$|V_{us}|$ from kaon semileptonic form factor in $N_f = 2+1$ QCD at the physical point on $(10\ \text{fm})^4$

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Refs: PRD101,9,094504(2020), PRD106,9,094501(2022)

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Introduction

Urgent task: search for signal beyond standard model (BSM)

Muon $g-2$ @ FNAL 2021 : 4.2σ away from SM

$|V_{us}|$: a candidate of BSM signal

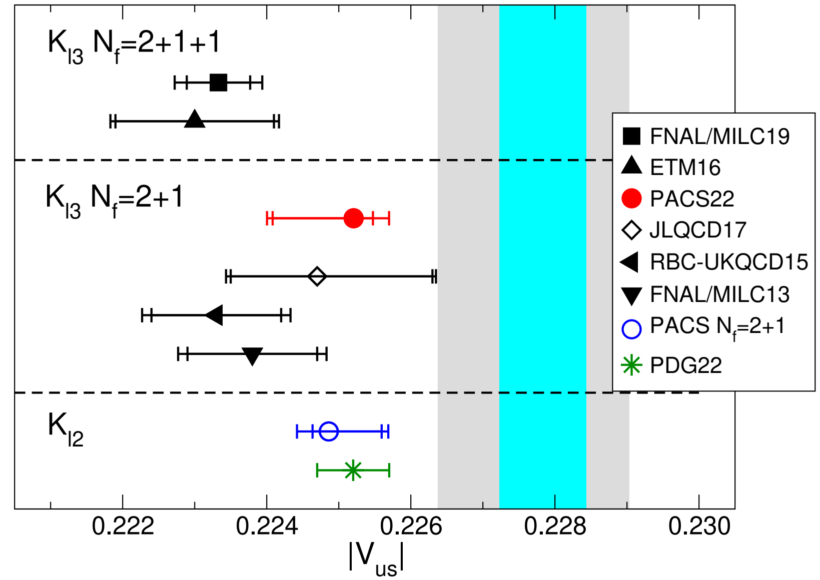
Most accurate $|V_{us}|$ from $K_{\ell 3}$ decay
['19 FNAL/MILC]

$\sim 5\sigma$ from CKM unitarity (cyan band)

$|V_{us}| \approx \sqrt{1-|V_{ud}|^2}$ w/ $|V_{ud}|$ ['18 Seng *et al.*]

$\sim 3\sigma$ (grey band) w/ $|V_{ud}|$ ['20 Hardy, Towner]

$\sim 2\sigma$ from $K_{\ell 2}$ decay [PDG22]



Important to confirm by several independent calculations

$K_{\ell 3}$ form factors with two PACS10 configurations ['20 PACS, '22 PACS]

$L \gtrsim 10$[fm] at physical point

Negligible finite L effect, tiny q^2 region, without chiral extrapolation

Largest uncertainty form finite a effect

This talk: preliminary result with 3rd PACS10 configuration

Simulation parameters

PACS10 configurations: $L \gtrsim 10$[fm] at physical point

$N_f = 2+1$ six-stout-smeared non-perturbative $O(a)$ Wilson action
+ Iwasaki gauge action

β	$L^3 \cdot T$	L[fm]	a[fm]	a^{-1}[GeV]	M_π[MeV]	M_K[MeV]	$N_{\rm conf}$	$t_{\rm sep}$[fm]
2.20	256^4	10.5	0.041	4.792	142	514	20	3.5, 4.0
2.00	160^4	10.2	0.063	3.111	137	501	20	2.3–4.1
1.82	128^4	10.9	0.085	2.316	135	497	20	3.4–3.9

(1000–2500 measurements in each $t_{\rm sep}$)

All the results on 256^4 are preliminary.

$K_{\ell 3}$ form factors $f_+(q^2), f_0(q^2)$ from 3-point function

w/ $Z(2)\otimes Z(2)$ random source spread in L^3, color, spin ['08 RBC-UKQCD]

$$C_{V_\mu}(t,p) = \langle 0|O_K(t_{\rm sep},\mathbf{0})V_\mu(t,\mathbf{p})O_\pi^\dagger(0,\mathbf{p})|0\rangle$$

V_μ: Local vector current renormalized by Z_V
Conserved vector current

$$\langle\pi(p)|V_\mu|K(0)\rangle = (p_K+p_\pi)_\mu f_+(q^2) + (p_K-p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$p_K = (M_K, \mathbf{0}), p_\pi = (E_\pi, \vec{p})$
$q^2 = -(M_K - E_\pi)^2 + p^2$

Resources: Fugaku in HPCI System Research Project
(hp200062, hp200167, hp210112, hp220079, hp230199)

Calculation method w/ local vector current

Details in PACS:PRD101,9,094504(2020)

3-point function*

$$C_{V_\mu}(t, t_{\rm sep}, p) = \langle 0|O_K(t_{\rm sep}, \mathbf{0}) V_\mu(t, \mathbf{p}) O_\pi^\dagger(0, \mathbf{p})|0\rangle = \frac{Z_\pi Z_K}{Z_V}\frac{M_\mu(p)}{4E_\pi M_K} e^{-E_\pi t} e^{-M_K(t_{\rm sep}-t)} + \cdots$$

$p \equiv |\mathbf{p}| = \left|\frac{2\pi}{L}\mathbf{n}\right|$, $|\mathbf{n}| = 0$–6

with periodic boundary

2-point function* $X = \pi, K$

$$C_X(t, p) = \langle 0|O_X(t, \mathbf{p}) O_X^\dagger(0, \mathbf{p})|0\rangle = \frac{Z_X^2}{2E_X}\left(e^{-E_X t} + e^{-E_X(2T-t)}\right) + \cdots$$

