Renyi Entropy due to the Presence of Static Quarks

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Motivation

- QCD string is a pure spatially-extended state of gluons
- Dynamics of QCD string are important to hadronization in heavy-ion collisions & DIS, e.g. Lund model in Pythia
- Goal: study quantum correlations between parts of a static QCD string
  - Pure 4D Yang Mills at $T = \frac{1}{2} T_c$
  - Static heavy quarks as sources
  - Renyi entanglement entropy as measure of quantum correlations
Entanglement Entropy (Renyi Entropy)

\[ \hat{\rho}_A = \text{tr}_A(\hat{\rho}) \]

Example:

\[ \psi = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \]

\[ \rho = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \]

\[ \hat{\rho}_A = \text{Tr}_A \rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \]

\[ S_{EE} = -\text{tr}_A (\hat{\rho}_A \log (\hat{\rho}_A)) \]

\[ S_{EE} = -\frac{d}{dq} \log(\text{tr}_A (\hat{\rho}_A^q))|_{q=1} \]

\[ S(q) = \frac{1}{1-q} \log (\text{tr}_A (\hat{\rho}_A^q)) \forall q \in \mathbb{N}, \ q \geq 2. \]

\[ S_{EE} = \lim_{q \to 1} S(q). \]
Reduced Density Matrix in Lattice Field Theory

\[ \hat{\rho}_A = \text{tr}_A (\hat{\rho}) \]

\[ \langle \psi'_1 | \hat{\rho} | \psi'_2 \rangle = \frac{1}{Z} \langle \psi'_1 | e^{-\beta \hat{H}} | \psi'_2 \rangle \]

(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)
Reduced Density Matrix squared

$$\langle \psi_1 | \hat{\rho}_{\mathcal{A}}^2 | \psi_2 \rangle = \langle \psi_1 | \hat{\rho}_{\mathcal{A}} | \psi_k \rangle \langle \psi_k | \hat{\rho}_{\mathcal{A}} | \psi_2 \rangle = \sum_{|\psi_k\rangle, |\psi_{\mathcal{A}_1}\rangle, |\psi_{\mathcal{A}_2}\rangle} \langle \psi_{\mathcal{A}_1} | \hat{\rho}_{\mathcal{A}} | \psi_k \rangle \langle \psi_k | \hat{\rho}_{\mathcal{A}} | \psi_{\mathcal{A}_2} \rangle L_T$$
Reduced Density Matrix squared

$$\text{Tr} \hat{\rho}_A^2 = \sum_{\psi_A} \langle \psi_A | \hat{\rho}_A^2 | \psi_A \rangle = \frac{Z_2}{(Z_1)^2}$$
Polyakov Lines
Polyakov Lines: Reduced Density Matrix
Polyakov Lines: Reduced Density Matrix squared
UV-finite Entanglement Entropy

\[ \frac{1}{|\partial A|} S = \frac{1}{|\partial A|} S_{UV} + \frac{1}{|\partial A|} S_f \]

(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)

\[ \tilde{S}_{\bar{q}Q}^E(q) \equiv S_{\bar{q}Q}^E(q) - S^E(q) \]

(Excess entropy due to QCD string)

\[ S_X^E(q) = -\frac{1}{q-1} \log \text{Tr}_A \left[ (\text{Tr}_A \rho_X)^q \right] = -\frac{1}{q-1} \log \frac{Z_X^{(q)}}{(Z_X)^q} \]

\[ -\frac{1}{q-1} \log \left( \frac{\prod_{r=1}^q \text{Tr}P_0^{(r)} \text{Tr}P_x^{(r)\dagger}}{\langle \text{Tr}P_0 \text{Tr}P_x \rangle^q} \right) = -\frac{1}{q-1} \log \frac{Z_X^{(q)}}{(Z_{\bar{q}Q}/Z)^q} = -\frac{1}{q-1} \log \left( \frac{Z_{\bar{q}Q}^{(q)}}{(Z_{\bar{q}Q})^q} \cdot \frac{Z^q}{Z^{(q)}} \right) = S_{\bar{q}Q}^E(q) - S^E(q) \]
A = Half-slab
Entropy Location along the String Dependence

Entropy Location Dependence (.420fm string)

Renyi Entropy ($q=2$)

- $y=[0.084\text{fm}, 0.168\text{fm}]$
- $y=[0.168\text{fm}, 0.252\text{fm}]$
- $y=[0.252\text{fm}, 0.336\text{fm}]$
Entropy String Length Dependence

![Graph showing entropy string length dependence]
Entropy Region Width Dependence

Entropy Region Width Dependence (.420fm string)

Renyi Entropy ($q=2$)

- $w=.084$ fm
- $w=.168$ fm
- $w=.252$ fm

$x$ (fm)

-4 -3 -2 -1 0 1 2 3 4 5 6 7 8

-4 -3 -2 -1 0 1 2 3 4 5 6 7 8
Entropy Region Width Dependence

![Graph showing the relationship between Renyi Entropy Difference and x (fm) for different widths of the entropy region, labeled S(w=0.252fm) - S(w=0.168fm) and S(w=0.168fm) - S(w=0.084fm).]
Preliminary Scaling Behavior

Entropy $a$-scaling ($w=.336\text{fm}$)

Renyi Entropy ($q=2$)

- $a=.112\text{fm}$
- $a=.084\text{fm}$

$x(\text{fm})$
Conclusions

Introduced UV-finite measure of entanglement entropy due to QCD string using Polyakov lines and replicas.

Used half-slab geometry to explore entanglement entropy of different regions of the QCD string.

Found entanglement entropy does not depend strongly on location along the string*, string length*, or region A width in the large positive and large negative x limits*.

Further Directions:
- Scaling behavior
- Extrapolation to Von Neumann entropy using q>2
- Better statistics
- Deconfinement

* must confirm in continuum with robust scaling study
Backup
A = Full Slab
A = Full Slab

- Geometry of region A: infinite slab of constant physical width, quark and antiquark straddle region A
- 2 replicas
- $T = 1/2 \, T_c$
- Issues: Poor Signal to Noise
A = Slab Parallel to QQbar Separation

- Geometry of region A: infinite slab of constant physical width, quark pair separation vector parallel to the slab
- Total Volume: $(2.7\text{fm})^3$
- 2 replicas
- $T = 1/2 \ T_c$
- Lattice spacings vary from $.056\text{fm}$ to $.168\text{fm}$
Parallel Slab $a$-scaling
Parallel Slab Continuum Limit
Parallel Slab Mutual Information Symmetry