Quark orbital angular momentum in the proton from a twist-3 generalized parton distribution

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Quark OAM and parton distributions

Ji sum rule:

$$L_3 = J_3 - S_3 = \frac{1}{2} \int dx\, x\,(H + E) - \frac{1}{2} \int dx\, \widetilde{H}$$

Twist-2 GTMD F_{14}:

$$L_3 = -\int dx \int d^2k_T\, \frac{k_T^2}{M^2}\, F_{14}$$

$\longrightarrow$ from Ji to Jaffe-Manohar OAM

There is a third way to access Ji OAM: Twist-3 GPD $\widetilde{E}_{2T}$

$$L_3 = (L_3 + 2S_3) - 2S_3 = -\int dx\, x \widetilde{E}_{2T} - \int dx\, \widetilde{H}$$

Note: A version of this approach was first noted by M. Polyakov, denoting the relevant twist-3 GPD variously as G_2 or G_3. Relation to OAM derived using OPE. Here, will review alternative approach based on GTMDs.

Note: All distribution functions above taken in the forward limit!

Equation of motion relation

$$-x\widetilde{E}_{2T} - \widetilde{H} = -\int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14} + \widetilde{M} \qquad \Longrightarrow \qquad L_3 = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14} = -\int dx \, x \, \widetilde{E}_{2T} - \int dx \, \widetilde{H}$$

(forward limit)

How? Consider correlator (straight gauge link $\mathcal{U}$)

$$W^{\Gamma}_{\Lambda'\Lambda} = \frac{1}{2} \int \frac{dz^- \, d^2z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T \cdot z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2) \, \Gamma \, \mathcal{U} \, \psi(z/2) \mid p, \Lambda \rangle \big|_{z^+=0}$$

Use $\Gamma = i\sigma^{i+}\gamma^5$ and quark field equation of motion $\longrightarrow$ $(i\not{D} - m)\psi = 0$

to generate (note zero skewness throughout)

$$ik^+ \epsilon^{ij} W^{\gamma^j}_{\Lambda'\Lambda} + \frac{\Delta^i}{2} W^{\gamma^+\gamma^5}_{\Lambda'\Lambda} - i\epsilon^{ij} k^j W^{\gamma^+}_{\Lambda'\Lambda} + \mathcal{M}^i_{\Lambda'\Lambda} = 0$$

Form combination $\dfrac{\Delta^i}{\Delta_T^2} \left(W^{\Gamma}_{++} - W^{\Gamma}_{--} \right)$, insert parametrizations of $W^{\Gamma}_{\Lambda'\Lambda}$ in terms of GTMDs

Integrate over k_T, identify GPDs

Extraction from QCD matrix element

$$L_3 + 2S_3 = -\int dx\, x\widetilde{E}_{2T} = 2\int dx\, x \int d^2k_T \left(\frac{k_T \cdot \Delta_T}{\Delta_T^2} F_{27} + F_{28} \right) = -iP^+ \int dx\, x \int d^2k_T \epsilon_{ij} \frac{\Delta^i}{\Delta_T^2} \left(W_{++}^{\gamma^j} - W_{--}^{\gamma^j} \right)$$

(all in forward limit – note $W_{\Lambda'\Lambda}^{\gamma^i}$ is parametrized in terms of 8 twist-3 GTMDs $F_{21}, \ldots, F_{28}$).

$$L_3 + 2S_3 = \epsilon_{ij} \, \frac{1}{2} \, \frac{\partial}{\partial(z \cdot P)} \, \frac{\partial}{\partial \Delta^i} \, \langle P + \Delta_T/2, +| \, \bar{\psi}(-z/2)\gamma^j \, \mathcal{U} \, \psi(z/2) \, |P - \Delta_T/2, +\rangle|_{z^+=z^-=0, \Delta_T=0, z_T \to 0}$$

Renormalize using number of valence quarks n

$$2\, P^j \, n = \langle P, +| \, \bar{\psi}(-z/2)\gamma^j \, \mathcal{U} \, \psi(z/2) \, |P, +\rangle \Big|_{\, z^+=z^-=0, z_T \to 0}$$

