Nucleon Electromagnetic Polarizabilities from Four-point Correlation Functions

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August 4th, 2023

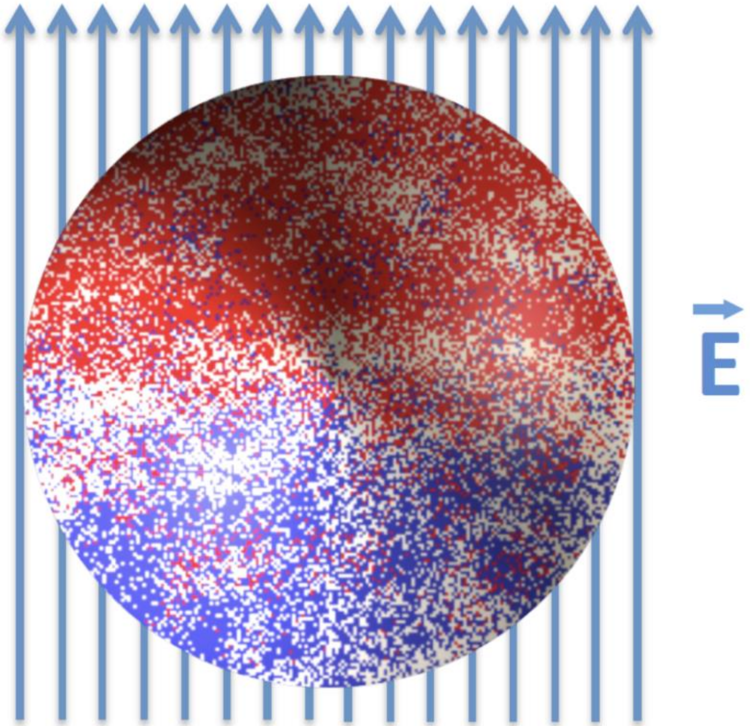
Motivations

- Polarizabilities are fundamental parameters of hadron structure

- Characterize the second-order response of a proton to an EM field,

$$H_{eff}^{(2)} = -\frac{4\pi}{2}\alpha_E E^2 - \frac{4\pi}{2}\beta_M B^2$$

- Our Method: calculating a proton 4-point correlation function on lattice, can be used in multiple applications[1, 2].

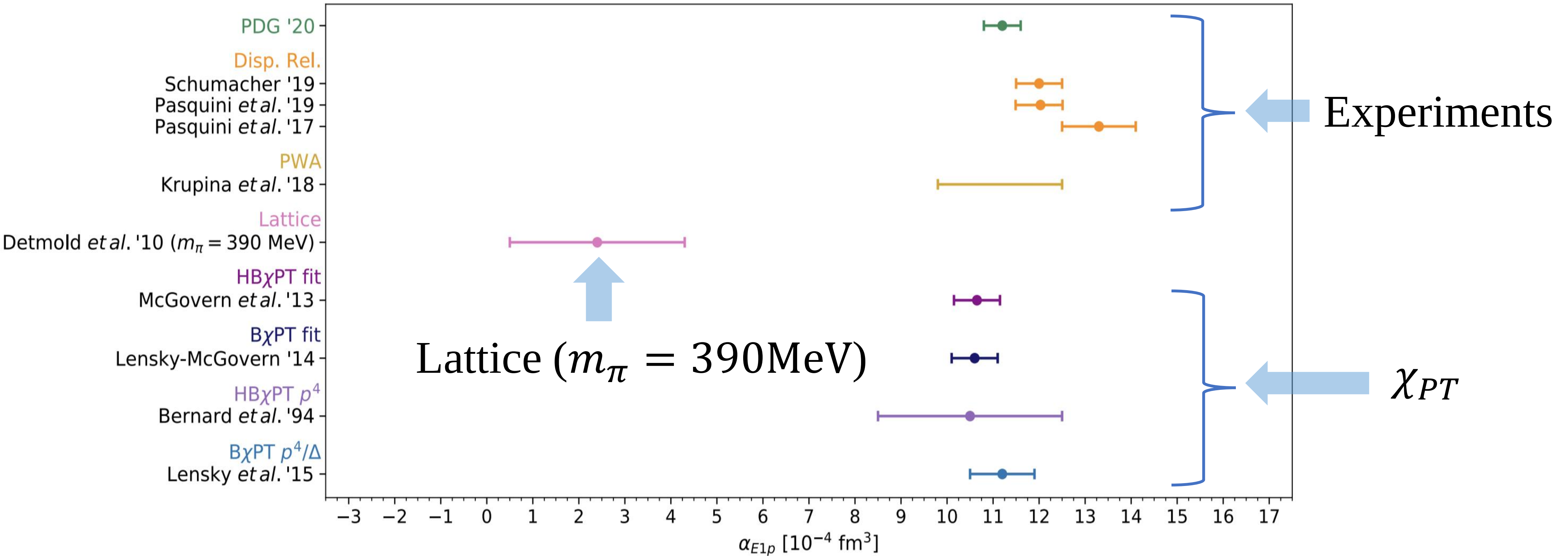


[1] Y Fu et al. Lattice QCD Calculation of the Two-Photon Exchange Contribution to the Muonic-Hydrogen Lamb Shift[J]. 2022

[2] P X Ma et al. Lattice Calculation of Electroweak Radiative Corrections of Neutron Beta Decays, arXiv: 2308.xxxx

Recent Researches on Polarizabilities

- Summary for electric polarizability α_E of the proton[1]



Only one former Lattice result for proton α_E^p

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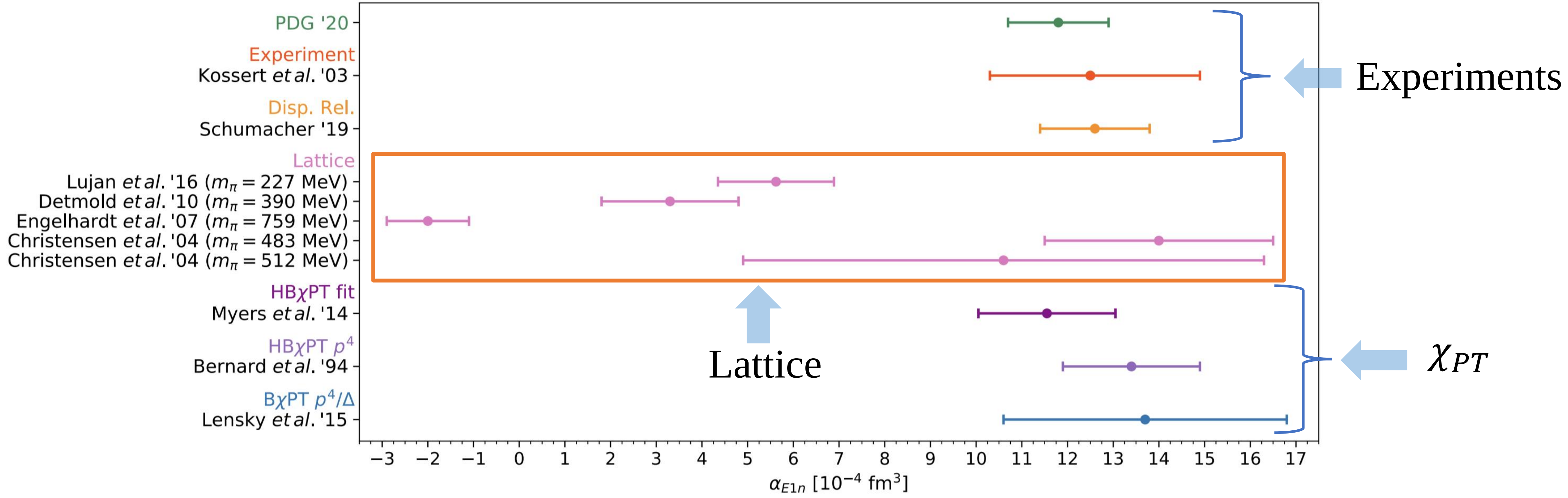
[1]F Hagelstein, Nucleon Polarizabilities and Compton Scattering as Playground for Chiral Perturbation Theory[J]. 2020

