Scalar QED with Rydberg atoms

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QuLAT collaboration
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Hybrid hadronization in event generators?
Toy model: the compact Abelian Higgs Model (Scalar QED)
Model building (literally!) with Arrays of Rydberg atoms
Matching the target model with the simulator (or vice-versa!)
Effective theory for the simulator
Practical examples and work with QuERa
Phase diagram of the simulator
Towards an hybrid event generator (QuPYTH), with K. Heitritter and S. Mrenna, arxiv:2212.02476
Hybrid hadronization in event generators?

Hadronization of a particle-antiparticle pair with the Lund model/Pythia (left), the Abelian Higgs model in 1+1 dimensions (middle), and a Rydberg atom simulator for this model (right).
Compact Abelian Higgs Model (CAHM)

The lattice compact Abelian Higgs model is a non-perturbative regularized formulation of scalar quantum electrodynamics (scalar electrons-positrons + photons with compact fields).

\[
Z_{\text{CAHM}} = \prod_x \int_{-\pi}^{\pi} \frac{d\phi_x}{2\pi} \prod_{x,\mu} \int_{-\pi}^{\pi} \frac{dA_{x,\mu}}{2\pi} e^{-S_{\text{gauge}} - S_{\text{matter}}},
\]

\[
S_{\text{gauge}} = \beta_{\text{plaquette}} \sum_{x,\mu<\nu} (1 - \cos(A_{x,\mu} + A_{x+\hat{\mu},\nu} - A_{x+\hat{\nu},\mu} - A_{x,\nu})),
\]

\[
S_{\text{matter}} = \beta_{\text{link}} \sum_{x,\mu} (1 - \cos(\phi_{x+\hat{\mu}} - \phi_x + A_{x,\mu})).
\]

- local invariance: \( \phi'_x = \phi_x + \alpha_x \) and \( A'_{x,\mu} = A_{x,\mu} - (\alpha_{x+\hat{\mu}} - \alpha_x) \).
- \( \phi \) is the Nambu-Goldstone mode of the original model. The Brout-Englert-Higgs mode is decoupled (heavy).
Transfer matrix and Gauss’s law with NISQ machines

The continuous-time limit yields the Hamiltonian

\[ H = \frac{U}{2} \sum_{i=1}^{N_s} (L^z_i)^2 + \frac{Y}{2} \sum_{i} (L^z_{i+1} - L^z_i)^2 - X \sum_{i=1}^{N_s} U^x_i \]

with \( U^x \equiv \frac{1}{2} (U^+ + U^-) \) and \( L^z |m\rangle = m|m\rangle \) and \( U^\pm |m\rangle = |m \pm 1\rangle \).

- \( m \) is a discrete electric field quantum number \((-\infty < m < +\infty)\)
- In practice, we need to apply truncations: \( U^\pm |\pm m_{\text{max}}\rangle = 0 \).
- We focus on the spin-1 truncation \((m = \pm 1, 0\text{ and } U^x = L^x/\sqrt{2})\).
- \( U \)-term: electric field energy.
- \( Y \)-term: matter charges (determined by Gauss’s law)
- \( X \)-term: currents inducing temporal changes in the electric field.
Target simulations (E-field, spin-1, 5 sites)

\[ H = \frac{U}{2} \sum_{i=1}^{N_s} (L^z_i)^2 + \frac{Y}{2} \sum_i (L^z_{i+1} - L^z_i)^2 - X \sum_{i=1}^{N_s} U_i^x \]

Initial state: particle-antiparticle (connected by an electric field +1)
One can adapt (Y.M., PRD 104) the optical lattice construction (J. Zhang et al. PRL 121) using $^87Rb$ atoms separated by controllable (but not too small) distances, coupled to the excited Rydberg state $|r\rangle$ with a detuning $\Delta$.

The ground state is denoted $|g\rangle$ and the two possible states $|g\rangle$ and $|r\rangle$ can be seen as a qubit. $n|g\rangle = 0$, $n|r\rangle = |r\rangle$.

The (effective) Hamiltonian reads

$$H = \frac{\Omega}{2} \sum_i (|g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|) - \Delta \sum_i n_i + \sum_{i<j} V_{ij}n_in_j,$$

with

$$V_{ij} = \Omega R_b^6/r_{ij}^6,$$

for a distance $r_{ij}$ between the atoms labelled as $i$ and $j$. Note: when $r = R_b$, $V = \Omega$.

