

# The determination of $r_0$ and $r_1$ in $N_f = 2 + 1$ QCD

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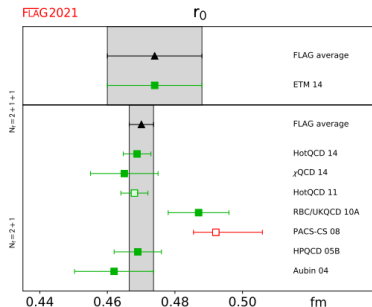
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**LATTICE**

FOR 5269



# Motivation

- ▶  $r_0$  as reference scale
  - Weak quark mass dependence
  - Reasonable statistical precision
  - Mild lattice artifacts
- ▶ Add to existing measurements with state of the art simulated data



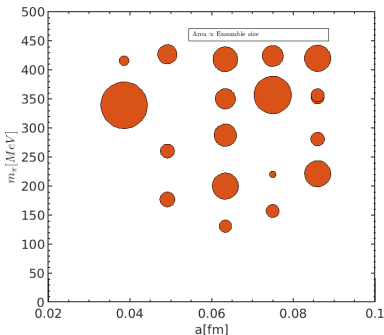
[ FLAG Review 2021, 2111.09849 ]



# CLS - Coordinated Lattice Simulations ( $N_f = 2 + 1$ )

“Simulation of QCD with  $N_f = 2 + 1$  flavors of non-perturbatively improved Wilson fermions” [M.Bruno et al. (2014) 1411.3982]

- ▶ 2+1 flavors of improved Wilson fermions using Lüscher-Weisz gauge action with open boundary conditions in time.
- ▶ Chiral trajectory with  $m_u + m_d + m_s = \text{const}$  while making sure that  $m_\pi L > 4$
- ▶ 5 lattice spacings from 0.085 fm to 0.037 fm
- ▶  $m_\pi$  from 430 MeV to 134 MeV

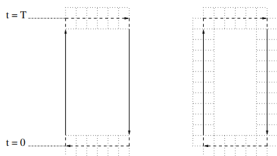


# Wilson loops

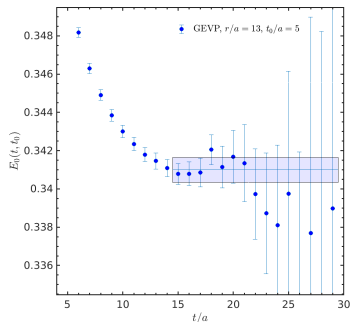
## “Determination of the Static Potential with Dynamical Fermions”

[ M.Donnellan, F.Knechtli, B.Leder, R.Sommer, 1012.3037 ]

- ▶ Operator basis with different levels of HYP smearing.



- ▶ Energy levels calculated using a GEVP



# Scale setting $r_0$

We are using an improved definition of the force:

$$F(r_I) = [V(r) - V(r - a)]/a$$

where  $r_I = r - a/2 + \mathcal{O}(a^2)$

[ S. Necco and R. Sommer, hep-lat/0108008 ]

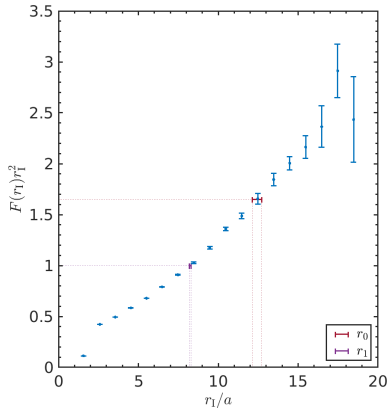
Using that we find the following scales:

$$r_I^2 F(r_I)|_{r_I=r_0} = 1.65$$

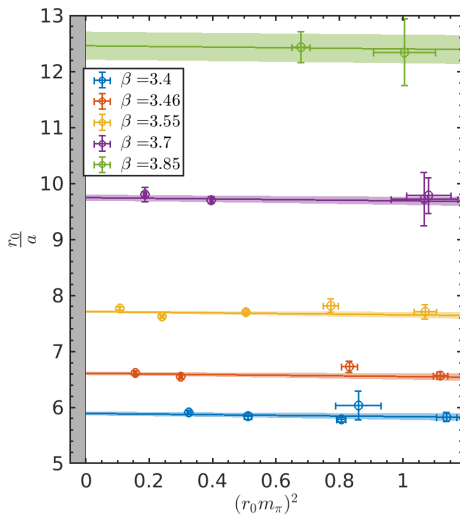
[ R. Sommer, hep-lat/9310022 ]

$$r_I^2 F(r_I)|_{r_I=r_1} = 1$$

[ C. Bernard et al, (2000) ]