*Averaging ones with periodic, anti-periodic temporal boundary conditions reducing wrapping around effect in 3pt, and doubling periodicity in 2pt

$Z_V = 1/\sqrt{F_\pi^{\rm bare}(0) F_K^{\rm bare}(0)}$ determined w/ electromagnetic form factor $F_{\pi,K}(0) = 1$

Ratio $(0 \ll t \ll t_{\rm sep})$

$$\frac{Z_\pi Z_K Z_V C_{V_\mu}(t, t_{\rm sep}, p)}{C_\pi(t, p) C_K(t_{\rm sep} - t, 0)} = M_\mu(p) + \underbrace{A(p) e^{-\Delta_\pi(p) t} + B(p) e^{-\Delta_K(t_{\rm sep}-t)}}_{\text{1st excited state, } \pi', K', \text{ contributions}}$$

$\Delta_\pi(p) = E_{\pi'}(p) - E_\pi(p), \Delta_K = M_{K'} - M_K$ fixed $M_{\pi'}, M_{K'}$

Extract $M_\mu(p) = \langle \pi(p)|V_\mu|K(0)\rangle$ w/ fit including 1st excited states

Conserved current case: $V_\mu \to \widetilde{V}_\mu$ and $Z_V = 1$

Calculation method

Details in PACS:PRD101,9,094504(2020)

$M_\mu(p) = \langle \pi(p) | V_\mu | K(0) \rangle$ extracted from ratio

$K_{\ell 3}$ form factors $f_+(q^2), f_0(q^2)$

$$\langle \pi(p) | V_\mu | K(0) \rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$p_K = (M_K, \mathbf{0}), p_\pi = (E_\pi, \vec{p})$

$q^2 = -(M_K - E_\pi)^2 + p^2$

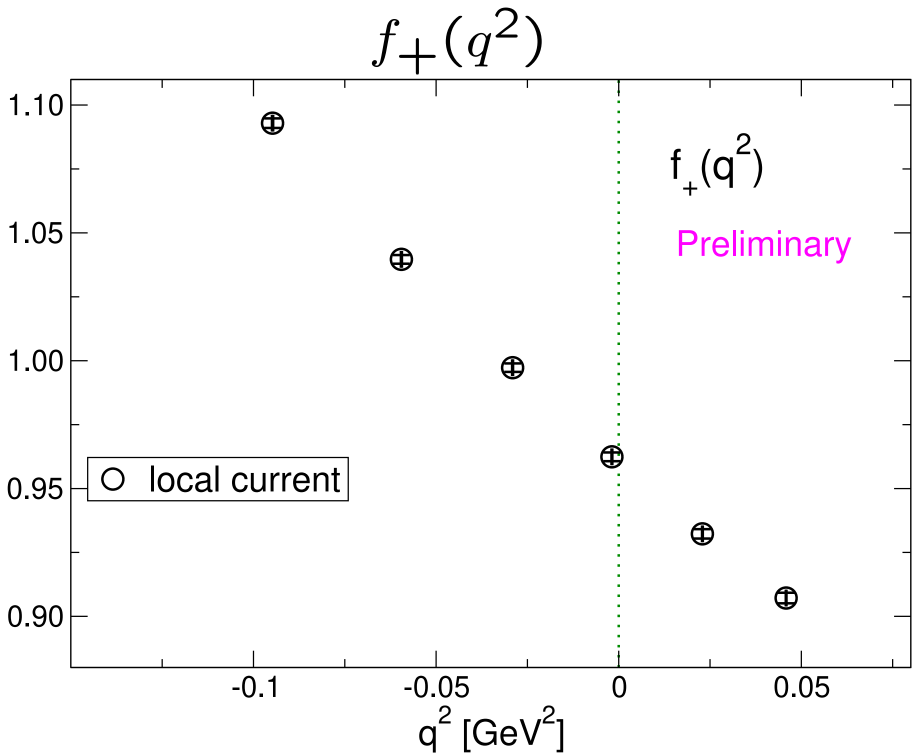
Determination of $|V_{us}|$

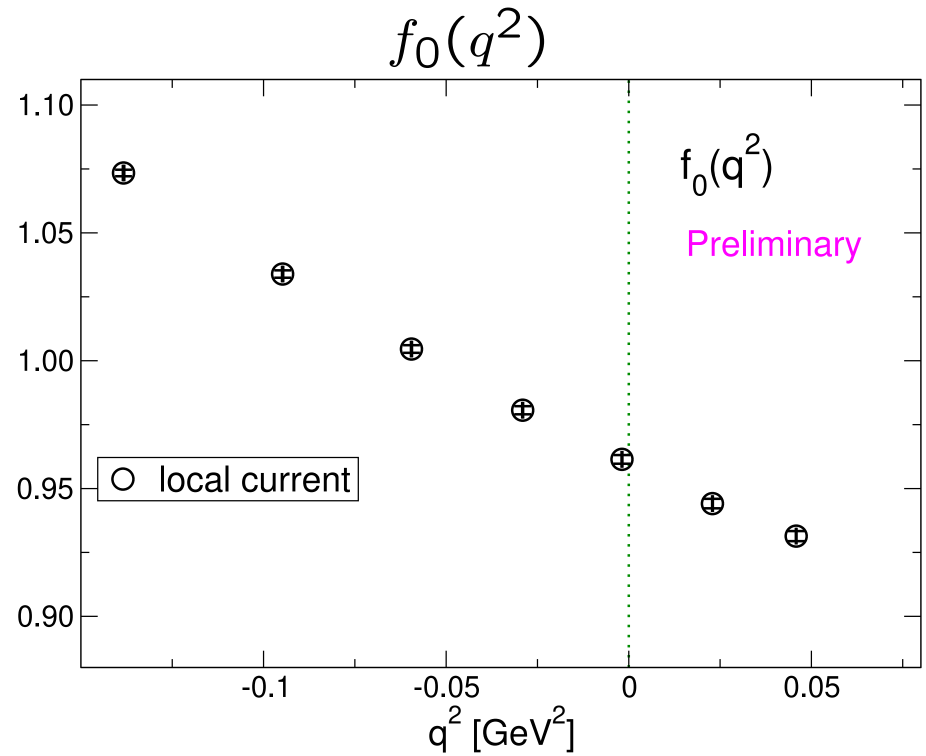
$M_4(p), M_i(p) \to f_+(q^2), f_0(q^2)$ at each q^2 except for $p = 0$, where only $f_0(q^2)$