Recent Researches on Polarizabilities

- Summary for electric polarizability α_E of the neutron[1]

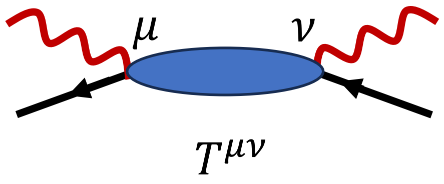


Lattice results seems lower than other?

- Calculate on physical pion mass
- Calculate with new method (4-point function)

[1]F Hagelstein, Nucleon Polarizabilities and Compton Scattering as Playground for Chiral Perturbation Theory[J]. 2020

Doubly Virtual Compton Scattering



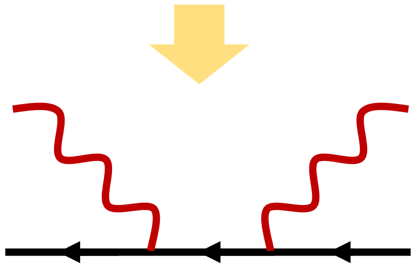
- Unpolarizaed doubly virtual Compton scattering

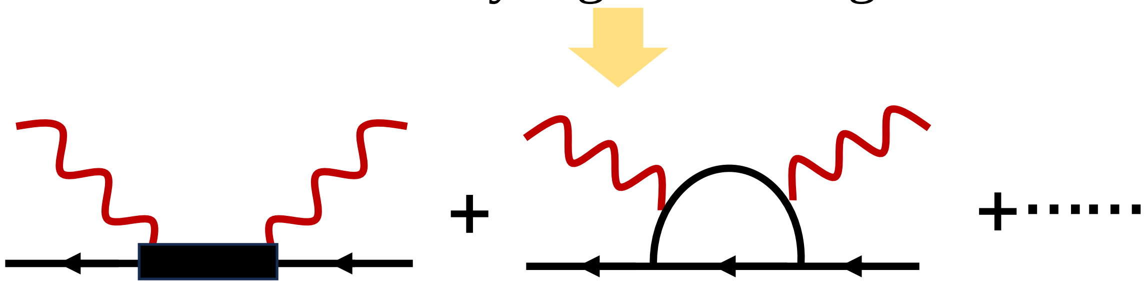
$$T^{\mu\nu}(P,q) = T^{\mu\nu}_{Born} + \frac{8\pi M}{e^2}\left[-\beta_M K_1^{\mu\nu} + (\alpha_E + \beta_M) K_2^{\mu\nu}\right]$$

$$K_1^{\mu\nu} = q^\mu q^\nu - g^{\mu\nu} q^2,\ K_2^{\mu\nu} = \frac{1}{M^2}\left[(P^\mu q^\nu + P^\nu q^\mu) P\cdot q - g^{\mu\nu}(P\cdot q)^2 - P^\mu P^\nu q^2\right]$$

- Born term:
  Thomson scattering

- Polarizability terms:
  Rayleigh scattering



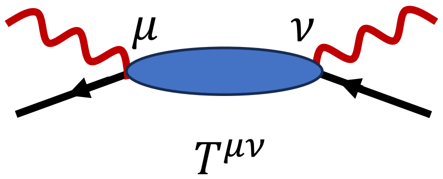


Intermeidate states: N

Intermeidate states: $N^*, N\pi, \Delta, \cdots$

- Polarizabilities: pure excited states contribution.

Extraction from 4-point Function



- Compton tensor

Lattice QCD input

$$T^{\mu\nu} = \int d^4x\, e^{iqx} \langle N|J^\mu(x,t)J^\nu(0)|N\rangle = T^{\mu\nu}_{Born} + \frac{8\pi M}{e^2}\left[-\beta_M K_1^{\mu\nu} + (\alpha_E + \beta_M)K_2^{\mu\nu}\right]$$

- With nucleon momentum $P = (M, 0)$ and photon momentum $q = (q_0, 0)$:

$$\alpha_E = \frac{1}{3}\left(\frac{\partial T^{ii}}{\partial q_0^2} - \frac{\partial T^{ii}_{Born}}{\partial q_0^2}\right)\Bigg|_{q_0\to 0}$$

$$= \frac{e^2}{4\pi}\left(\frac{1+\kappa^2}{4M^3} + \frac{\langle r_E^2\rangle}{3M}\right) + \frac{e^2}{4\pi}\int_{|t|<t_s} d^4x \left(-\frac{t^2}{12M}\right) H_{ii}(\mathbf{x}, t)$$

magnetic moment

charge radius

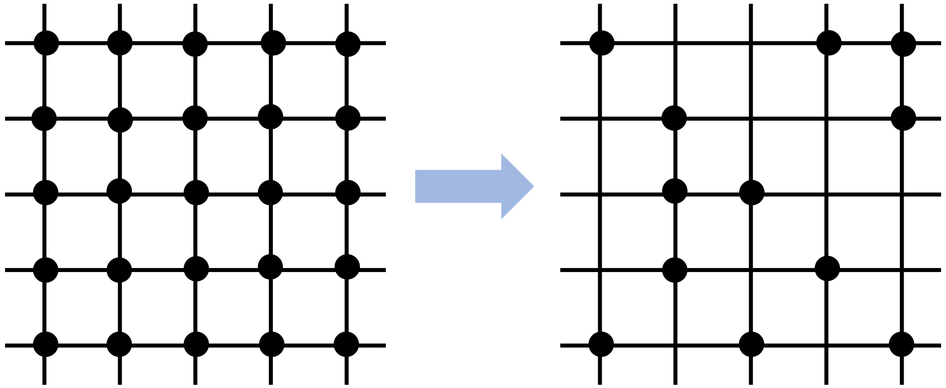
$H_{ii}(x,t) = \langle N|J^i(x,t)J^i(0)|N\rangle$

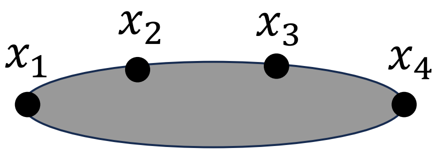
➢ For neutron： $\alpha_E^{(n)} = \frac{e^2}{4\pi}\frac{\kappa_{(n)}^2}{4M^3} + \frac{e^2}{4\pi}\int_{|t|<t_s} d^4x \left(-\frac{t^2}{12M}\right) H_{ii}^{(n)}(\mathbf{x}, t)$

Ensemble for Calculation

Ensembles	m_π[MeV]	m_p[MeV]	L/a	T/a	a[fm]	N_{conf}
24D	141.7(2)	935(5)	24	64	0.1944	207

- Domain Wall Fermion ensemble generated by RBC/UKQCD[1].
- Random field selection method[2,3] is used.





