Renyi Entropy due to the Presence of Static Quarks

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Motivation

- QCD string is a pure spatially-extended state of gluons
- Dynamics of QCD string are important to hadronization in heavy-ion collisions & DIS, e.g. Lund model in Pythia
- Goal: study quantum correlations between parts of a static QCD string
    - Pure 4D Yang Mills at $T=\frac{1}{2}T_c$
    - Static heavy quarks as sources
    - Renyi entanglement entropy as measure of quantum correlations

Entanglement Entropy (Renyi Entropy)

$$\hat{\rho}_{\mathcal{A}} = \mathrm{tr}_{\bar{\mathcal{A}}}\left(\hat{\rho}\right)$$

Example:

$$\psi = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$\rho = \begin{array}{c} \begin{array}{cccc} |\uparrow\uparrow\rangle & |\uparrow\downarrow\rangle & |\downarrow\uparrow\rangle & |\downarrow\downarrow\rangle \end{array} \\ \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \end{array}$$

$$\hat{\rho}_{\mathcal{A}} = \mathrm{Tr}_{\bar{\mathcal{A}}}\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$S_{\mathrm{EE}} = -\,\mathrm{tr}_{\mathcal{A}}\left(\hat{\rho}_{\mathcal{A}} \log\left(\hat{\rho}_{\mathcal{A}}\right)\right)$$

$$S_{\mathrm{EE}} = -\frac{d}{dq}\log(\mathrm{tr}_{\mathcal{A}}(\hat{\rho}^{q}_{\mathcal{A}}))|_{q=1}$$

$$S^{(q)} = \frac{1}{1-q}\log\left(\mathrm{tr}_{\mathcal{A}}\left(\hat{\rho}^{q}_{\mathcal{A}}\right)\right) \forall\ q \in \mathbb{N},\ \ q \geq 2.$$

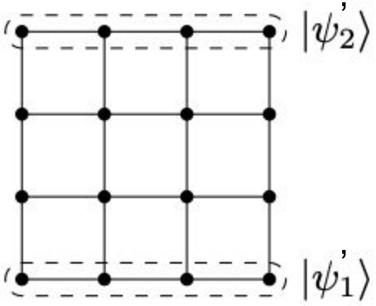
$$S_{\mathrm{EE}} = \lim_{q\to 1} S^{(q)}.$$

Reduced Density Matrix in Lattice Field Theory

$$\hat{\rho}_{\mathcal{A}} = \mathrm{tr}_{\bar{\mathcal{A}}}\left(\hat{\rho}\right)$$

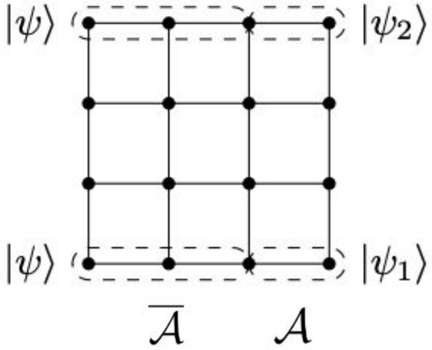
$$\langle\psi_1^{'}|\,\hat{\rho}\,|\psi_2^{'}\rangle = \frac{1}{Z}\,\langle\psi_1^{'}|\,e^{-\beta\hat{H}}\,|\psi_2^{'}\rangle$$

$$=$$



$$\langle\psi_1|\,\hat{\rho}_{\mathcal{A}}\,|\psi_2\rangle$$

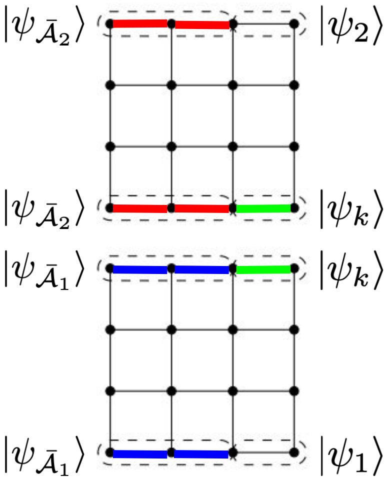
$$= \sum_{|\psi\rangle}$$



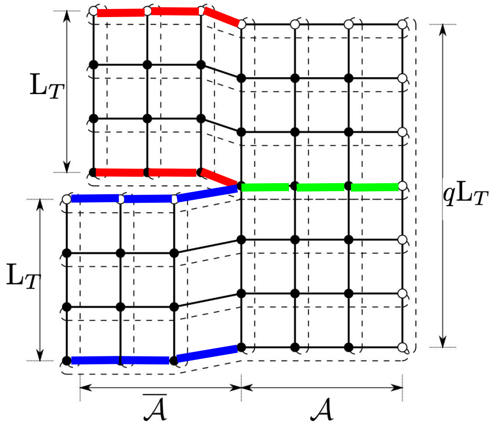
(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)

Reduced Density Matrix squared

$$\langle\psi_1|\hat{\rho}^2_{\mathcal{A}}|\psi_2\rangle = \langle\psi_1|\hat{\rho}_{\mathcal{A}}|\psi_k\rangle\langle\psi_k|\hat{\rho}_{\mathcal{A}}|\psi_2\rangle = \sum_{|\psi_k\rangle,|\psi_{\bar{\mathcal{A}}_1}\rangle,|\psi_{\bar{\mathcal{A}}_2}\rangle}$$



