The determination of r_0 and r_1 in $N_f = 2+1$ QCD

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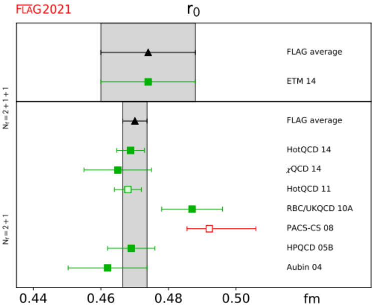
FOR 5269

Motivation

- r_0 as reference scale
  - Weak quark mass dependence
  - Reasonable statistical precision
  - Mild lattice artifacts
- Add to existing measurements with state of the art simulated data

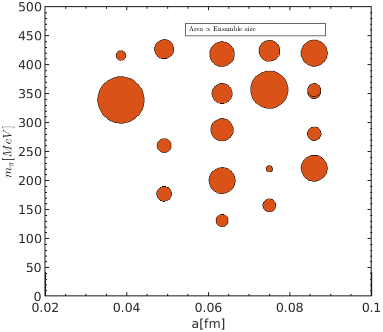


[FLAG Review 2021, 2111.09849]

CLS - Coordinated Lattice Simulations ($N_f = 2 + 1$)

"Simulation of QCD with $N_f = 2 + 1$ flavors of non-perturbatively improved Wilson fermions" [M.Bruno et al, (2014) 1411.3982]

- 2+1 flavors of improved Wilson fermions using Lüscher-Weisz gauge action with open boundary conditions in time.
- Chiral trajectory with $m_u + m_d + m_s = const$ while making sure that $m_\pi L > 4$
- 5 lattice spacings from 0.085 fm to 0.037 fm
- m_π from 430 MeV to 134 MeV

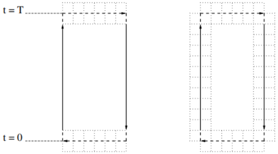


Wilson loops

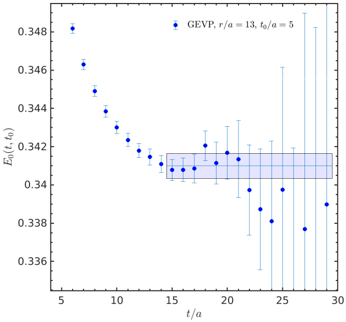
"Determination of the Static Potential with Dynamical Fermions"

[M.Donnellan, F.Knechtli, B.Leder, R.Sommer, 1012.3037]

- Operator basis with different levels of HYP smearing.



- Energy levels calculated using a GEVP



Scale setting r_0

We are using an improved definition of the force:
$F(r_I) = [V(r) - V(r-a)]/a$
where $r_I = r - a/2 + \mathcal{O}(a^2)$

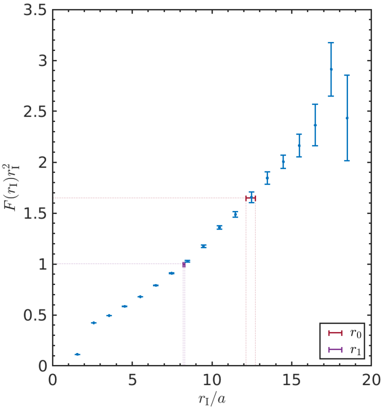
[S. Necco and R. Sommer, `hep-lat/0108008`]

Using that we find the following scales:
$r_I^2 F(r_I)|_{r_I=r_0} = 1.65$

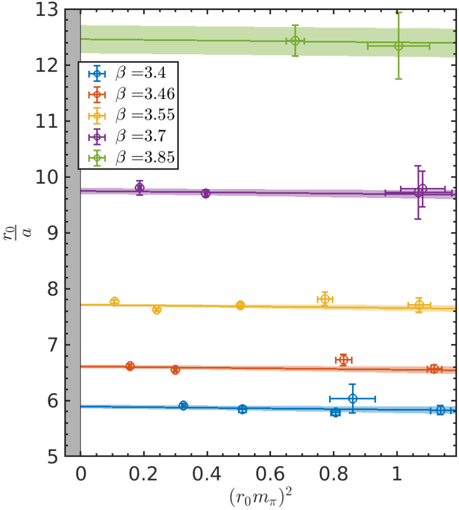
[R. Sommer, `hep-lat/9310022`]