$\to q^2$ interpolation for $f_+(q^2), f_0(q^2) \to f_+(0) \, (= f_0(0))$

$\to |V_{us}|$ through $|V_{us}| f_+(0) = 0.21654(41)$ [Moulson:PoS(CKM2016)033(2017)]

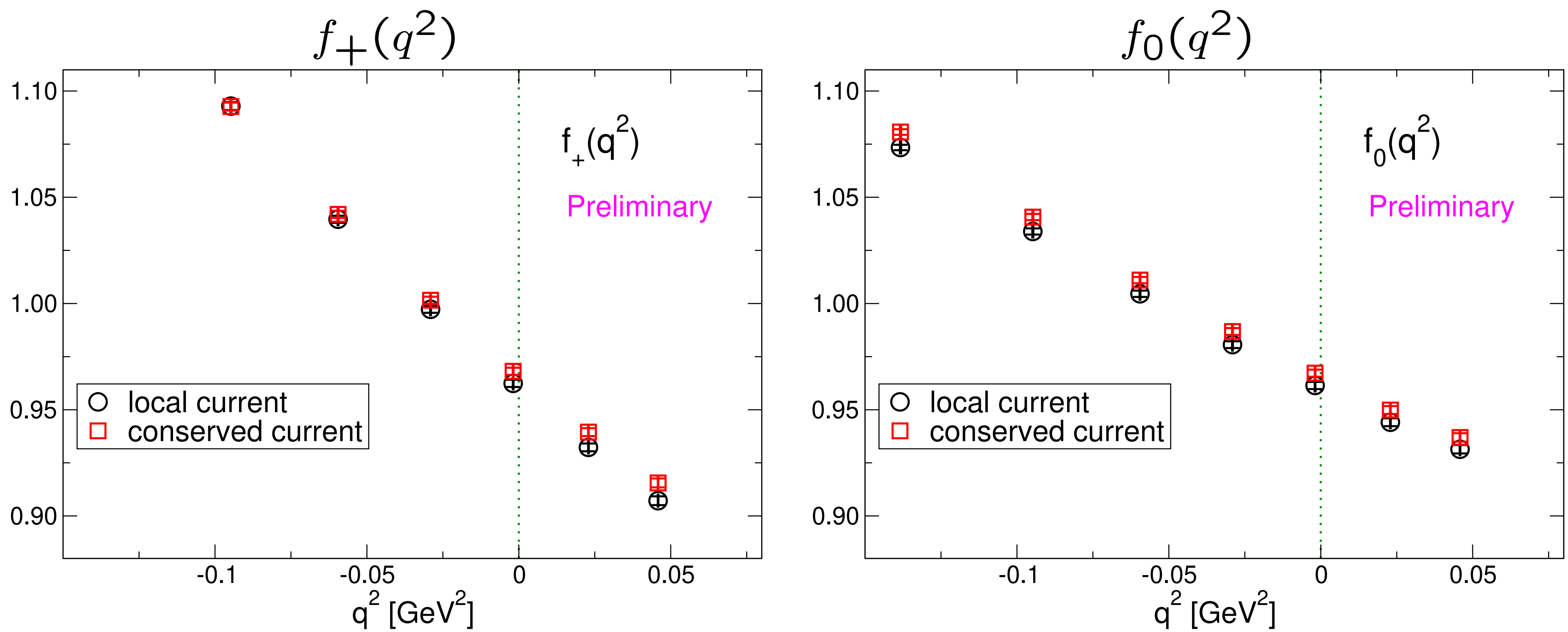
$f_+(q^2)$ and $f_0(q^2)$ on 256^4





Clear signal in tiny q^2 region thanks to $L \sim 10$ [fm]

$f_+(q^2)$ and $f_0(q^2)$ on 256^4



Clear signal in tiny q^2 region thanks to $L \sim 10$ [fm]