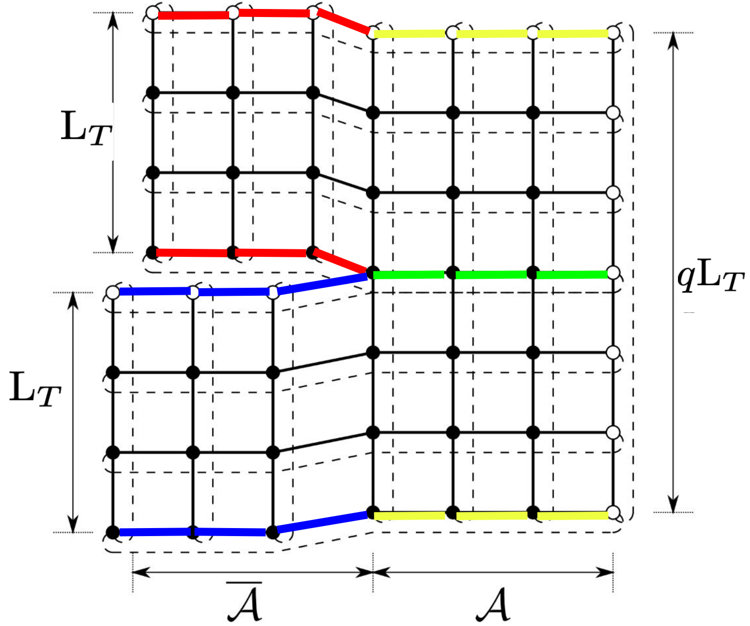
$$=$$



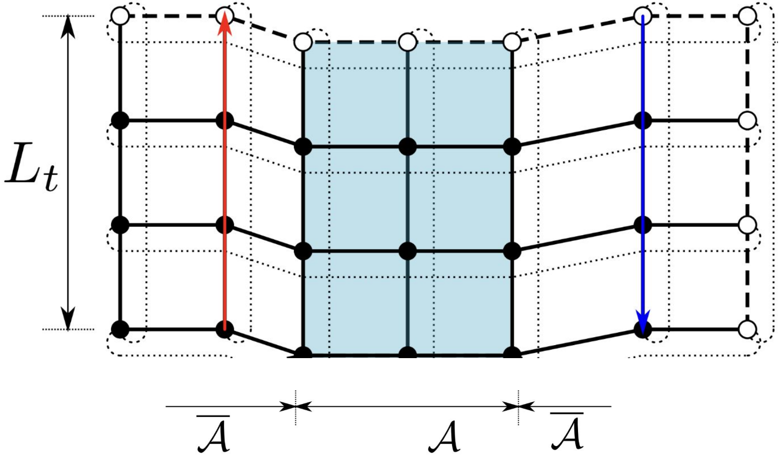
Reduced Density Matrix squared

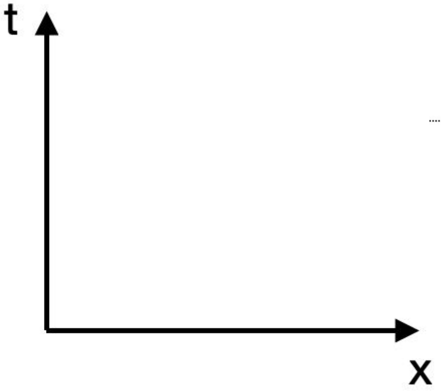
$$\mathrm{Tr}\hat{\rho}_{\mathcal{A}}^2 = \sum_{\psi_{\mathcal{A}}} \langle \psi_{\mathcal{A}} | \hat{\rho}_{\mathcal{A}}^2 | \psi_{\mathcal{A}} \rangle = $$