# Chiral extrapolations in lattice units



► Global fit using:

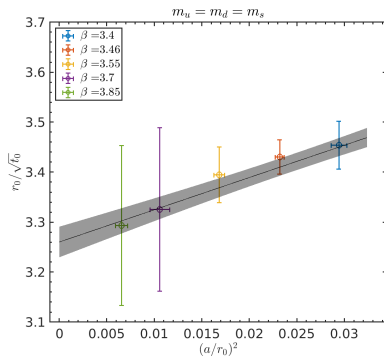
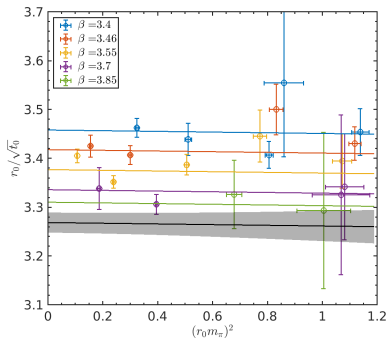
$$\frac{r_0}{a} = c_1|\beta + c_2(r_0 m_\pi)^2$$

► Mild mass dependence



# Global Fit, continuum and chiral extrapolation

Global fit using:  $\frac{r_0}{\sqrt{t_0}} = c_1 + c_2 \left( \frac{a}{r_{0, \text{sym}}} \right)^2 + c_3 (r_0 m_\pi)^2$



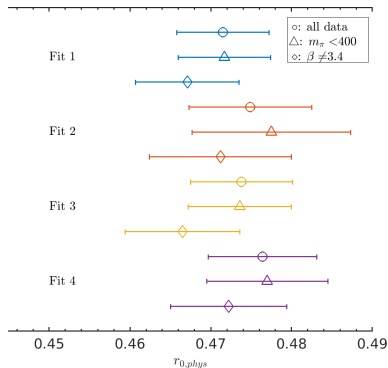
# Fit Comparison

Using  $\sqrt{t_0} = 0.1443(7)\text{fm}$  [ B.Straßberger et al, 2112.06696 ]

Method	$r_0$	$\chi^2$ /d.o.f.
Fit 1: $c_1 + c_2(a/r_{0,sym})^2 + c_3(r_0 m_\pi)^2$		
all	0.4715(57)	19.5/17
$m_\pi < 400$	0.4717(57)	18.3/11
$\beta \neq 3.4$	0.4671(64)	14.2/12
Fit 2: $c_1 + c_2(a/r_{0,sym})^2 + c_3(r_0 m_\pi)^2 + c_4(m_\pi a)^2$		
all	0.4749(76)	11.8/16
$m_\pi < 400$	0.4775(98)	9.1/10
$\beta \neq 3.4$	0.4712(88)	9.8/11
Fit 3: $c_1 + c_2(a/r_{0,sym})^2 + c_3\phi_2 + c_4(1.098 - \phi_4)$		
all	0.4738(63)	17.1/16
$m_\pi < 400$	0.4736(64)	16.8/10
$\beta \neq 3.4$	0.4665(71)	9.1/11
Fit 4: $c_1 + c_2(a^2/t_{0,sym}) + c_3(t_0 m_\pi^2)$		
all	0.4764(67)	17.4/16
$m_\pi < 400$	0.4770(75)	15.6/10
$\beta \neq 3.4$	0.4722(72)	12.4/11