i.e., ultimately determine ratio $(L_3 + 2S_3)/n$

Setting up a lattice calculation

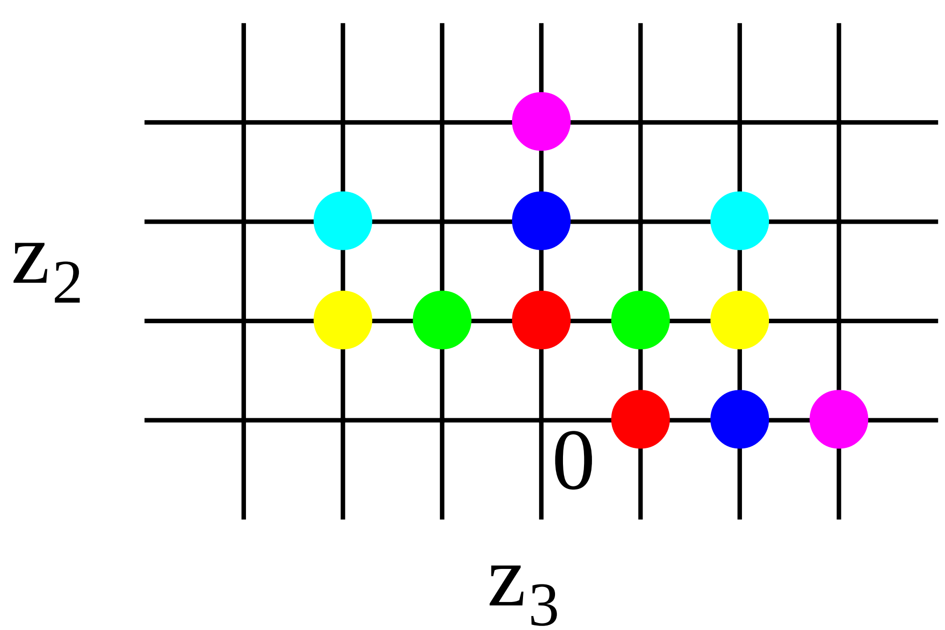
Lattice frame is boosted with respect to phenomenology frame; use invariants $z \cdot P$, z^2

Original frame: $z^+ = 0, \quad z \cdot P = z^- P^+, \quad z^2 = -z_T^2$

Lattice frame: $z_0 = 0, \quad z \cdot P = -z_3 P_3, \quad z^2 = -z_3^2 - z_T^2$

Use Rome direct derivative method to perform $\partial / \partial \Delta^i$

Perform $\partial / \partial z_3$ via finite difference at constant z^2



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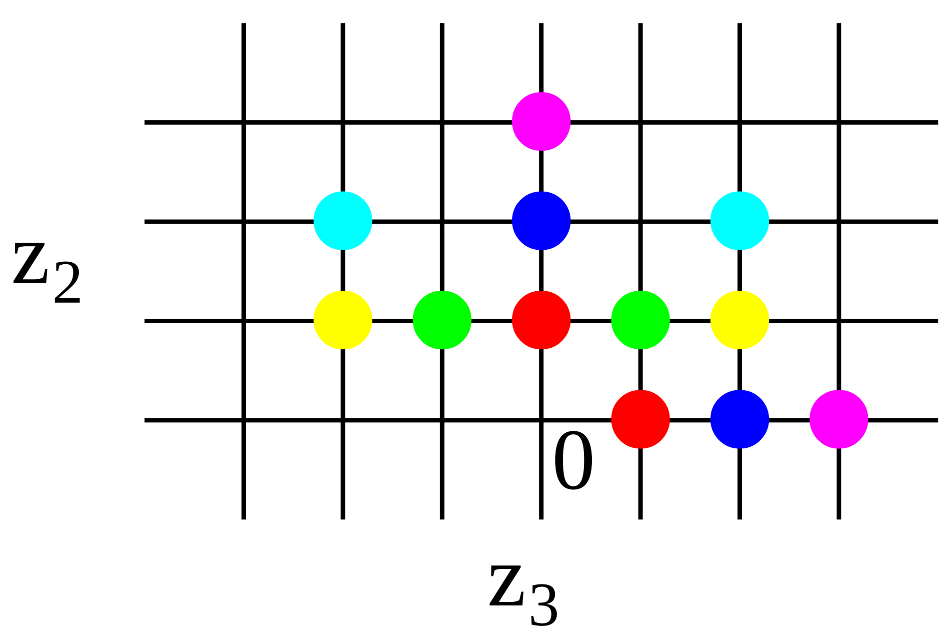
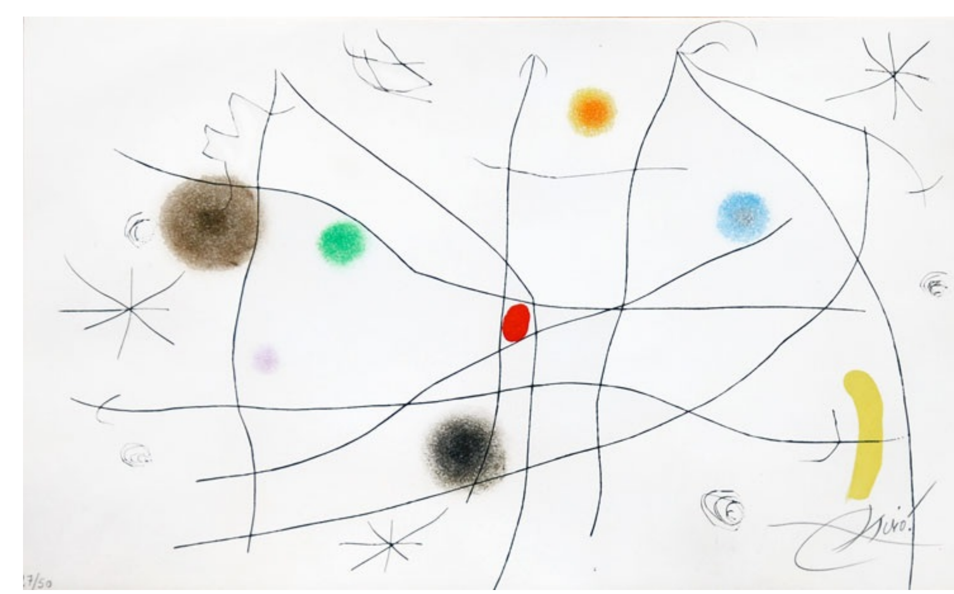