$$= \frac{Z_2}{(Z_1)^2}$$

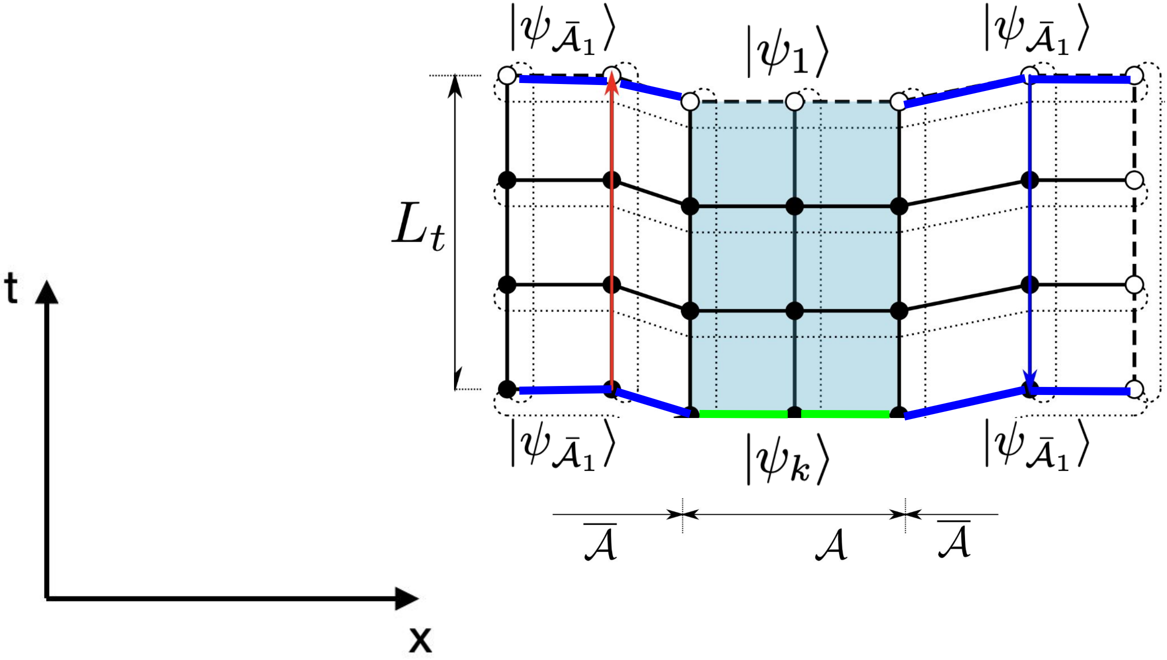


Polyakov Lines

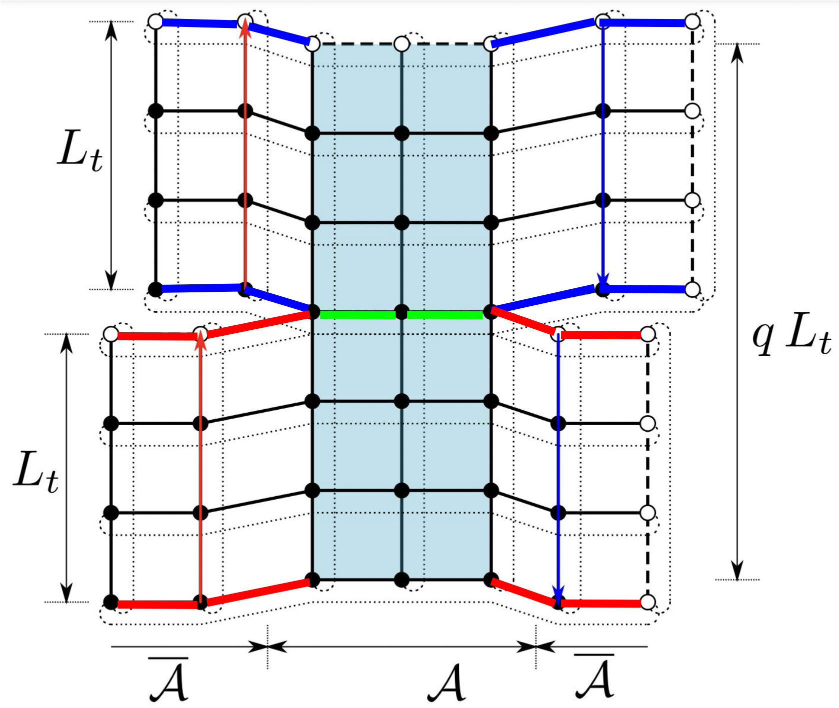


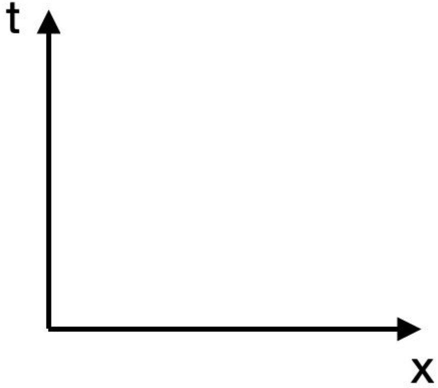


Polyakov Lines: Reduced Density Matrix



Polyakov Lines: Reduced Density Matrix squared





UV-finite Entanglement Entropy

$$\frac{1}{|\partial A|} S = \frac{1}{|\partial A|} S_{UV} + \frac{1}{|\partial A|} S_f$$

(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)
(T. Nishioka, T Takayanagi arXiv:hep-th/0611035)

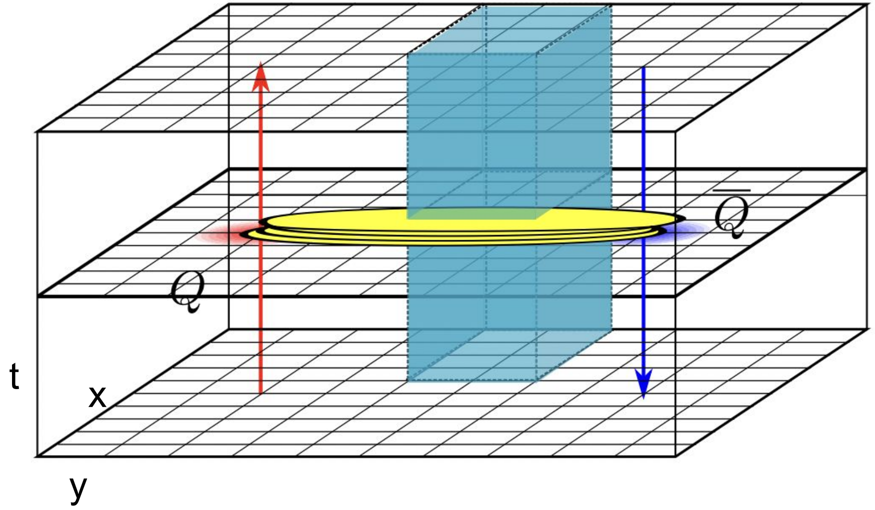
$$\tilde{S}^{E(q)}_{|Q\bar{Q}} \equiv S^{E(q)}_{|Q\bar{Q}} - S^{E(q)}$$