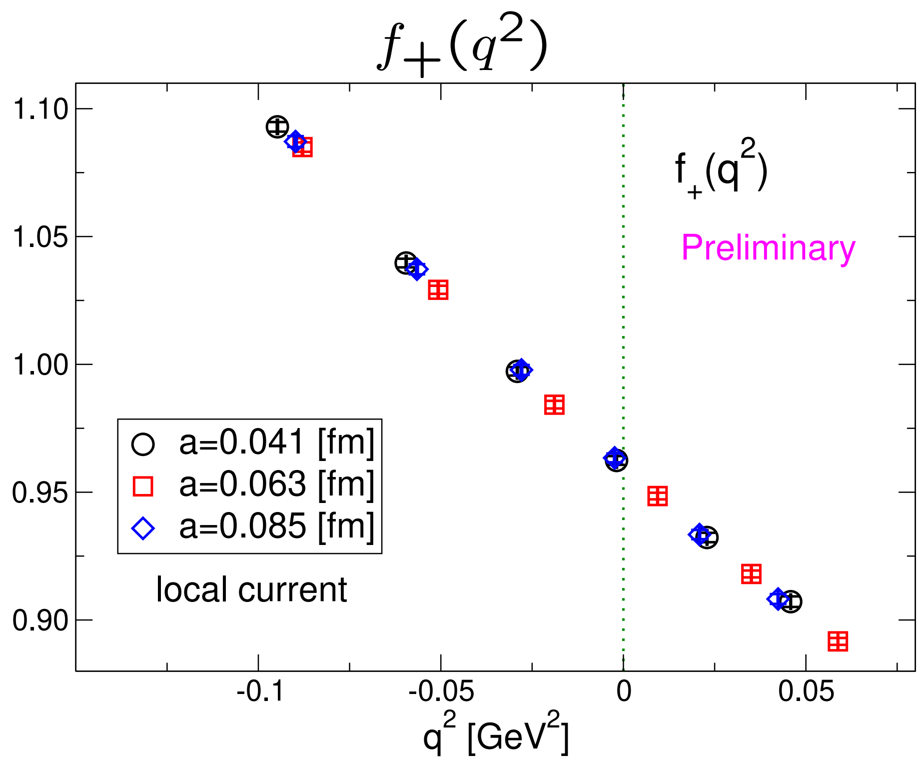
in both currents

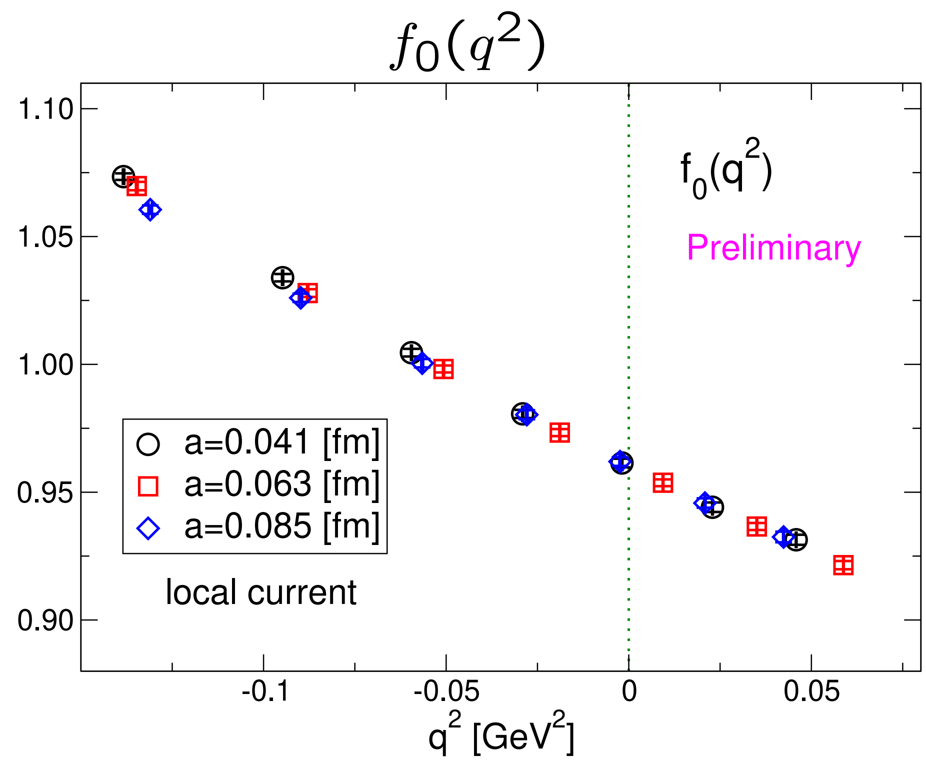
Conserved data systematically larger than local data

$\rightarrow$ Similar trend in larger lattice spacings

but smaller difference between local and conserved data

$f_+(q^2)$ and $f_0(q^2)$ at three lattice spacings (local current)