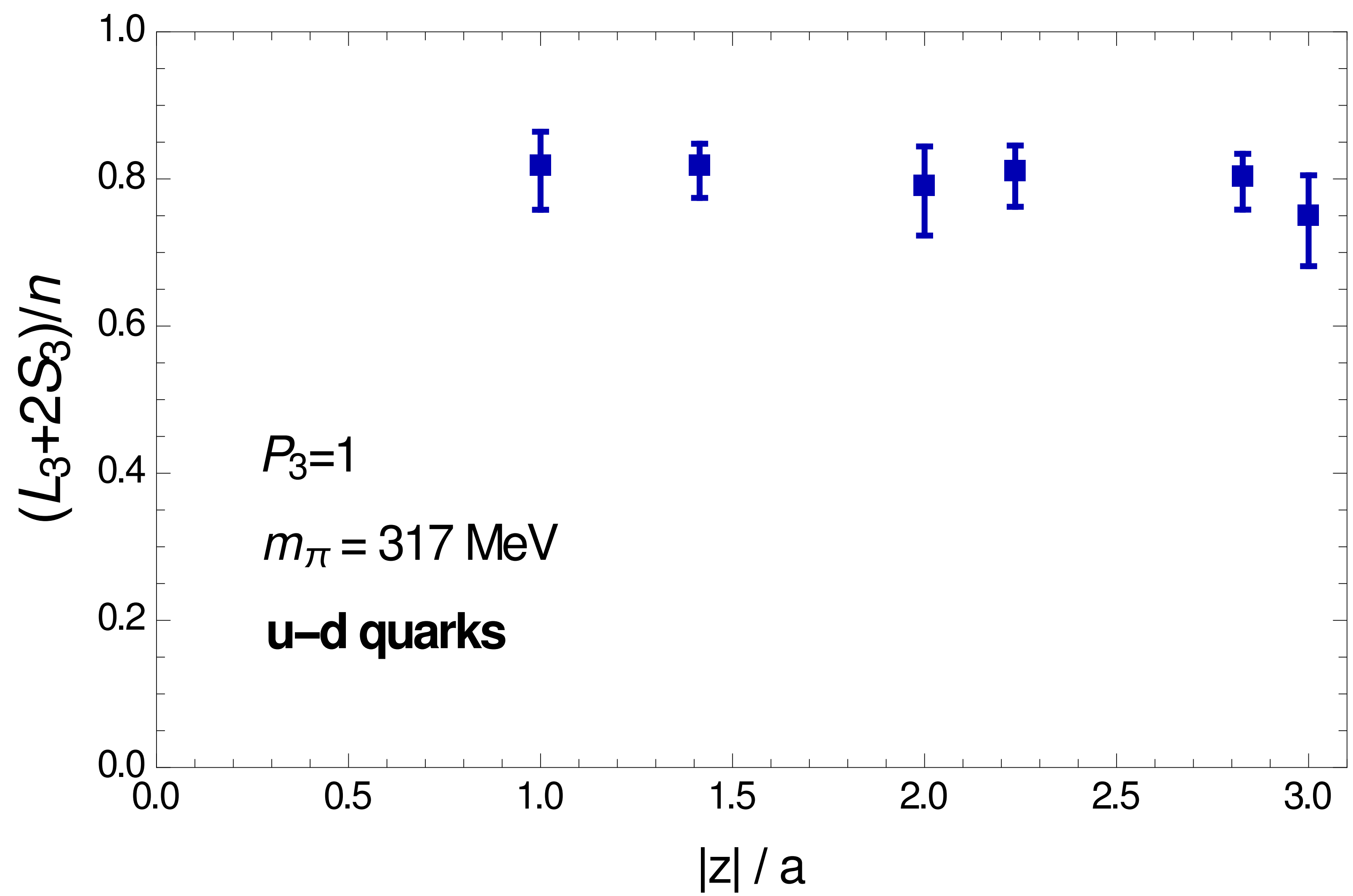
Joan Miró, print, 1974,
www.masterworksfineart.com