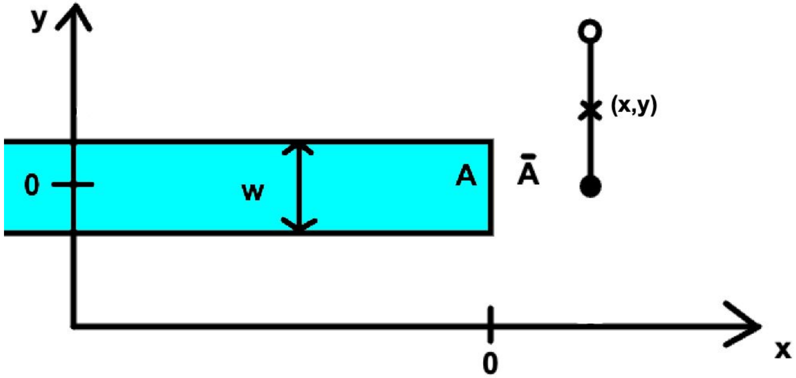
(Excess entropy due to QCD string)

$$S_X^{E(q)} = -\frac{1}{q-1} \log \mathrm{Tr}_A \left[\left(\mathrm{Tr}_{\bar{A}} \rho_X \right)^q \right] = -\frac{1}{q-1} \log \frac{Z_X^{(q)}}{(Z_X)^q}$$

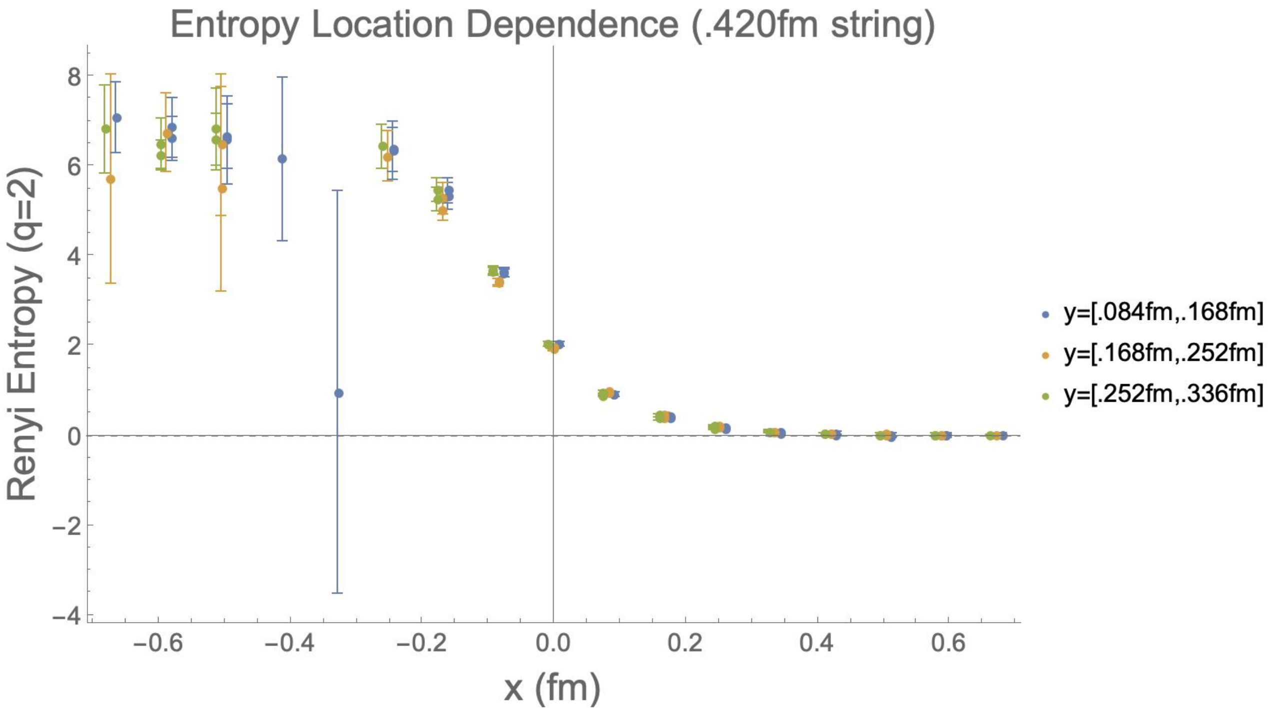
$$\boxed{-\frac{1}{q-1} \log \frac{\langle \prod_{r=1}^{q} \mathrm{Tr} P_0^{(r)} \mathrm{Tr} P_{\vec{x}}^{(r)\dagger} \rangle}{\langle \mathrm{Tr} P_0 \mathrm{Tr} P_{\vec{x}}^{\dagger} \rangle^q}} = -\frac{1}{q-1} \log \frac{Z^{(q)}_{|Q\bar{Q}} / Z^{(q)}}{(Z_{|Q\bar{Q}}/Z)^q} = -\frac{1}{q-1} \log \left(\frac{Z^{(q)}_{|Q\bar{Q}}}{(Z_{|Q\bar{Q}})^q} \cdot \frac{Z^q}{Z^{(q)}} \right) = S^{E(q)}_{|Q\bar{Q}} - S^{E(q)}$$