4-points: $\sum_{\{x_1,x_2,x_3\}} \sim L^9$

Lattice data: highly correlated

1000 times less points yields similar precision

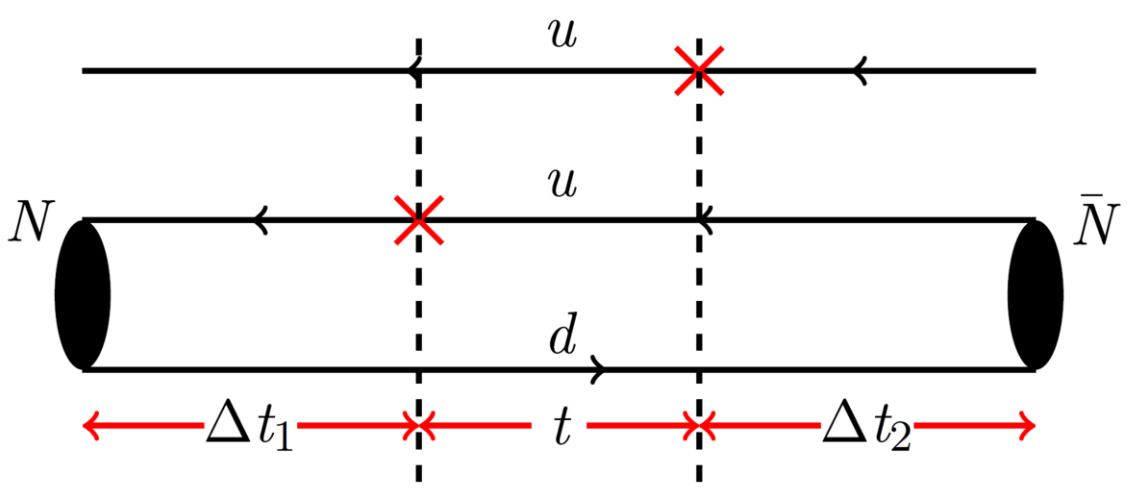
[1] Blum T et al. Domain wall QCD with physical quark masses[J]. Physical Review D, 2016, 93(7):074505.
[2] Y Li et al. Field sparsening for the construction of correlation functions in lattice QCD[J]. 2021.
[3] W Detmold et al. Sparsening Algorithm for Multihadron Lattice QCD Correlation Function[J]. 2021.

4-point Function Calculated on Lattice

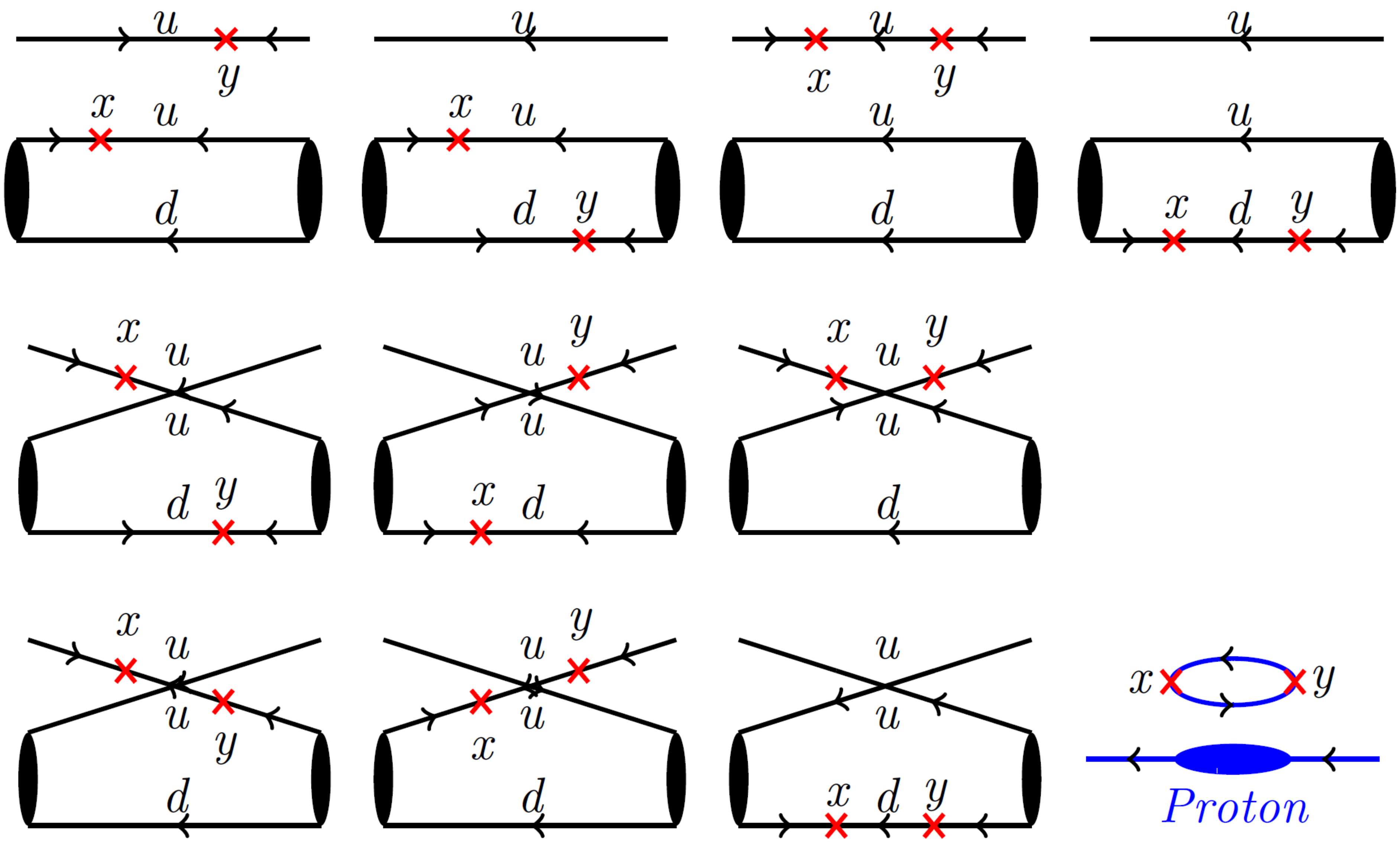
- Compton tensor extracted from nucleon 4-point function

$$\langle p|J^{\mu}(x)J^{\nu}(0)|p\rangle \Rightarrow \langle N(t+\Delta t_1)J^{\mu}(t,\mathbf{x})J^{\nu}(0)\bar{N}(-\Delta t_2)\rangle$$

- Two nucleon operators and two vector current operators placed on different time slices, separated by Δt and t.



