Precision Determination of Baryon Masses including Isospin-breaking

Alexander Segner
Andreas Risch, Hartmut Wittig

July 31, 2023

Motivation

- Increased interest in high-precision observables
- Discrepancy between theory and experiment on $a_\mu = \frac{(g-2)_\mu}{2}$
- Theoretical uncertainty is dominated by QCD
  $\rightarrow$ Strongly influenced by the uncertainty of the lattice scale
- Scale setting on CLS ensembles currently done using f_π and f_K [1]
  $\rightarrow$ Difficult to determine IB corrections[2] reliably
  $\rightarrow$ Use baryon masses instead

[1] Bruno et al. 2017, *Phys. Rev. D* **95** no. 7, p. 074504.
[2] Carrasco et al. 2015, *Phys. Rev. D* **91** no. 7, p. 074506.

Operator Basis

- Construction based on isospin-symmetric QCD following a procedure introduced by the *Lattice Hadron Physics Collaboration*[3]
- Classification by symmetries in flavor and spin (and parity eigenvalues)
- Example for symmetric spin and flavor indices (e.g. $\Omega^-_{ijk} = s_i s_j s_k$, $\Delta^+_{ijk} = \frac{1}{\sqrt{3}}(u_i u_j d_k + u_i d_j u_k + d_i u_j u_k)$, etc.)
(Operators in Dirac-Pauli basis)

Embedding	S_z	gerade (even)	ungerade (odd)
1	$\frac{3}{2}$	Ω_{111}	$\sqrt{3}\Omega_{113}$
1	$\frac{1}{2}$	$\sqrt{3}\Omega_{112}$	$\Omega_{114} + 2\Omega_{123}$
1	$-\frac{1}{2}$	$\sqrt{3}\Omega_{122}$	$2\Omega_{124} + \Omega_{223}$
1	$-\frac{3}{2}$	Ω_{222}	$\sqrt{3}\Omega_{224}$
2	$\frac{3}{2}$	$\sqrt{3}\Omega_{133}$	Ω_{333}
2	$\frac{1}{2}$	$2\Omega_{134} + \Omega_{233}$	$\sqrt{3}\Omega_{334}$
2	$-\frac{1}{2}$	$\Omega_{144} + 2\Omega_{234}$	$\sqrt{3}\Omega_{344}$
2	$-\frac{3}{2}$	$\sqrt{3}\Omega_{244}$	Ω_{444}

H irrep for symmetric spin and flavor indices

[3] Basak et al. 2005, *Phys. Rev. D* **72**, p. 074501.

Simulation Setup

- Setup:
  - $\mathcal{O}(a)$-improved Wilson fermions
  - Tree-level Lüscher–Weisz gauge action
  - Interpolators as described before
  - Wuppertal smeared point sources with smearing radius $\sim 0.5\,\text{fm}$
  - APE smeared gauge links

- Correlation functions averaged over
  - Different S_z
  - Forward propagator and backwards parity partner

  e.g. From previous table Ω_{111}, $\sqrt{3}\Omega_{112}$, $\sqrt{3}\Omega_{122}$, Ω_{222}, and the time reversed Ω_{333}, $\sqrt{3}\Omega_{334}$, $\sqrt{3}\Omega_{344}$, Ω_{444} can be combined

- Have access to correlator matrices allowing for GEVP
  - $\rightarrow$ Found that one operator is much less noisy than the others for each state
  - $\rightarrow$ GEVP mostly projects on least noisy correlator
  - $\rightarrow$ Focus on correlation function of operator that gives the best signal

QCD+QED vs. QCD$_{\text{iso}}$

- Method based on approach introduced by the *RM123 collaboration*[4,5]
- Consider a QCD+QED action S with parameters:

$$\varepsilon = \left(\beta, e^2, m_u, m_d, m_s\right)$$

- Expand around isosymmetric action $S^{(0)}$ with parameters

$$\varepsilon^{(0)} = \left(\beta^{(0)}, 0, m_{ud}^{(0)}, m_{ud}^{(0)}, m_s^{(0)}\right)$$

- Dividing S into three parts, write

$$S[U, A, \psi, \bar{\psi}] = S_g[U] + S_\gamma[A] + S_q[U, A, \psi, \bar{\psi}]$$

- QED$_L$ prescription[6] in Coulomb gauge

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[4] Divitiis et al. 2012, *JHEP* **04**, p. 124.
[5] Divitiis et al. 2013, *Phys. Rev. D* **87** no. 11, p. 114505.
[6] Hayakawa and Uno 2008, *Prog. Theor. Phys.* **120**, pp. 413–441.