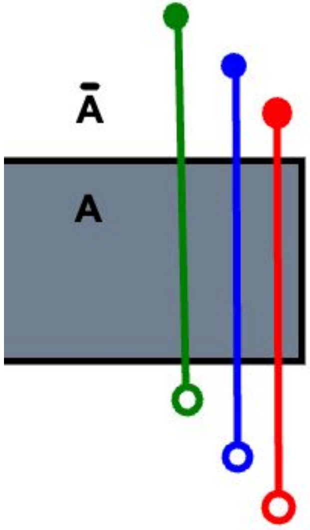
A = Half-slab



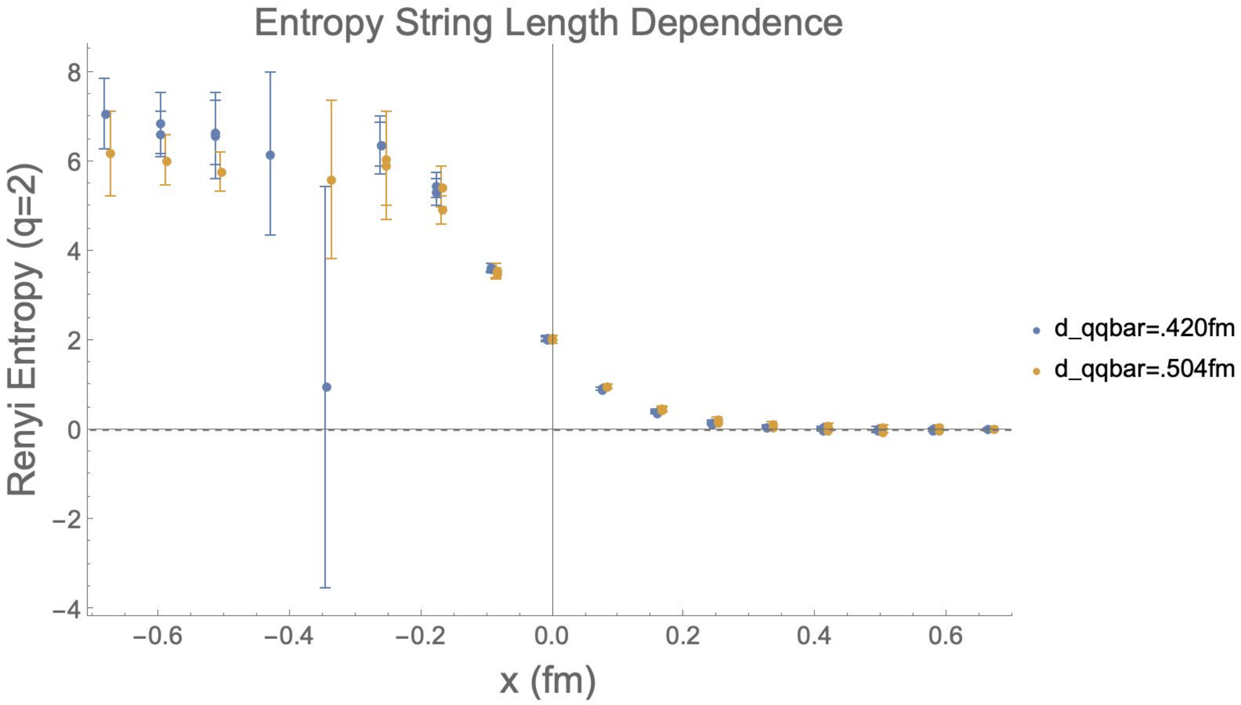


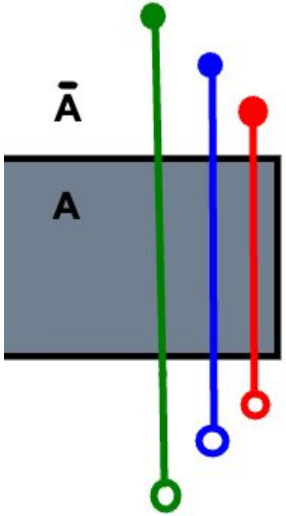
Entropy Location along the String Dependence