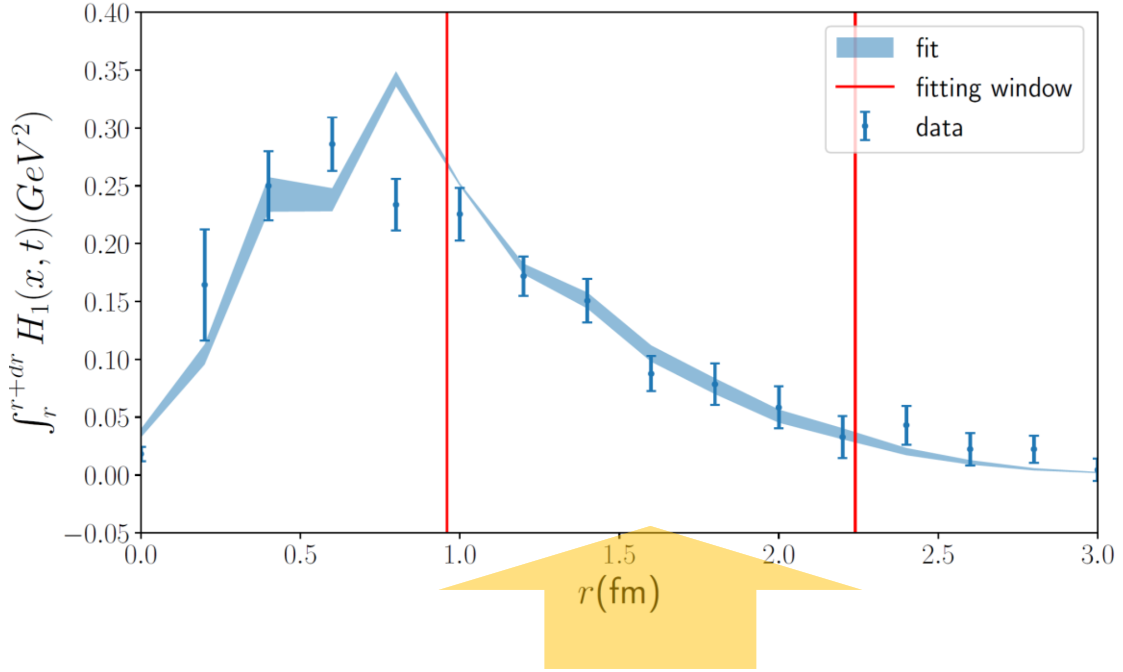
Feynman Diagrams in 4-point Function

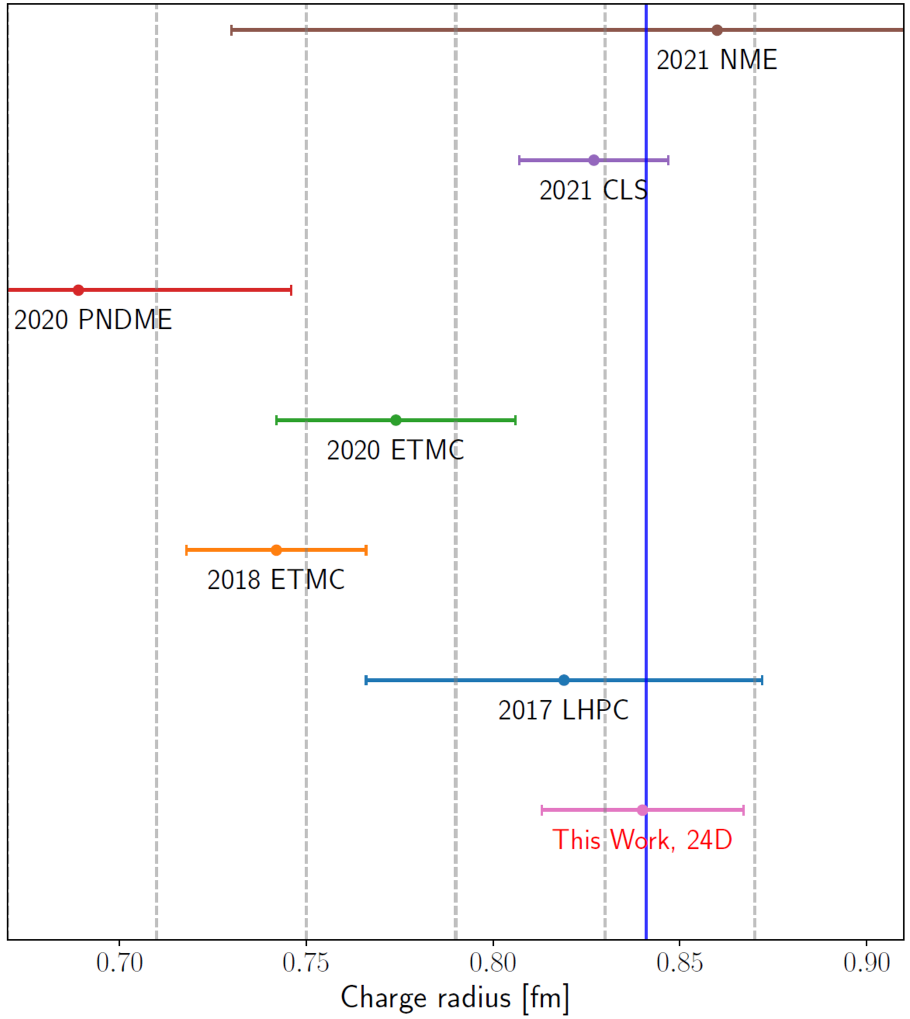


Examination of 4-point Function: Charge Radius

$$\langle N|J^0(x,t)J^0(0)|N\rangle \xrightarrow{\text{long distance}} \int \frac{d^3Q}{(2\pi)^3}\frac{M(E+M)}{E} G_E^2(Q^2)e^{ipx}e^{-(E-M)t}$$

- $\langle N|J^0(x,t)J^0(0)|N\rangle$ as a function of $|x|$





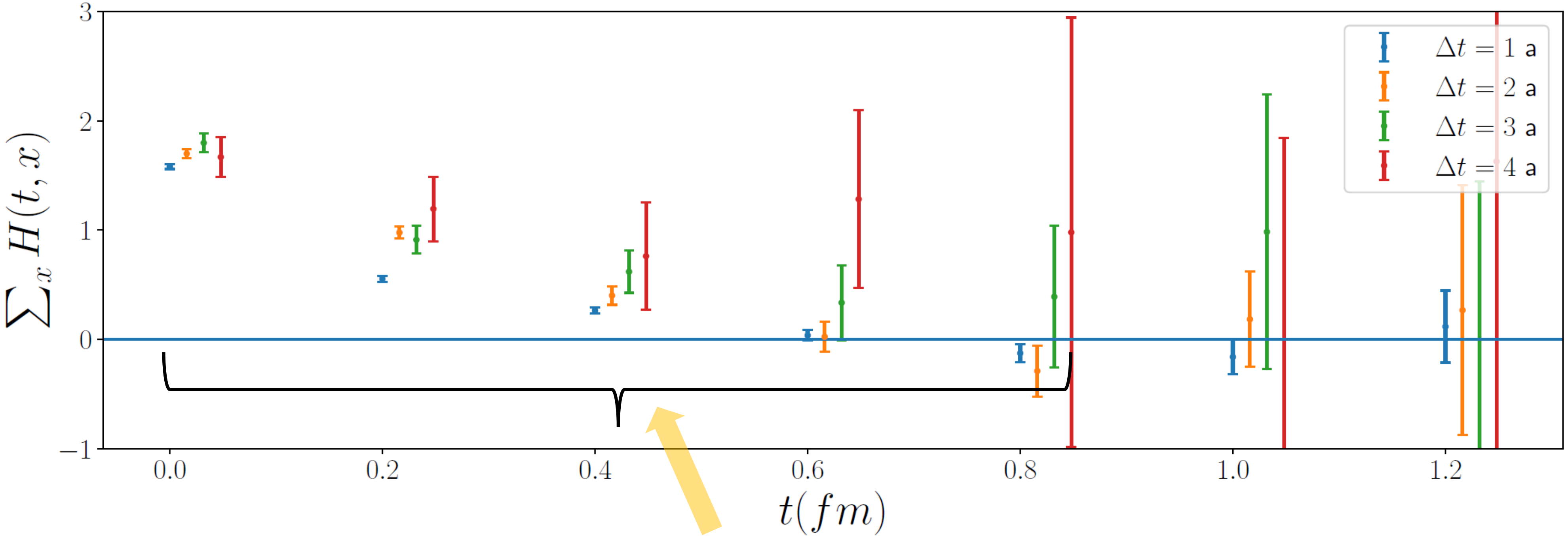
Charge radius fitted at long distance $r > 1fm$, with dipole model:

$$G_E(Q^2) = 1/\left(1 + Q^2\langle r_E^2\rangle/12\right)^2$$

Signal of 4-point Function i-i Component： $H_{ii}(t,x)$

$$\alpha_E = \frac{e^2}{4\pi}\left(\frac{1+\kappa^2}{4M^3} + \frac{\langle r_E^2 \rangle}{3M}\right) + \frac{e^2}{4\pi}\int_{|t|<t_s} d^4x \left(-\frac{t^2}{12M}\right) H_{ii}(\mathbf{x}, t)$$