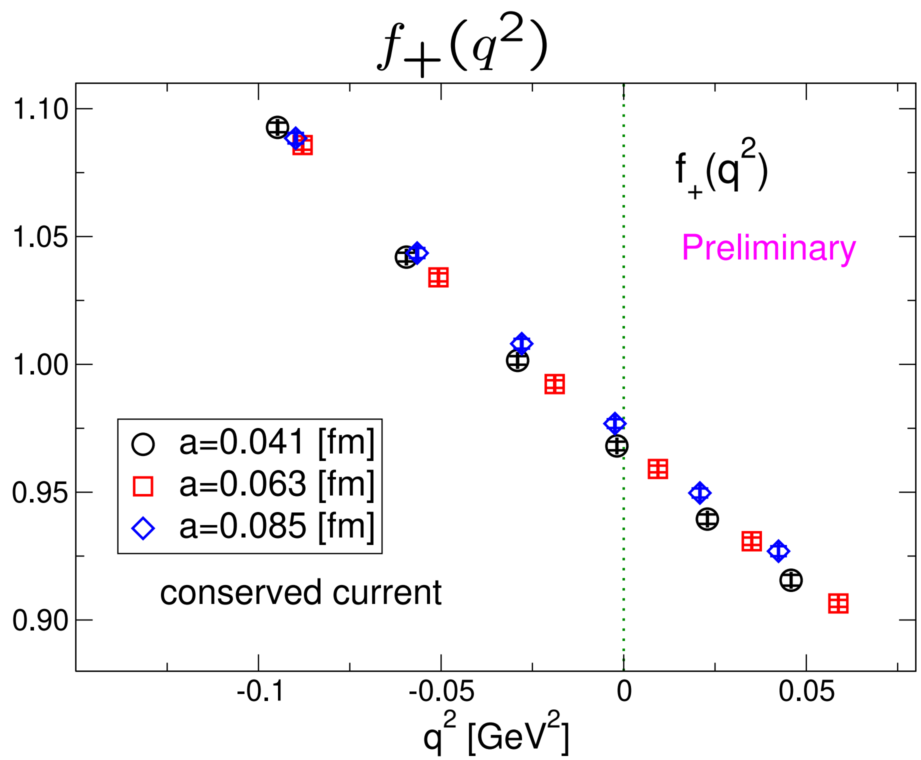


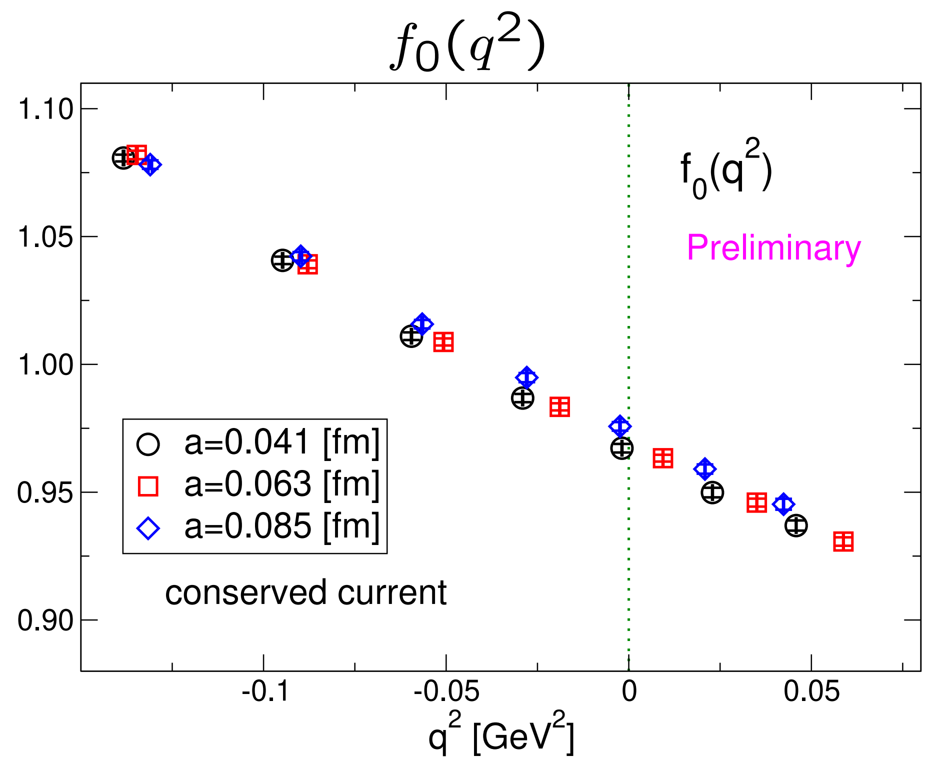


local : Little a dependence in small and large q^2 region

$\rightarrow$ Small a effect in $q^2 \sim 0$

$f_+(q^2)$ and $f_0(q^2)$ at three lattice spacings (conserved current)





local : Little a dependence in small and large q^2 region

$\rightarrow$ Small a effect in $q^2 \sim 0$

conserved : Relatively larger difference in all q^2 region

$f_+(q^2)$ and $f_0(q^2)$ decrease with a.

