

Lattice QCD calculation of the invisible decay $J/\psi \rightarrow \gamma\nu\bar{\nu}$

Yu Meng (Zhengzhou University)

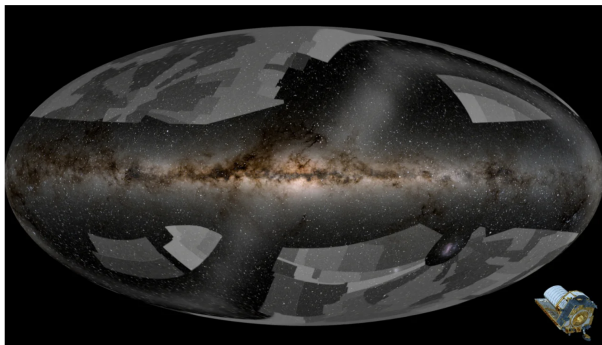
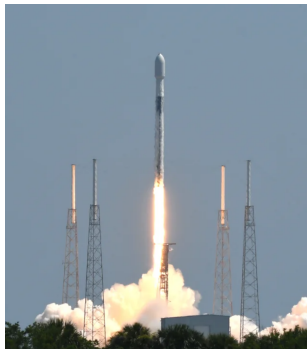
In collaboration with Xu Feng and Chuan Liu

Aug 1, 2023

The International Symposium on Lattice Field Theory (LATTICE 23)
Fermilab, USA, 2023

- Motivation
- Methodology: scalar function method
 - An example: $\eta_c \rightarrow \gamma\gamma$
- Lattice studies on $J/\psi \rightarrow \gamma\nu\bar{\nu}$
- Summary

Euclid space telescope



- Launching: 11:12 a.m, Sat July 1, 2023
- Mission: 3D map of the universe, with billions of galaxies that stretch 10 billion light-years away

511keV γ -Ray in Galactic Center

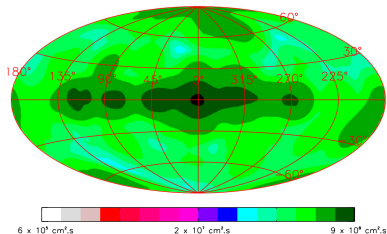
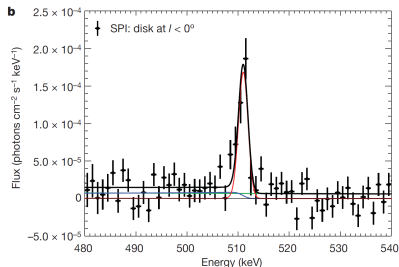


Figure 1. 508.25–513.75 keV *INTEGRAL* SPI exposure map. Units are in $\text{cm}^2 \times \text{s}$. This map takes into account the differential sensitivity of SPI across its field of view.

Astrophys.J. 720,1772(2010)



Nature, 451,159(2008)

- First observation, *Astrophys.J.* 172, L1(1972).
- One of the views: **light dark matter** (1-100MeV) annihilating into e^+e^- pairs
PRL, 92,101301(2004)

Current status of dark matter search on the collider

- Remarkable candidates
 - **Dark photon A'** : coupling to the photon through the kinetic mixing $\frac{\epsilon}{2}F^{\mu\nu}F'_{\mu\nu}$, free parameters are $m_{A'}$ and ϵ
 - **CP-odd Higgs Boson A^0** : m_{A^0} and $g_{a\bar{l}l}$
 - **Axion-like particle a** : m_a and $g_{a\gamma\gamma}$
- Dark matter can be produced on the collider
 - **Direct production: $e^+e^- \rightarrow \gamma X$**
BABAR,PRL119,131804(2017) BESIII,PLB839,137785(2023)
 - **Resonant production: $e^+e^- \rightarrow J/\psi/\Upsilon(1S) \rightarrow \gamma + X$**
CLEO,PRD81,091101(2010) BABAR,PRL107,021804(2011)
Belle,PRL122,011801(2019) Belle,PRL128,081804 (2022)
BESIII,PRD101,112005(2020) BESIII,PRD105,012008(2022)
BESIII,PLB838,137698(2023)

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CLEO 2010

PHYSICAL REVIEW D 81, 091101(R) (2010)

Search for the decay $J/\psi \rightarrow \gamma + \text{invisible}$

(CLEO Collaboration)

$$J/\psi \rightarrow \gamma + X$$

invisible to detector

light dark matter ?

