Form factors for the charm-baryon semileptonic decay $\Xi_c \to \Xi \ell \nu$ from domain-wall lattice QCD

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$\Xi_c^0(dsc) \to \Xi^-(dss)\ell^+\nu_\ell$: Recent experimental progress

In 2018:

$$B_{\text{Belle}}(\Xi_c^0 \to \Xi^-\pi^+) = (1.80 \pm 0.50 \pm 0.14)\%$$

In 2021:

$$\frac{B_{\text{Belle}}(\Xi_c^0 \to \Xi^- e^+\nu_e)}{B_{\text{Belle}}(\Xi_c^0 \to \Xi^-\pi^+)} = (0.730 \pm 0.021 \pm 0.039)\%$$

$$\frac{B_{\text{ALICE}}(\Xi_c^0 \to \Xi^- e^+\nu_e)}{B_{\text{ALICE}}(\Xi_c^0 \to \Xi^-\pi^+)} = (1.38 \pm 0.14 \pm 0.22)\%$$

Giving the absolute branching fractions:

$$B_{\text{Belle}}(\Xi_c^0 \to \Xi^- e^+\nu_e) = (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\%$$

$$B_{\text{ALICE}}(\Xi_c^0 \to \Xi^- e^+\nu_e) = (2.48 \pm 0.25 \pm 0.40 \pm 0.72)\%$$

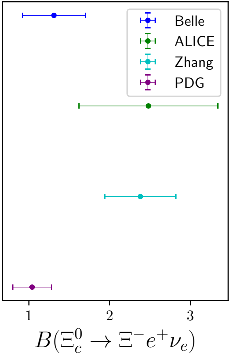
Computed on the lattice: [Zhang et al., 2103.07064, Chin. Phys. C, 2022]:

$$B_{\text{Lattice}}(\Xi_c^0 \to \Xi^- e^+\nu_e) = (2.38 \pm 0.30 \pm 0.32)\%$$

and reported by the PDG:

$$B_{\text{PDG}}(\Xi_c^0 \to \Xi^- e^+\nu_e) = (1.04 \pm 0.24)\%!?$$

(Unclear exactly how this number is obtained by PDG.)



Model-dependent Calculations of the Branching Ratio

A (probably incomplete) list:

	Method	$B(\Xi_c^0 \to \Xi^- e^+ \nu_e)$
Geng, Liu and Tsai, 2012.04147, Phys.Rev.D 103 (2021)	light-front quark model	$(3.49 \pm 0.95)\%$
Zhao, 2103.09436, (2021)	QCD sum rules	$(3.4 \pm 1.7)\%$
Faustov and Galkin, 1905.08652, Eur.Phys.J.C 79 (2019)	rel. quark model	$(2.38)\%^*$
Geng et al., 1901.05610, Phys.Lett.B 792 (2019)	SU(3)	$(3.0 \pm 0.3)\%^*$
Zhao, 1803.02292, Chin.Phys.C 42 (2018)	light-front quark model	$(1.35)\%^*$
Geng et al., 1801.03276, Phys.Rev.D 97 (2018)	SU(3)	$(4.87 \pm 1.74)\%^*$
Geng et al., 1709.00808, JHEP 11 (2017)	SU(3)	$(11.9 \pm 1.6)\%^*$
Azizi, Sarac and Sundu, 1107.5925, Eur.Phys.J.A (2012)	light-cone QCD sum rules	$(7.26 \pm 2.54)\%^*$
Liu and Huang, 1102.4245, J.Phys.G:Nucl.Part.Phys (2010)	QCD sum rules	$(2.4)\%^*$

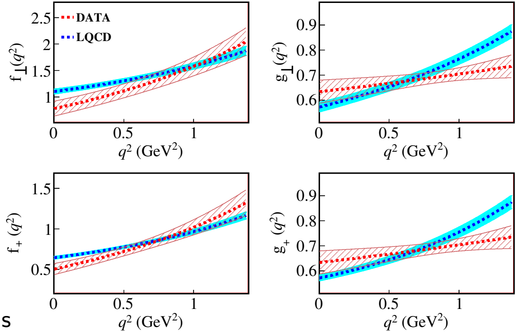
$(*) \to$ Calculation used substantially different Ξ_c^0 lifetime