Ensemble details

Clover fermion ensemble provided by R. Edwards, B. Joó and K. Orginos (JLab / W&M Collaboration)

$L^3 \times T$	a(fm)	m_π (MeV)	m_N (GeV)	#conf.	#meas.
$32^3 \times 96$	0.11403(77)	317(2)(2)	1.077(8)	967	23224

Dependence on z^2



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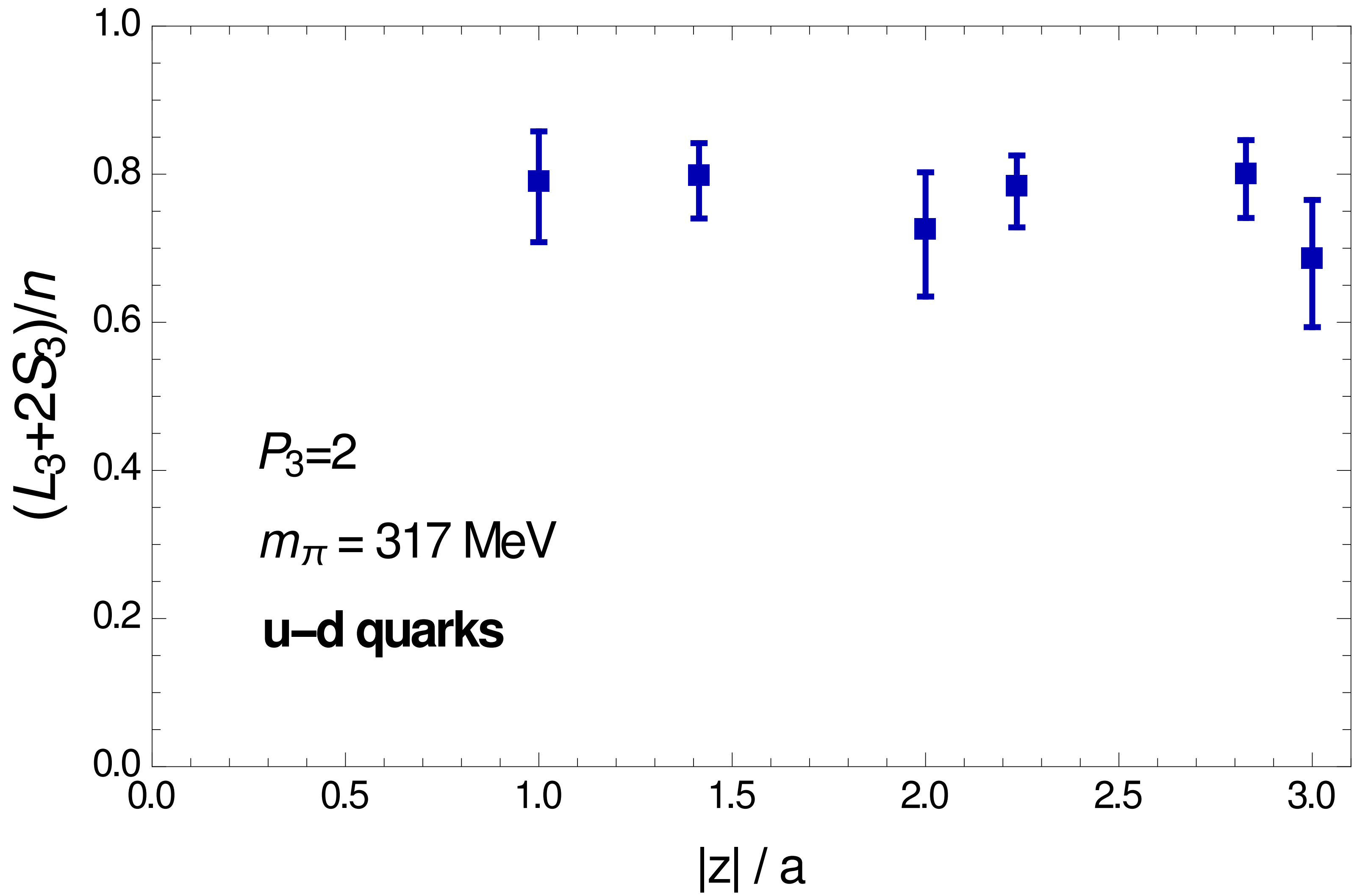
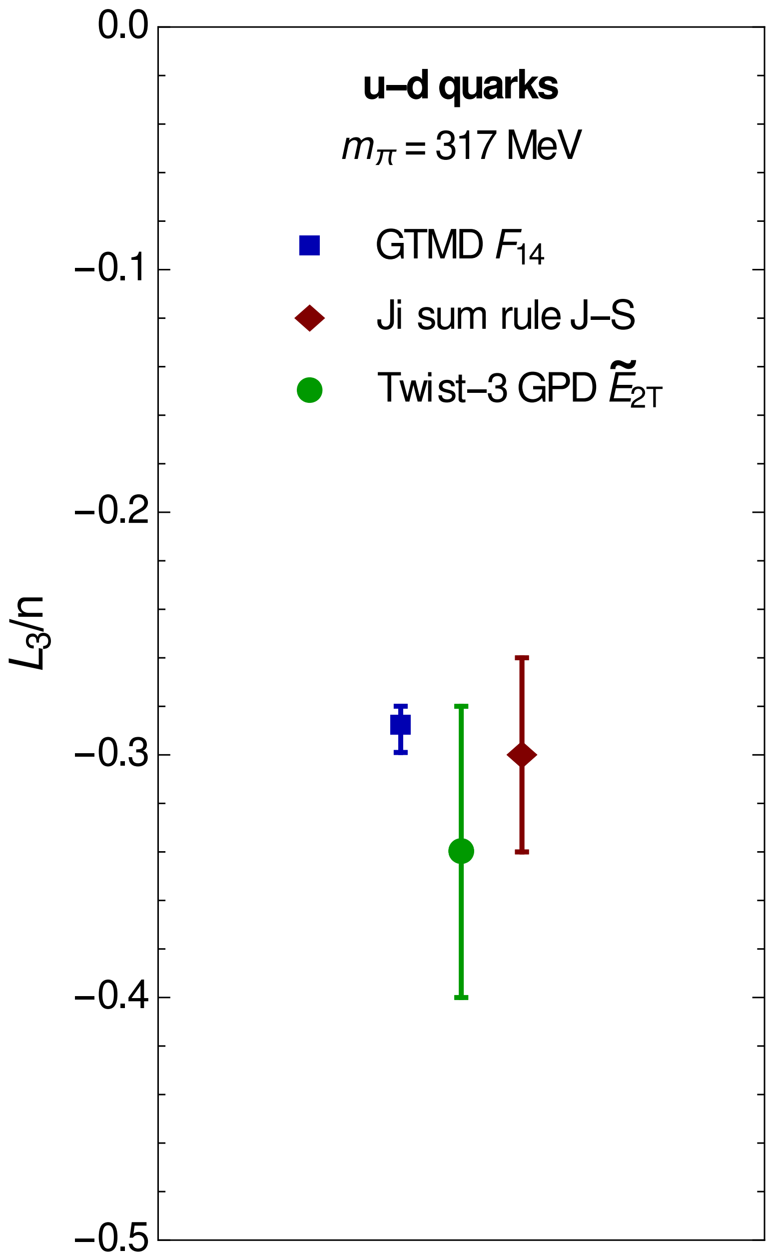


Result for L_3 and comparison to other approaches; conclusions



- Combine with $2S_3$ previously obtained on same ensemble, same parameters (arXiv:1703.06703): $2S_3 = 1.18(2)$ (extrapolate $\widetilde{E}_{2T}$ to $P = 0$ to match).
- Different systematics within the various approaches appear to remain smaller than statistical uncertainty.
- Larger uncertainty of twist-3 GPD approach: Difference of large numbers, incomplete cancellation of fluctuations due to use of rotational symmetry in normalization.
- It is feasible to extract OAM from twist-3 GPDs, but higher effort compared to other approaches for comparable accuracy.