The upper limit corresponding to $m_X = 0$

is 4.3×10^{-6} at the 90% confidence level

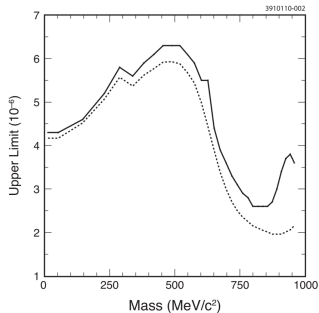


FIG. 4. The 90% confidence-level upper limits for $J/\psi \rightarrow \gamma X$, where X is invisible to the detector. The dashed line shows the results for statistical uncertainties alone, and the solid line includes systematic and statistical uncertainties.

Light dark matter search on the collider

BABAR
Belle

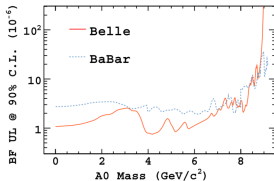
$$\Upsilon(1S) \rightarrow \gamma + X$$

PRL 107, 021804 (2011)

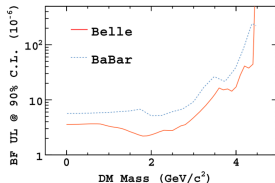
Search for Production of Invisible Final States in Single-Photon Decays of $\Upsilon(1S)$
(BABAR Collaboration)

PRL 122, 011801 (2019)

Search for a Light CP-odd Higgs Boson and Low-Mass Dark Matter at the Belle Experiment
(Belle Collaboration)



$$\mathcal{B}(\Upsilon \rightarrow \gamma A^0) \times \mathcal{B}(A^0 \rightarrow \text{invisible})$$



$$\mathcal{B}(\Upsilon(1S) \rightarrow \gamma \chi \bar{\chi})$$

BESIII

PHYSICAL REVIEW D 101, 112005 (2020)

Search for the decay $J/\psi \rightarrow \gamma + \text{invisible}$

Upper bound $\mathcal{B} \sim 7.0 \times 10^{-7}$

PHYSICAL REVIEW D 105, 012008 (2022)

Search for a CP-odd light Higgs boson in $J/\psi \rightarrow \gamma A^0$

Upper bound $\mathcal{B}(J/\psi \rightarrow \gamma A^0) \times \mathcal{B}(A^0 \rightarrow \mu^+ \mu^-) \sim (1.2 - 778.0) \times 10^{-9}$

PHYSICS LETTERS B 838, 137698 (2023)

Search for an axion-like particle in radiative J/ψ decays

We search for an axion-like particle (ALP) a through the process $\psi(3686) \rightarrow \pi^+ \pi^- J/\psi$, $J/\psi \rightarrow \gamma a$, $a \rightarrow \gamma\gamma$ in a data sample of $(2.71 \pm 0.01) \times 10^9$ $\psi(3686)$ events collected by the BESIII detector. No significant ALP signal is observed over the expected background, and the upper limits on the branching fraction of the decay $J/\psi \rightarrow \gamma a$ and the ALP-photon coupling constant $g_{a\gamma\gamma}$ are set at 95% confidence level in the mass range of $0.165 \leq m_a \leq 2.84$ GeV/ c^2 . The limits on $\mathcal{B}(J/\psi \rightarrow \gamma a)$ range from 8.3×10^{-8} to 1.8×10^{-6} over the search region, and the constraints on the ALP-photon coupling are the most stringent to date for $0.165 \leq m_a \leq 1.468$ GeV/ c^2 .

Future experiments in the search for dark matter

- Super τ - Charm Facility (STCF)

arXiv:2303.15790

Samples about 100 larger than the present largest τ -charm factory - BESIII

2.6	New light particles beyond the SM	52
2.6.1	Particles in the dark sector	52
2.6.2	Millicharged particles	54

- Belle II: The Belle II Physics Book

arXiv:1808.10567

16.3	Experiment: Quarkonium Decay	569
16.3.1	Searches for BSM physics in invisible $\Upsilon(1S)$ decays	569
16.3.2	Probe of new light CP even Higgs bosons from bottomonium χ_{b0} decay	571
16.3.3	Search for a CP -odd Higgs boson in radiative $\Upsilon(3S)$ decays	572

- LHCb II: Physics case for an LHCb Upgrade II

arXiv:1808.08865

8.6	Searches for prompt and detached dark photons	97
8.7	Searches for semileptonic and hadronic decays of long-lived particles	98

Standard model background in J/ψ invisible decay

Only after we exactly determine the standard model background can the precise experimental investigations of the $J/\psi \rightarrow \gamma + \text{invisible}$ decay possibly provide us rigorous constraints on new physics.