- Pre-2019 PDG value: $\tau_{\Xi_c^0} = (112 \pm 13)$ fs
- [2109.01334, LHCb-PAPER-2021-021]: $\tau_{\Xi_c^0} = (152.0 \pm 2.0)$ fs

	$B(\Xi_c^0 \to \Xi^- e^+ \nu_e)$
Belle	$(1.31 \pm 0.39)\%$
ALICE	$(2.48 \pm 0.86)\%$
Zhang et al.	$(2.38 \pm 0.44)\%$

Why do we care?

- Flavor physics?
  $\Xi_c^0 \to \Xi^- \ell^+ \nu_\ell$ is not particularly relevant, but it is affected by the same systematics as more important calculations!
- $\Lambda_c^+ \to \Lambda e^+ \nu_e$
  - Recent BES III measurement of the total decay rate is in good agreement with the previous lattice calculation [Meinel, 1611.09696, Phys.Rev.Lett. 118 (2017)]
  - Some deviations present in form factor slopes



[BESIII, 2306.02624, (2023)]

- Test model-dep. calculations
- $\Xi_c - \Xi_c'$ mixing and $SU(3)$ expectations:
  - Surprisingly high $SU(3)$ symmetry breaking in $B(\Xi_c^0 \to \Xi^- e^+ \nu_e)$? [He et al., 2110.04179, Phys.Lett.B (2021)]
  - Can be resolved with large $\Xi_c - \Xi_c'$ mixing angle [Geng, Jin and Liu, 2210.07211, Phys.Lett. B (2023)]
  - Lattice calculation suggest a negligible mixing angle [Liu et al., 2303.17865, Phys.Lett.B 841 (2023)]

Form Factor Definitions

The weak effective Hamilitonian for the $c \to s$ decays is given by:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cs}^* [\overline{s}\gamma^\mu (1-\gamma_5) c \; \overline{\nu}_\ell \gamma_\mu (1-\gamma_5)\ell] + h.c.$$

We use the helicity based definition of the form factors [Feldmann and Yip, 1111.1844, Phys. Rev. D, 2012]:

$$\begin{aligned}
\langle \Xi(p',s')|\overline{s}\,\gamma^\mu\, c|\Xi_c(p,s)\rangle =& \overline{u}_\Xi(p',s')\Big[f_0(q^2)\,(m_{\Xi_c}-m_\Xi)\frac{q^\mu}{q^2} \\
&+ f_+(q^2)\frac{m_{\Xi_c}+m_\Xi}{s_+}\left(p^\mu + p'^\mu - (m_{\Xi_c}^2 - m_\Xi^2)\frac{q^\mu}{q^2}\right) \\
&+ f_\perp(q^2)\left(\gamma^\mu - \frac{2m_\Xi}{s_+}p^\mu - \frac{2m_{\Xi_c}}{s_+}p'^\mu\right)\Big] u_{\Xi_c}(p,s), \\
\langle \Xi(p',s')|\overline{s}\,\gamma^\mu\gamma_5\, c|\Xi_c(p,s)\rangle =& -\overline{u}_\Xi(p',s')\,\gamma_5 \Big[g_0(q^2)\,(m_{\Xi_c}+m_\Xi)\frac{q^\mu}{q^2} \\
&+ g_+(q^2)\frac{m_{\Xi_c}-m_\Xi}{s_-}\left(p^\mu + p'^\mu - (m_{\Xi_c}^2 - m_\Xi^2)\frac{q^\mu}{q^2}\right) \\
&+ g_\perp(q^2)\left(\gamma^\mu + \frac{2m_\Xi}{s_-}p^\mu - \frac{2m_{\Xi_c}}{s_-}p'^\mu\right)\Big] u_{\Xi_c}(p,s)
\end{aligned}$$

The Zhang et al. Calculation

From [Zhang et al., 2103.07064, Chin. Phys. C, 2022]:

TABLE I. Parameters of the 2+1 flavor clover fermion ensembles used in this calculation. The π/η_s masses and the lattice spacings are given in units of MeV, and fm, respectively.

