$B_s \to K\ell\nu$ form factors from lattice QCD with domain-wall heavy quarks

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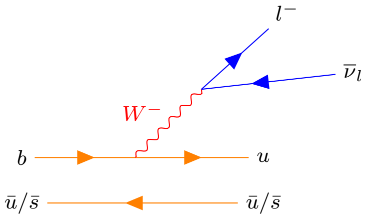
Introduction and Motivation

- There exists 2.2σ tension between exclusive and inclusive determination of $|V_{ub}|$.[PDG, Review of CKM Matrix elements]
- As s quark is heavier than light u/d quarks we can expect that the statistical uncertainty should be smaller than $B \to \pi \ell \nu$ determination.[Parisi ('84), Lepage ('89)]



- LHCb observed ${B_s}^0 \to K^- \mu^+ \nu_\mu$ decay. [R. Aaij *et al.*, PRL 126, 081804 (2021)]
- In this talk we report our on going study of form factors with domain-wall heavy quark.

Form factor extraction w/ zero-momentum

- The two-point and three-point functions used are -

$$\begin{aligned}
C_{3,\mu}^{K\to B_s}(t,T) &= \sum_{\vec{x},\vec{y}} \langle \mathcal{O}_{B_s}(\vec{x},T) V_{bl}^{\mu}(\vec{y},t) \mathcal{O}_K(\vec{0},0)^\dagger \rangle \exp\left[-i(\vec{q}.\vec{y})\right] \\
C_{3,\mu}^{B_s\to K}(t,T) &= \sum_{\vec{x},\vec{y}} \langle \mathcal{O}_{K}(\vec{x},T) V_{lb}^{\mu}(\vec{y},t) \mathcal{O}_{B_s}(\vec{0},0)^\dagger \rangle \\
C_{3,\mu}^{K\to K}(t,T) &= \sum_{\vec{x},\vec{y}} \langle \mathcal{O}_{K}(\vec{x},T) V_{ll}^{\mu}(\vec{y},t) \mathcal{O}_{K}(\vec{0},0)^\dagger \rangle \\
C_{3,\mu}^{B_s\to B_s}(t,T) &= \sum_{\vec{x},\vec{y}} \langle \mathcal{O}_{B_s}(\vec{x},T) V_{bb}^{\mu}(\vec{y},t) \mathcal{O}_{B_s}(\vec{0},0)^\dagger \rangle \\
C_2^{K}(t) &= \sum_{\vec{x}} \langle \mathcal{O}_{K}(\vec{x},t) \mathcal{O}_{K}(\vec{0},0)^\dagger \rangle \exp\left[-i(\vec{p}_K.\vec{x})\right] \\
C_2^{B_s}(t) &= \sum_{\vec{x}} \langle \mathcal{O}_{B_s}(\vec{x},t) \mathcal{O}_{B_s}(\vec{0},0)^\dagger \rangle
\end{aligned}$$

  With $V_{bl}^{\mu}(x) = \bar{b}(x)\gamma^{\mu} l(x)$, $V_{lb}^{\mu}(x) = \bar{l}(x)\gamma^{\mu} b(x)$, $V_{ll}^{\mu}(x) = \bar{l}(x)\gamma^{\mu} l(x)$ and $V_{bb}^{\mu}(x) = \bar{b}(x)\gamma^{\mu} b(x)$.

- $C_{3,\mu}^{B_s\to K}(t,T)$, $C_{3,\mu}^{K\to K}(t,T)$, $C_{3,\mu}^{B_s\to B_s}(t,T)$ are calculated with zero momentum insertion, as they are needed for zero momentum analysis only.
- B_s meson kept at rest.