In the standard model, the background is

$$J/\psi \rightarrow \gamma + \nu + \bar{\nu}$$

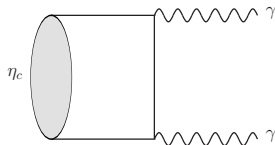
Since the neutrinos are only invisible particles

We provide an exact theoretical predication for the invisible decay.

The scalar function method has been widely and successfully applied to various physical processes

- **π -physics** $\pi^0 \rightarrow e^+e^-$ Norman Christ et al, PRL**130**,191901(2023)
 π mass splitting Xu Feng et al, PRL**128**,052003(2022)
 $0\nu 2\beta$ decay Xin-Yu Tuo et al, PRD**106**,074510(2022)
- **K -physics** K_{l4} decay Xin-Yu Tuo et al, PRD**105**,054508(2022)
 K_{l3} decay En-Hung Chao's talk, Aug 3rd, 13:30
 $K_L \rightarrow \mu^+\mu^-$ Norman Christ's talk, Aug 4th, 9:00
- **Charmonium physics** $\eta_c \rightarrow 2\gamma$ Yu Meng et al, Sci.Bull*(2023)
 $\chi_{c0} \rightarrow 2\gamma$ Chuan Liu et al, CPC**46**,053102(2022)
- **Nuclear physics**
Muonic-Hydrogen Lamb shift Yang Fu et al, PRL**128**,172002(2022)
Nucleon electromagnetic polarizability Xuan-He Wang's talk, Aug 4th, 9:00

Scalar function method: an example of $\eta_c \rightarrow 2\gamma$



- Amplitude:

$$\begin{aligned}\mathcal{M} &= e^2 \epsilon_\mu(p) \epsilon_\nu(p') \mathcal{F}_{\mu\nu}(p, q) \\ \mathcal{F}_{\mu\nu}(p, q) &= \int d^4 x e^{-ipx} \mathcal{H}_{\mu\nu}(x, q), \quad \mathcal{H}_{\mu\nu} = \langle 0 | \text{Tr}[J_\mu(x) J_\nu(0)] | \eta_c(q) \rangle\end{aligned}$$

- Form factor: $\mathcal{F}_{\mu\nu}(p, q) = \epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta F(p^2)$
- Decay width:

$$\Gamma_{\eta_c \gamma\gamma} = \alpha_{\text{em}}^2 \frac{\pi}{4} m_{\eta_c}^3 |F_{\eta_c \gamma\gamma}|^2, \quad F_{\eta_c \gamma\gamma} = F(0)$$

Scalar function method: an example of $\eta_c \rightarrow 2\gamma$

- Constructing a scalar function[infinite volume]

$$\begin{aligned}\mathcal{I} &\equiv \epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta \mathcal{F}_{\mu\nu}(p, q) \\ &= m_{\eta_c} \int dt e^{m_{\eta_c} t/2} \int d^3 \vec{x} e^{-i\vec{p}\cdot\vec{x}} \epsilon_{\mu\nu\alpha 0} \frac{\partial \mathcal{H}_{\mu\nu}(x, q)}{\partial x_\alpha}\end{aligned}$$

- Averaging the \vec{p} directions and projecting $|\vec{p}| = m_{\eta_c}/2$ [infinite volume]

$$\begin{aligned}F_{\eta_c \gamma\gamma} &\equiv \frac{\mathcal{I}}{[\epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta][\epsilon_{\mu\nu\rho\sigma} p_\rho q_\sigma]} \\ &= -\frac{1}{2m_{\eta_c}} \int dt e^{m_{\eta_c} t/2} \int d^3 \vec{x} \frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|} \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x, q)\end{aligned}$$

