Progresses on high-temperature QCD: Equation of State and energy-momentum tensor

M. Bresciani, M. Dalla Brida, L. Giusti, M. Pepe

University of Milano Bicocca
INFN Milano Bicocca

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Overview

1. **Finite temperature QCD: a tool for NP renormalization**
2. **Renormalization of the energy momentum tensor**
3. **Equation of State**
4. **Conclusions & outlook**

QCD in a moving frame

Hypercubic lattice of size $L_0 \times L^3$

- Shifted and twisted boundary conditions

$$U_\mu(x_0 + L_0, \boldsymbol{x}) = U_\mu(x_0, \boldsymbol{x} - L_0 \boldsymbol{\xi})$$

$$\psi(x_0 + L_0, \boldsymbol{x}) = -e^{i\theta_0}\, \psi(x_0, \boldsymbol{x} - L_0 \boldsymbol{\xi})$$
$$\bar{\psi}(x_0 + L_0, \boldsymbol{x}) = -e^{-i\theta_0}\, \bar{\psi}(x_0, \boldsymbol{x} - L_0 \boldsymbol{\xi})$$

$\boldsymbol{\xi} \rightarrow$ euclidean boost parameter, 3d vector [Giusti, Meyer, JHEP 01 (2013), 140]
$\theta_0 \rightarrow$ twist phase, imaginary chemical potential [Dalla Brida, Giusti, Pepe, JHEP 04 (2020) 043]

- $N_f = 3$ massless $\mathcal{O}(a)$-improved Wilson fermions

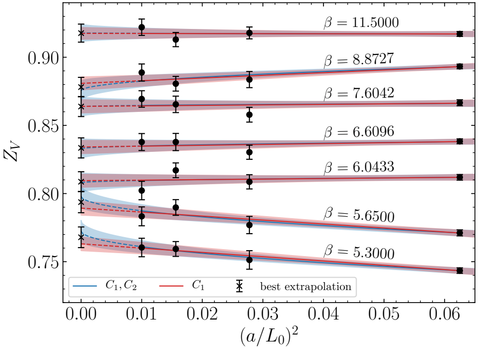
Finite temperature QCD: a tool for NP renormalization

- Ward Identity for Z_V

$$Z_V \langle V_\mu^l \rangle_{\boldsymbol{\xi},\theta_0} = \langle V_\mu^c \rangle_{\boldsymbol{\xi},\theta_0}$$

- Limit to zero temperature

$$Z_V\left(g_0^2, \frac{a}{L_0}\right) = Z_V(g_0^2) + C_1 \cdot \left(\frac{a}{L_0}\right)^2 + C_2 \cdot (a\Lambda_{\text{QCD}})\left(\frac{a}{L_0}\right) + C_3 \cdot (a\Lambda_{\text{QCD}})^2 + \ \ldots$$



[Bresciani, Dalla Brida, Giusti, Pepe, Rapuano, PLB 835 (2022), 137579]

Renormalization of the energy momentum tensor

- WIs for the non-singlet energy momentum tensor

$$\langle T_{0k}^{\{6\},R}\rangle_{\boldsymbol{\xi},\theta_0} = Z_G^{\{6\}}\langle T_{0k}^{G,\{6\}}\rangle_{\boldsymbol{\xi},\theta_0} + Z_F^{\{6\}}\langle T_{0k}^{F,\{6\}}\rangle_{\boldsymbol{\xi},\theta_0}$$

$$\langle T_{0k}^{R,\{6\}}\rangle_{\boldsymbol{\xi},\theta_0} = \xi_k\langle T_{0j}^{R,\{3\}}\rangle_{\boldsymbol{\xi},\theta_0} \quad (j \neq k, \xi_j = 0)$$

[Dalla Brida, Giusti, Pepe, JHEP 04 (2020) 043]

- Compute at two values of θ_0: θ_0^A, θ_0^B

$$\langle T_{0k}\rangle_{\boldsymbol{\xi},\theta_0} = -\frac{\partial f_{\boldsymbol{\xi},\theta_0}}{\partial \xi_k}$$

$$\frac{\partial f_{\boldsymbol{\xi},\theta_0^B}}{\partial \xi_k} = \frac{\partial f_{\boldsymbol{\xi},\theta_0^A}}{\partial \xi_k} + \frac{i}{L_0}\int_{\theta_0^A}^{\theta_0^B} d\theta_0 \frac{\partial}{\partial \xi_k}\langle V_0\rangle_{\boldsymbol{\xi},\theta_0} \equiv \frac{\partial f_{\boldsymbol{\xi},\theta_0^A}}{\partial \xi_k} - \mathcal{V}_{0,k}^{AB}$$

Renormalization of the energy momentum tensor

- WIs for the non-singlet energy momentum tensor

$$\langle T_{0k}^{\{6\},R} \rangle_{\boldsymbol{\xi},\theta_0} = Z_G^{\{6\}} \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0} + Z_F^{\{6\}} \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0}$$

$$\langle T_{0k}^{R,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0} = \xi_k \langle T_{0j}^{R,\{3\}} \rangle_{\boldsymbol{\xi},\theta_0} \quad (j \neq k, \xi_j = 0)$$

[Dalla Brida, Giusti, Pepe, JHEP 04 (2020) 043]

- Compute at two values of θ_0: θ_0^A, θ_0^B

$$\begin{pmatrix} \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^A} & \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^A} \\ \\ \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^B} & \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^B} \end{pmatrix} \begin{pmatrix} Z_G^{\{6\}} \\ \\ Z_F^{\{6\}} \end{pmatrix} = \begin{pmatrix} -\dfrac{\Delta f_{\boldsymbol{\xi},\theta_0^A}}{\Delta \xi_k} \\ \\ -\dfrac{\Delta f_{\boldsymbol{\xi},\theta_0^A}}{\Delta \xi_k} + \mathcal{V}_{0,k}^{AB} \end{pmatrix} + \mathcal{O}(a^2)$$

Integral over the mass

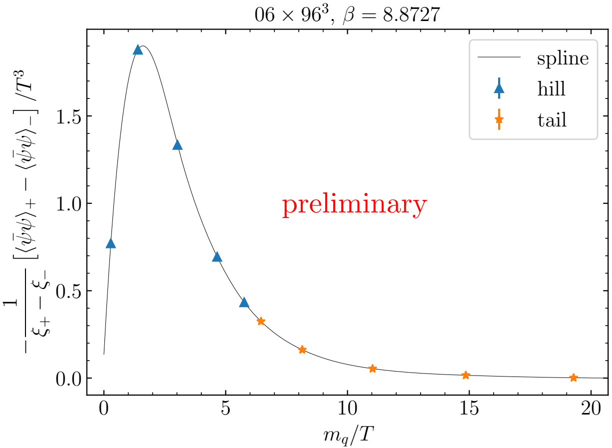
$$\begin{pmatrix} \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^A} & \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^A} \\ \\ \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^B} & \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^B} \end{pmatrix} \begin{pmatrix} Z_G^{\{6\}} \\ \\ Z_F^{\{6\}} \end{pmatrix} = \begin{pmatrix} -\frac{\Delta f_{\boldsymbol{\xi},\theta_0^A}}{\Delta \xi_k} \\ \\ -\frac{\Delta f_{\boldsymbol{\xi},\theta_0^A}}{\Delta \xi_k} + \mathcal{V}_{0,k}^{AB} \end{pmatrix} + \mathcal{O}(a^2)$$

Integral over the mass

$$\frac{\Delta f_{\boldsymbol{\xi},\theta_0}}{\Delta \xi_k} = \frac{\Delta}{\Delta \xi_k}\left(f^{\rm YM}_{\boldsymbol{\xi},\theta_0} + f_{\boldsymbol{\xi},\theta_0} - f^{\rm YM}_{\boldsymbol{\xi},\theta_0}\right)$$

$$= \frac{\Delta}{\Delta \xi_k}\left(f^{\rm YM}_{\boldsymbol{\xi},\theta_0} - \int_0^\infty dm_q \, \frac{\partial f_{\boldsymbol{\xi},\theta_0}}{\partial m_q}\right)$$

$$= \frac{\Delta}{\Delta \xi_k}\left(f^{\rm YM}_{\boldsymbol{\xi},\theta_0} - \int_0^\infty dm_q \, \langle \bar\psi\psi\rangle_{\boldsymbol{\xi},\theta_0}\right)$$



- YM free-energy computed as in [Giusti, Pepe, PRD **91** (2015), 114504]

Integral over θ

$$\begin{pmatrix} \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^A} & \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^A} \\ \\ \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^B} & \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^B} \end{pmatrix} \begin{pmatrix} Z_G^{\{6\}} \\ \\ Z_F^{\{6\}} \end{pmatrix} = \begin{pmatrix} -\dfrac{\Delta f_{\boldsymbol{\xi},\theta_0^A}}{\Delta \xi_k} \\ \\ -\dfrac{\Delta f_{\boldsymbol{\xi},\theta_0^A}}{\Delta \xi_k} + \mathcal{V}_{0,k}^{AB} \end{pmatrix} + \mathcal{O}(a^2)$$