$r_I^2 F(r_I)|_{r_I=r_1} = 1$

[C.Bernard et al, (2000)]



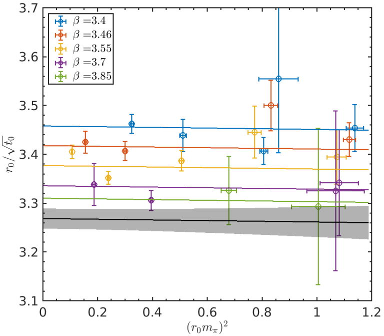
Chiral extrapolations in lattice units

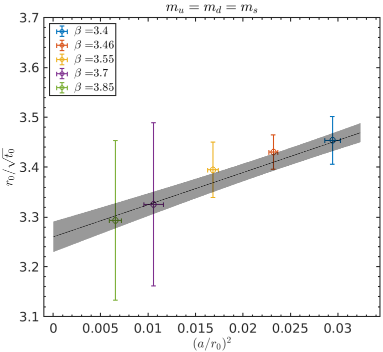


- Global fit using: $\frac{r_0}{a} = c_1|_\beta + c_2(r_0 m_\pi)^2$
- Mild mass dependence

Global Fit, continuum and chiral extrapolation

Global fit using: $\frac{r_0}{\sqrt{t_0}} = c_1 + c_2\left(\frac{a}{r_{0,sym}}\right)^2 + c_3(r_0 m_\pi)^2$

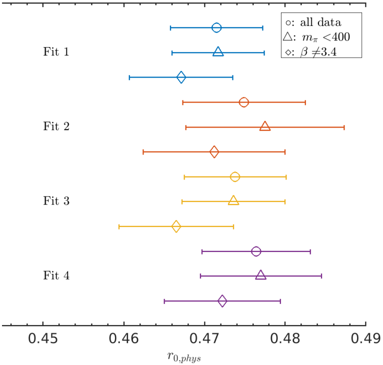




Fit Comparision

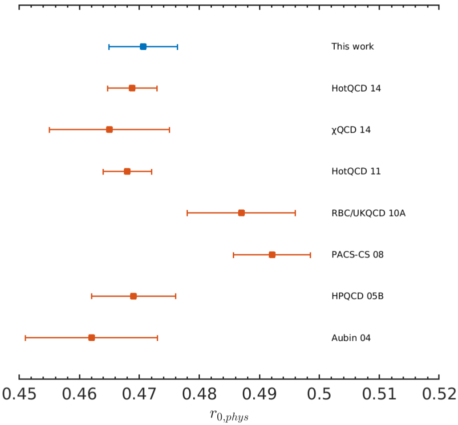
Using $\sqrt{t_0} = 0.1443(7)$fm [B.Straßberger et al, 2112.06696]

Method	r_0	χ^2 /d.o.f.
Fit 1: $c_1 + c_2(a/r_{0,sym})^2 + c_3(r_0 m_\pi)^2$		
all	0.4715(57)	19.5/17
m_π <400	0.4717(57)	18.3/11
$\beta \neq 3.4$	0.4671(64)	14.2/12
Fit 2: $c_1 + c_2(a/r_{0,sym})^2 + c_3(r_0 m_\pi)^2 + c_4(m_\pi a)^2$		
all	0.4749(76)	11.8/16
m_π <400	0.4775(98)	9.1/10
$\beta \neq 3.4$	0.4712(88)	9.8/11
Fit 3: $c_1 + c_2(a/r_{0,sym})^2 + c_3\phi_2 + c_4(1.098 - \phi_4)$		
all	0.4738(63)	17.1/16
m_π <400	0.4736(64)	16.8/10
$\beta \neq 3.4$	0.4665(71)	9.1/11
Fit 4: $c_1 + c_2(a^2/t_{0,sym}) + c_3(t_0 m_\pi^2)$		
all	0.4764(67)	17.4/16
m_π <400	0.4770(75)	15.6/10
$\beta \neq 3.4$	0.4722(72)	12.4/11