$\rightarrow$ expect to converge to local current data toward $a \rightarrow 0$

q^2 interpolation + tiny chiral extrapolation

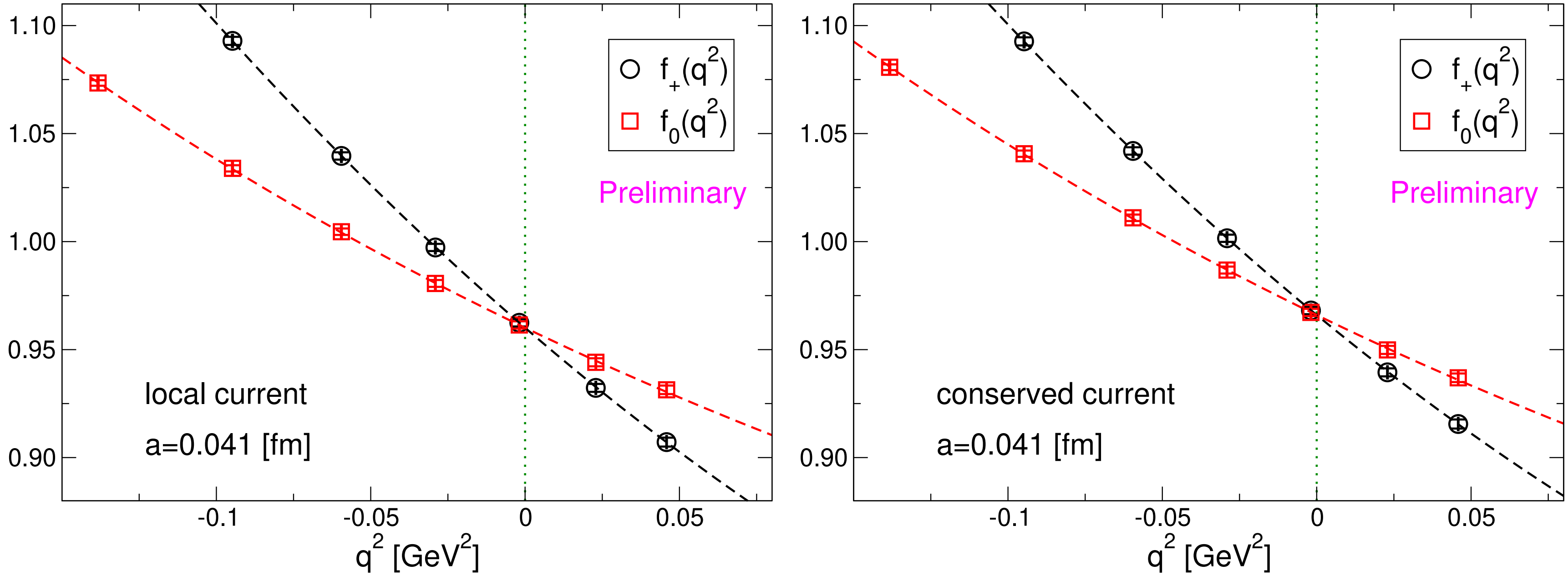
Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ [PACS, PRD101,9,094504(2020)]

$$f_+(q^2) = 1 - \frac{4}{F_0^2} L_9(\mu) q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^+ q^4$$

$$f_0(q^2) = 1 - \frac{8}{F_0^2} L_5(\mu) q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^0 q^4$$

Fit parameters : $L_9(\mu), L_5(\mu), c_0, c_2^+, c_2^0$; Known functions : K_+, K_0 ['85 Gasser, Leutwyler]

using $\mu = 0.77$ GeV, $F_0 = 0.11205$ GeV (estimated from FLAG $F^{\mathrm{SU(2)}}/F_0$ w/ $F^{\mathrm{SU(2)}} = 0.129$ GeV)

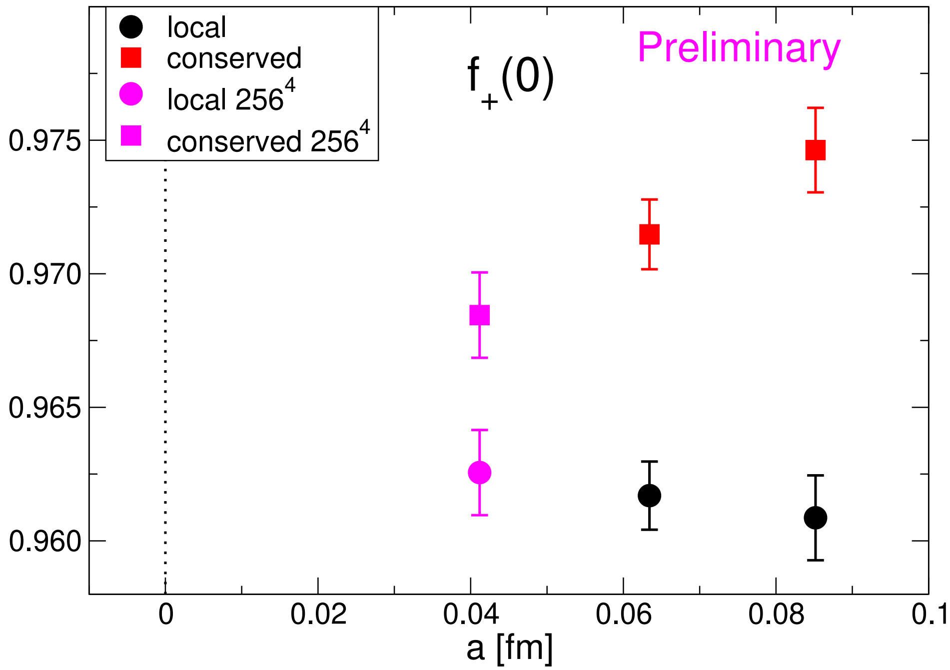


q^2 fits for (f_+, f_0) in each (local,conserved) work well.

Tiny extrapolation to physical M_{π^-} and M_{K^0} using same formulas

$M_\pi - M_{\pi^-}^{\mathrm{phys}} \sim 2$ MeV, $M_K - M_{K^0}^{\mathrm{phys}} \sim 16$ MeV