- Two-point functions are always smeared at the source but are local or smeared at the sink while the three-point functions are always smeared at both source and sink. Needed to cancel overlap factors in the ratio.
- We express the three-point functions as

$$C_{3,4}^{K\to B_s}(t,T) = \sum_{n,m} A_n^{B_s}\, A_m^{K^*}\, D_{4,nm}^{K\to B_s} \exp\left[-E_n^{B_s}(T-t)\right] \exp\left[-E_m^{K} t\right]$$

Here $A_n^{B_s} = \frac{\langle 0|\mathcal{O}_{B_s}(\vec{0},0)|B_s^n\rangle}{\sqrt{2E_n^{B_s}}}$ and $D_{4,nm}^{K\to B_s} = \frac{\langle B_s^n|V_{bl}^4(\vec{0},0)|K^m\rangle}{2\sqrt{E_n^{B_s} E_m^K}}$.

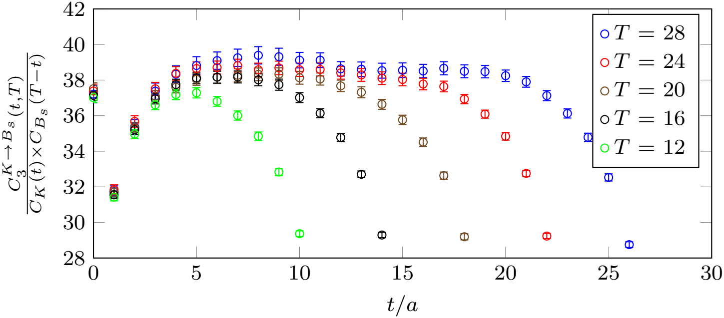
- We studied the ground state saturation

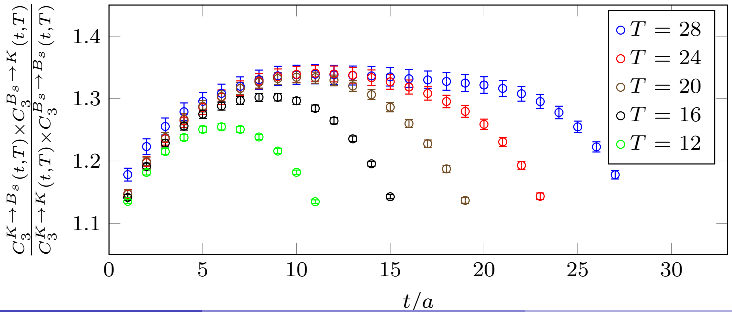
$$R_{3p2p} = \frac{C_{3,4}^{K\to B_s}(t,T)}{C_2^K(t)\, C_2^{B_s}(T-t)} \to \frac{D_{4,00}^{K\to B_s}}{A_0^K\, A_0^{B_s^*}}$$

- Double Ratio : $\frac{C_{3,4}^{K\to B_s}(t,T)\, C_{3,4}^{B_s\to K}(t,T)}{C_{3,4}^{K\to K}(t,T)\, C_{3,4}^{B_s\to B_s}(t,T)} \longrightarrow \frac{D_{4,00}^{K\to B_s}\, D_{4,00}^{B_s\to K}}{D_{4,00}^{K\to K}\, D_{4,00}^{B_s\to B_s}}$

  $\Rightarrow$ No need for current renormalization. [Hashimoto *et al.*, PRD 61, 014502 (1999)]