	$\beta = \frac{10}{g^2}$	$L^3 \times T$	a	$c_{\rm sw}$	κ_l	m_π	κ_s	m_{η_s}
s108	6.20	$24^3 \times 72$	0.108	1.161	0.1343	290	0.1330	640
s080	6.41	$32^3 \times 96$	0.080	1.141	0.1326	300	0.1318	650

Charm quark masses: $m_c^{s108} a = 0.485$ and $m_c^{s080} a = 0.235$.

Lattice Setup

We use ensembles generated by the RBC and UKQCD collaborations:

Label	$N_s^3 \times N_t \times N_5$	a [fm]	$am_{u,d}$	$am_s^{\text{(sea)}}$	$am_s^{\text{(val)}}$	m_π [MeV]	N_{meas}
C01	$24^3 \times 64 \times 16$	0.111	0.01	0.04	0.0323	420	2264 sl, 283 ex
C005	$24^3 \times 64 \times 16$	0.111	0.005	0.04	0.0323	340	2488 sl, 311 ex
F004	$32^3 \times 64 \times 16$	0.083	0.004	0.03	0.0248	300	2008 sl, 251 ex
F1M	$48^3 \times 96 \times 12$	0.073	0.002144	0.02144	0.02217	230	1808 sl, 113 ex

- The ensembles were generated with an Iwasaki action for the gauge fields and a 2+1 flavor domain-wall action for the fermion fields.
- On F1M, a Möbius domain-wall action is used, on all other ensembles a Shamir domain-wall action is used for both the sea and valence quarks.
- For the light quarks, the sea and valence quark masses are identical. The valence strange quark masses are tuned to the physical value and therefore slightly different.
- All-mode-averaging is used on every ensemble. [Shintani et al., 1402.0244, Phys.Rev.D 91 (2015)]

Heavy Quark Action

For the charm quark we use an anisotropic clover action:

$$S_Q = a^4 \sum_x \bar{Q} \left[m_Q + \gamma_0 \nabla_0 - \frac{a}{2} \nabla_0^{(2)} + \nu \sum_{i=1}^{3} \left(\gamma_i \nabla_i - \frac{a}{2} \nabla_i^{(2)} \right) - c_E \frac{a}{2} \sum_{i=1}^{3} \sigma_{0i} F_{0i} - c_B \frac{a}{4} \sum_{i,\, j=1}^{3} \sigma_{ij} F_{ij} \right] Q$$

We tune the parameters to match the experimental values of the D_s meson rest mass, kinetic mass and hyperfine splitting.

Ensemble	$am_Q^{(c)}$	$\nu^{(c)}$	$c_{E,B}^{(c)}$
C005, C01	0.15410	1.2004	1.8407
F004	−0.05167	1.1021	1.4483
F1M	−0.05874	1.0941	1.5345

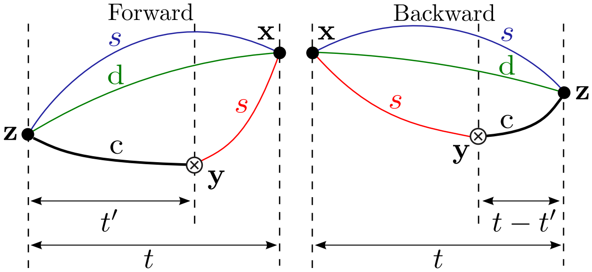
Current Renormalization:

$$
\begin{aligned}
V_0 =& \sqrt{Z_V^{(ss)} Z_V^{(cc)}}\, \rho_{V_0} \left[\bar{s}\gamma_0 c + 2a \left(c^R_{V_0}\ \bar{s}\gamma_0\gamma_j \overrightarrow{\nabla}_j c + c^L_{V_0}\ \bar{s}\overleftarrow{\nabla}_j \gamma_0\gamma_j c \right) \right], \\
A_0 =& \sqrt{Z_V^{(ss)} Z_V^{(cc)}}\, \rho_{A_0} \left[\bar{s}\gamma_0\gamma_5 c + 2a \left(c^R_{A_0}\ \bar{s}\gamma_0\gamma_5\gamma_j \overrightarrow{\nabla}_j c + c^L_{A_0}\ \bar{s}\overleftarrow{\nabla}_j \gamma_0\gamma_5\gamma_j c \right) \right], \\
V_i =& \sqrt{Z_V^{(ss)} Z_V^{(cc)}}\, \rho_{V_i} \Big[\bar{s}\gamma_i c + 2a \Big(c^R_{V_i}\ \bar{s}\gamma_i\gamma_j \overrightarrow{\nabla}_j c + c^L_{V_i}\ \bar{s}\overleftarrow{\nabla}_j \gamma_i\gamma_j c \\
& \qquad + d^R_{V_i}\ \bar{s}\overrightarrow{\nabla}_i c + d^L_{V_i}\ \bar{s}\overleftarrow{\nabla}_i c \Big) \Big] \\
A_i =& \sqrt{Z_V^{(ss)} Z_V^{(cc)}}\, \rho_{A_i} \Big[\bar{s}\gamma_i\gamma_5 c + 2a \Big(c^R_{A_i}\ \bar{s}\gamma_i\gamma_5\gamma_j \overrightarrow{\nabla}_j c + c^L_{A_i}\ \bar{s}\overleftarrow{\nabla}_j \gamma_i\gamma_5\gamma_j c \\
& \qquad + d^R_{A_i}\ \bar{s}\gamma_5 \overrightarrow{\nabla}_i c + d^L_{A_i}\ \bar{s}\overleftarrow{\nabla}_i \gamma_5 c \Big) \Big],
\end{aligned}
$$

We use a mostly nonperturbative method to compute $Z_V^{(cc)}$. $Z_V^{(ss)}$ is taken from RBC/UKQCD [RBC/UKQCD Collab., [1011.0892], Phys.Rev.D 83 (2011)]. The residual matching factors ρ_Γ and $\mathcal{O}(a)$ improvement terms are computed by at 1-loop using lattice perturbation theory.

(See preceding talk by Stefan Meinel for more details.)

Correlation functions



$$C^{(3,\mathrm{fw})}_{\delta\alpha}(\Gamma,\mathbf{p}',t,t') = \sum_{\mathbf{y},\mathbf{z}} e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})}\left\langle \Xi_\delta(x_0,\mathbf{x})\ J^\dagger_\Gamma(x_0-t+t',\mathbf{y})\ \overline{\Xi}_{c\alpha}(x_0-t,\mathbf{z})\right\rangle$$

$$C^{(3,\mathrm{bw})}_{\delta\alpha}(\Gamma,\mathbf{p}',t,t-t') = \sum_{\mathbf{y},\mathbf{z}} e^{-i\mathbf{p}'\cdot(\mathbf{y}-\mathbf{x})}\left\langle \Xi_{c\delta}(x_0+t,\mathbf{z})\ J_\Gamma(x_0+t',\mathbf{y})\ \overline{\Xi}_{\alpha}(x_0,\mathbf{x})\right\rangle$$

For $J^P = \frac{1}{2}^+$ we use the interpolating fields operators:

$$\Xi_{c\alpha} = \epsilon_{abc}(C\gamma_5)_{\beta\gamma}u^a_\beta s^b_\gamma c^c_\alpha \qquad \Xi_{\alpha} = \epsilon_{abc}(C\gamma_5)_{\beta\gamma}u^a_\beta s^b_\gamma s^c_\alpha$$

Diagram modified from [Detmold, Lehner and Meinel, 1503.01421, Phys. Rev. D (2015)].

Extracting the form factors:

The form factors can be isolated from the overlap factors and exponential time-dependence of the ground-state by taking ratios of correlation functions:

$$\mathscr{R}_{+}^{V}(\mathbf{p}', t, t') = r_\mu[(1,\mathbf{0})]\, r_\nu[(1,\mathbf{0})] \frac{\mathrm{Tr}\left[C^{(3,\mathrm{fw})}(\mathbf{p}',\ \gamma^\mu, t, t')\, C^{(3,\mathrm{bw})}(\mathbf{p}',\ \gamma^\nu, t, t-t')\right]}{\mathrm{Tr}\left[C^{(2,\Xi,\mathrm{av})}(\mathbf{p}', t)\right] \mathrm{Tr}\left[C^{(2,\Xi_c,\mathrm{av})}(t)\right]}$$