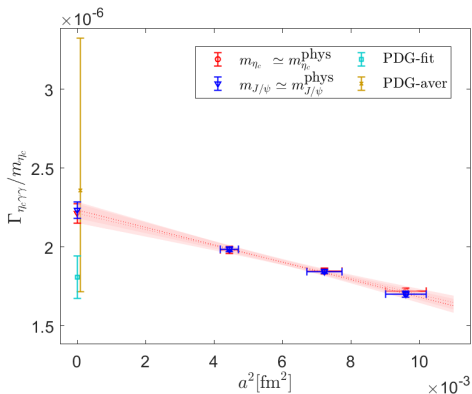
- Replaced by finite-volume version

$$\epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x, q) \rightarrow \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}^L(x, q)$$

- Exponentially suppressed with the distance

Scalar function method: an example of $\eta_c \rightarrow 2\gamma$

- NRQCD(NLO) ~ 6.2
K.-T.Chao et al, PRD56,368(1997)
- NRQCD(NNLO) ~ 10
F.Feng et al, PRL119,252001(2017)
- Lattice QCD $\sim 6.0(7)$
CLQCD(2020)
- DSE ~ 6.4
J.Chen et al, PRD95,016010(2017)



$$\Gamma(\eta_c \rightarrow 2\gamma) = \begin{cases} 6.67(16)(6) \text{ keV} \\ 5.4(4) \text{ keV} & \text{PDG-fit} \\ 7.04_{-1.9}^{+2.9} \text{ keV} & \text{PDG-aver} \end{cases}$$

YM et al, Sci Bull*(2023), 2109.09381

- HPQCD, $\Gamma_{\eta_c \gamma \gamma} = 6.788(45)_{\text{fit}}(41)_{\text{syst}}$ keV, PRD108,014513(2023),2305.06231

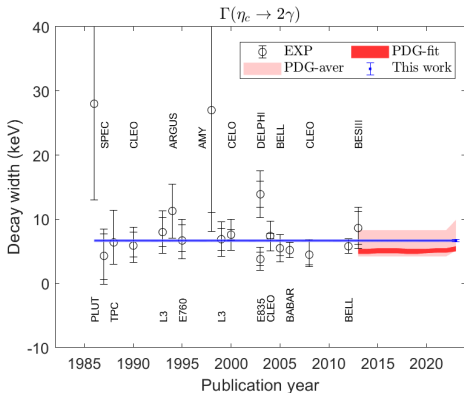
Discussion: $\eta_c \rightarrow 2\gamma$

- PDG-fit:
 - (i) Highly related with other channels
 - (ii) Large errors in CLEO and BESIII by $J/\psi \rightarrow \gamma\eta_c \rightarrow 3\gamma$

- Experiments: dependence on $J/\psi \rightarrow \gamma\eta_c$, $\mathcal{B}^{\text{PDG}} = 1.7(4)\%$

- $\mathcal{B}^{\text{lat}}(J/\psi \rightarrow \gamma\eta_c) = 2.31(13)\%$
[YM et al, to appear]

which is well consistent with other lattice calculations



- Combining the BESIII(13), it gives

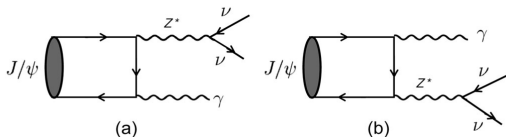
$$\Gamma(\eta_c \rightarrow 2\gamma) = 6.2(1.7)(0.9)\text{keV} \Leftarrow 8.64(2.56)(1.92)\text{keV} \text{ BESIII}$$

- A new BESIII result with no-dependence on $J/\psi \rightarrow \gamma\eta_c$ is coming soon.

$$J/\psi \rightarrow \gamma \nu \bar{\nu}$$

$J/\psi \rightarrow \gamma \nu \bar{\nu}$ decay from lattice QCD

$J/\psi \rightarrow \gamma\nu\bar{\nu}$: Formalism



- Amplitude

$$i\mathcal{M} = -i\frac{(q_c e)G_F}{\sqrt{2}} [H_{\mu\nu\alpha}(q, p)\epsilon_{J/\psi}^\alpha(p)\epsilon^{\nu*}(q)\bar{u}(q_1)\gamma^\mu(1-\gamma_5)v(q_2)]$$

- Hadronic function

$$H_{\mu\nu\alpha}(q, p) = \int d^4x e^{-iqx} \langle 0 | T \{ J_\mu^{\text{em}}(x) J_\nu^Z(0) \} | J/\psi(p)_\alpha \rangle$$

$$\equiv \epsilon_{\mu\nu\alpha\beta} q_\beta F_{\gamma\nu\bar{\nu}} \quad \text{PRD } \mathbf{90}, 077501(2014)$$