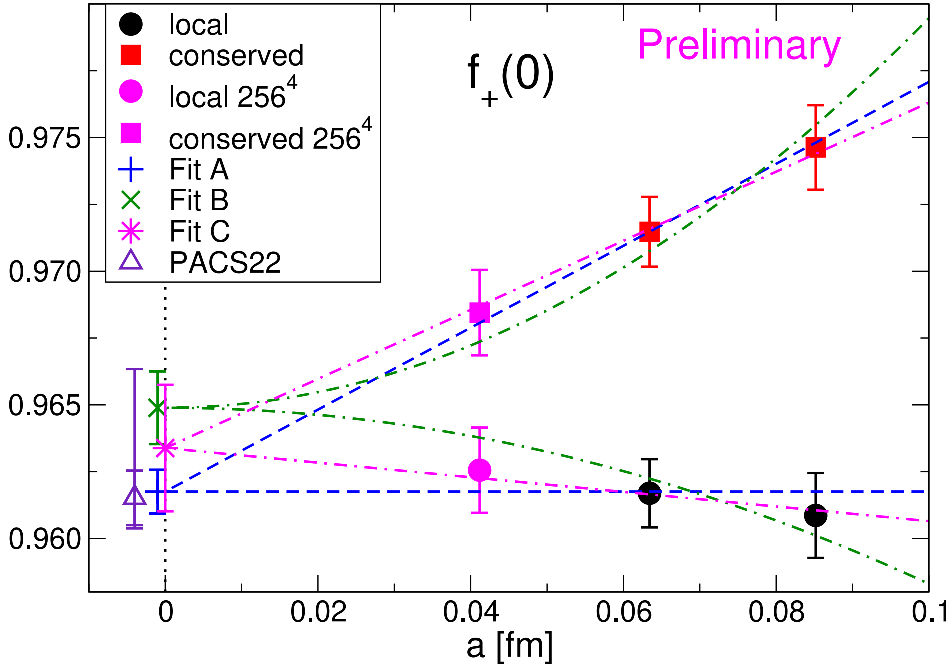
Continuum extrapolation of $f_+(0)$



local current: little a dependence after tiny chiral extrapolation

conserved current: clear a dependence

Continuum extrapolation of $f_+(0)$



local current: little a dependence after tiny chiral extrapolation
conserved current: clear a dependence

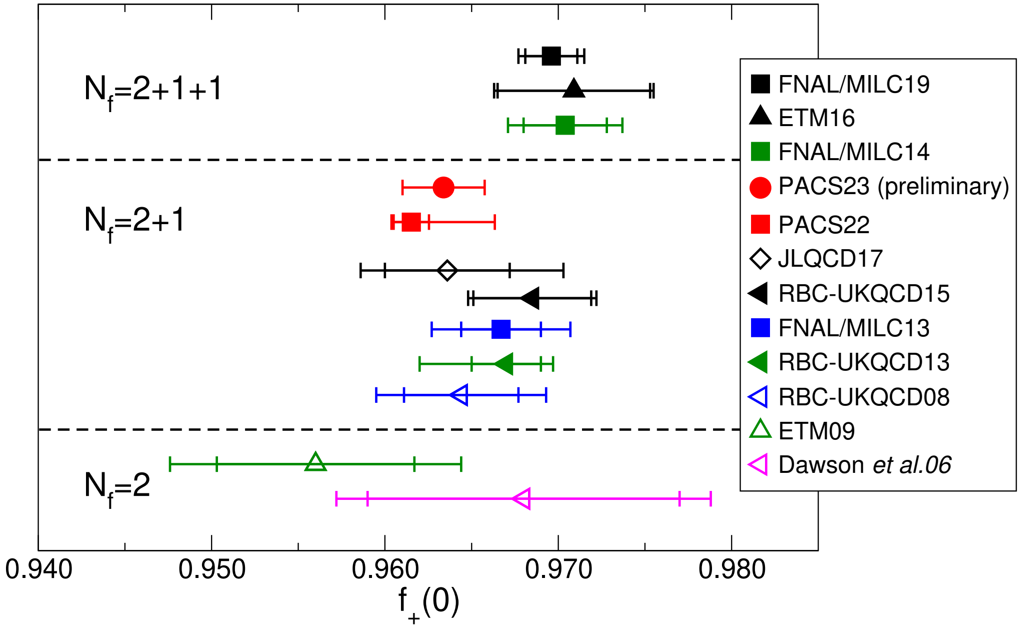
fit form	Fit A	Fit B	Fit C
local	C_0	$C_0 + C_2 a^2$	$C_0 + C_1 a$
conserved	$C_0 + C_1' a$	$C_0 + C_2' a^2$	$C_0 + C_1' a$

Result of Fit C covers other two fit results within the error.

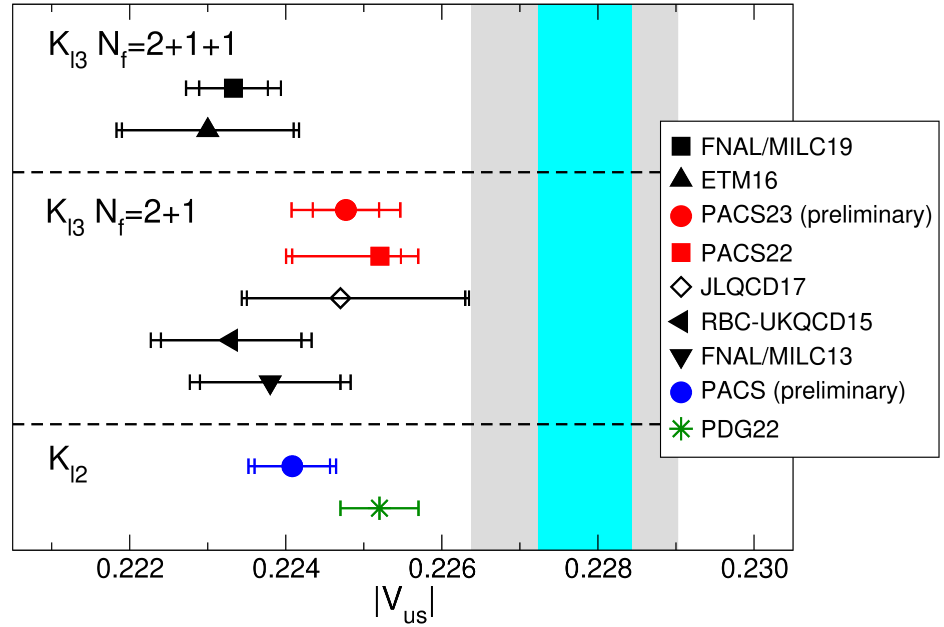
Fit C adopted as our preliminary result

Consistent with PACS22 result

$f_+(0)$ and $|V_{us}|$



inner, outer = statistical, total(stat.+sys.)



inner, outer = lattice, total(lat.+exp.)