$$\mathscr{R}_{\perp}^{V}(\mathbf{p}', t, t') = r_\mu[(0, \mathbf{e}_j \times \mathbf{p}')] r_\nu[(0, \mathbf{e}_k \times \mathbf{p}')] \times \frac{\mathrm{Tr}\left[C^{(3,\mathrm{fw})}(\mathbf{p}',\ \gamma^\mu, t, t')\gamma_5\gamma^j\, C^{(3,\mathrm{bw})}(\mathbf{p}',\ \gamma^\nu, t, t-t')\gamma_5\gamma^k\right]}{\mathrm{Tr}\left[C^{(2,\Xi,\mathrm{av})}(\mathbf{p}', t)\right] \mathrm{Tr}\left[C^{(2,\Xi_c,\mathrm{av})}(t)\right]}$$

$$\mathscr{R}_{0}^{V}(\mathbf{p}', t, t') = q_\mu q_\nu \frac{\mathrm{Tr}\left[C^{(3,\mathrm{fw})}(\mathbf{p}',\ \gamma^\mu, t, t')\, C^{(3,\mathrm{bw})}(\mathbf{p}',\ \gamma^\nu, t, t-t')\right]}{\mathrm{Tr}\left[C^{(2,\Xi,\mathrm{av})}(\mathbf{p}', t)\right] \mathrm{Tr}\left[C^{(2,\Xi_c,\mathrm{av})}(t)\right]}$$

To obtain definite helicities we contract with virtual polarization vectors, where

$$r[n] = n - \frac{(q \cdot n)}{q^2} q$$

[Detmold, Lehner and Meinel, 1503.01421, Phys. Rev. D (2015)].

Extracting the form factors:

The form factors can be isolated from the overlap factors and exponential time-dependence of the ground-state by taking ratios of correlation functions:

$$\mathscr{R}_{+}^{V}(\mathbf{p}', t, t') = \frac{(E_{\Xi} - m_{\Xi})^2 (E_{\Xi} + m_{\Xi}) \Big[m_{\Xi_c} (m_{\Xi_c} + m_{\Xi}) f_{+} \Big]^2}{4\, m_{\Xi_c}^2\, E_{\Xi}\, q^4} + (\text{excited-state contributions}),$$

$$\mathscr{R}_{\perp}^{V}(\mathbf{p}', t, t') = \frac{(E_{\Xi} - m_{\Xi})^2 (E_{\Xi} + m_{\Xi}) \Big[m_{\Xi_c} f_{\perp} \Big]^2}{m_{\Xi_c}^2\, E_{\Xi}} + (\text{excited-state contributions}),$$

$$\mathscr{R}_{0}^{V}(\mathbf{p}', t, t') = \frac{(E_{\Xi} + m_{\Xi}) \Big[m_{\Xi_c} (m_{\Xi_c} - m_{\Xi}) f_{0} \Big]^2}{4 E_{\Xi} m_{\Xi_c}^2} + (\text{excited-state contributions}).$$

[Detmold, Lehner and Meinel, 1503.01421, Phys. Rev. D (2015)].

Extracting the form factors:

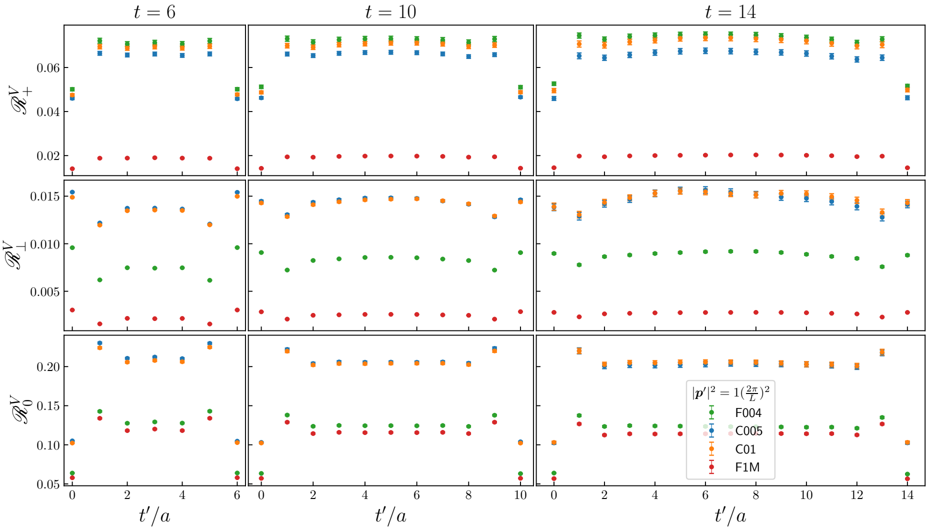
The form factors can be isolated from the overlap factors and exponential time-dependence of the ground-state by taking ratios of correlation functions:

$$
\begin{aligned}
R_{f_+}(|\mathbf{p}'|,t) =& \frac{2\,q^2}{(E_\Xi - m_\Xi)(m_{\Xi_c} + m_\Xi)}\sqrt{\frac{E_\Xi}{E_\Xi + m_\Xi}\mathscr{R}_+^V(|\mathbf{p}'|,t,t/2)} \\
=& f_+ + (\text{excited-state contributions}) \\
R_{f_\perp}(|\mathbf{p}'|,t) =& \frac{1}{E_\Xi - m_\Xi}\sqrt{\frac{E_\Xi}{E_\Xi + m_\Xi}\mathscr{R}_\perp^V(|\mathbf{p}'|,t,t/2)} \\
=& f_\perp + (\text{excited-state contributions}) \\
R_{f_0}(|\mathbf{p}'|,t) =& \frac{2}{m_{\Xi_c} - m_\Xi}\sqrt{\frac{E_\Xi}{E_\Xi + m_\Xi}\mathscr{R}_0^V(|\mathbf{p}'|,t,t/2)} \\
=& f_0 + (\text{excited-state contributions})
\end{aligned}
$$

Where we have evaluated $\mathscr{R}^V(\mathbf{p}', t, t' = t/2)$, to remove excited-state contamination.
[Detmold, Lehner and Meinel, 1503.01421, Phys. Rev. D (2015)].

Extracting the form factors:

The form factors can be isolated from the overlap factors and exponential time-dependence of the ground-state by taking ratios of correlation functions:

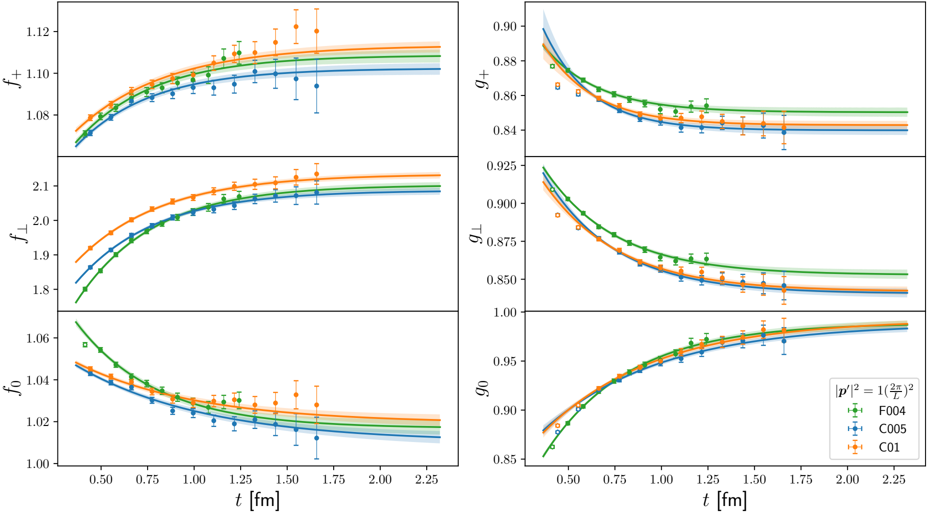


[Detmold, Lehner and Meinel, 1503.01421, Phys. Rev. D (2015)].