- Decay width

$$\Gamma(J/\psi \rightarrow \gamma\nu\bar{\nu}) = \frac{\alpha G_F^2}{3\pi^2} \int_0^{\frac{m_{J/\psi}}{2}} |\vec{q}|^3 (m_{J/\psi} - |\vec{q}|) |F_{\gamma\nu\bar{\nu}}|^2 d|\vec{q}|$$

Hadronic function in Minkowski and Euclidean space

- Minkowski

$$H_{\mu\nu\alpha}(q, p) = i \sum_{n, \vec{q}} \frac{1}{E_\gamma - E_n + i\epsilon} \langle 0 | J_\mu^{\text{em}}(0) | n(\vec{q}) \rangle \langle n(\vec{q}) | J_\nu^Z(0) | J/\psi(p) \rangle_\alpha$$
$$- i \sum_{n', \vec{q}} \frac{1}{E_\gamma + E_{n'} - m_{J/\psi} - i\epsilon} \langle 0 | J_\nu^Z(0) | n'(-\vec{q}) \rangle \langle n'(-\vec{q}) | J_\mu^{\text{em}}(0) | J/\psi(p) \rangle_\alpha$$

- Euclidean

$$H_{\mu\nu\alpha}^E(q, p) = i \sum_{n, \vec{q}} \frac{1 - e^{-(E_n - E_\gamma)T/2}}{E_\gamma - E_n + i\epsilon} \langle 0 | J_\mu^{\text{em}}(0) | n(\vec{q}) \rangle \langle n(\vec{q}) | J_\nu^Z(0) | J/\psi(p) \rangle_\alpha$$
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- A naive relation requires

- $\delta E_n \equiv E_n - E_\gamma > 0$
- $\delta E_{n'} \equiv E_\gamma + E_{n'} - m_{J/\psi} > 0$

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- A naive relation requires

- $\delta E_n \equiv E_n - E_\gamma > 0$ $|n\rangle \underline{\underline{=}}_{\text{lowest}} |J/\psi\rangle$
- $\delta E_{n'} \equiv E_\gamma + E_{n'} - m_{J/\psi} > 0$ $|n'\rangle \underline{\underline{=}}_{\text{lowest}} |\eta_c\rangle$

Note: a general approach to exponentially growing term

Norman H. Christ et al, PRD 103,014507(2021)

$N_f = 2$ twisted mass ensembles

Ens	a (fm)	$L^3 \times T$	$N_{\text{conf}} \times T$	Δt	Z_V	Z_A
a67	0.0667(20)	$32^3 \times 64$	197×64	12-18	0.6516(15)	0.772(06)
a85	0.085(2)	$24^3 \times 48$	200×48	10-14	0.6257(21)	0.746(06)
a98	0.098(3)	$24^3 \times 48$	236×48	9-13	0.6047(19)	0.746(11)

- All ensembles have similar physical spatial volume: $2.04 \sim 2.35$ fm.
- All ensembles have similar pion mass: $300 \sim 360$ MeV.
- A series of Δt to extract the ground-state contribution.
- Charm quark mass is tuned by physical J/ψ mass.
- Connected diagram only.

Energy levels of η_c

Ensemble	a67	a85	a98
$aE_{\eta_c}(\vec{n} ^2 = 0)$	1.0142(2)	1.2958(3)	1.4995(3)
$aE_{\eta_c}(\vec{n} ^2 = 1)$	1.0302(2)	1.3157(3)	1.5144(4)
$aE_{\eta_c}(\vec{n} ^2 = 2)$	1.0467(2)	1.3354(3)	1.5290(4)
$aE_{\eta_c}(\vec{n} ^2 = 3)$	1.0629(3)	1.3546(4)	1.5434(4)
$aE_{\eta_c}(\vec{n} ^2 = 4)$	1.0782(4)	1.3729(5)	1.5572(5)
$a\delta E_{\eta_c}(\vec{n} ^2 = 0)$	-0.0343(2)	-0.0372(3)	-0.0387(3)
$a\delta E_{\eta_c}(\vec{n} ^2 = 1)$	0.1781(2)	0.2446(3)	0.2380(4)
$a\delta E_{\eta_c}(\vec{n} ^2 = 2)$	0.2758(3)	0.3737(3)	0.3611(4)
$a\delta E_{\eta_c}(\vec{n} ^2 = 3)$	0.3544(3)	0.4751(4)	0.4587(4)
$a\delta E_{\eta_c}(\vec{n} ^2 = 4)$	0.4223(4)	0.5636(5)	0.5426(5)