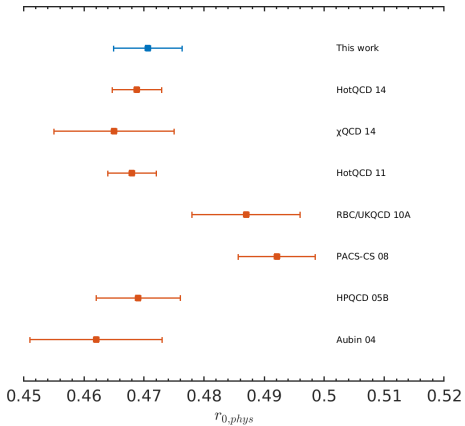
$$\phi_2 \propto m_u + m_d$$

$$\phi_4 \propto m_u + m_d + m_s$$

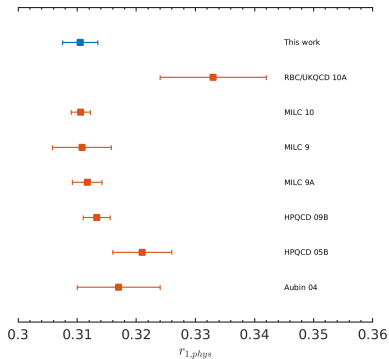
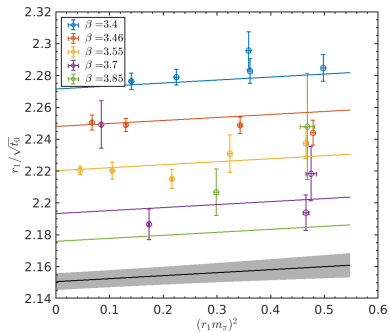




# $r_0$ at the physical point



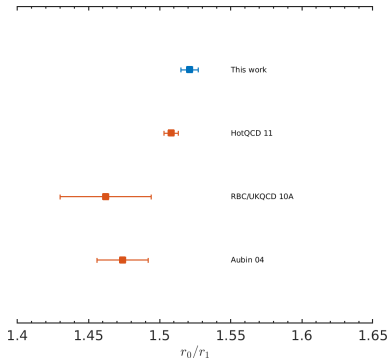
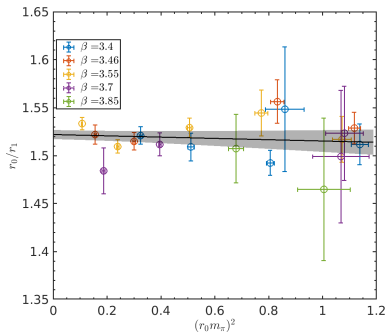
# $r_1$ preliminary



- ▶ a-extrapolation is harder for  $r_1$
- ▶  $\chi^2$  is a good deal larger



# $r_0/r_1$ preliminary



- Lattice artifacts seem in good agreement



# Structure of the static potential

Through the motivation of the Cornell potential:

$$V = \sigma r - \frac{K}{r}$$

$$K = 0.52 \quad [\text{E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane}$$

and T.M. Yan, *Phys. Rev. D* 21 (1980) 203 ]

Building following quantity

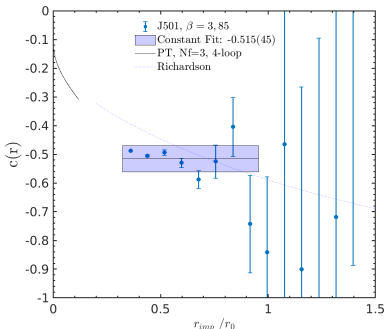
$$c(\tilde{r}) = \frac{1}{2} \tilde{r}^3 F'(r)$$

with  $\tilde{r} = r + \mathcal{O}(a^2)$  such that

$$c_{tree}(\tilde{r}) = -C_F \frac{g_0^2}{4\pi}$$

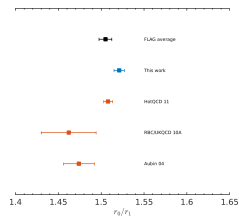
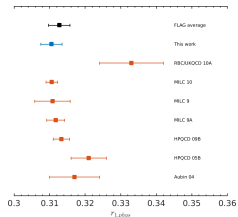
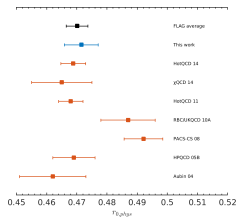
[ M. Lüscher and P. Weisz, [hep-lat/0207003](#) ]

[ J.L. Richardson, *Phys. Lett.* B82 (1979) 272 ]



# Conclusion

- Good agreement
- Competitive error
- very well controlled systematics



# Thats it

Thanks and bye!

this is how i finish a presentation:

