Study on $\Lambda(1405)$ in the flavor SU(3) limit in the HAL QCD method

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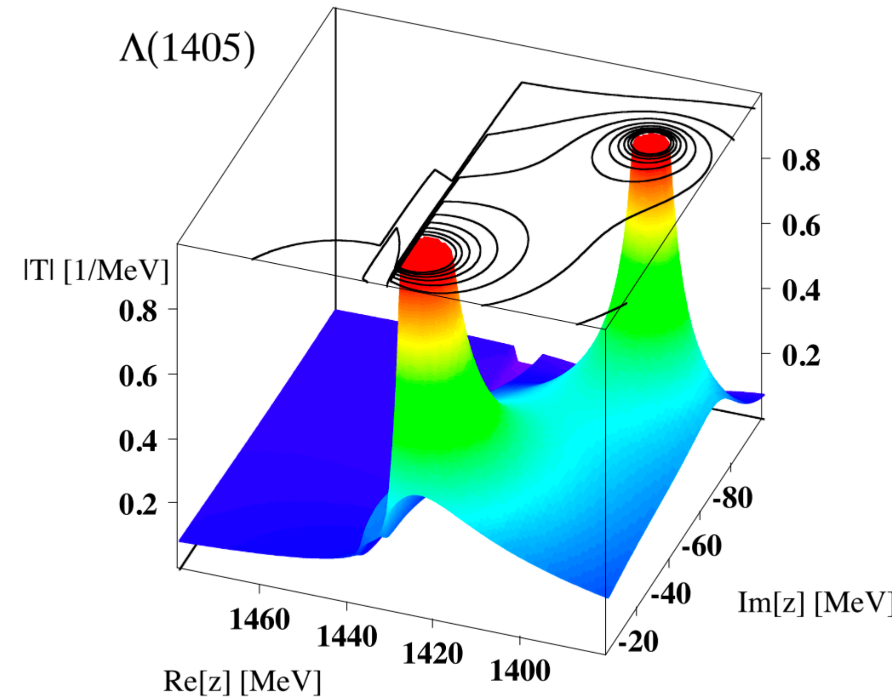
Motivation

- $\Lambda(1405)$: not a simple Λ baryon (**exotic hadron**)

- chiral unitary model

  ➡ two poles in $\bar{K}N$-$\pi\Sigma$ scattering amplitudes?

  [Oller and Meissner, 2001]



(Hyodo and Jido, Prog. Part. Nucl. Phys. **67** (2012), 55-98)

- studies through the scattering processes using lattice QCD

  - very recent simulation in finite-volume method at $m_\pi \approx 200$ MeV

    → virtual state below $\pi\Sigma$ + resonance below $\bar{K}N$

    [J. Bulava et al., 2023]

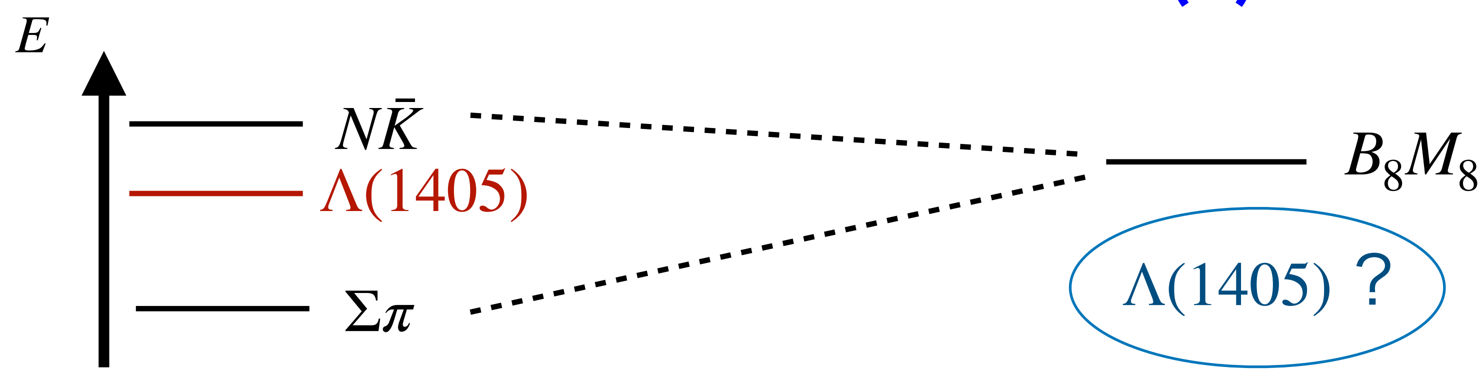
  - HAL QCD method: **this talk**

$\Lambda(1405)$ in flavor SU(3) limit

- we study $\Lambda(1405)$ in **flavor SU(3) limit** $m_u = m_d = m_s$

Physical point

SU(3) limit



single-channel analysis

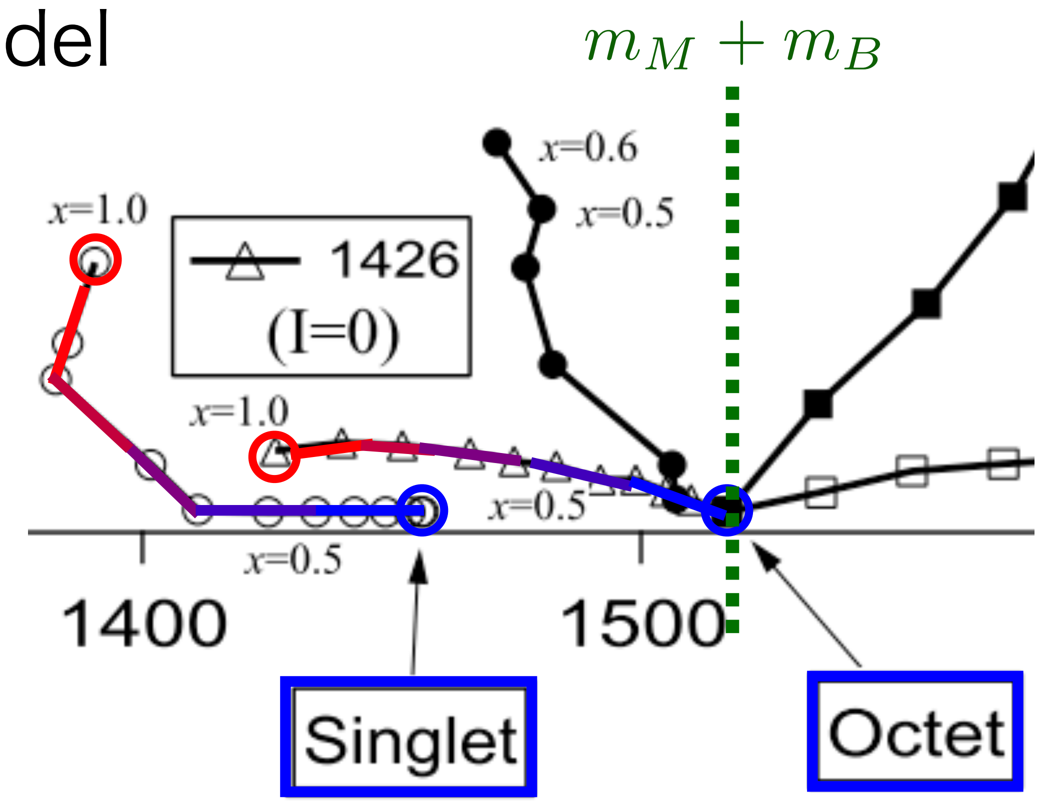
- previous study in the chiral unitary model

Physical point

two poles constituting $\Lambda(1405)$

SU(3) limit

each pole in singlet and octet channels



(Jido et al., Nucl. Phys. A **725** (2003), 181-200)

investigate these states in lattice QCD via HAL QCD method

(Time-dependent) HAL QCD method

[Ishii, Aoki, Hatsuda 2007]
[Ishii et al. 2011]

($m_1 = m_2 = m$ for simplicity)

- 3-point (4-point) function

$$F(\mathbf{r}, t) = \langle 0|\hat{O}(\mathbf{r}, t)\hat{O}(\mathbf{0}, t)\underbrace{\bar{J}(0)}_{\text{source operator of 2-body states}}|0\rangle$$

- time-dependent equation

$$\int d^3 r' \underbrace{U(\mathbf{r}, \mathbf{r}')}_{\approx V^{LO}(r)\delta^{(3)}(\mathbf{r}-\mathbf{r}')} R(\mathbf{r}', t) \simeq \left(-\frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right) R(\mathbf{r}, t)$$

(leading-order approximation)

$$R(\mathbf{r}, t) = \frac{F(\mathbf{r}, t)}{C(t)C(t)}$$: R-correlator

2-point function

$$\Rightarrow \quad V^{LO}(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r}, t)}\left(-\frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2} + \frac{\nabla^2}{2\mu}\right) R(\mathbf{r}, t)$$

Target

- $J^P = 1/2^-$ ➡ S-wave
- SU(3) reps.: $\underbrace{\underline{8}}_{\text{meson}} \otimes \underbrace{\underline{8}}_{\text{baryon}} = 27 \oplus 10 \oplus 10^* \oplus \underline{8_1 \oplus 8_2 \oplus 1}$

3-point functions

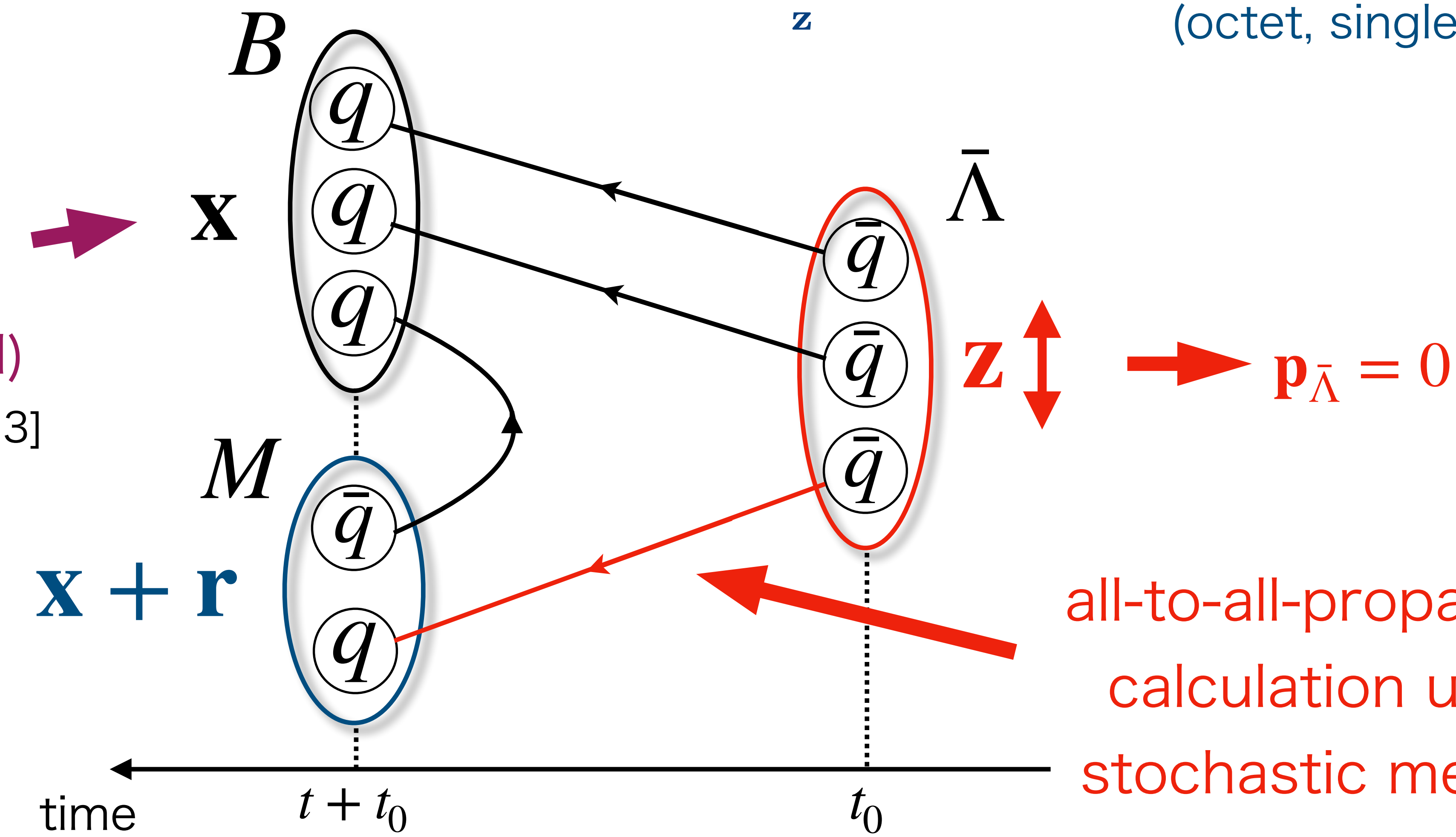
$$F_\alpha^{(\text{rep})}(\mathbf{r}, t) = \langle (M(\mathbf{r}+\mathbf{x}, t) B_\alpha(\mathbf{x}, t))_{(\text{rep})} \underline{\bar{\Lambda}_{\bar{\alpha}}(t_0)} \rangle \quad (\text{rep} = (8_1, 8_2, 1))$$

$\sim \sum_{\mathbf{z}} \bar{u}(\mathbf{z})\bar{d}(\mathbf{z})\bar{s}(\mathbf{z})$: 3-quark type (octet, singlet)

- (one of) quark contractions

move $\mathbf{x}$ to increase statistics (CAA + TSM)

[Blum, Izubuchi, Shintani 2013]
[Bali, Collins, Schäfer 2010]



all-to-all-propagator calculation using stochastic method

Numerical setups

- use SU(3) configurations by Inoue (HAL QCD Collab.) (360 confs.)

(Inoue (HAL QCD), PoS **CD15** (2016), 020)

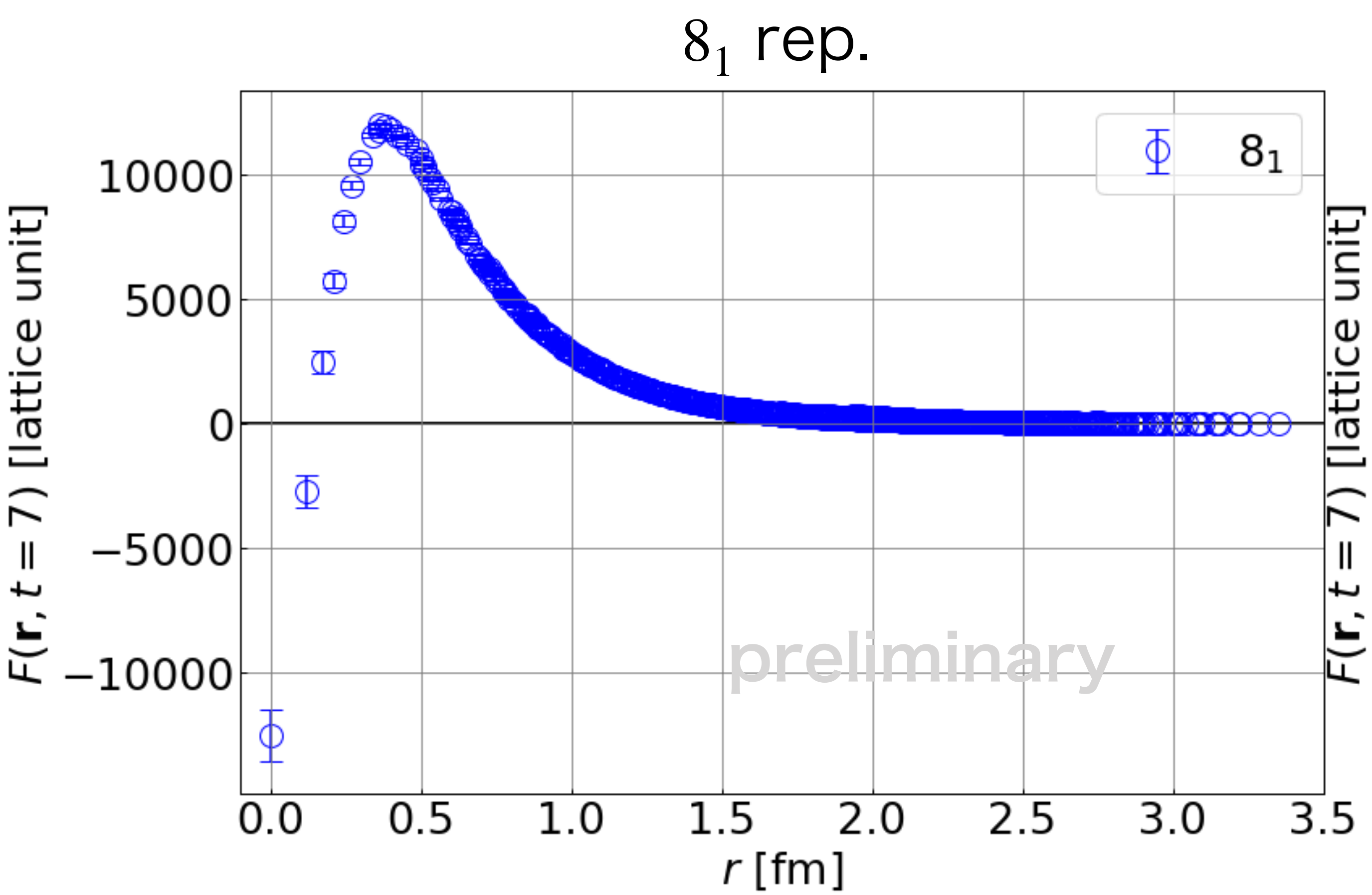
- $a \approx 0.12$ fm, 32^4 lattices ($L \approx 3.87$ fm)

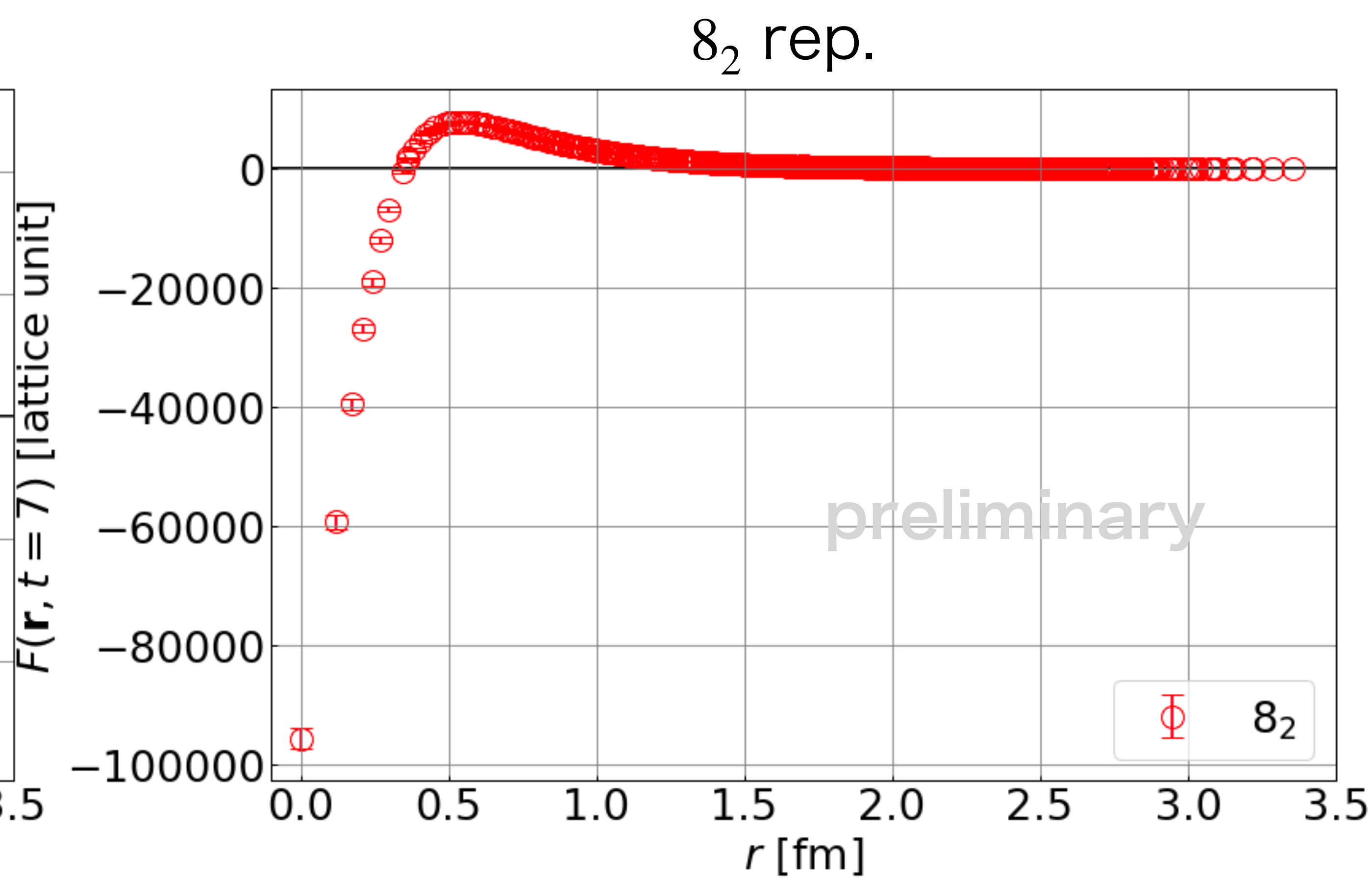
- $m_M \approx 670$ MeV, $m_B \approx 1489$ MeV

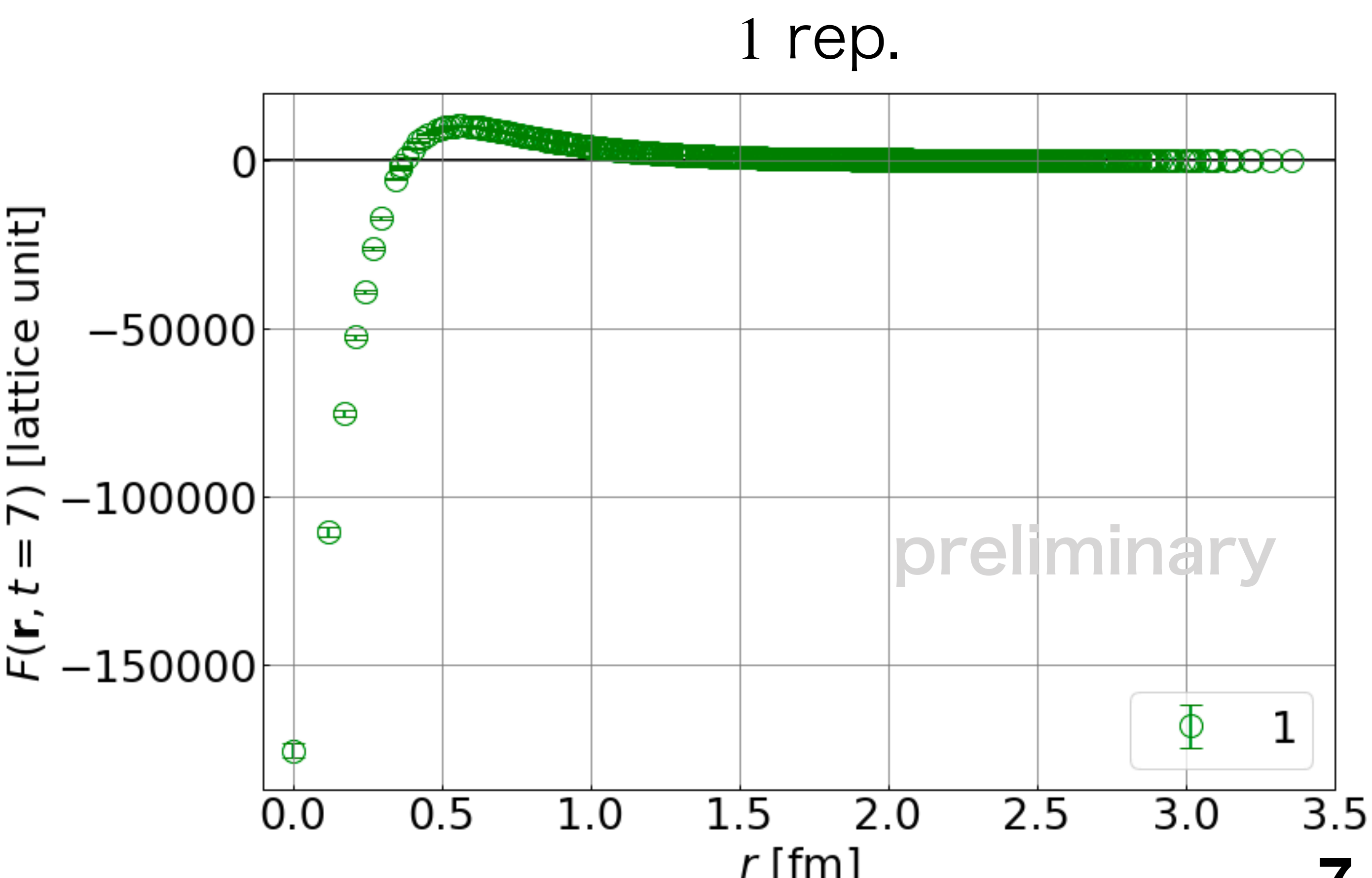
(cf. $m_M = 368$ MeV, $m_B = 1151$ MeV in chiral unitary model)

3-point functions

$$F_{\alpha}^{(\mathrm{rep})}(\mathbf{r},t) = \langle (M(\mathbf{r}+\mathbf{x},t)B_{\alpha}(\mathbf{x},t))_{(\mathrm{rep})}\bar{\Lambda}_{\bar{\alpha}}(t_0)\rangle \quad (\mathrm{rep} = (8_1, 8_2, 1))$$





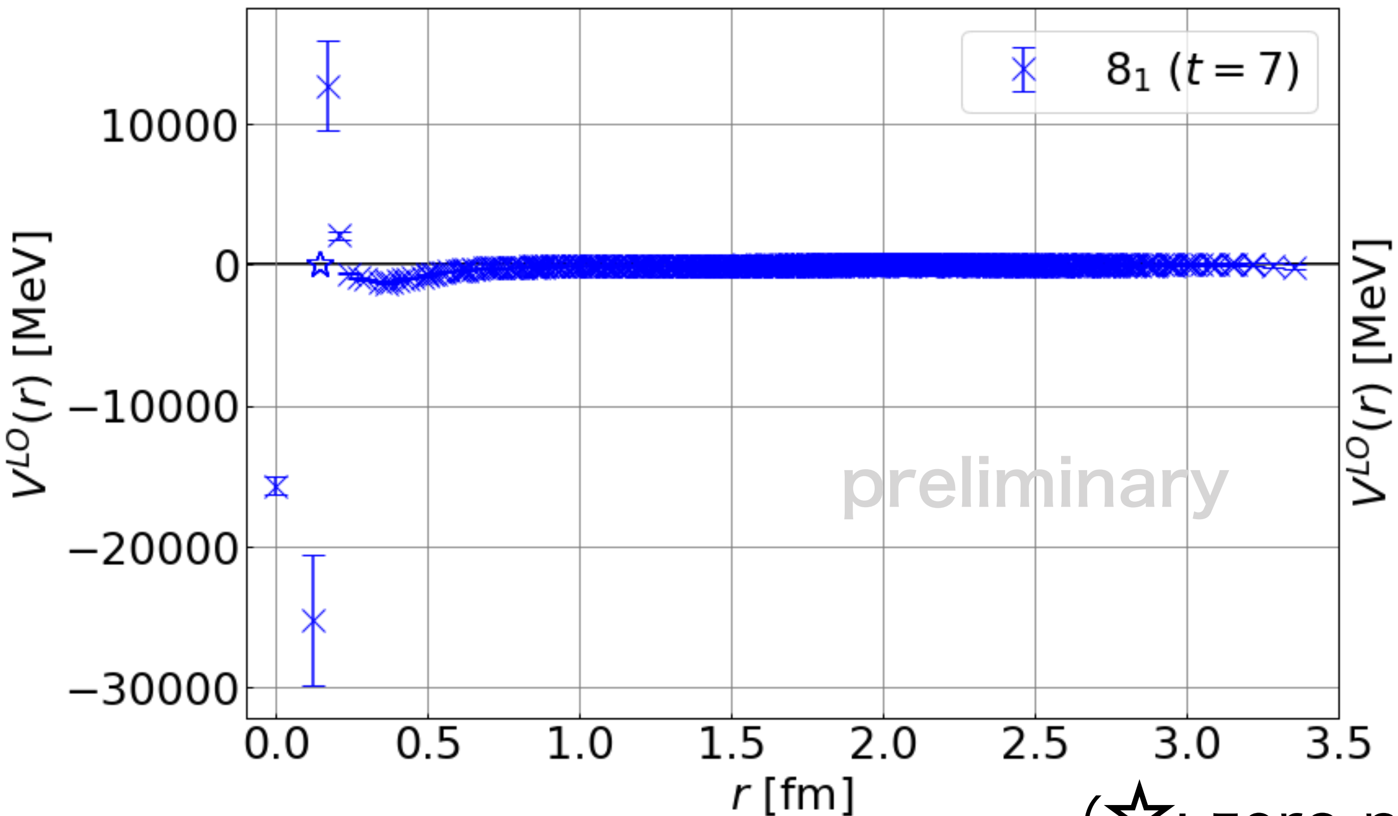


- crossing zero at finite r

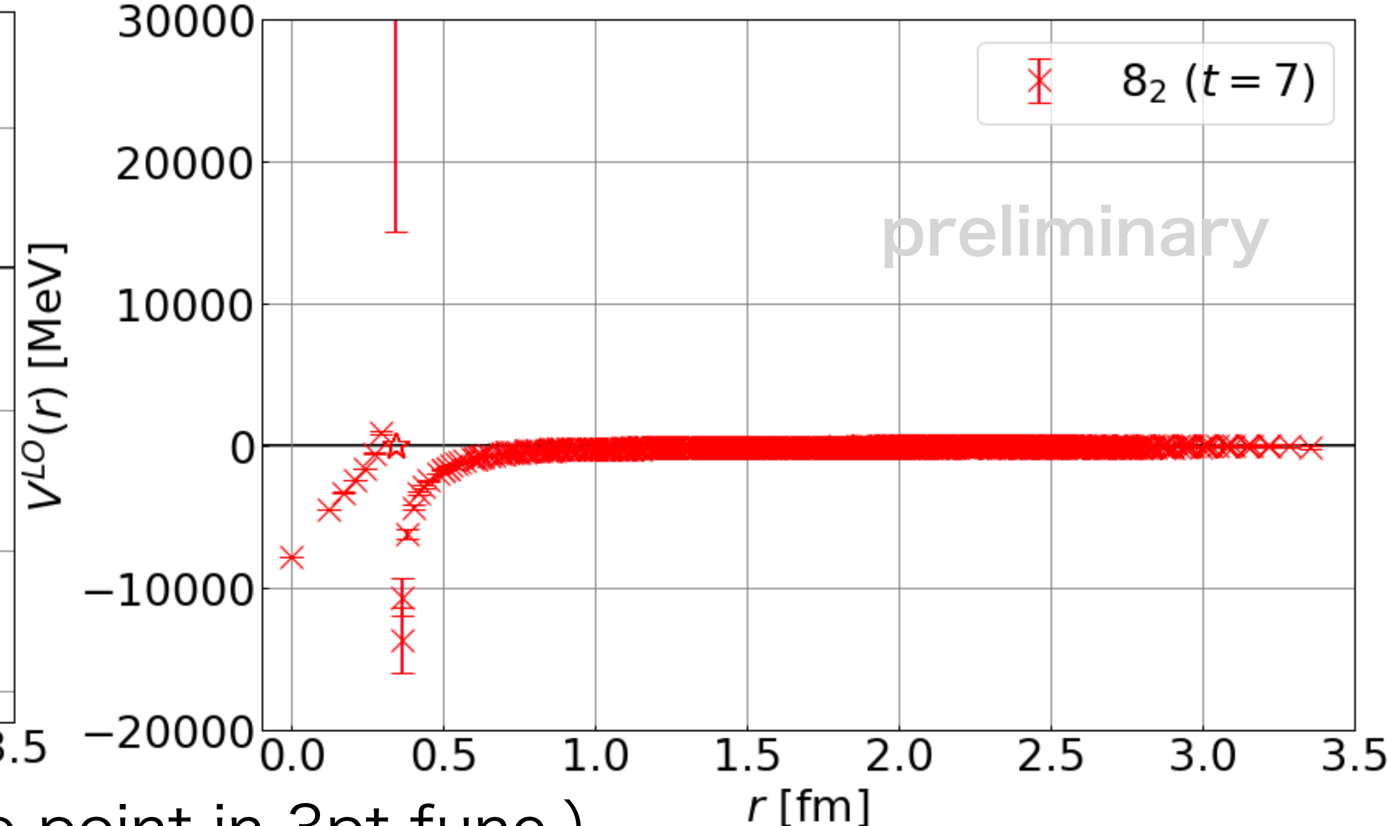
LO potentials

$$V^{LO}(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(-\frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right)R(\mathbf{r},t),\ \ R(\mathbf{r},t) = \frac{F(\mathbf{r},t)}{C(t)C(t)}$$

8_1 rep.

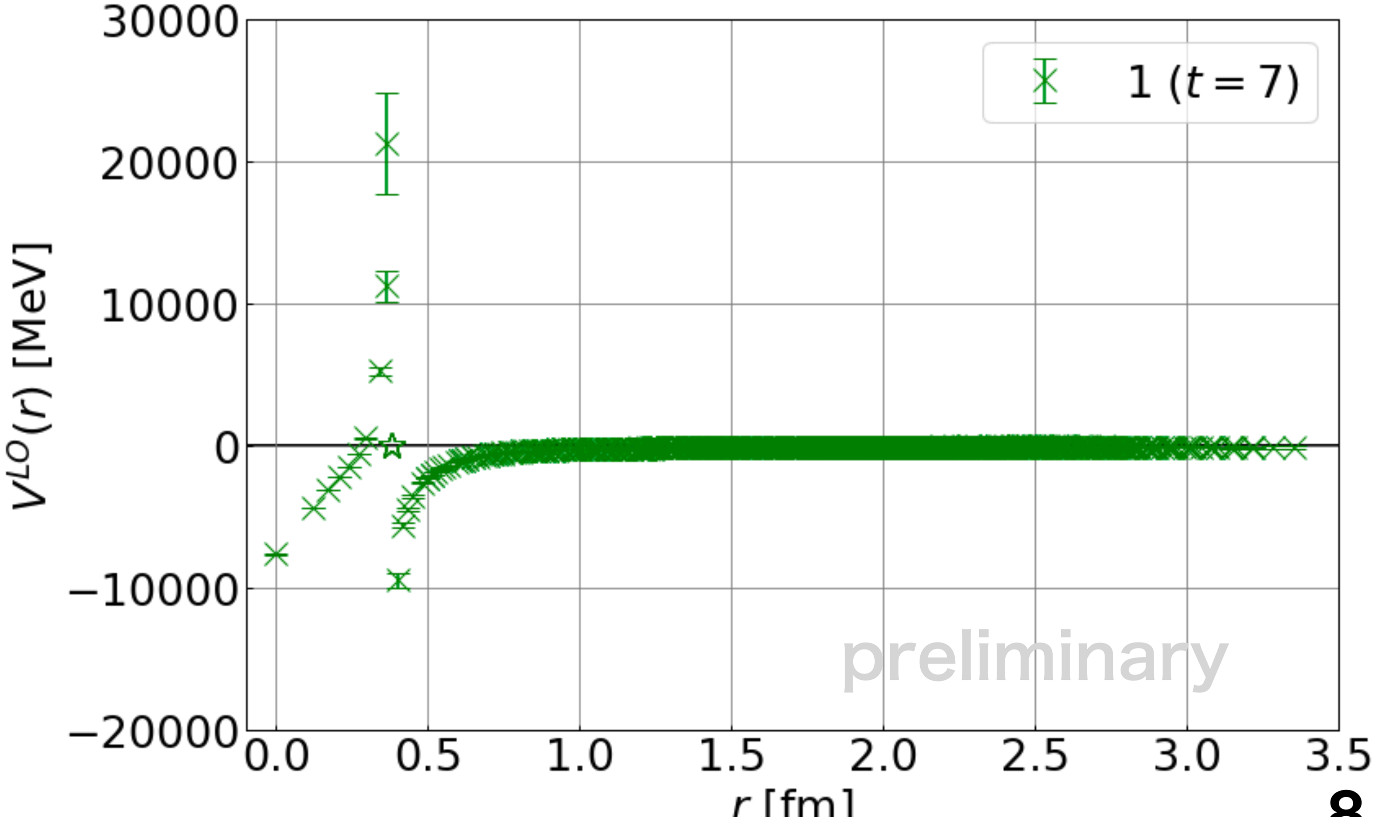


8_2 rep.



(☆: zero point in 3pt func.)

1 rep.



- singular behavior because of the 3pt func. crossing zero

non-locality effect?
(LO is not enough)

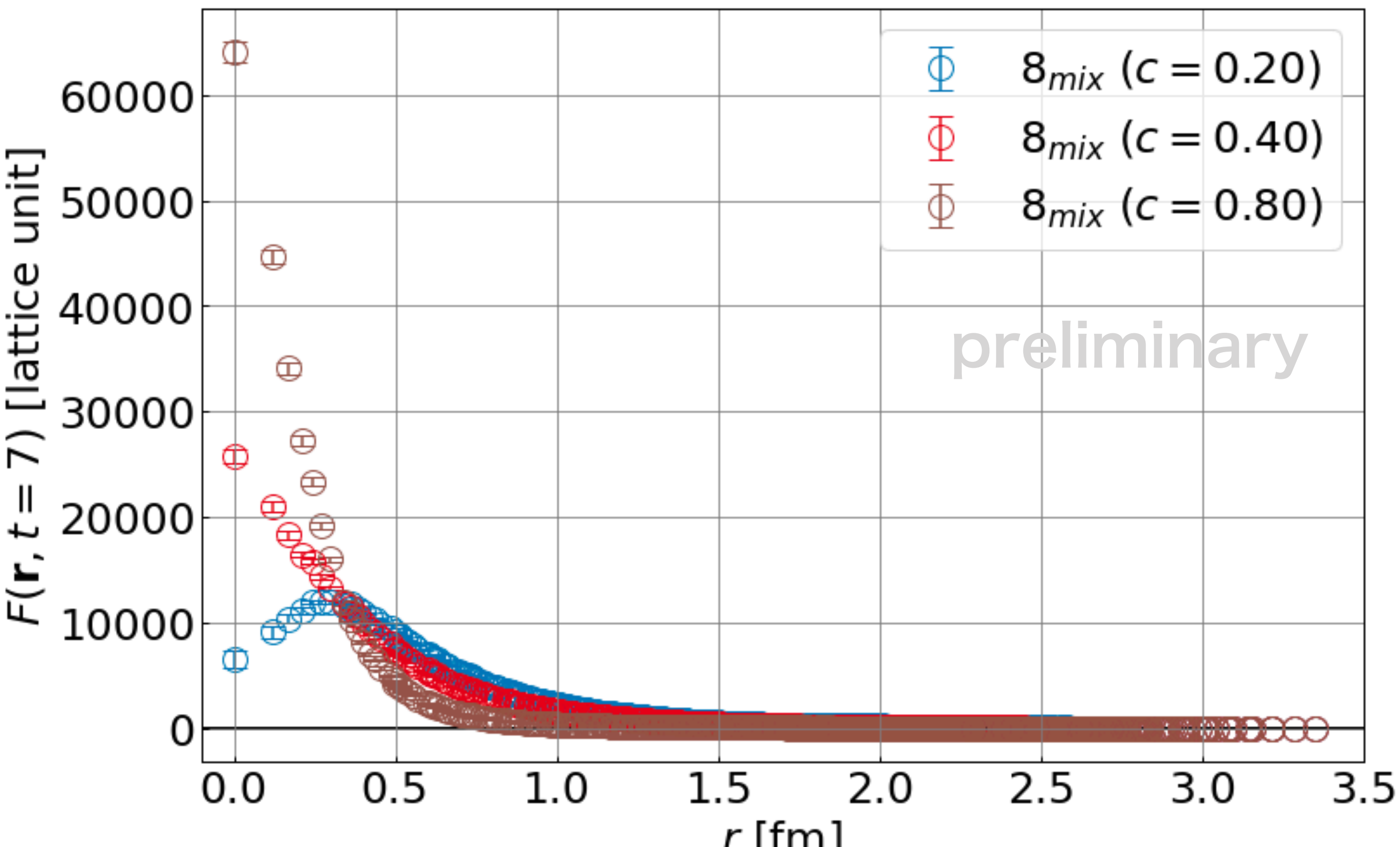
Two meson-baryon operators in the octet channel

- sink operators $(M(\mathbf{r}+\mathbf{x},t)B_\alpha(\mathbf{x},t))_{8_1}$ and $(M(\mathbf{r}+\mathbf{x},t)B_\alpha(\mathbf{x},t))_{8_2}$ cannot be distinguished by the symmetry

➡ we can generate “better” operators by taking linear combination without changing physical observables in principle

$$(M(\mathbf{r}+\mathbf{x},t)B_\alpha(\mathbf{x},t))_{8_1} - c(M(\mathbf{r}+\mathbf{x},t)B_\alpha(\mathbf{x},t))_{8_2}$$

- we set c such that the 3-point function does not cross zero

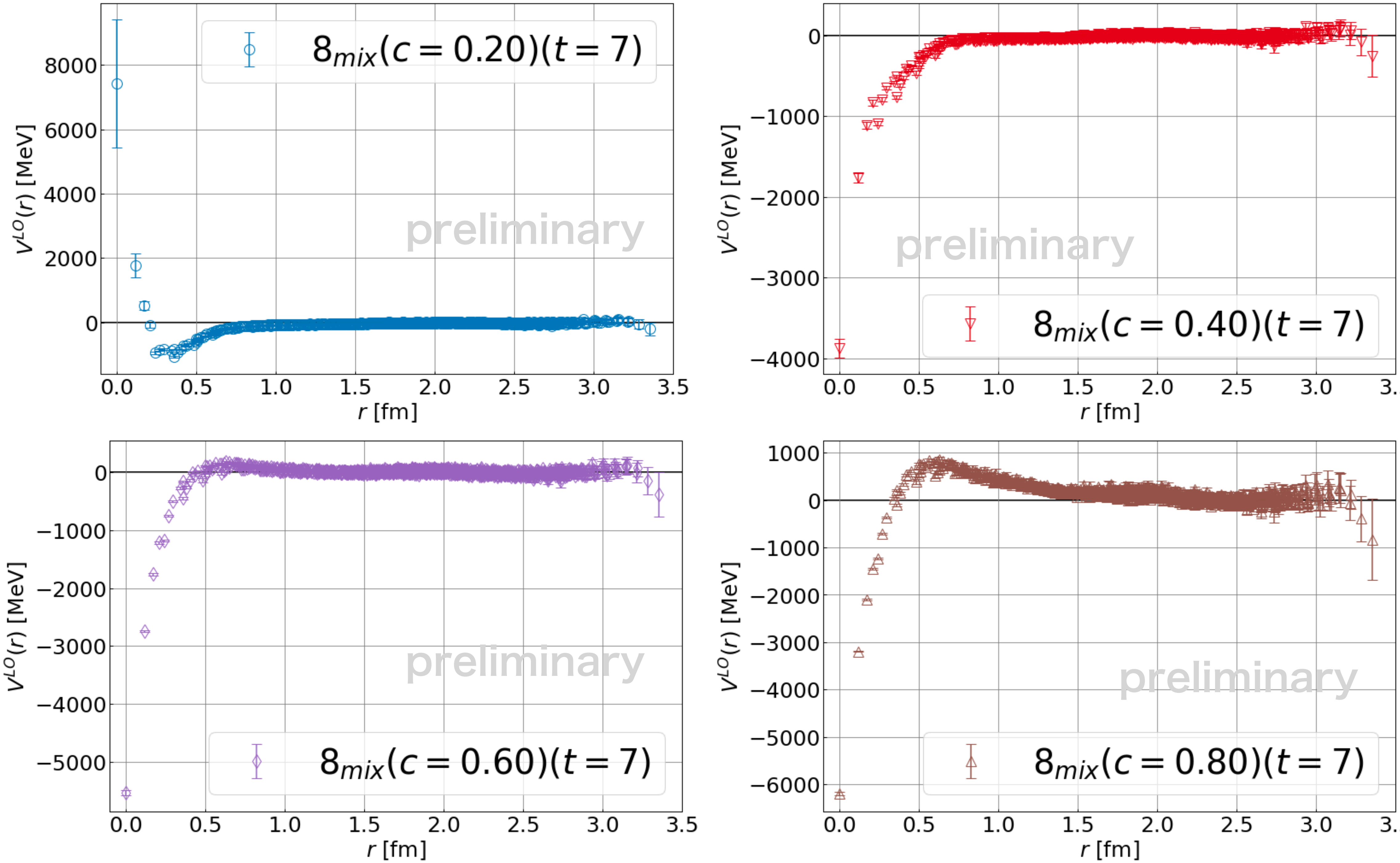


- the zero point disappear for $0.2 \lesssim c \lesssim 0.8$

➡ non-singular potentials

Potentials from the mixed operators

$$(MB)_{8_{\rm mix}} = (MB)_{8_1} - c(MB)_{8_2}$$



- the shape drastically changes for different c

➡ **physical observables?**

c-dependence of the binding energy in octet channel

$$(MB)_{8_{\mathrm{mix}}} = (MB)_{8_1} - c(MB)_{8_2}$$

- binding energy for each c (t=7)

c	0.2	0.25	0.3	0.4	0.6	0.8
E_{bind} [MeV]	182(10)	187(10)	182(11)	174(12)	152(14)	129(14)

$$\Rightarrow \quad E_{\mathrm{bind}}^{(\mathrm{octet})} = 174(12)_{\mathrm{stat}} \begin{pmatrix} +13 \\ -45 \end{pmatrix}_{\mathrm{sys}} \ \mathrm{MeV}$$

from non-locality effect

consistent within error bar

- cf. from 2-point function $\langle \Lambda_{\bar{\alpha}}(t)\bar{\Lambda}_{\bar{\alpha}}(0)\rangle \Rightarrow 156(8)_{\mathrm{stat}}$ MeV

Summary

- we study $\Lambda(1405)$ in flavor SU(3) limit from the meson-baryon scatterings using the HAL QCD method

- 3-point functions with the individual sink operators have zero points, which produce the singular behavior of the potentials

- for the **octet channel,** we can obtain the non-singular potential using the mixed sink operators.

not physical observable

- the potentials with different mixed operators change the shapes, but give similar binding energies

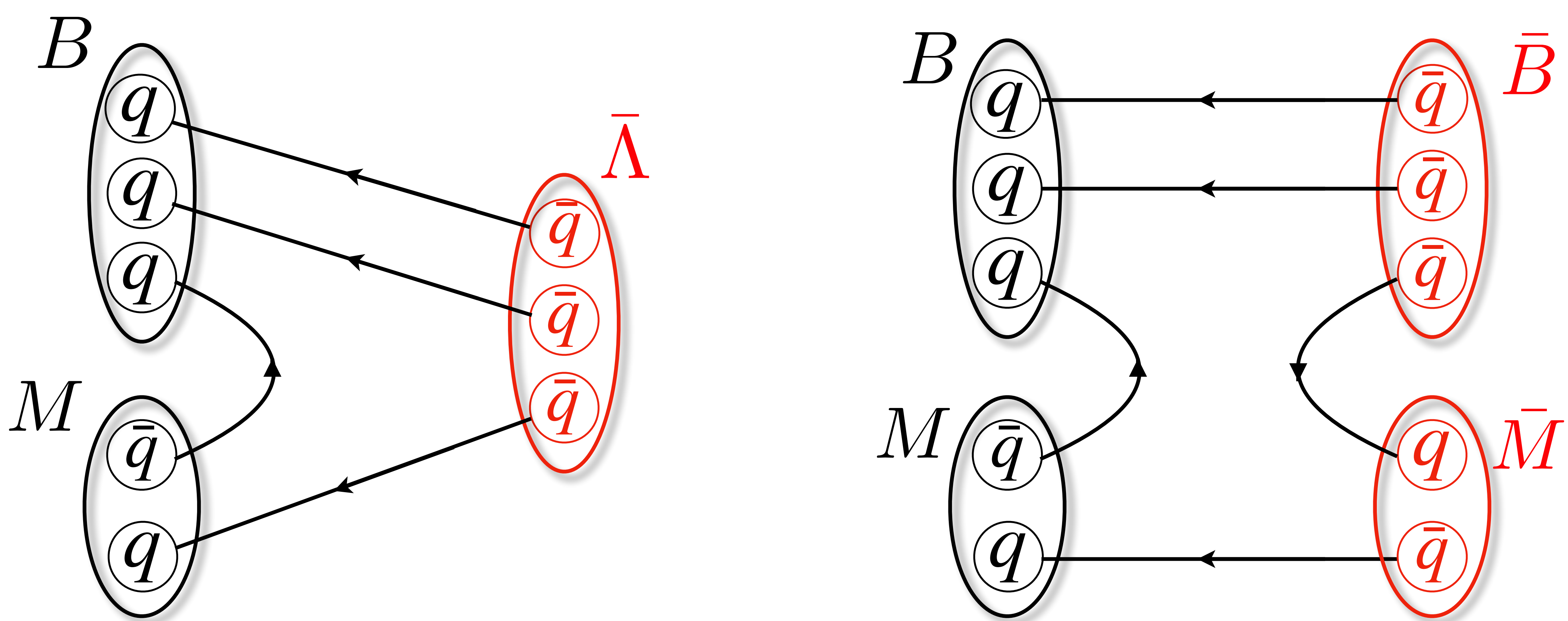
- our results of the binding energy in octet channel:

$$E_{\mathrm{bind}}^{(\mathrm{octet})} = 174(12)_{\mathrm{stat}} \begin{pmatrix} +13 \\ -45 \end{pmatrix}_{\mathrm{sys}} \ \mathrm{MeV}$$

Future works

- **singlet**: get rid of the zero point of 3-point function by using multiple **source operators**

$$\langle (M(\mathbf{r}+\mathbf{x},t)B(\mathbf{x},t))_1 \bar{J}^{\mathrm{mix}}(0)\rangle, \quad \bar{J}^{\mathrm{mix}}(0) = \bar{\Lambda}(0) - c(\bar{M}(0)\bar{B}(0))_1$$



Back up

Lattice QCD and hadron resonances

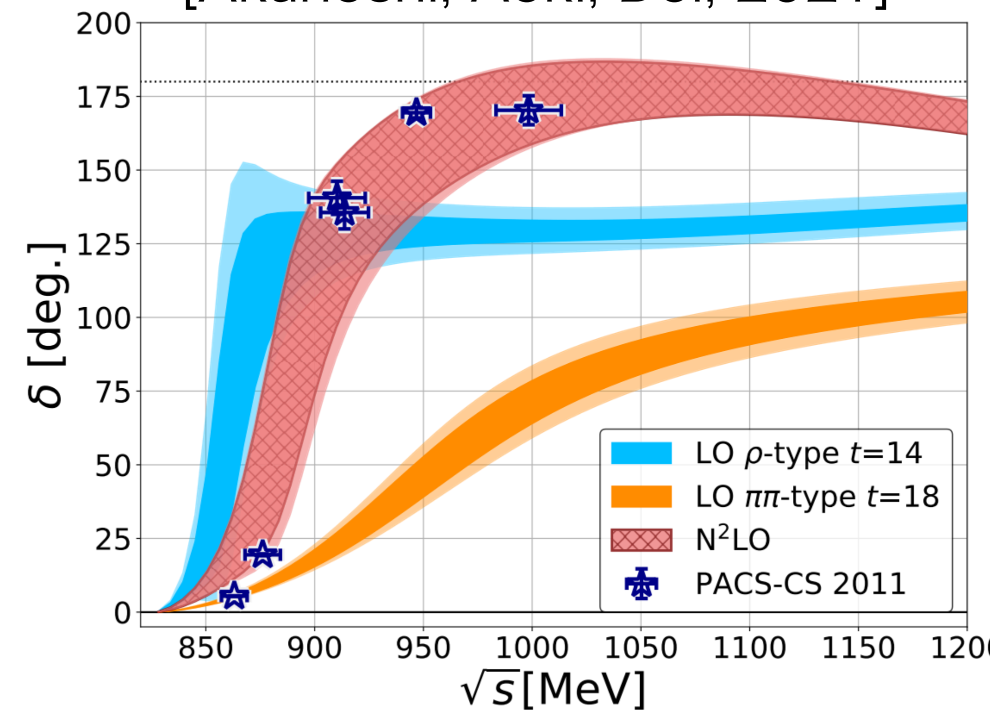
- hadron resonance: pole of S-matrix for **hadron scatterings**

in lattice QCD,

- **Finite-volume method**: use energies in finite volume [Lüscher 1991]
- **HAL QCD method**: extract interaction potentials [Ishii, Aoki, Hatsuda 2007]

- hadron resonances in the HAL QCD method: started **very recently**
  - $\pi\pi \to \rho$ (stable) [Akahoshi et al., 2020]
  - $\pi\pi \to \rho$ (resonance) [Akahoshi, Aoki, Doi, 2021]
    (First successful analysis of resonances in HAL QCD)
  - P-wave $N\pi$, $\Xi\bar{K} \to \Delta$, Ω (stable) [KM, Akahoshi, Aoki, Doi, Sasaki, 2023]
  - $\Lambda(1405)$ in flavor SU(3) limit (Today's talk)

[Akahoshi, Aoki, Doi, 2021]



Hadron scatterings in lattice QCD

- key quantity: Equal-time Nambu-Bethe-Salpeter (NBS) wave function

$$\Psi^W(\mathbf{r}) = \langle 0|O_1(\mathbf{r},0)O_2(\mathbf{0},0)|1,2;W\rangle$$

2-body hadron state with energy W

hadron operators

$$\left(W = \sqrt{k^2+m_1^2}+\sqrt{k^2+m_2^2}\right)$$

$$\begin{cases} \xrightarrow[r\to\infty]{} A\dfrac{\sin(kr-\frac{l}{2}\pi+\delta^l(k))}{kr}e^{i\delta^l(k)}Y_{l,m}(\Omega) & \text{(for scattering states)} \\ \xrightarrow[r\to\infty]{} A'\dfrac{e^{-\kappa_B r}}{r}Y_{l,m}(\Omega) & \text{(for bound states)} \end{cases}$$

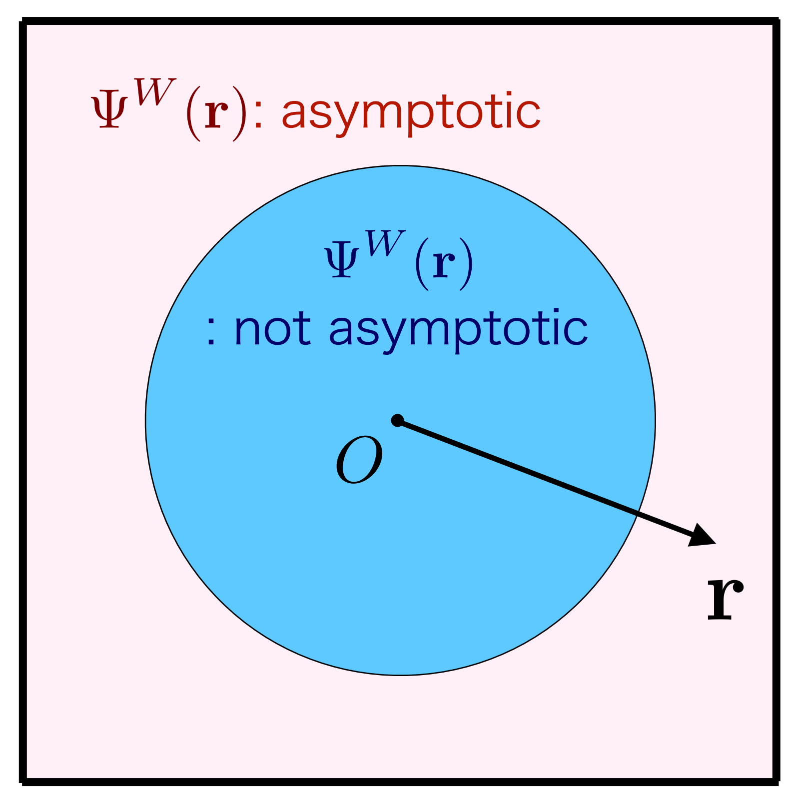
phase shift

[Lin, Martinelli, Sachrajda, Testa, 2001]

$$m_B = \sqrt{-\kappa_B^2+m_1^2}+\sqrt{-\kappa_B^2+m_2^2}$$

[Gongyo Aoki, 2018]

- methods based on NBS wave function:
  - **Finite-volume method**
    : use periodic boundary condition in finite-volume system
  - **HAL QCD method**
    : extract potentials from NBS functions



HAL QCD method

- from the previous discussion,

$$(W = \sqrt{k^2 + m_1^2} + \sqrt{k^2 + m_2^2})$$

$$\left(\frac{k^2}{2\mu} + \frac{\nabla^2}{2\mu}\right)\Psi^W(\mathbf{r}) \underset{r\to\infty}{\to} 0 \qquad (\mu = \frac{m_1 m_2}{m_1 + m_2})$$

➡ we extract **interaction potential** $U(\mathbf{r}, \mathbf{r}')$ for finite r

[Ishii, Aoki, Hatsuda 2007]

$$\int d^3r' \; U(\mathbf{r}, \mathbf{r}')\Psi^W(\mathbf{r}') = \left(\frac{k^2}{2\mu} + \frac{\nabla^2}{2\mu}\right)\Psi^W(\mathbf{r})$$

non-local but independent of energy

- Naive way in lattice QCD: use n**-point function** $F(t, \mathbf{r})$ **(**$n \geq 3$**)**

$$F(t, \mathbf{r}) = \langle 0 | \overbrace{O_1(\mathbf{r}, t) O_2(\mathbf{0}, t) \; \underline{\bar{J}(0)}}^{\mathbf{1} = \sum_n |n\rangle\langle n|} |0\rangle$$

operator to generate $|1, 2; W\rangle$ (source operator)

$$= \sum_n \underbrace{\langle 0 | O_1(\mathbf{r}, 0) O_2(\mathbf{0}, 0) | 1, 2; W_n\rangle}_{= \Psi^{W_n}(\mathbf{r})} \langle 1, 2; W_n | \bar{J}(0) | 0\rangle e^{-W_n t} + \ldots$$

$$\underset{t\to\infty}{\to} \Psi^{W_0}(\mathbf{r}) \langle 1, 2; W_0 | \bar{J}(0) | 0\rangle \; e^{-W_0 t}$$

- small $W_1 - W_0$
- exponential growth of gauge fluctuation

difficult when we consider baryons

[Iritani et al. 2016]

Time-dependent HAL QCD method

[Ishii et al. 2011]

- R-correlator (For simplicity, $m_1 = m_2 = m$)

$$R(t,\mathbf{r}) = \frac{F(t,\mathbf{r})}{C_1(t)C_2(t)} \simeq \sum_n A_n\ \Psi^{W_n}(\mathbf{r})\ e^{-\underbrace{(W_n - 2m)}_{=\Delta W_n}t} + \ldots$$

2-point function

- each term satisfies the Schrödinger equation

$$\int d^3r'\ U(\mathbf{r},\mathbf{r}')\ A_n \Psi^{W_n}(\mathbf{r})\ e^{-\Delta W_n t} = \left(\frac{k_n^2}{2\mu} + \frac{\nabla^2}{2\mu}\right) A_n \Psi^{W_n}(\mathbf{r})\ e^{-\Delta W_n t}$$

$\approx V^{LO}(r)\delta^{(3)}(\mathbf{r}-\mathbf{r}')$

(leading-order approximation)

$= \Delta W_n + \frac{1}{8\mu}\Delta W_n^2 = -\frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}$

$\times \sum_n$

$$\boxed{V^{LO}(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(-\frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2} + \frac{\nabla^2}{2\mu}\right)R(\mathbf{r},t)}$$

- no need to pick up only ground state ➡ **applicable to baryons**

Details of the setups

$$(MB_\alpha)_{8_1}^{S=-1,I=0} = \frac{\sqrt{10}}{10}(K\Xi_\alpha)^{I=0} - \frac{\sqrt{10}}{10}(\bar{K}N_\alpha)^{I=0} - \frac{\sqrt{15}}{5}(\pi\Sigma_\alpha)^{I=0} - \frac{\sqrt{5}}{5}\eta^8\Lambda_\alpha^8$$

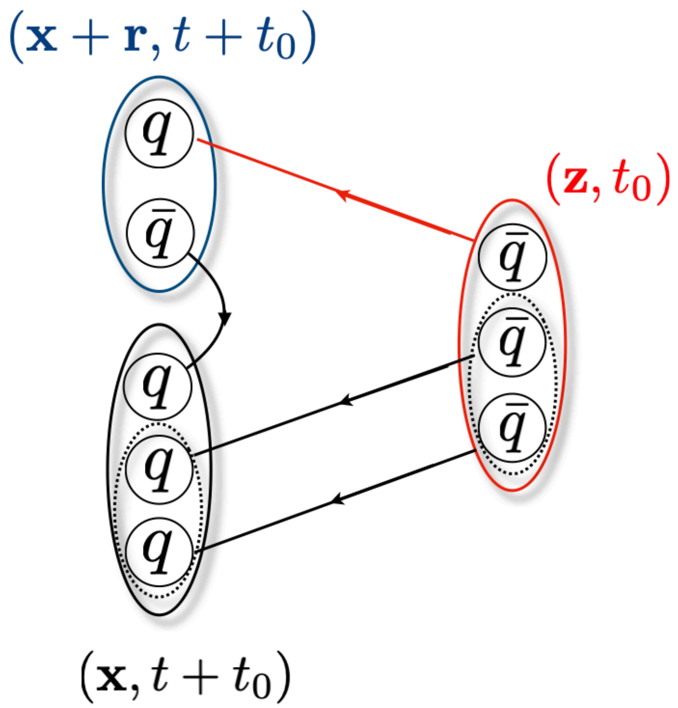
$$(MB_\alpha)_{8_2}^{S=-1,I=0} = \frac{\sqrt{2}}{2}(K\Xi_\alpha)^{I=0} + \frac{\sqrt{2}}{2}(\bar{K}N_\alpha)^{I=0}$$

$$(MB_\alpha)_{1}^{S=-1,I=0} = \frac{1}{2}(K\Xi_\alpha)^{I=0} - \frac{1}{2}(\bar{K}N_\alpha)^{I=0} + \frac{\sqrt{6}}{4}(\pi\Sigma_\alpha)^{I=0} - \frac{\sqrt{2}}{4}\eta^8\Lambda_\alpha^8$$

- conf: Inoue conf. (Coulomb gauge fixed)
- $a \approx 0.121$ fm, Size = 32^4, $\kappa_u = \kappa_s = 0.1380$ (SU(3) limit)
- timeslice = 32, #AMA = 64
- dilution: time, color, spinor, s4 [Akahoshi et al. 2019]
- smeared source, smeared sink (range=0.7)

Quark contractions

(1)



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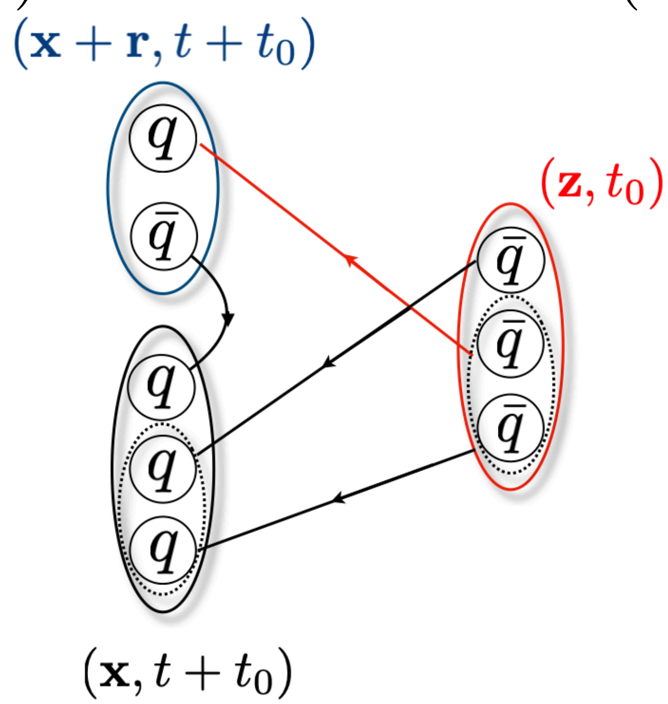

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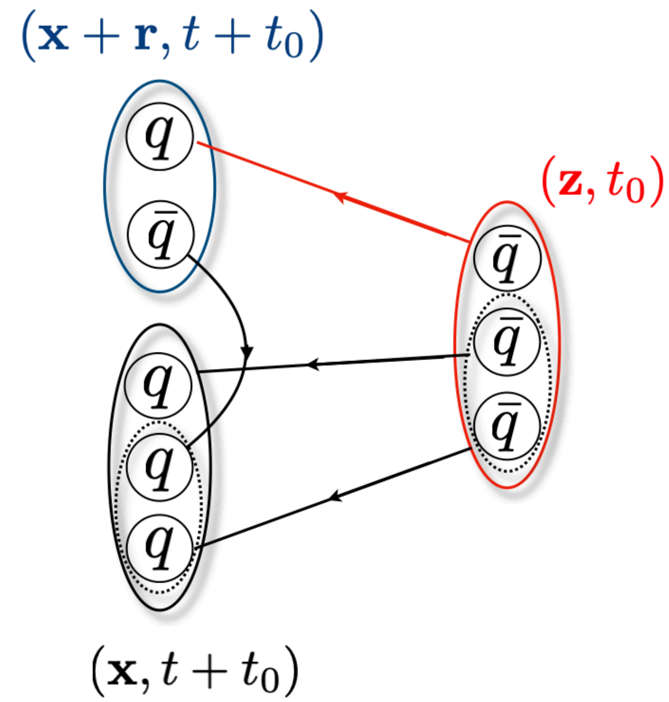

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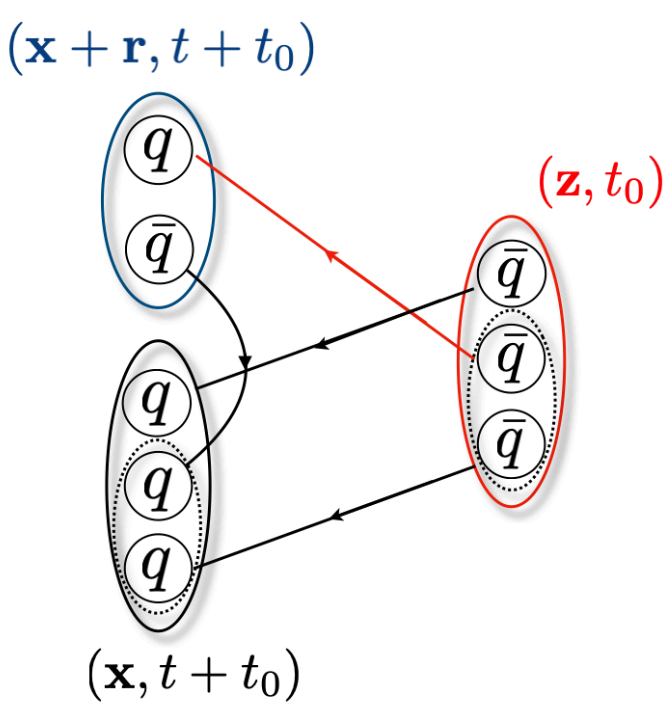
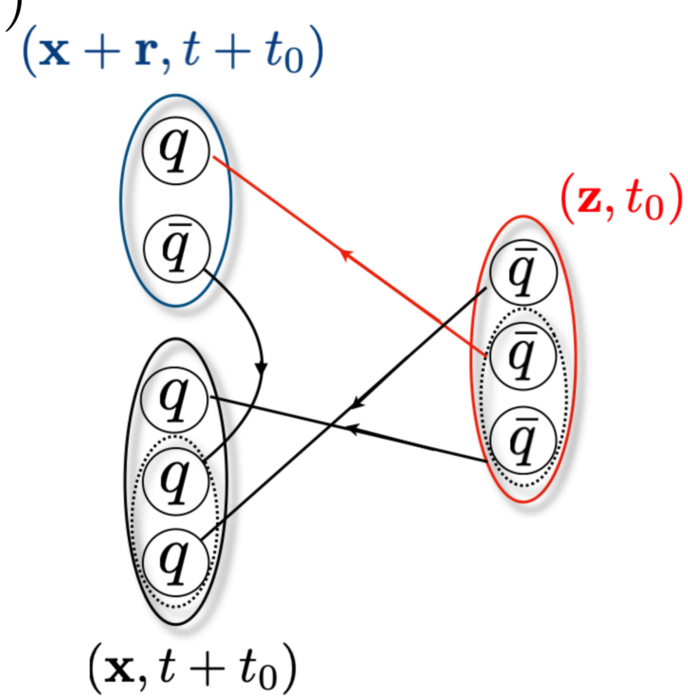


(5)