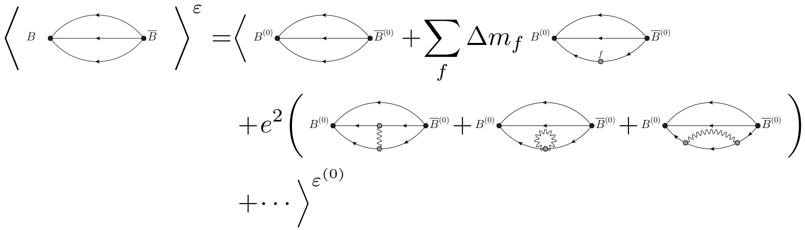
Perturbative Expansion – Expectation Values

Expand expectation values

$$\langle \mathcal{O} \rangle^{\varepsilon} = \langle \mathcal{O} \rangle^{\varepsilon^{(0)}} + \sum_{\varepsilon_i \in \varepsilon} \underbrace{\left(\varepsilon_i - \varepsilon_i^{(0)} \right)}_{=:\Delta\varepsilon_i} \frac{\partial \langle \mathcal{O} \rangle^{\varepsilon}}{\partial \varepsilon_i} \Big|_{\varepsilon=\varepsilon^{(0)}} + O\left(\Delta\varepsilon^2\right)$$

$$\left.\begin{aligned} \varepsilon &= \left(\beta, e^2, m_u, m_d, m_s\right) \\ \varepsilon^{(0)} &= \left(\beta, 0, m_{ud}^{(0)}, m_{ud}^{(0)}, m_s^{(0)}\right) \end{aligned}\right\} \Delta\varepsilon = \left(0, e^2, \Delta m_u, \Delta m_d, \Delta m_s\right)$$

Baryon correlation function:



Perturbative Expansion – Spectroscopy

- Correlation functions asymptotically behave like

$$C(t) = ce^{-mt}$$

- Expansion in isospin breaking parameters ($\Delta\varepsilon_i := \varepsilon_i - \varepsilon_i^{(0)}$)

$$\begin{aligned}\to C(t) =& c^{(0)} e^{-m^{(0)} t} \\ &+ \sum_i \Delta\varepsilon_i \left(c_i^{(1)} - c^{(0)} m_i^{(1)} t \right) e^{-m^{(0)} t} \\ &+ \mathcal{O}(\Delta\varepsilon^2)\end{aligned}$$

$\to$ Can define effective mass at first order via

$$(am_{\text{eff}})^{(1)}_{\Delta\varepsilon_i} := -a \frac{\mathrm{d}}{\mathrm{d}t} \frac{C_i^{(1)}(t)}{C^{(0)}(t)} = \frac{C_i^{(1)}(t)}{C^{(0)}(t)} - \frac{C_i^{(1)}(t+a)}{C^{(0)}(t+a)}$$

- $\Delta\varepsilon_i$ can be set by matching different average multiplet masses and mass splittings

Model Averaging

- Choice of fit interval and fit function is subjective and to some degree ambiguous
  $\rightarrow$ Use model average

- Fit models (combination of fit function and -interval) are averaged with weights from Akaike information criterion[7,8,9] (AIC) with penalty term

$$\mathrm{pr}(M|D) \propto \exp\left(-\frac{\chi^2}{2} - k - n\right)$$

where k: number of fit parameters, n: number of data points **not** considered in fit

[7] Akaike 1998.
[8] Jay and Neil 2021, *Phys. Rev. D* **103**, p. 114502.
[9] Neil and Sitison 2022.