- $\sum_x H(x,t)$ as a function of time separation t



Hadronic function mainly contribute in the region of $t < 0.8$ fm

Polarizability α_E from $H_{ii}(t,x)$

$$\alpha_E = \frac{e^2}{4\pi}\left(\frac{1+\kappa^2}{4M^3} + \frac{\langle r_E^2\rangle}{3M}\right) + \frac{e^2}{4\pi}\int_{|t|<t_s} d^4x\left(-\frac{t^2}{12M}\right)H_{ii}(\mathbf{x},t)$$

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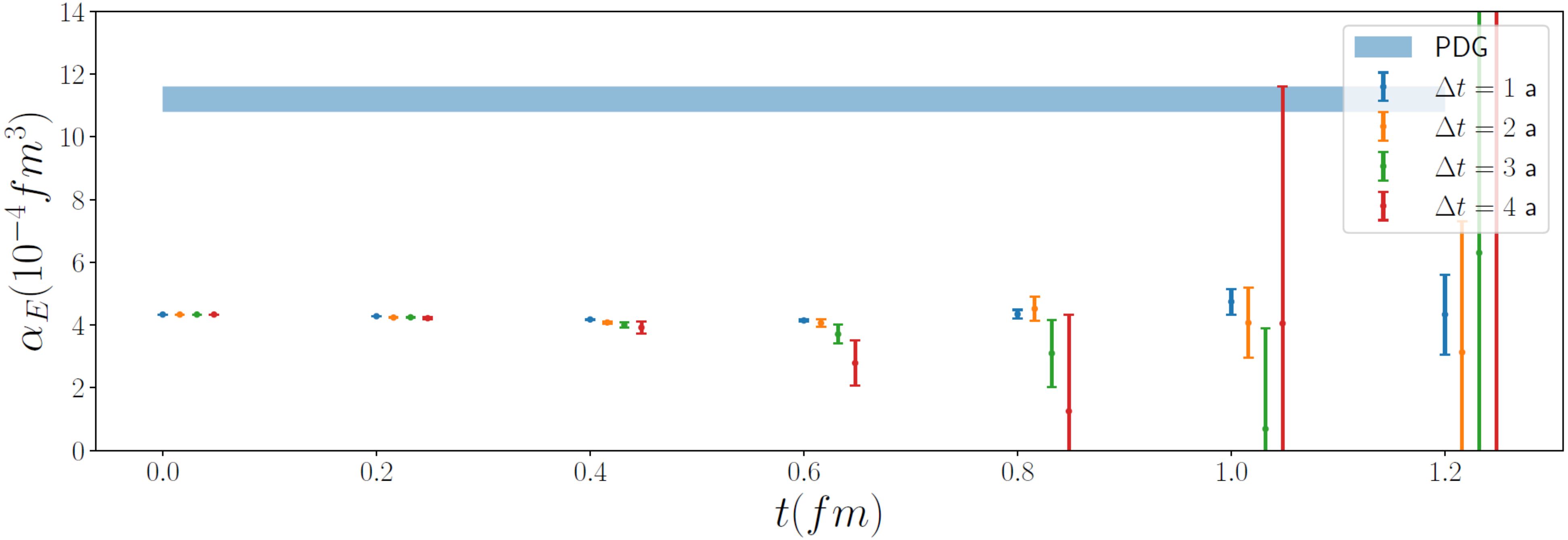
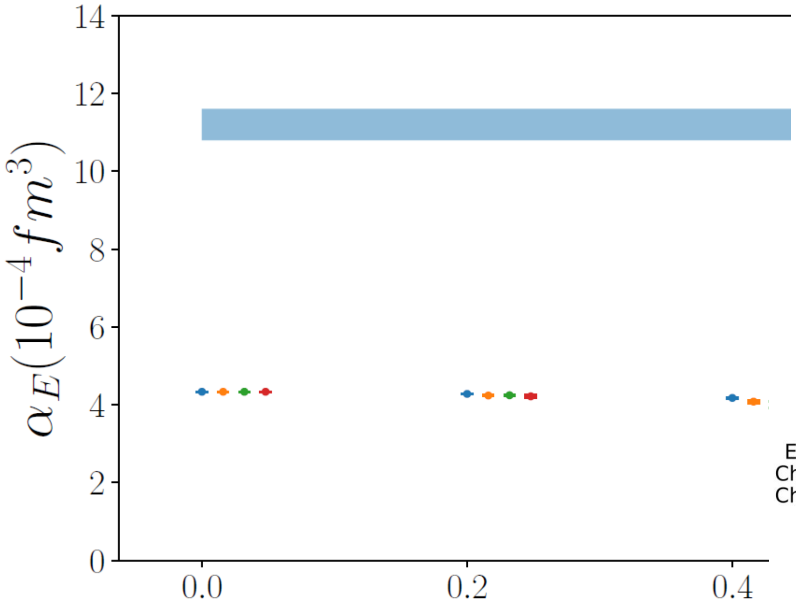