- Numerical results: $\delta E_{\eta_c}(\vec{n} \neq 0) > 0$
- Note that $\langle 0 | J_\nu^Z(0) | \eta_c(\vec{0}) \rangle \langle \eta_c(\vec{0}) | J_\mu^{\text{em}}(0) | J/\psi(\vec{0})_\alpha \rangle = 0$

$$\sum_{\vec{q}} e^{-(E_\gamma + E_{n'} - m_{J/\psi})T/2} \langle 0 | J_\nu^Z(0) | n'(-\vec{q}) \rangle \langle n'(-\vec{q}) | J_\mu^{\text{em}}(0) | J/\psi(p)_\alpha \rangle \xrightarrow{T \rightarrow \infty} 0$$

Hadronic function in Minkowski and Euclidean space

- Minkowski

$$H_{\mu\nu\alpha}(q, p) = i \sum_{n, \vec{q}} \frac{1}{E_\gamma - E_n + i\epsilon} \langle 0 | J_\mu^{\text{em}}(0) | n(\vec{q}) \rangle \langle n(\vec{q}) | J_\nu^Z(0) | J/\psi(p) \rangle_\alpha$$

$$- i \sum_{n', \vec{q}} \frac{1}{E_\gamma + E_{n'} - m_{J/\psi} - i\epsilon} \langle 0 | J_\nu^Z(0) | n'(-\vec{q}) \rangle \langle n'(-\vec{q}) | J_\mu^{\text{em}}(0) | J/\psi(p) \rangle_\alpha$$

- Euclidean

$$H_{\mu\nu\alpha}^E(q, p) = i \sum_{n, \vec{q}} \frac{1 - e^{-(E_n - E_\gamma)T/2}}{E_\gamma - E_n + i\epsilon} \langle 0 | J_\mu^{\text{em}}(0) | n(\vec{q}) \rangle \langle n(\vec{q}) | J_\nu^Z(0) | J/\psi(p) \rangle_\alpha$$

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- **A naive relation** requires

- $\delta E_n \equiv E_n - E_\gamma > 0$ $|n\rangle \underline{\underline{\text{lowest}}}$ $|J/\psi\rangle$
- $\delta E_{n'} \equiv E_\gamma + E_{n'} - m_{J/\psi} > 0$ $|n'\rangle \underline{\underline{\text{lowest}}}$ $|\eta_c\rangle$

- Form factor

$$\begin{aligned} F_{\gamma \nu \bar{\nu}}(E_\gamma, \Delta t) &= \frac{1}{6p \cdot q} \epsilon_{\mu\nu\alpha\beta} p_\beta H_{\mu\nu\alpha}(q, p) \\ &= -\frac{i}{6E_\gamma} \int e^{E_\gamma t} dt \int d^3 \vec{x} j_0(E_\gamma |\vec{x}|) \epsilon_{\mu\nu\alpha 0} \mathcal{H}_{\mu\nu\alpha}(x, \Delta t) \end{aligned}$$

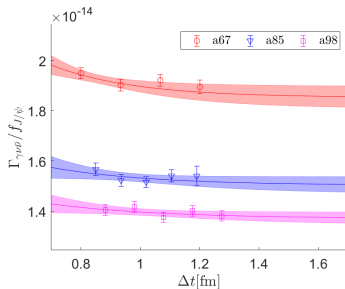
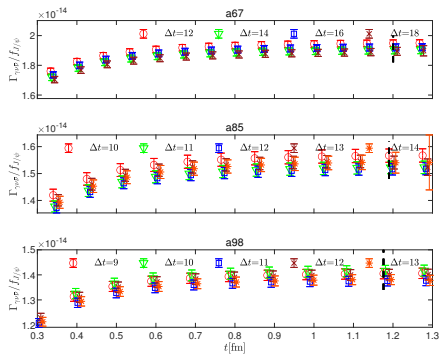
- Decay width evaluated by a Monte-Carlo method

$$\Gamma_{\gamma \nu \bar{\nu}}(\Delta t) = \frac{\alpha G_F^2}{3\pi^2} \frac{m_{J/\psi}}{2N_{MC}} \sum_{i=1}^{N_{MC}} \left(E_\gamma^3 (m_{J/\psi} - E_\gamma) |F_{\gamma \nu \bar{\nu}}(E_\gamma, \Delta t)|^2 \right)_i$$