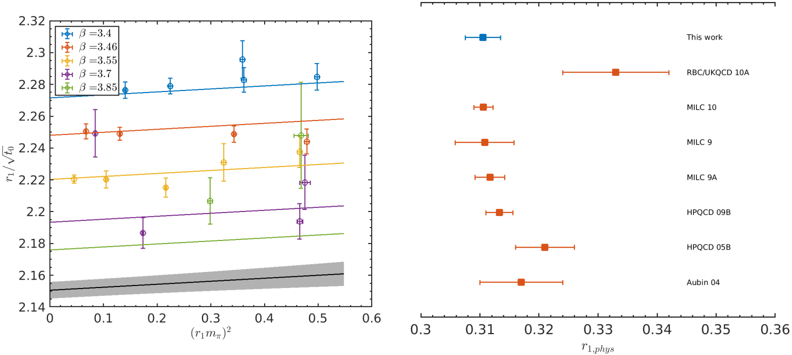


$$\phi_2 \propto m_u + m_d$$
$$\phi_4 \propto m_u + m_d + m_s$$

r_0 at the physical point

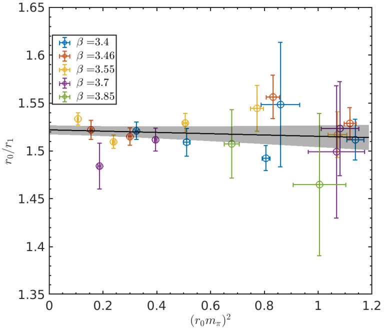


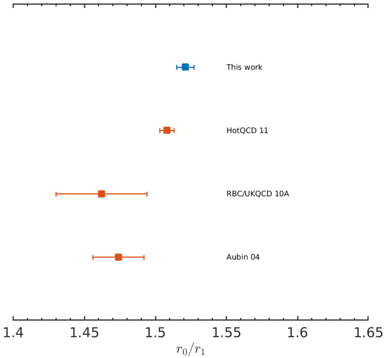
r_1 preliminary



- a-extrapolation is harder for r_1
- χ^2 is a good deal larger

r_0/r_1 preliminary





- Lattice artifacts seem in good agreement

Structure of the static potential

Through the motivation of the Cornell potential:

$V = \sigma r - \frac{K}{r}$

$K = 0.52$ [E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D21 (1980) 203]

Building following quantity

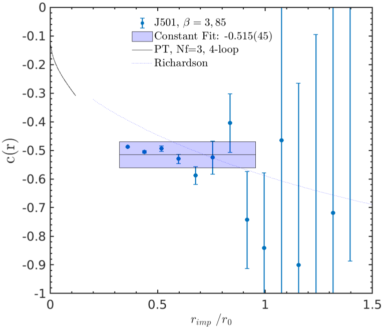
$c(\tilde{r}) = \frac{1}{2}\tilde{r}^3 F'(r)$

with $\tilde{r} = r + \mathcal{O}(a^2)$ such that

$c_{tree}(\tilde{r}) = -C_F \frac{g_0^2}{4\pi}$

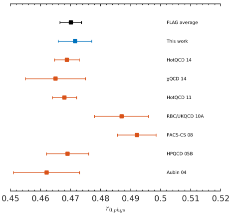
[M. Lüscher and P. Weisz, hep-lat/0207003]

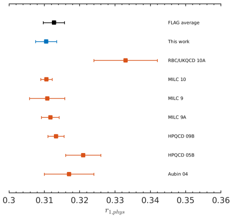
[J.L. Richardson, Phys. Lett. B82 (1979) 272]

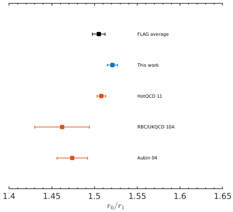


Conclusion

- Good agreement
- Competitive error
- very well controlled systematics







Thats it

Thanks and bye!