This repulsive interaction prevents two atoms close enough to each other to be both in the $|r\rangle$ state. This is the so-called blockade mechanism.
One site spin-1 with 2 and 3 atoms (Y.M. PRD 104)

Solid line: target, Symbols: simulator
1 site, 2 atoms: exact up to $|rr\rangle$ transitions (when $\Omega = 0$, can be implemented by setting $\Delta = -U/2$ and $\Omega = -X$).

2 sites, 4 atoms: when $X = \Omega = 0$ reads $\Delta = -U/2 - \frac{Y}{2}$, $V_1 = Y$, $V_2 = -Y$. No solution with current technology (homogeneous setup).

1 site, 3 atoms: it’s complicated! (ideally: inhomogeneous $\Delta$ to split $m = 0$ and $m = \pm 1$, otherwise use degenerate perturbation theory as a guide, James Corona’s work in progress).

A better approach may be to study all the continuum limits (where correlation lengths become large) that can be obtained with the simulator.
Yannick Meurice (U. of Iowa)

LGT with Rydberg atoms

Fermilab, August 2, 2023
10 atoms (5 sites) with QuEra and $H_{\text{eff}}$.

Weakly interacting pairs with strong Rydberg blockade

Values of $< L_z^2 >$ for five sites (10 atoms) starting in the $|ggggggggggg\rangle$ state, $\Omega = 4\pi MHz$, $\Delta = 2\Omega$, for 50 steps of $10^{-8}$s, $a_x = 1R_b \simeq 8.7 \mu m$; $a_y = 0.5R_b$ so $|rr\rangle$ at that site are unlikely. Left: exact diagonalization. Middle: QuEra (local simulator). Right: Effective Hamiltonian.
The basic spin relation is $L^z_i = n_{i,2} - n_{i,1}$ for atoms 1 and 2 at a site.

$(L^z_i)^2 \approx n_{i,2} + n_{i,1}$ if we ignore $|rr\rangle$.

This implies $n_{i,2(1)} \approx \left[(L^z_i)^2 \pm L^z_i\right]/2$.

Plugging into the simulator Hamiltonian we get the effective Hamiltonian

$$\hat{H}_{2LR}^{\text{eff}} = -\Delta \sum_{i=1}^{N_s} (L^z_i)^2 + \sum_{k} \left(\frac{V_1^{(k)} - V_2^{(k)}}{2} \sum_{i=1}^{N_s-1} L^z_i L^z_{i+k} \right)$$

$$+ \frac{V_1^{(k)} + V_2^{(k)}}{2} \sum_{i=1}^{N_s-1} (L^z_i)^2 (L^z_{i+k})^2$$

$$+ \frac{\Omega}{2} \sum_{i=1}^{N_s} \left(\hat{U}^+_i + \hat{U}^-_i\right).$$

If the distance between the sites is of order of 1.0 $R_b$ or more we can neglect the NNN interactions (which is done in the following).
Exact diagonalization, QuEra and $H_{\text{eff}}$. 

10 atoms; $a_x = 8.7 \mu; a_y = a_x/2$; site 0
$\Omega = 4\pi \text{ MHz}; \Delta = 8\pi \text{ MHz}$

10 atoms; $a_x = 8.7 \mu; a_y = a_x/2$; site 1
$\Omega = 4\pi \text{ MHz}; \Delta = 8\pi \text{ MHz}$

10 atoms; $a_x = 8.7 \mu; a_y = a_x/2$; site 2
$\Omega = 4\pi \text{ MHz}; \Delta = 8\pi \text{ MHz}$

$\langle I^2 \rangle$ vs. time (0.1$\mu$ sec)
Comparison between $H_{\text{eff.}}$ and $H_{\text{CAHM}}$

\[ H_{\text{CAHM}} = \frac{U}{2} \sum_{i=1}^{N_s} (L_i^z)^2 - Y \sum_{i=1}^{N_s-1} L_i^z L_{i+1}^z - X \sum_{i=1}^{N_s} U_i^x \]

\[ H_{\text{eff.}} = -\Delta \sum_{i=1}^{N_s} (L_i^z)^2 + \frac{V_1 - V_2}{2} \sum_{i=1}^{N_s-1} L_i^z L_{i+1}^z + \Omega \sum_{i=1}^{N_s} U_i^x + H_{\text{quartic}} \]

\[ H_{\text{quartic}} = \frac{V_1 + V_2}{2} \sum_{i=1}^{N_s-1} (L_i^z)^2 (L_{i+1}^z)^2. \]