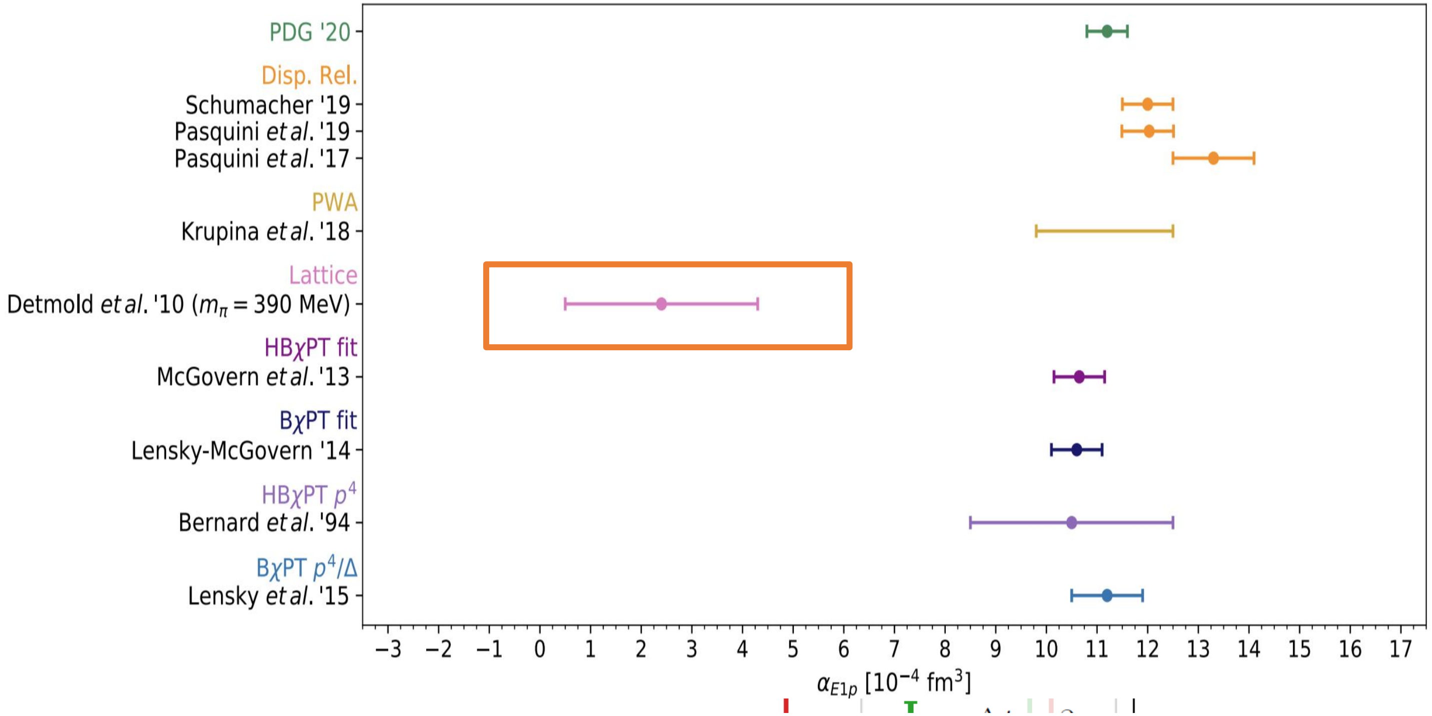
However, lattice predictions are significantly below the PDG value. Why?

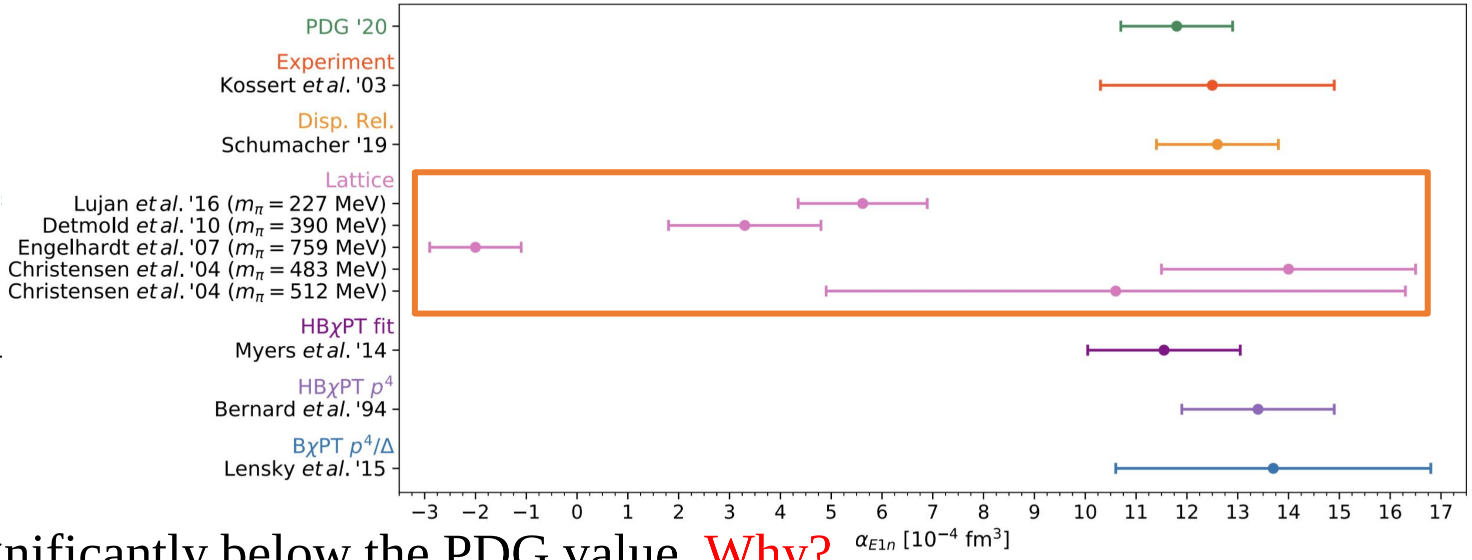
Polarizability α_E fron

$$\alpha_E = \frac{e^2}{4\pi}\left(\frac{1+\kappa^2}{4M^3} + \frac{\langle r_E^2\rangle}{3M}\right) + \frac{e^2}{4\pi}\int_{|t|<}$$

- $\sum_x H(x,t)$ as a function of time sepa





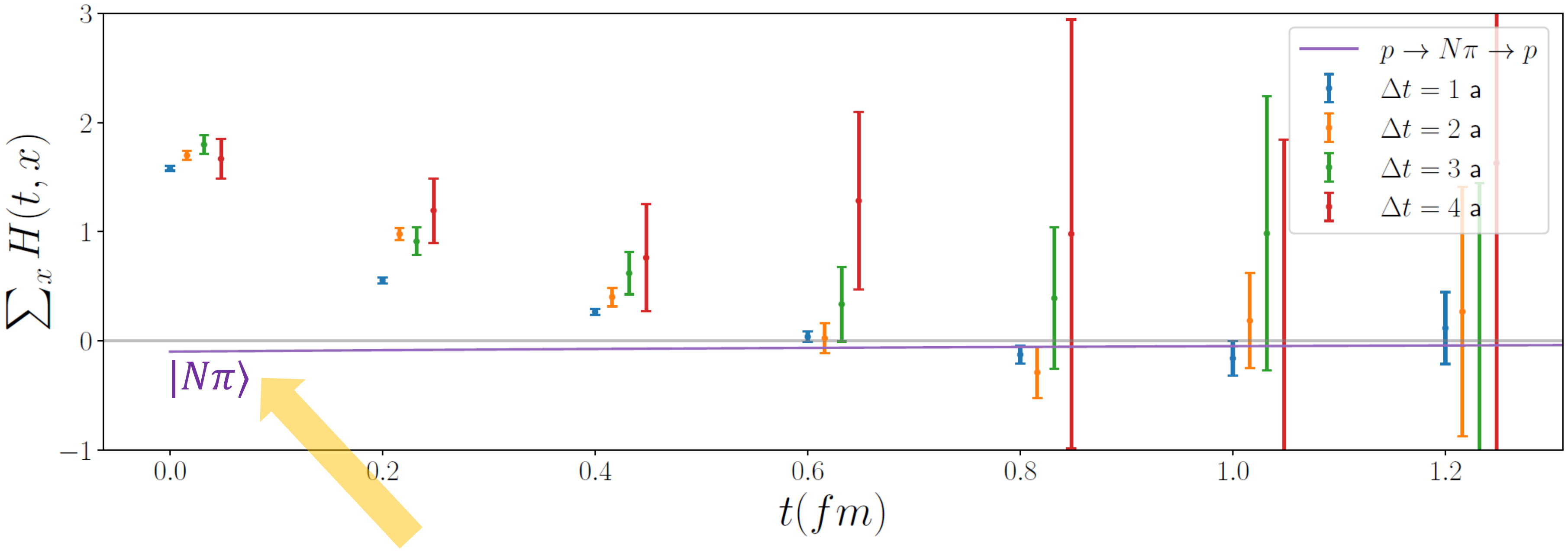


However, lattice predictions are significantly below the PDG value. Why?