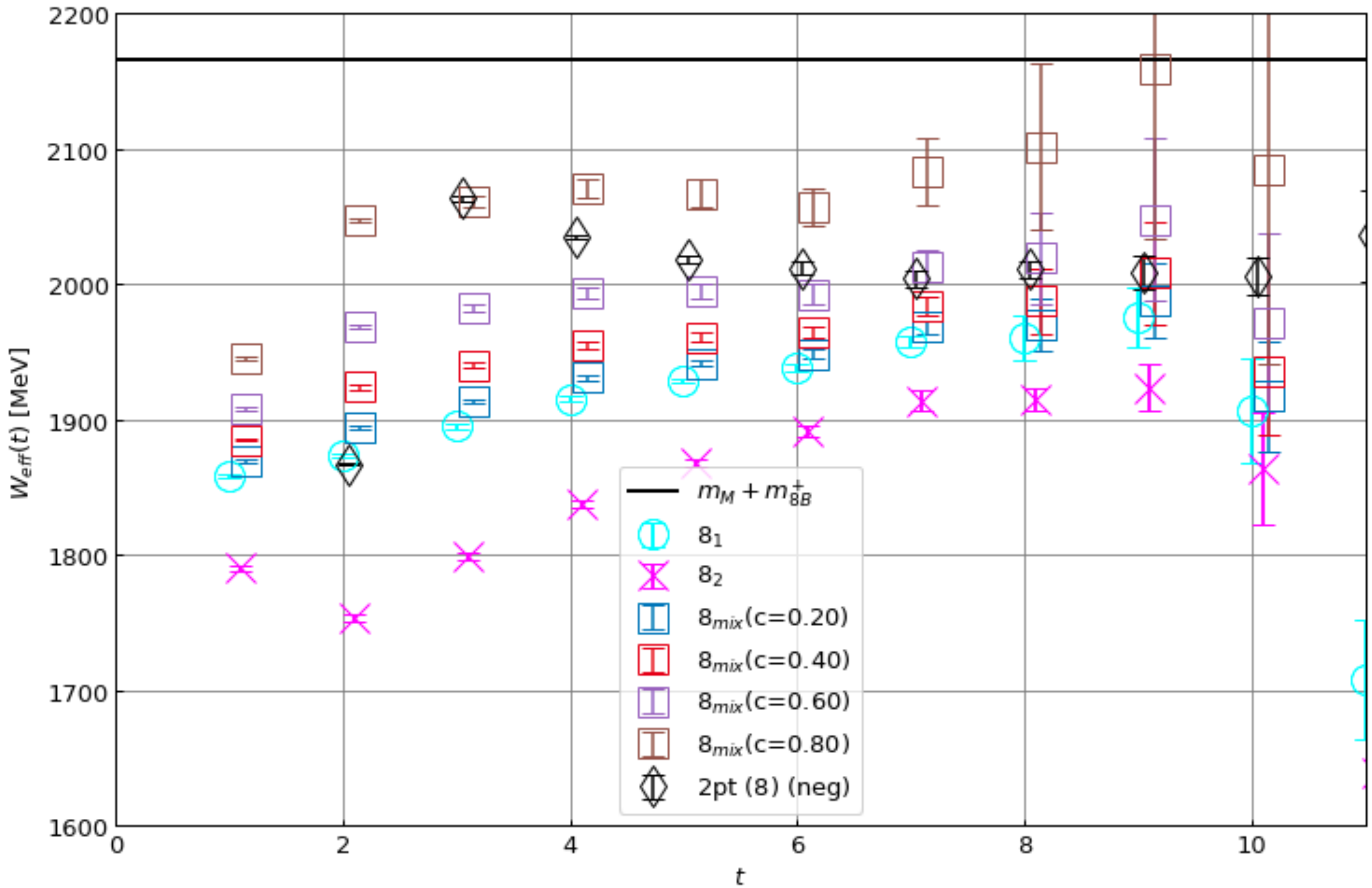
$$F_\alpha^{8_1}(\mathbf{r}, t) = -\sqrt{10}[(1) + (2) + (3) - (4) - 2(5)]$$

$$F_\alpha^{8_2}(\mathbf{r}, t) = -\frac{\sqrt{6}}{2}[(1) + (2) + (3) + (4)]$$

$$F_\alpha^{1}(\mathbf{r}, t) = -\frac{4}{\sqrt{3}}[(1) - 2(2) + (3) - (4) + (5)]$$

time dep. of 3pt functions

$$(MB)_{8_{mix}} = (MB)_{8_1} - c(MB)_{8_2}$$

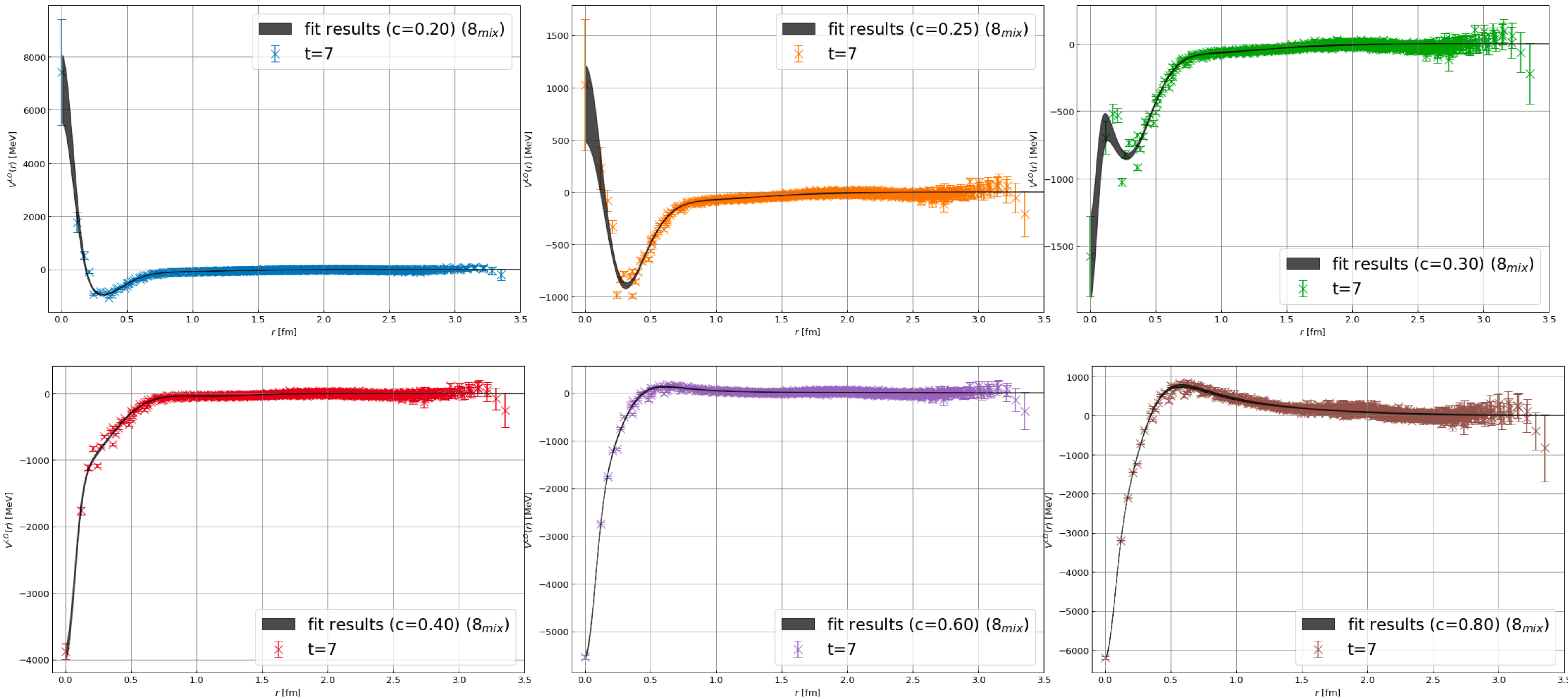


- get close to 2pt results except for $c = 0.8$

fitting results (t=7)

$$(MB)_{8_{mix}} = (MB)_{8_1} - c(MB)_{8_2}$$

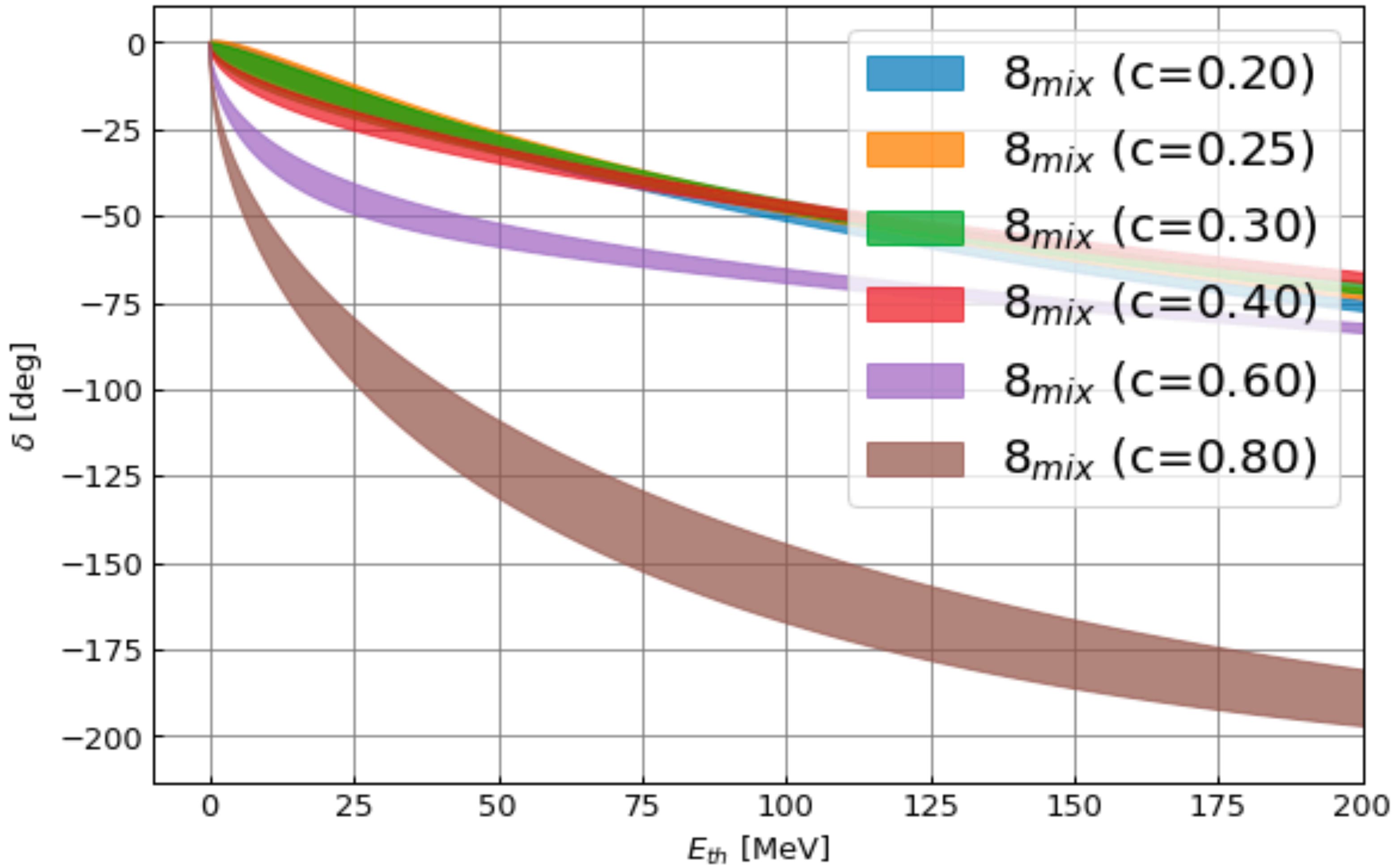
fit function: 5 Gaussians (c=0.2, 0.3)
4 Gaussians (others)



- fitting works well

phase shifts (t=7)

$$(MB)_{8_{mix}} = (MB)_{8_1} - c(MB)_{8_2}$$



- not drastically changed except for c=0.6 and 0.8