Comparison of R_{3p2p} and Double Ratio for $K \to B_s$ data





- The ground state contribution of R_{3p2p} and Double Ration are related as

$$R_{3p2p} A_0^K \, A_0^{B_s} \sqrt{Z_{V_{bl}}} = \sqrt{\text{Double Ratio}}$$

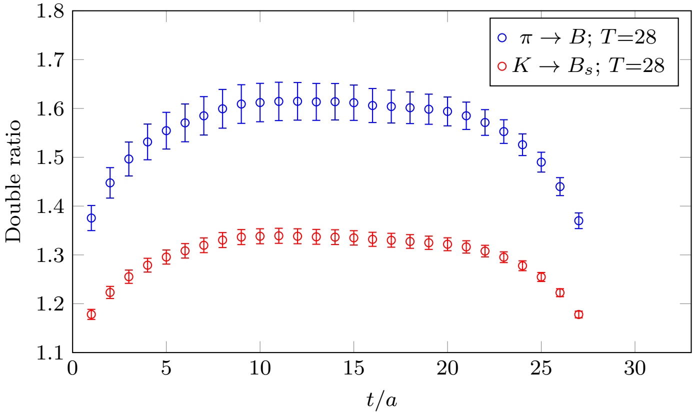
Here, $\mathcal{V}_{bl}^{\mu} = Z_{V_{bl}} V_{bl}^{\mu}; \quad Z_{V_{bl}} = \sqrt{Z_{V_{ll}} Z_{V_{bb}}}$

- Comparison of numbers obtained by these two methods on 100, $32^3 \times 64 \times 12$ configurations with $\beta = 4.17$, $a \sim 0.08$fm, m_s=0.04, m_l=0.019. For the heavy quark mass(m_Q) we have three-points $m_c(0.44037)$, $1.25 \times m_c(0.55046)$ and $1.25^2 \times m_c(0.68808)$.

m_Q	0.44037	0.55046	0.68808
$R_{3p2p} A_0^K \, A_0^{B_s} \sqrt{Z_{V_{bl}}}$	1.154(12)	1.177(12)	1.209(13)
$\sqrt{\text{Double Ratio}}$	1.1546(10)	1.1802(16)	1.2061(25)

- In the R_{3p2p} ratio method the largest errors come from the measurement of $Z_{V_{bl}}$.

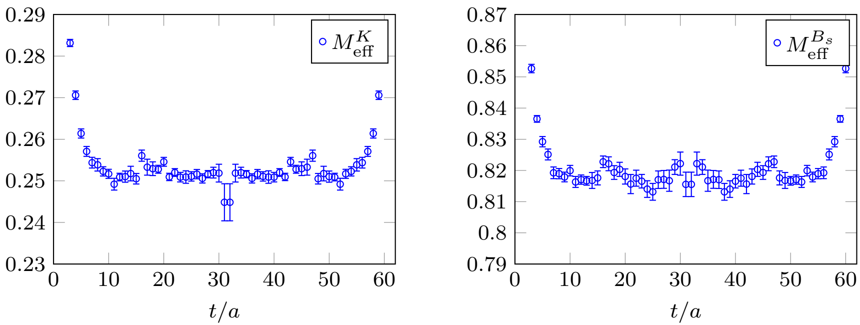
- Comparison of $\pi \to B$ and $K \to B_s$ data. The data is generated at four source positions and then averaged over for both the cases.



- The error bars are much smaller for $K \to B_s$ data.

Fitting

- Here we show the effective masses of K and B_s. The ground state contribution dominates around $t/a \geq 10$.



- The covariance matrix becomes singular for larger fit range. Apart from few eigen-values the statistical fluctuations of the eigen-values are of the same order as the eigen-values themselves.
- We report the result of uncorrelated fit in this talk.

- We included only the first excited state in our analysis.
- Fit formula for double ratio –

