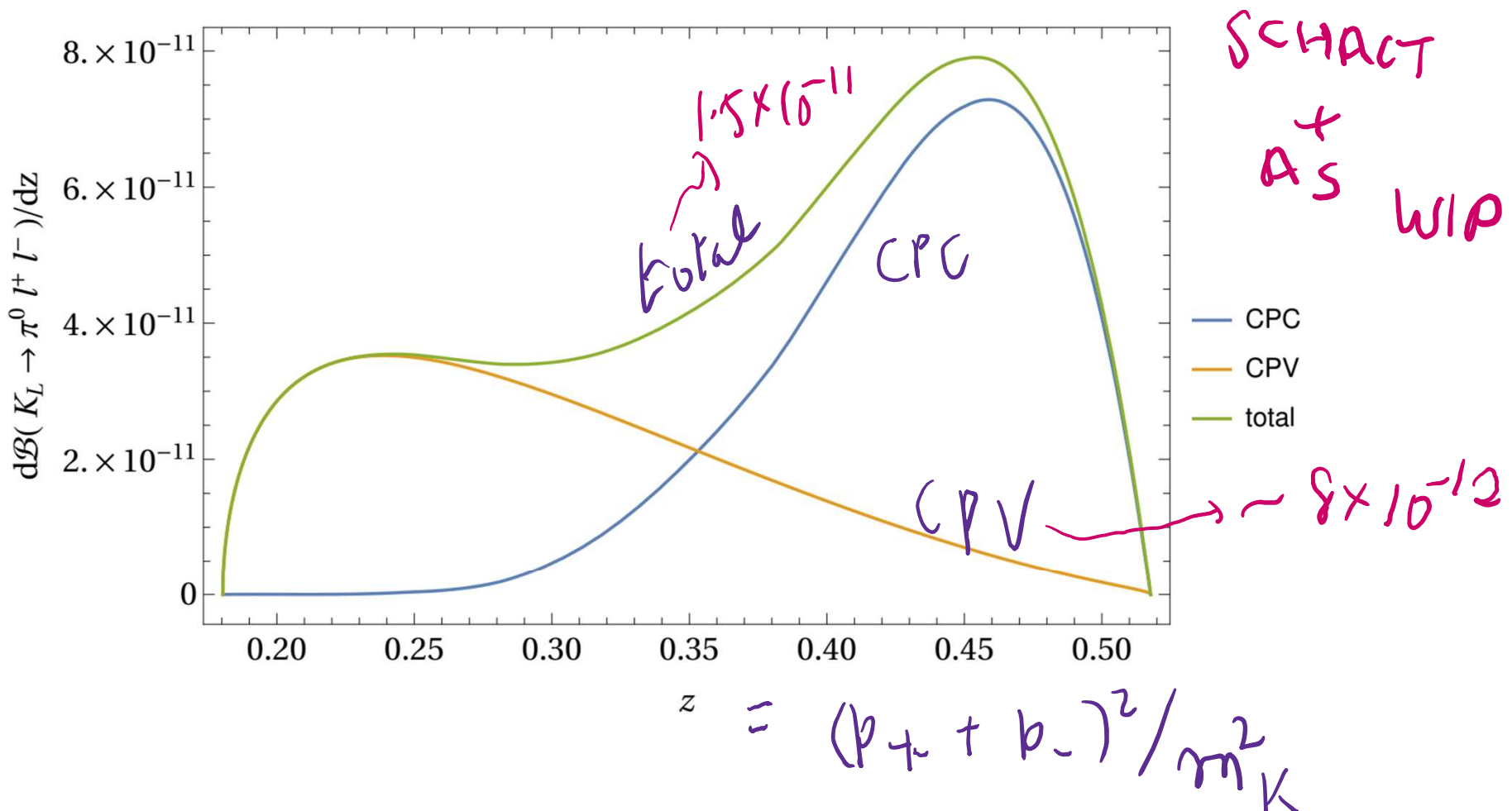


FIG. 2. The CPC and CPV contributions.

Summary

- Considerable progress on the experimental and the theory front on $K^+ \Rightarrow \pi^+ \nu \nu$This is one important pillar in Kaon UT
 - $K_L \Rightarrow \pi^0 \nu \nu$ while gold plated, experimental progress near the SM prediction is highly unlikely to come for many years perhaps for 10 years
 - Meantime theoretical progress on ϵ_s' will continually improve and provide a very useful constraint on the KUT.
 - Experimental progress on K^0 [both K_L and K_S] to $\pi^0 \mu^+ \mu^-$ by KOTO at JPARC as well as LHCb is strongly advocated
 - In particular precision study by the lattice of $K^0 \Rightarrow \pi^0 \mu^+ \mu^-$ is strongly suggested....In this regard existing and ongoing work of RBC-UKQCD on $K^+ \Rightarrow \pi^+ \nu \nu$ is noted. Noted is also En-Hung Chao and Norman Christ et al's work on a related mode $K_L \Rightarrow \mu^+ \mu^-$
- In fact if you can do precisely K_Lyou should seriously consider K^0 to $\pi^0 \mu^+ \mu^-$