Fitting the ratios

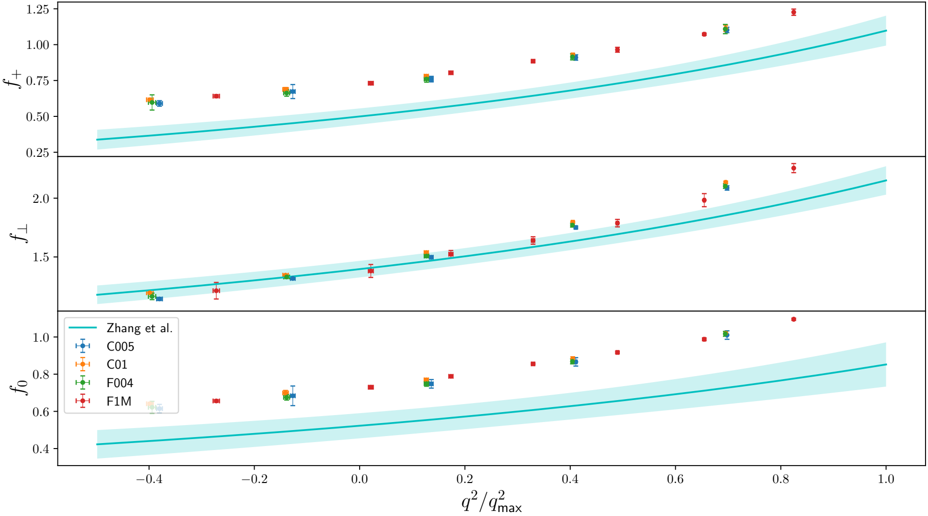
To obtain the form factors at infinite source-sink separation, we fit:

$$R_f(t) = f + A_f\, e^{-\delta_f\, t}, \quad \delta_f = \delta_{\min} + e^{\,l_f} \;\text{GeV}, \quad \delta_{\min} = 100\;\text{MeV}$$

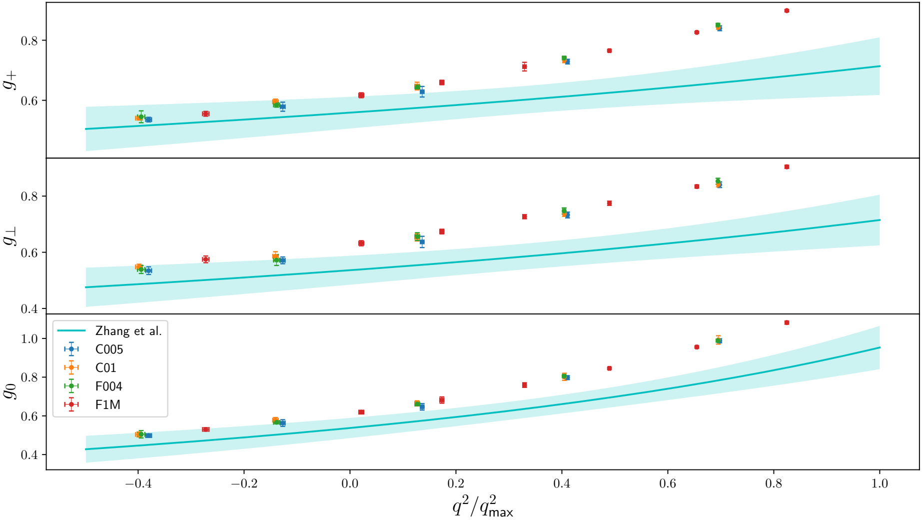


Vector Form Factors

We have not yet performed our chiral/continuum extrapolation. Here we compare our data points to the Zhang et al. extrapolated form factors.



Axial-Vector Form Factors



Conclusions

- We expect our extrapolation to stay close to the data, and therefore be in slight tension with the Zhang et al results.
- Our preliminary "estimates" of the branching ratio are substantially higher than current experimental results:

	$B(\Xi_c^0 \to \Xi^- e^+ \nu_e)$
Belle	$(1.31 \pm 0.39)\%$
ALICE	$(2.48 \pm 0.86)\%$
Zhang et al.	$(2.38 \pm 0.44)\%$

Thank you!