Standard model (SM) prediction using $|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$
cyan band: ['18 Seng *et al.*]; grey band: ['20 Hardy, Towner]

$f_+(0)$: Reasonably agree with previous lattice calculations $\lesssim 2\sigma$
Systematic error not estimated yet

$|V_{us}|$ using $|V_{us}|f_+(0) = 0.21654(41)$ ['17 Moulson]

agree with $|V_{us}|$ from K_{l2} using f_K/f_π

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(35)$$
['19 Di Carlo *et al.*]

$2 \sim 3\sigma$ difference from CKM unitarity (grey and cyan bands)

Summary

Preliminary result of $K_{\ell 3}$ form factors on 3rd PACS10 configuration

$a = 0.041$ fm, $L \gtrsim 10$[fm] at physical point

- Clear signal for $f_+(q^2), f_0(q^2)$ in tiny q^2 region
- Continuum extrapolation of $f_+(0)$ with local and conserved data

  Reasonably consistent with PACS22 and previous results
- $|V_{us}|$ from our preliminary result

  Reasonably consistent with previous $K_{\ell 3}$ determinations

  Consistent with $K_{\ell 2}$ determinations

  Different from CKM unitarity by $2 \sim 3$ σ

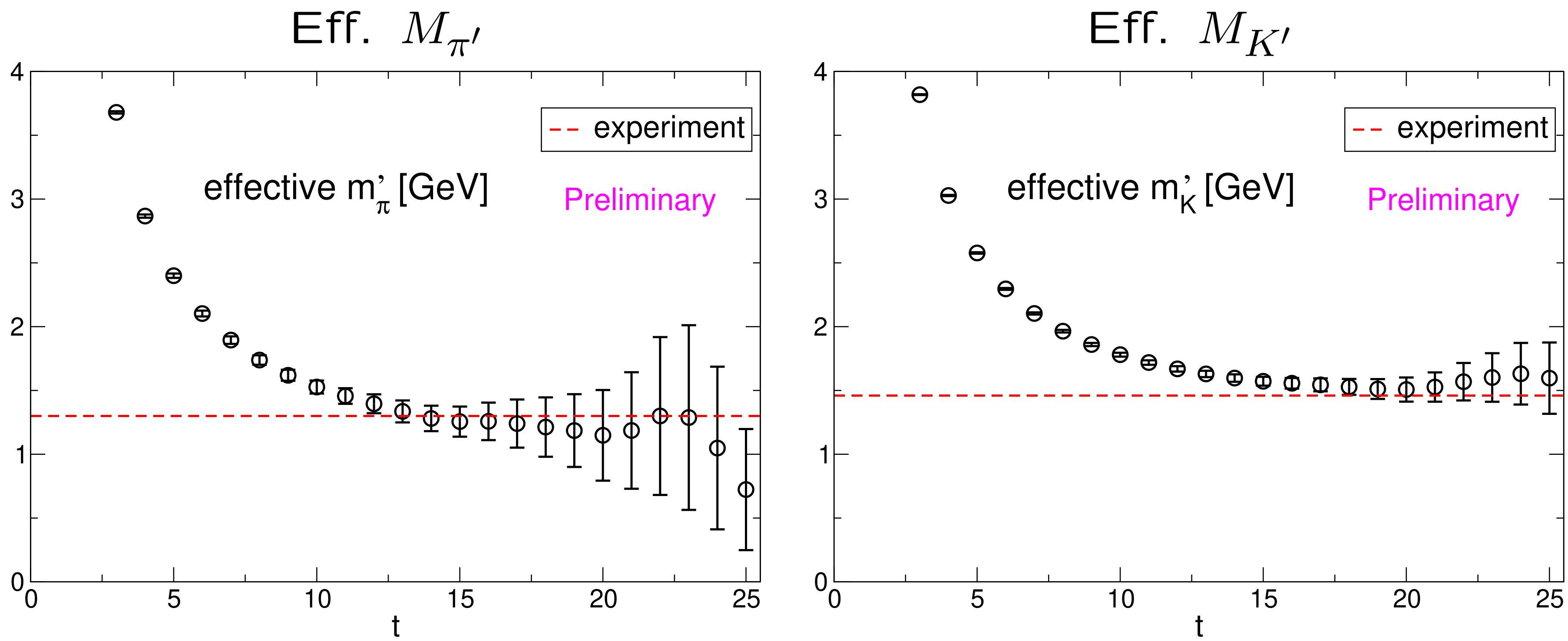
Future works

Calculation with different source operator

Estimate systematic error

Back up

Effective mass for 1st excited states π', K' on 256^4



Effective mass of 1st excited state

$$M'(t) = \log\left(\frac{C'(t)}{C'(t+1)}\right), \quad C'(t) = C(t) - \underline{A_0 e^{-M_0 t}}$$

ground state contribution from fit in $t \gg 1$

a dependence of form factors