Nucleon polarizabilities and $N\pi$ scattering

Structure of hadronic function $$\int d^4x \left(-\frac{t^2}{6}\right) H_{ii}(x,t) = \int dt \left(-\frac{t^2}{6}\right) \sum_k \langle p|J_i(0)|k\rangle e^{-(E_k-M)t} \langle k|J_i(0)|p\rangle$$
$$= -\frac{2}{3}\sum_k \frac{\langle p|J_i(0)|k\rangle\langle k|J_i(0)|p\rangle}{(E_k - M)^3}$$

The dominant contribution is given by $|k\rangle = |N\pi\rangle$ ground intermediate states



$N\pi$ contribution seems negligible and is completely hidden by noise at long distance

Nucleon polarizabilities and $N\pi$ scattering

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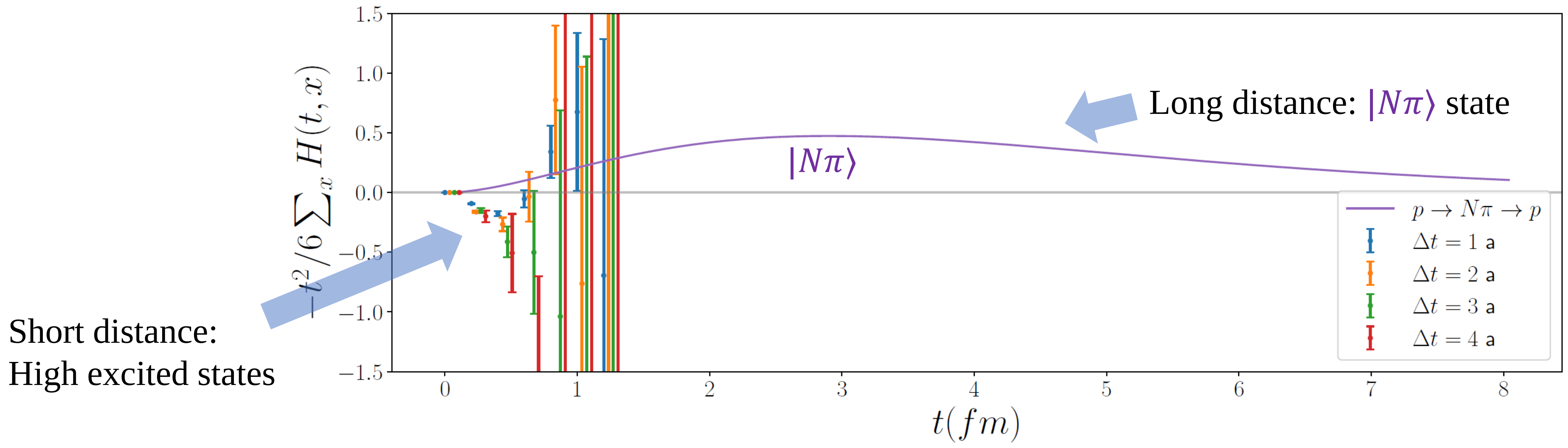
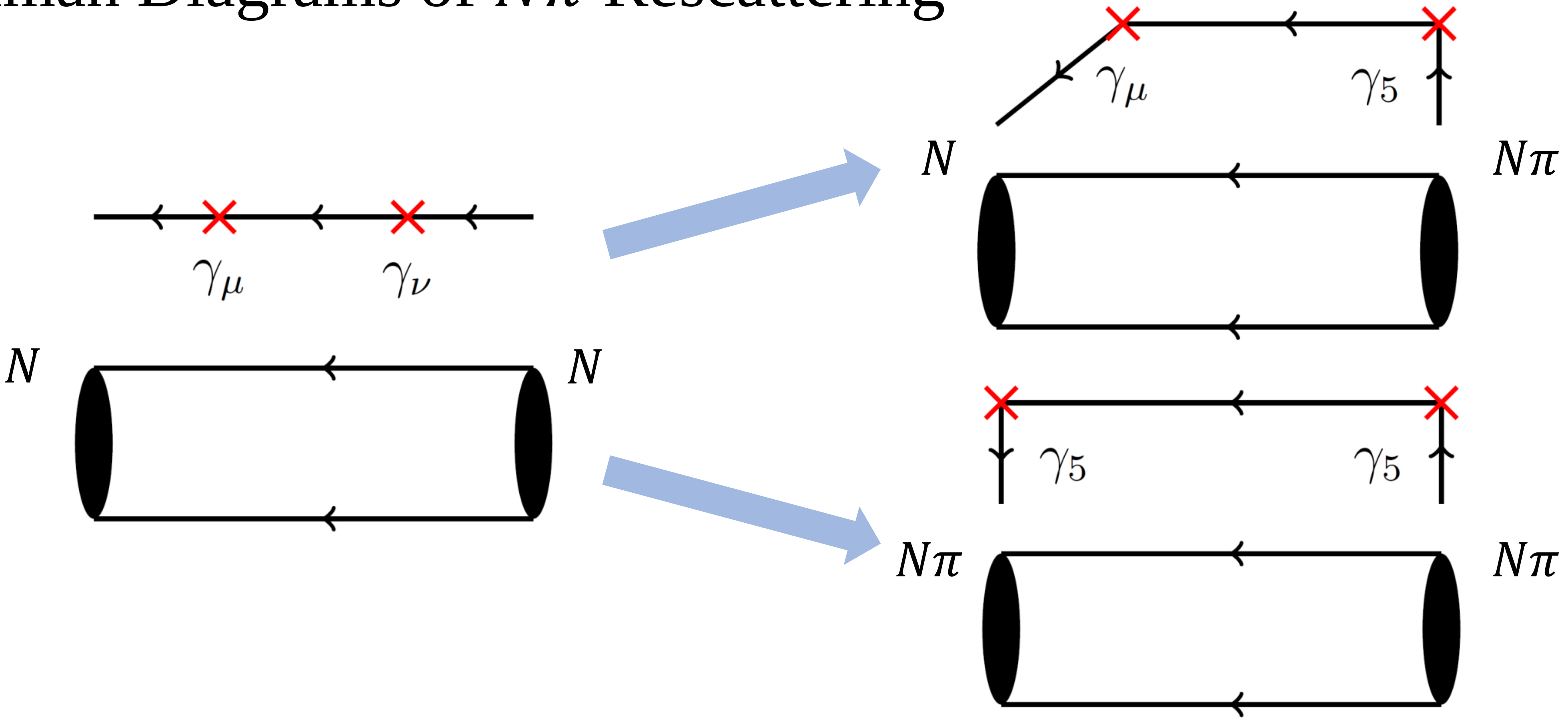


$N\pi$ contribution significantly enhanced by factor of t^2 → Need to calculate $N\pi$ rescattering on lattice