- Dimensionless quantity $R_f \equiv \Gamma_{\gamma \nu \bar{\nu}}/f_{J/\psi}$ and Δt dependence

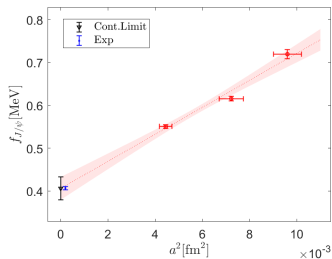
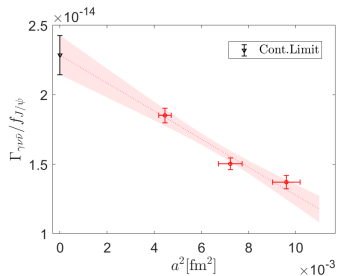
$$R_f(\Delta t) = R_f + \zeta \cdot e^{-(m_{J/\psi}^{(1)} - m_{J/\psi})\Delta t}$$

Excited-state contamination



- A slight dependence on the excited-state of J/ψ (Δt -dependence).
- Dashed black line: a suitable time truncation $t_{\text{cut}} \sim 1.2$ fm.
- The right: an extrapolation for Δt at t_{cut} .

Continuous limit



- The first lattice QCD calculation YM et al, in preparation

$$\text{Br}[J/\psi \rightarrow \gamma\nu\bar{\nu}] = 1.00(9)(7) \times 10^{-10}$$

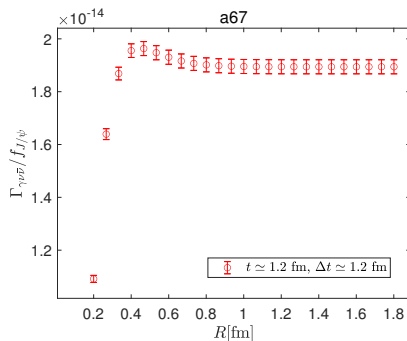
- Phenomenological estimation $\sim 0.7 \times 10^{-10}$

Dao-Neng Gao, PRD 90,077501(2014)

Finite-volume effects

- Examining the finite-volume effects ($R \equiv |\vec{x}|$)

$$F_{\gamma\nu\bar{\nu}}(E_\gamma, \Delta t) = -\frac{i}{6E_\gamma} \int e^{E_\gamma t} dt \int_0^R d^3\vec{x} j_0(E_\gamma|\vec{x}|) \epsilon_{\mu\nu\alpha 0} \mathcal{H}_{\mu\nu\alpha}(x, \Delta t)$$



Discussion: next generation experiments

- Dark matter search by $J/\psi \rightarrow \gamma + \text{invisible}$ [STCF]
 - STCF has the potential to improve the upper limit to 10^{-8} , even possibly 10^{-9} level.
 - Standard model background $J/\psi \rightarrow \gamma\nu\bar{\nu}$ is calculated on the lattice with a branching fraction of $1.00(9)(7) \times 10^{-10}$.
- Dark matter search by $\Upsilon(1S) \rightarrow \gamma + \text{invisible}$ [Belle II]
 - The design luminosity of Belle II is 80 times larger than Belle, which has an upper limit of 10^{-6} .
 - Naive phenomenological estimation $\mathcal{B}(\Upsilon(1S) \rightarrow \gamma\nu\bar{\nu}) \sim 10^{-9}$ considering $\mathcal{B} \propto M^2$.
 - No lattice calculation on $\Upsilon(1S) \rightarrow \gamma\nu\bar{\nu}$.

Summary

- We present first lattice calculation on the invisible decay $J/\psi \rightarrow \gamma\nu\bar{\nu}$ using a scalar function method.
- Various systematics are examined, including finite-volume effects, excited-state contamination, and discretization effects.
- **An exact branching fraction $\text{Br}[J/\psi \rightarrow \gamma\nu\bar{\nu}] = 1.00(9)(7) \times 10^{-10}$** is determined, providing theoretical support for the dark matter search in future experiments.
- The present and near future experiments fail to reach the upper limit of the standard model background, either for $J/\psi \rightarrow \gamma\nu\bar{\nu}$ or $\Upsilon(1S) \rightarrow \gamma\nu\bar{\nu}$.

Thank you for attention!