- Fit parameters a_0 can be estimated from model fit

$$\langle a_0 \rangle = \sum_i \langle a_0 \rangle_{M_i} \, \mathrm{pr}(M_i|D)$$

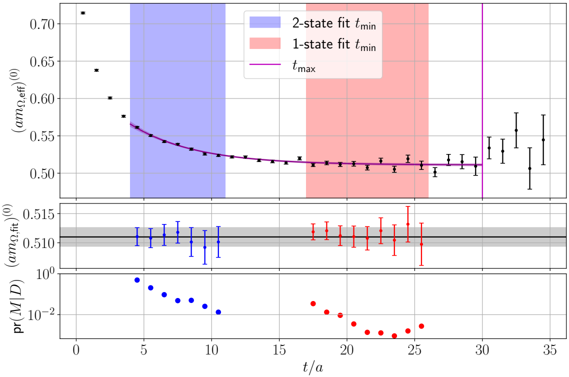
- Covariance matrix given by

$$\begin{aligned} C = & \sum_i C_i \mathrm{pr}(M_i|D) \\ & + \sum_i \langle a_0 \rangle_{M_i} \langle a_0 \rangle_{M_i}^T \, \mathrm{pr}(M_i|D) \\ & - \left(\sum_i \langle a_0 \rangle_{M_i} \, \mathrm{pr}(M_i|D) \right) \left(\sum_i \langle a_0 \rangle_{M_i}^T \, \mathrm{pr}(M_i|D) \right) \end{aligned}$$

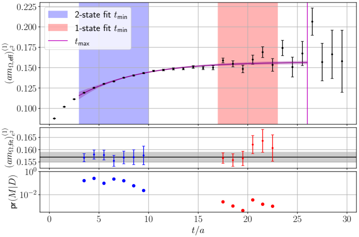
(statistic and systematic contributions)

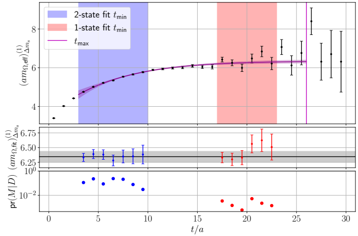
- Systematic contributions added to Jackknife/Bootstrap distribution as Gaussian noise

Model Averaging Example Isospin-Symmetric



- Constant fits for single-state ansatz
- 2-state fit function $am_{\text{eff}}(t) = m + \gamma e^{-\Delta M t}$
- At lower t_{min}, pr$(M|D)$ drops rapidly





- 2-state fit function

$$am_{\text{eff}}(t) = m + (\alpha - \beta t)e^{-\Delta M^{(0)} t}$$

- Data much more noisy in the region where one expects a plateau
- Model average helps with selection of fit ranges and provides estimate for uncertainty from choice of model/fit range
- Stable results from 2-state and single-state fits

Relative Precision of Baryon Masses

Relative uncertainties on asymptotic masses from AIC averages in the isospin-symmetric theory

Ensemble	N	Λ	Σ	Ξ	Ω
D450	0.49%	0.39%	0.80%	0.36%	0.28%
N200	1.46%	0.37%	0.40%	0.22%	0.33%
N203	0.35%	0.27%	0.28%	0.22%	0.44%
N451	1.20%	0.16%	0.16%	0.32%	0.23%
N452	0.80%	0.50%	0.74%	0.41%	1.14%

Relative uncertainties on the isospin-breaking corrections to the asymptotic mass for most promising candidates for scale-setting

Ensemble	Ξ^0			Ξ^-			Ω^-	
	e^2	Δm_u	Δm_s	e^2	Δm_d	Δm_s	e^2	Δm_s
D450	1.6%	2.2%	0.4%	0.8%	2.2%	0.4%	1.2%	1.2%
N200	1.5%	1.9%	0.7%	1.1%	1.9%	0.7%	1.3%	1.4%
N203	0.9%	1.2%	0.7%	0.7%	1.2%	0.7%	1.2%	1.2%
N451	1.2%	1.2%	1.2%	0.7%	1.2%	1.2%	0.9%	1.0%
N452	1.0%	0.9%	0.6%	0.8%	0.9%	0.6%	1.6%	2.4%

- All error estimates are purely statistical and exclude IB corrections to the sea-quark sector
- IB contributions are multiplied by expansion coefficients of $\mathcal{O}(10^{-3})$ $\rightarrow$ uncertainties likely negligible

Conclusion

- We compute masses and their isospin-breaking corrections for the full baryon octet and decuplet
- We find 0.2% to 0.5% statistical precision on all ensembles thus far for the Ξ baryon and on most ensembles for the Ω for the pure QCD-contribution
- $\mathcal{O}(1\text{‰})$ precision in IB corrections likely to be negligible for the full QCD+QED result
  $\rightarrow$ Ignores sea-quark interactions which might increase error
- Ξ and Ω promising candidates for setting the scale on CLS ensembles with isospin-breaking corrections