Feynman Diagrams of $N\pi$ Rescattering



$$I = 1/2:\ O_{N\pi}^{I_3=+\frac{1}{2}} = O_p O_{\pi^0} - \sqrt{2} O_n O_{\pi^+},\ O_{N\pi}^{I_3=-\frac{1}{2}} = \sqrt{2} O_n O_{\pi^0} - O_p O_{\pi^-}$$

$$I = 3/2:\ O_{N\pi}^{I_3=+\frac{1}{2}} = \sqrt{2} O_p O_{\pi^0} + O_n O_{\pi^+},\ O_{N\pi}^{I_3=-\frac{1}{2}} = O_n O_{\pi^0} + \sqrt{2} O_p O_{\pi^-}$$

} Operators

Results of $N\pi$ Scattering

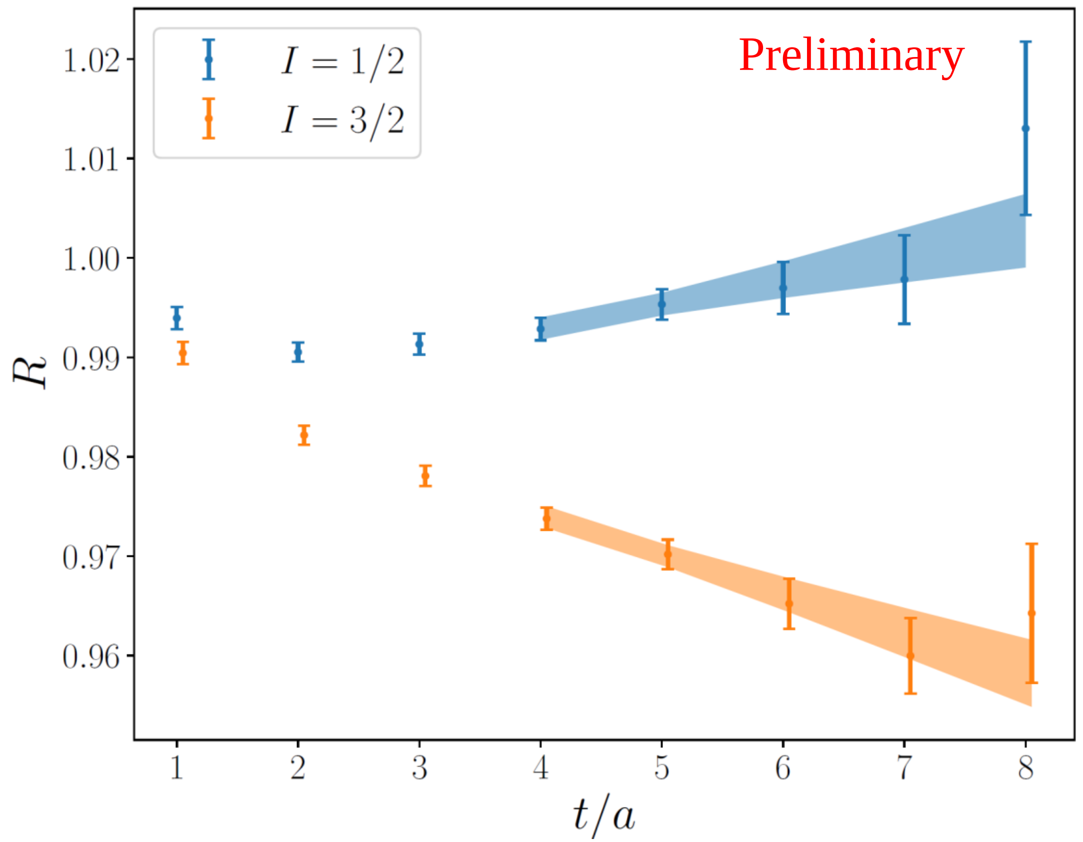
- $N\pi$ scattering for $I_3 = +1/2$

(similar for $I_3 = -1/2$ case)

$$R = \frac{C_2^{N\pi}(t)}{C_2^N(t)C_2^\pi(t)} = \frac{A_{N\pi}}{A_N A_\pi}\frac{e^{-E_{N\pi}t}}{e^{-(M_N+M_\pi)t}} \approx R_0(1-\Delta E t)$$

with $\Delta E = E_{N\pi} - M_N - M_\pi$

- Scattering for different isospin channel
  - $I = 1/2$, $\Delta E < 0$, attractive interaction
  - $I = 3/2$, $\Delta E > 0$, repulsive interaction



This work:

$a_0^{1/2}m_\pi = 0.101(45), a_0^{3/2}m_\pi = -0.131(26)$

Phenomenology[1]:

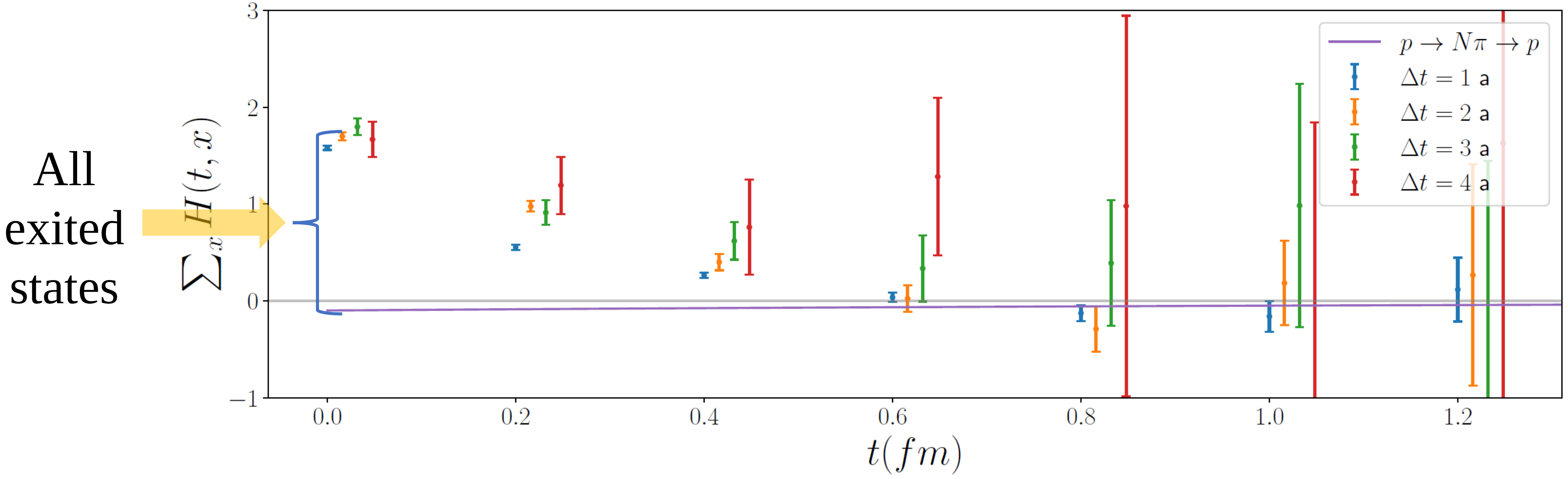
$a_0^{1/2}m_\pi = 0.1699(20), a_0^{3/2}m_\pi = -0.0865(18)$

$< 2\sigma$ consistent

[1]M. Hoferichter et al. On the role of isospin violation in the pion-nucleon σ-term[J]. Phys. Lett. B, 2023, 843:138001.

Summary

- All excited states calculated with 4-point correlation function, but the lowest $N\pi$ state contribution need to be calculated separately.
- Lattice QCD calculation of $N\gamma \to N\pi$ is necessary.



- Various other projects contains $N\pi$ scattering, like pion photoproduction……