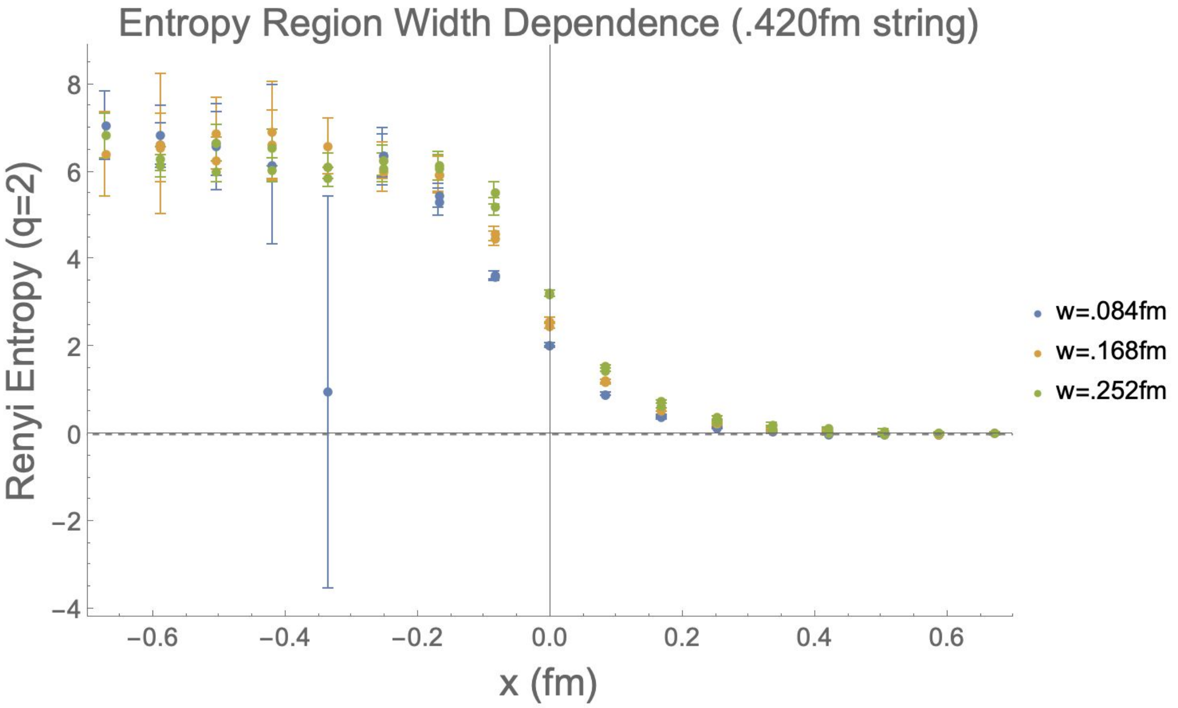


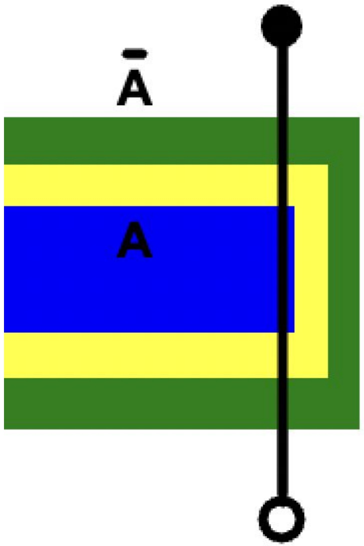
Entropy String Length Dependence



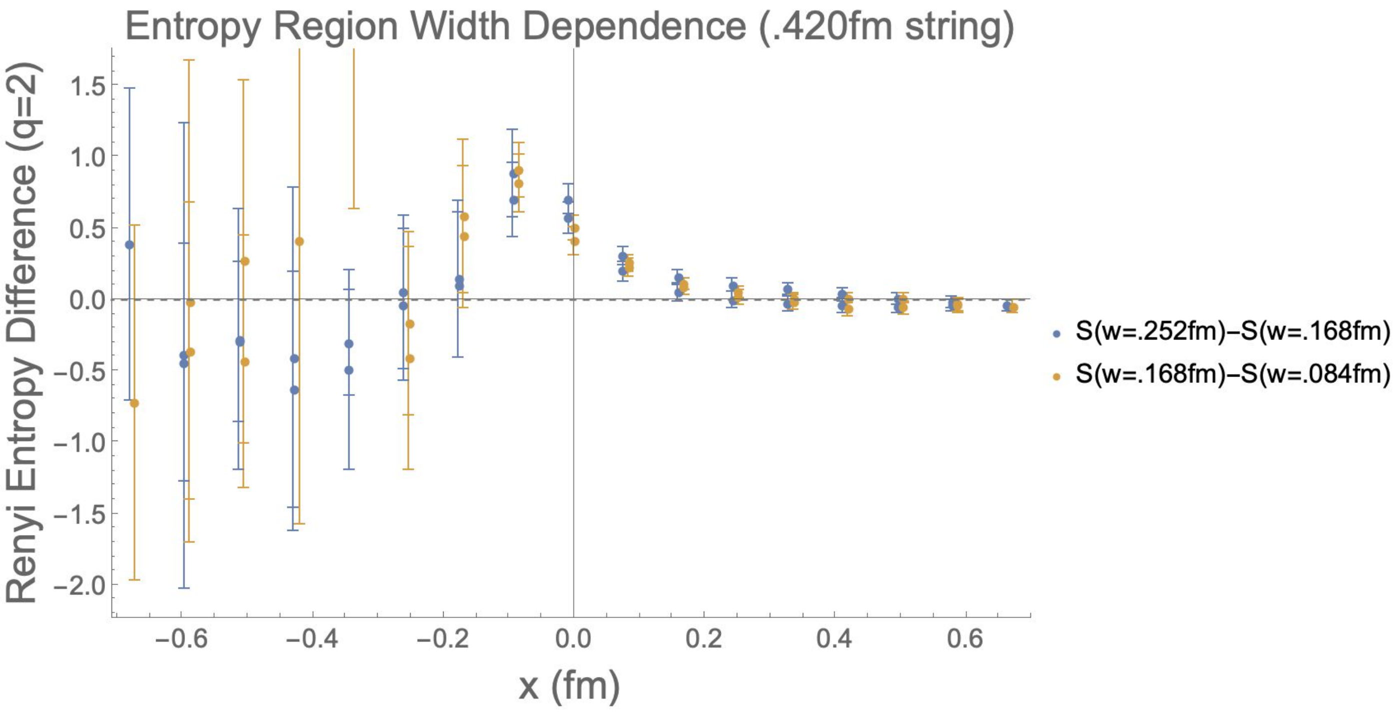


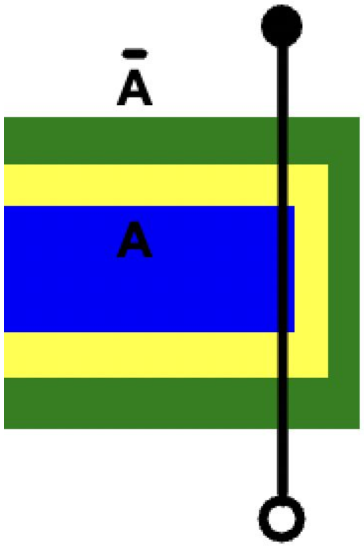
Entropy Region Width Dependence



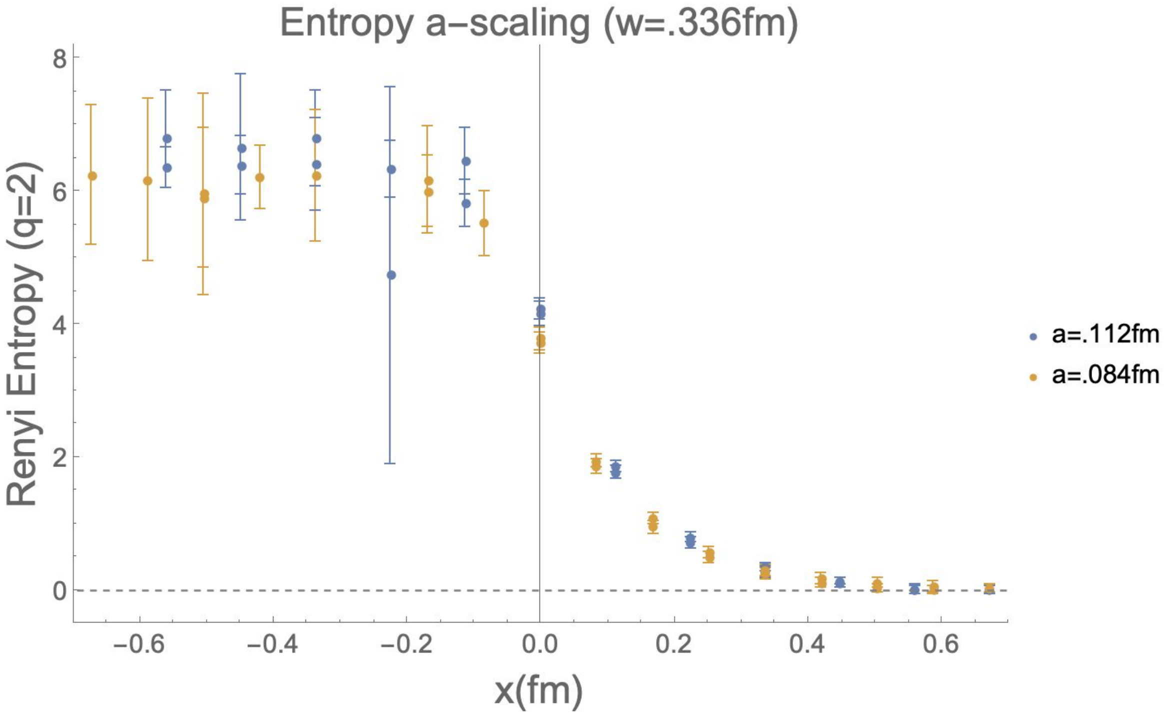


Entropy Region Width Dependence





Preliminary Scaling Behavior



Conclusions

Introduced UV-finite measure of entanglement entropy due to QCD string using Polyakov lines and replicas