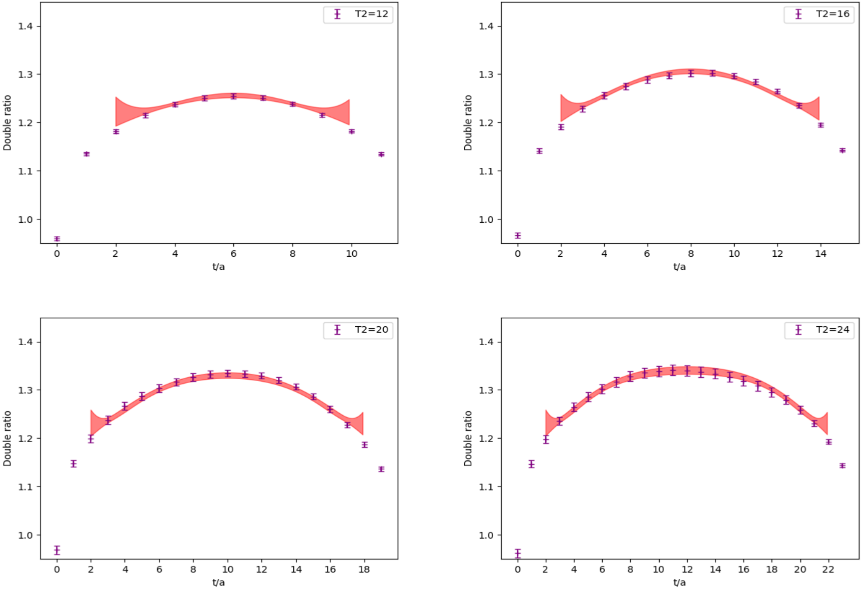
$$\frac{C_{3,4}^{K\to B_s}(t,T)\, C_{3,4}^{B_s\to K}(t,T)}{C_{3,4}^{K\to K}(t,T)\, C_{3,4}^{B_s\to B_s}(t,T)} = C_{00}\Big(1 + A'\Big[\exp\left[-\Delta E_K t\right] + \exp\left[-\Delta E_K (T-t)\right]\Big] + B'\Big[\exp\left[-\Delta E_{B_s} t\right] + \exp\left[-\Delta E_{B_s}(T-t)\right]\Big]\Big)$$

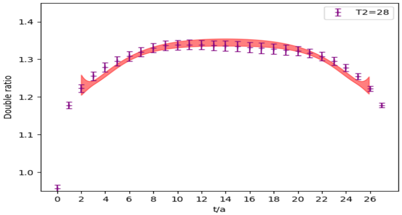
- Ground state contribution

$$C_{00} = \frac{\left(D_{4,00}\right)^2}{D_{4,00}^{K} D_{4,00}^{B_s}} = \frac{\langle B_s|(\bar{b}\gamma_4 u)|K\rangle \, \langle K|(\bar{u}\gamma_4 b)|B_s\rangle}{\langle K|(\bar{u}\gamma_4 u)|K\rangle \, \langle B_s|(\bar{b}\gamma_4 b)|B_s\rangle} = \frac{(M_{B_s} + M_K)^2 \left(f_0(q^2)\right)^2}{4 M_K M_{B_s}}$$

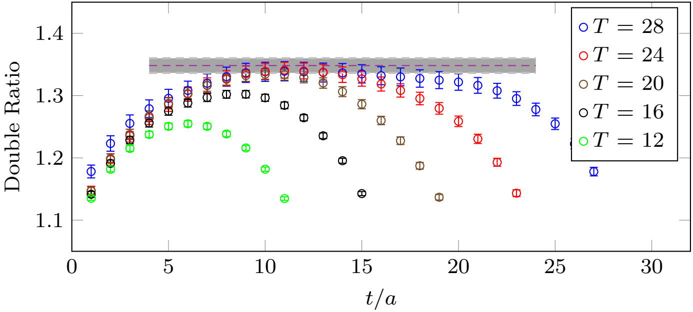
- We performed a simultaneous fit of K, B_s two-point functions and all the double ratios ($T = 12, 16, 20, 24, 28$) together.
- We used jackknife method for error calculation of the fit curve. We did binning of raw correlators with bin size $= 4$.

Fit results





C_{00} fit band



Form factor extraction for non-zero momenta

- In order to extract form factors at non zero momentum we consider the following ratio –

$$R_4(\vec{p}_K) = \frac{C_{3,4}^{K\to B_s}(t, T, \vec{p}_{B_s} = \vec{0}, \vec{p}_K)}{C_{3,4}^{K\to B_s}(t, T, \vec{p}_{B_s} = \vec{0}, \vec{p}_K = \vec{0})} \times \frac{C_2^K(t, \vec{p}_K = \vec{0})}{C_2^K(t, \vec{p}_K)}$$

$$f_{\parallel}(E_K) = \frac{\langle K|V^0|B_s\rangle}{\sqrt{2M_{B_s}}}$$

- We considered all possible combination (p_K^x, p_K^y, p_K^z) with $p_K^i = -1, 0, +1$
- $|\vec{p}_K|^2 = 1$**:** $(+1,0,0); (-1,0,0); (0,+1,0); (0,-1,0); (0,0,+1); (0,0,-1)$
- In order to extract $f_\perp$ we consider the following ratio

$$R_i(\vec{p}_K) = \frac{C_{3,i}^{K\to B_s}(t, T, \vec{p}_{B_s} = \vec{0}, \vec{p}_K)}{C_{3,4}^{K\to B_s}(t, T, \vec{p}_{B_s} = \vec{0}, \vec{p}_K)}$$

$$f_\perp(E_K) = \frac{1}{p_K^i} \frac{\langle K|V^i|B_s\rangle}{\sqrt{2M_{B_s}}}$$

Fit formulae

$$R_4(\vec{p}_K) = \frac{C_{3,4}^{K\to B_s}(t,T,\vec{p}_{B_s}=\vec{0},\vec{p}_K)}{C_{3,4}^{K\to B_s}(t,T,\vec{p}_{B_s}=\vec{0},\vec{p}_K=\vec{0})} \times \frac{C_2^K(t,\vec{p}_K=\vec{0})}{C_2^K(t,\vec{p}_K)}$$

$$= C_{44}\Big(1 + A\exp\left[-\Delta E_{B_s}(\vec{p}_{B_s}=\vec{0})(T-t)\right] + B\exp\left[-\Delta E_K(\vec{p}_K=\vec{0})t\right] + F\exp\left[-\Delta E_K(\vec{p}_K)t\right]\Big)$$

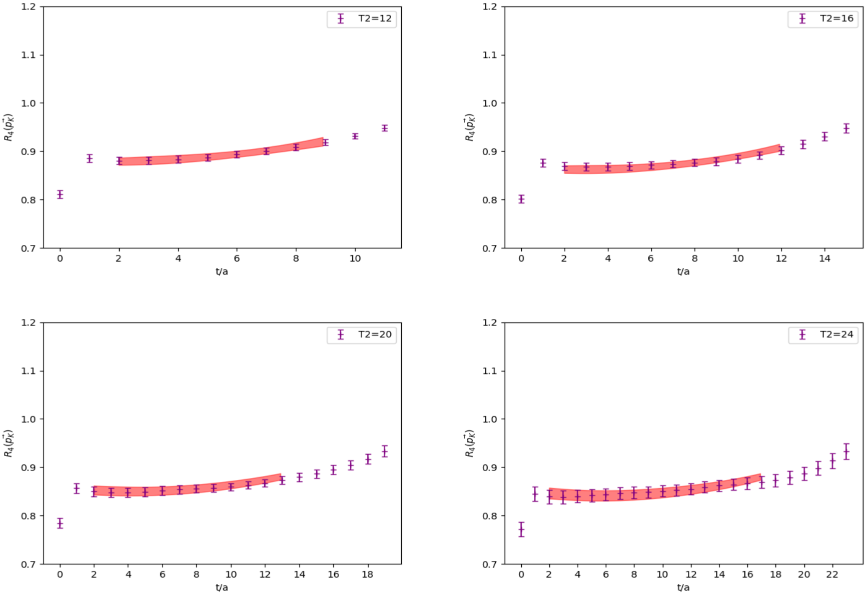
$$R_i(\vec{p}_K) = \frac{C_{3,i}^{K\to B_s}(t,T,\vec{p}_{B_s}=\vec{0},\vec{p}_K)}{C_{3,4}^{K\to B_s}(t,T,\vec{p}_{B_s}=\vec{0},\vec{p}_K)}$$