Matching

- $\Delta = -U/2$ (sign matters!)
- The coefficient for $L_i^z L_{i+1}^z$ is positive ($V_1 > V_2$) for the simulator (repulsive/antiferromagnetic) but the CAHM has ferromagnetic interactions. This can be remedied by redefining the observable $L_{2i+1}^z \rightarrow -L_{2i+1}^z$ (staggered)
- After redefinition $V_1 = -V_2 = Y > 0$ but $V_2 > 0$
- $\Omega = -X$ (sign does not matter)

Everything agrees with two-rung results (YM, PRD104)
**Effect of new quartic term (Jin Zhang)**

![Phase Diagram](image)

**Figure:** Ground-state phase diagram for the effective Hamiltonian of the two-leg Rydberg ladder. Here $L = 512$, $V_0 = 1000$, $\rho = \frac{dy}{dx} = 0.5$. The PRDW phase is disordered in even or odd sites, and the FRDW phase is FM in even or odd sites.
Phase diagram for a two-leg ladder with Sergio Cantu (QuEra), Fangli Liu (QuERA), Shan-Wen Tsai (UCR), Shengtao Wang (QuERA), Jin Zhang (ChongQing U.)

1.0
1.5
2.0
2.5
3.0
3.5
\( \Delta / \Omega \)

Floating
0.0
0.2
0.4
0.6
0.8

\( a_y = 2a_x \)
$\langle m_i m_j \rangle$ for two-leg ladder (prelim. exp. results)

First row: for $i$ and $j$; second row: versus $i - j$; third row: Fourier Tr.
Figure: Hadron multiplicity output of the Rydberg hadronization model for $R_b/a = 2.173$. $\Delta/\Omega$ plotted in decreasing order. We see by looking at multiplicity output of smaller ladders, that number of rungs is the main limiting factor on the measured hadron multiplicity. The recently public Rydberg simulator, Aquila, has the ability to investigate ladders with 128 atoms.
Conclusions

- QC/QIS in HEP and NP: big goals with many intermediate steps
- Tensor Lattice Field Theory (TLFT): generic tool to discretize path integral formulations of lattice model with compact variables (truncations preserve symmetries).
- Ladder-shaped Rydberg arrays with two (or three) atoms per site: simulators for the compact Abelian Higgs model.
- Matching between simulator and target model should be understood in the continuum limit (universal behavior).
- Effective Hamiltonians for the simulator: same three types of terms as the target model plus an extra quartic term.
- The two-leg ladder has a very rich phase diagram.
- Implementations with AWS/QuEra (ongoing).
- Progress with hybrid hadronization
- Thanks for listening!
- For questions, email: yannick-meurice@uiowa.edu.
Thanks for listening!

\[ GC=0.3; \ st=2; \ sth = 1.5; \ T=1.5 \]

**Figure:** Isingized version of Emmy Noether
Other work with Rydberg atoms

- For more refs. see: C. Bauer, Z. Davoudi et al., arXiv:2204.03381, PRX Quantum 4 (2023) 2, 027001.
Goals of the collaboration
Quantum computers are expected to exceed the capacity of classical computers and to revolutionize several aspects of computation especially for the simulation of quantum systems. We develop new methods for using quantum computers to study aspects of the evolution of strongly interacting particles in collisions, the quantum behavior of gravitational systems and the emergence of space-time which are beyond the reach of classical computing. Our goal is to design the building blocks of universal quantum computers relevant for these problems and develop algorithms which scale reasonably with the size of the system.

Principal Investigators
Alexei Bazavov, Michigan State University
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Stephen Jordan, University of Maryland/Microsoft
Seth Lloyd, MIT (consultant)
Yannick Meurice, University of Iowa (Spokesperson)
Symmetry Breaking and Clock Model Interpolation in 2D Classical O(2) Spin Systems Leon Hostetler (Michigan State University) 8/4/23, 9:00 AM

Simulations of the Hyperbolic Ising Model Goksu Toga (Syracuse University) 8/4/23, 10:00 AM

Simulating Field Theories with Quantum Computers Muhammad Asaduzzaman (University of Iowa) 8/4/23, 10:20 AM
Areas of either neutrino/very weakly interacting particle phenomenology in laboratory and astroparticle physics, or in the areas of applications of quantum information/computation in quantum field theory and particle physics.