Used half-slab geometry to explore entanglement entropy of different regions of the QCD string

Found entanglement entropy does not depend strongly on location along the string*, string length*, or region A width in the large positive and large negative x limits*

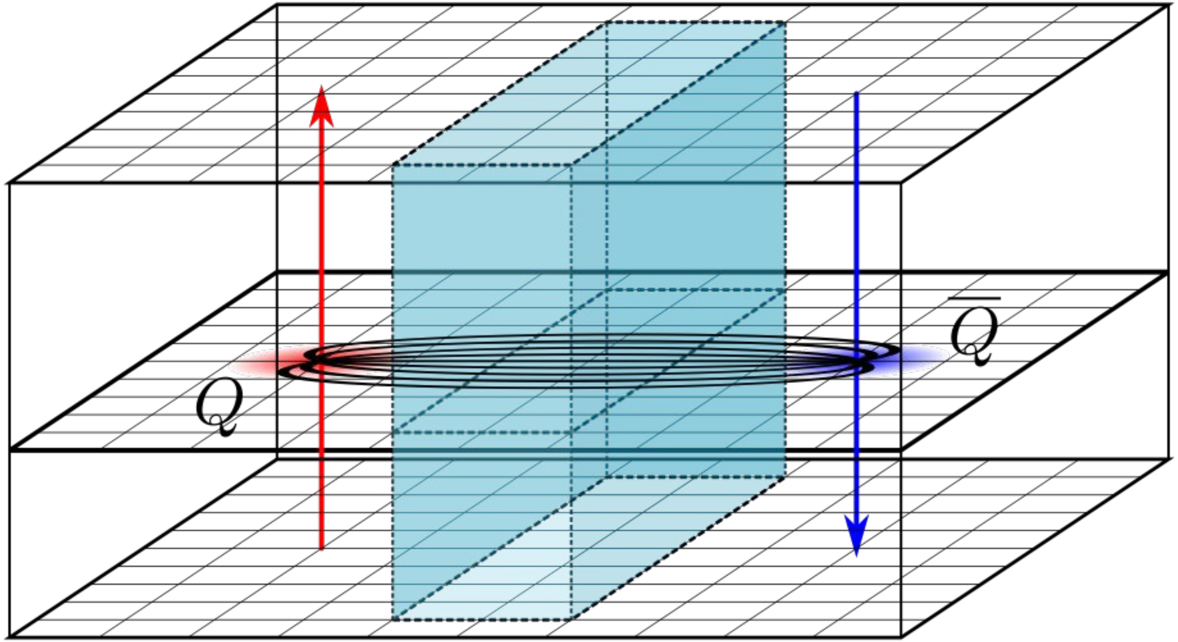
* must confirm in continuum with robust scaling study

Further Directions:

- Scaling behavior
- Extrapolation to Von Neumann entropy using q>2
- Better statistics
- Deconfinement