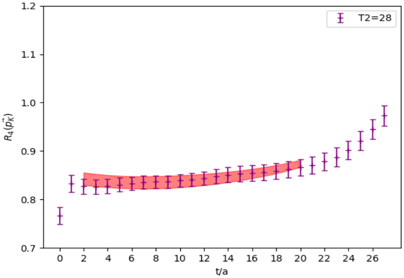
$$= C_{4i}\Big(1 + G\exp\left[-\Delta E_{B_s}(\vec{p}_{B_s}=\vec{0})(T-t)\right] + H\exp\left[-\Delta E_K(\vec{p}_K)t\right]\Big)$$

$$f_{\parallel}(E_K) = C_{44}\sqrt{2M_K C_{00}}; \quad f_{\perp}(E_K) = \frac{C_{4i}C_{44}\sqrt{2M_K C_{00}}}{p_K^i}$$

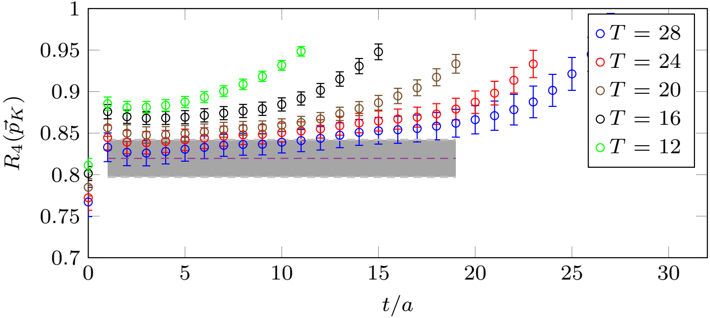
- We performed combined fit of $R_4(\vec{p}_K)$, $R_i(\vec{p}_K)$, $C_2^K(t,\vec{p}_K=\vec{0})$, $C_2^K(t,\vec{p}_K)$ and $C_2^{B_s}(t)$