Backup

A = Full Slab



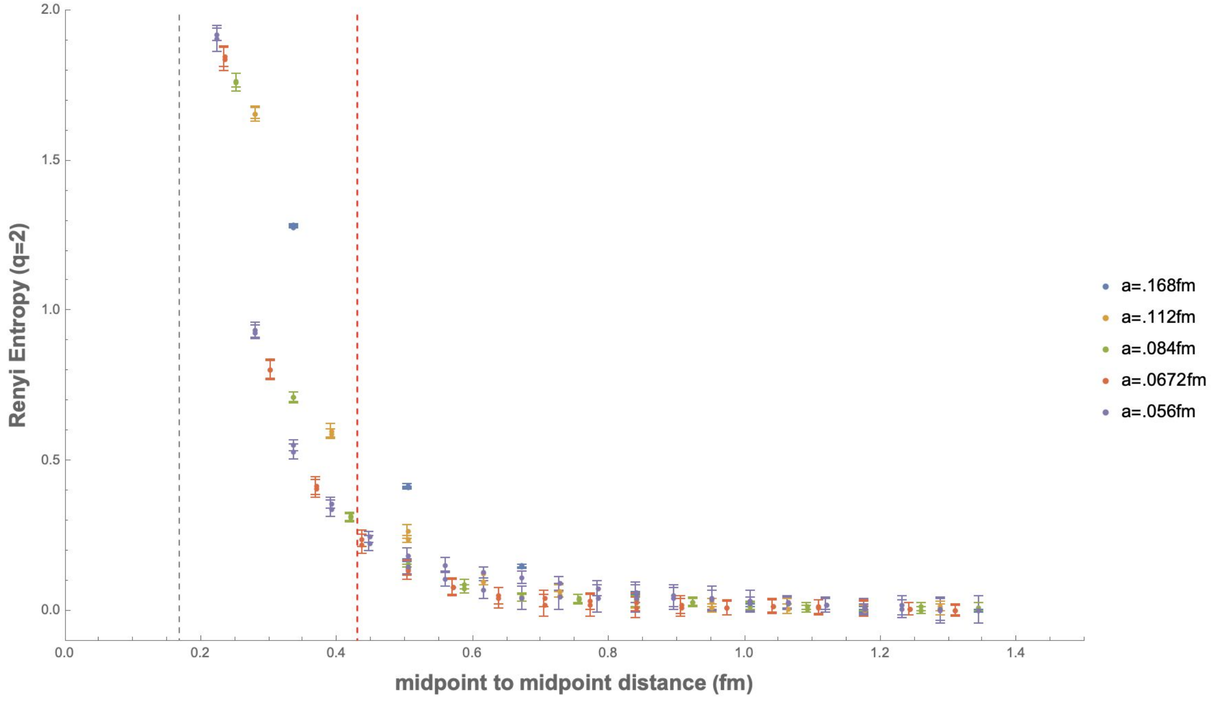
A = Full Slab

- Geometry of region A: infinite slab of constant physical width, quark and antiquark straddle region A
- 2 replicas
- T = 1/2 T_c
- Issues: Poor Signal to Noise

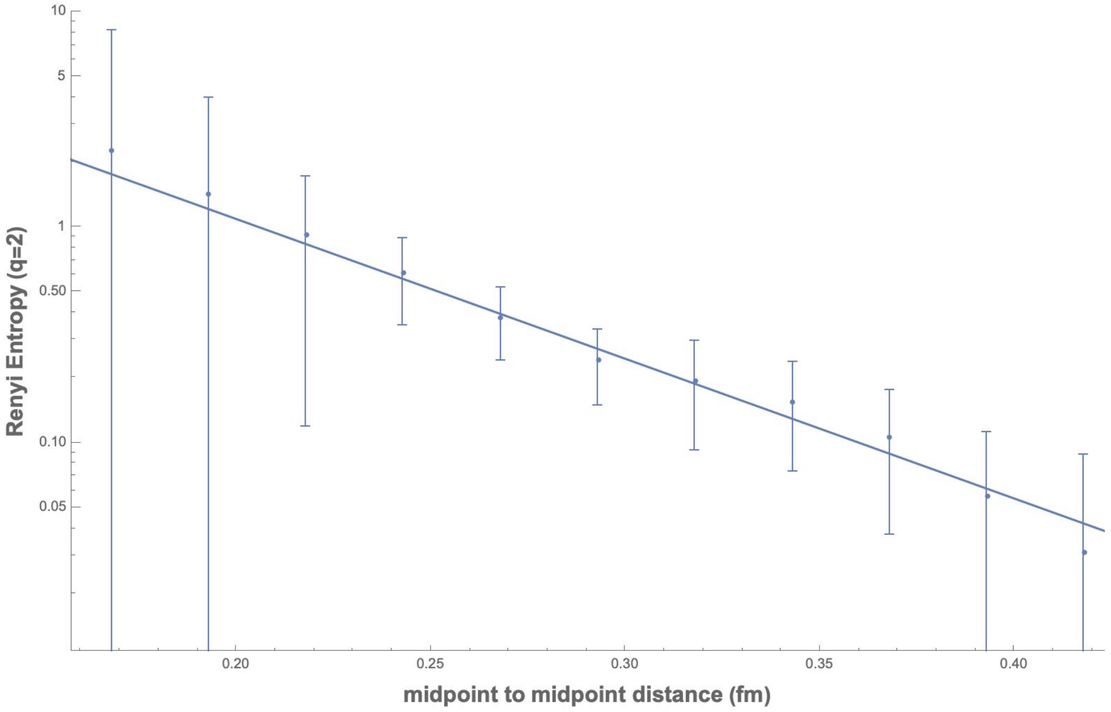
A = Slab Parallel to QQbar Separation

- Geometry of region A: infinite slab of constant physical width, quark pair separation vector parallel to the slab
- Total Volume: $(2.7fm)^3$
- 2 replicas
- $T = 1/2\, T_c$
- Lattice spacings vary from .056fm to .168fm

Parallel Slab a-scaling



Parallel Slab Continuum Limit



Parallel Slab Mutual Information Symmetry