Fit results for $|\vec{p}|^2 = 1, R_4(\vec{p}_K)$

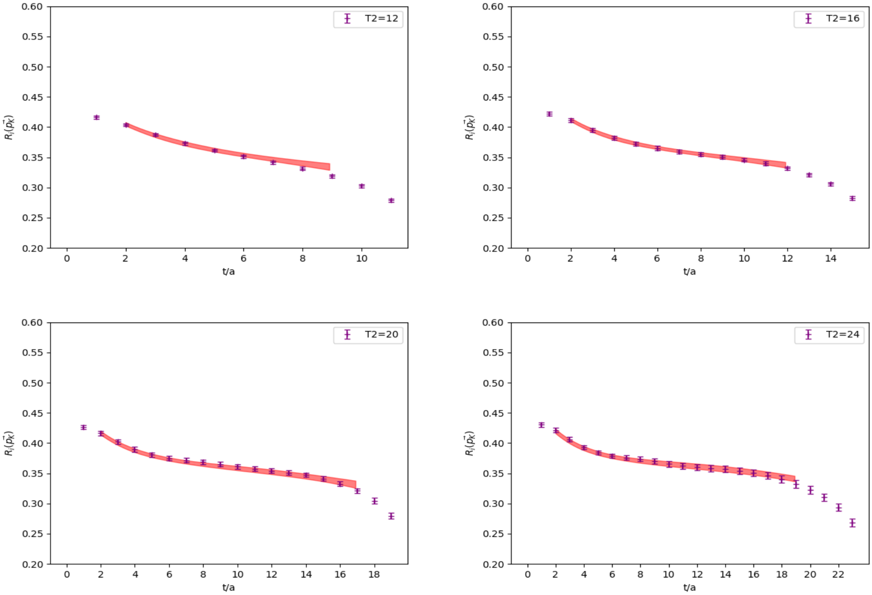


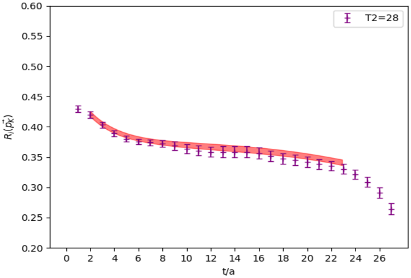


C_{44} fit band

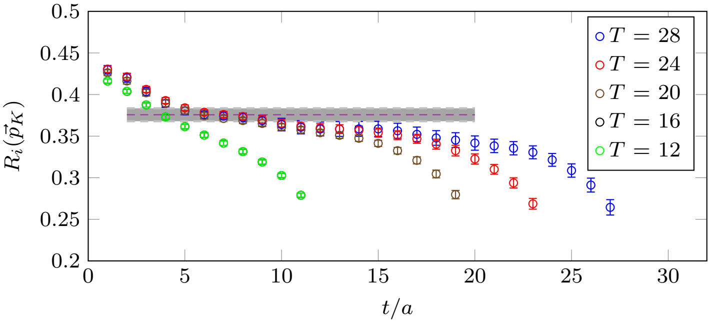


Fit results for $|\vec{p}|^2 = 1, R_i(\vec{p}_K)$





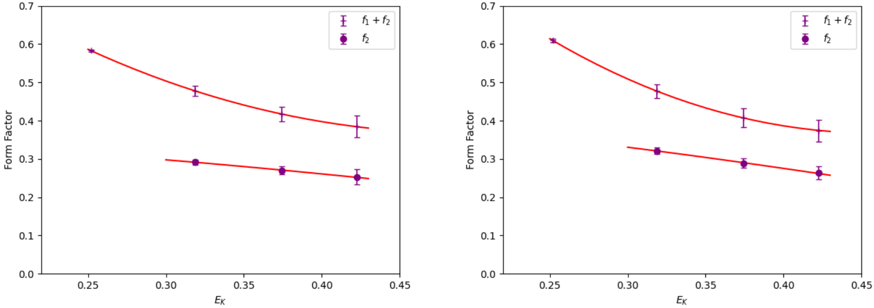
C_{4i} fit band



Form factors

- For comparison with $B \to \pi \ell \nu$ case [Colquhoun *et al.*, PRD 106, 054502 (2022)] we convert $f_{\|}$ and $f_{\perp}$ to HQET motivated definition of form factors f_1 and f_2:

$$f_1(E_K) + f_2(E_K) = \frac{f_{\|}(E_K)}{\sqrt{2}}; \quad f_2(E_K) = \frac{E_K\ f_{\perp}(E_K)}{\sqrt{2}}$$



(a) $m_Q = m_c$ (b) $m_Q = 1.25^2 m_c$

- Fit functions are assumed to be polynomial of E_K only.
- Statistical uncertainty is $\sim$1%. better than $B \to \pi \ell \nu$ by a factor of $\lesssim 3$.

Summary and Outlook

- We report JLQCD's study of $B_s \to K\ell\nu$ form factors with Möbius domain-wall heavy quarks
  $\to$ $m_b < 0.7a^{-1}$ to control discretization errors.
  $\to$ through correlator ratios $\Rightarrow$ no need for current renormalization.
  $\to$ preliminary results at $a^{-1} \sim 2.5$ GeV, $M_\pi \sim 500$ MeV
- Statistical accuracy $\sim 1\%$
  $\to$ averaged over 4 source-time slices
  $\to$ better than $B \to \pi\ell\nu$ (Lepage's arguments).
- Ground state saturation – studied by simulating 5 source-sink separations.
- On-going analysis at $a^{-1} \lesssim 4.5$ GeV and $M_\pi \gtrsim 230$ MeV.

$\mathcal{THANK \quad YOU}$