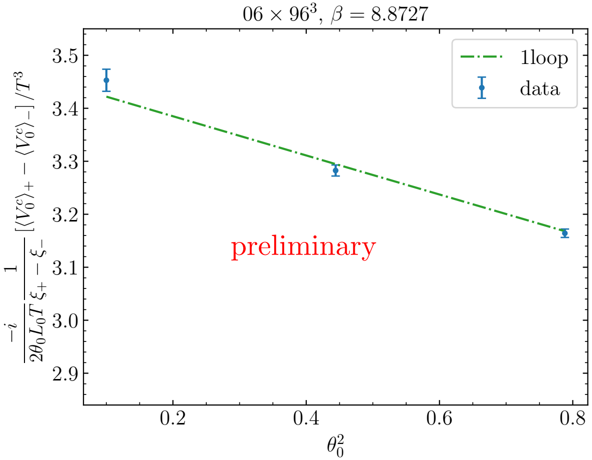
Integral over θ

$$\mathcal{V}_{0,k}^{AB} = \frac{\Delta f_{\boldsymbol{\xi},\theta_0^A}}{\Delta \xi_k} - \frac{\Delta f_{\boldsymbol{\xi},\theta_0^B}}{\Delta \xi_k}$$

$$= -\frac{\Delta}{\Delta \xi_k} \int_{\theta_0^A}^{\theta_0^B} d\theta_0 \, \frac{\partial}{\partial \theta_0} f_{\boldsymbol{\xi},\theta_0}$$

$$= -\frac{i}{L_0} \int_{\theta_0^A}^{\theta_0^B} d\theta_0 \, \frac{\Delta}{\Delta \xi_k} \langle V_0^c \rangle_{\boldsymbol{\xi},\theta_0}$$

- $\theta_0^A = 0$, $\theta_0^B = 3\pi/10$



One-point functions of the tensor

$$\begin{pmatrix} \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^A} & \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^A} \\ \\ \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^B} & \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta_0^B} \end{pmatrix} \begin{pmatrix} Z_G^{\{6\}} \\ \\ Z_F^{\{6\}} \end{pmatrix} = \begin{pmatrix} -\dfrac{\Delta f_{\boldsymbol{\xi},\theta_0^A}}{\Delta \xi_k} \\ \\ -\dfrac{\Delta f_{\boldsymbol{\xi},\theta_0^A}}{\Delta \xi_k} + \mathcal{V}_{0,k}^{AB} \end{pmatrix} + \mathcal{O}(a^2)$$

One-point functions of the tensor

- $L_0/a = 6$
- $\beta = 8.8727$
- $\boldsymbol{\xi} = (1, 0, 0)$

preliminary

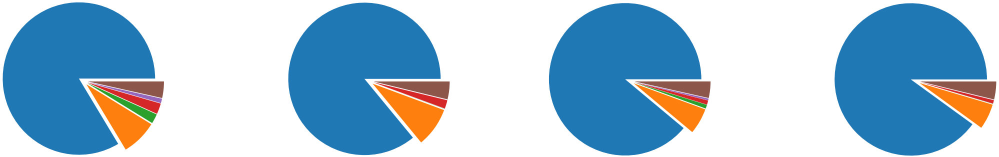
$\theta_0^A = 0$		
$\langle T_{01}^{F,\{6\}}\rangle/T^4$	-6.343(9)	0.15%
$\langle T_{01}^{G,\{6\}}\rangle/T^4$	-2.82(2)	0.85%
$\langle T_{02}^{F,\{3\}}\rangle/T^4$	-6.85(1)	0.17%
$\langle T_{02}^{G,\{3\}}\rangle/T^4$	-3.13(4)	1.20%
$L/a = 288$, $n_{\mathrm{trj}} = 100$		

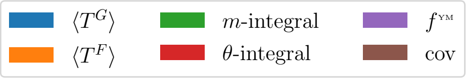
preliminary

$\theta_0^B = 3\pi/10$		
$\langle T_{01}^{F,\{6\}}\rangle/T^4$	-4.053(9)	0.21%
$\langle T_{01}^{G,\{6\}}\rangle/T^4$	-2.68(3)	1.00%
$\langle T_{02}^{F,\{3\}}\rangle/T^4$	-4.38(1)	0.34%
$\langle T_{02}^{G,\{3\}}\rangle/T^4$	-2.86(4)	1.53%
$L/a = 96$, $n_{\mathrm{trj}} = 2000$		

Renormalization constants

$Z_G^{\{6\}}$	$Z_F^{\{6\}}$	$Z_G^{\{3\}}$	$Z_F^{\{3\}}$
3.86%	2.18%	6.16%	4.12%





preliminary

Entropy density

- $N_f = 3$ massless $\mathcal{O}(a)$-improved Wilson fermions
- $L_0/a = 6$, $\beta = 8.8727$ (~ 140 GeV)
- $\boldsymbol{\xi} = (1, 0, 0)$, $\theta_0 = 0$

$$\frac{s}{T^3} = \frac{1+\boldsymbol{\xi}^2}{\xi_k} \frac{1}{T^4} \frac{\Delta f_{\boldsymbol{\xi}}}{\Delta \xi_k} = s^{\mathrm{YM}}/T^3 + s^f/T^3$$

[Giusti, Meyer, JHEP 01 (2013), 140]

[Dalla Brida, Giusti, Pepe, JHEP 04 (2020) 043]

$$s^{\mathrm{YM}} = \frac{1+\boldsymbol{\xi}^2}{\xi_k} \frac{1}{T} \frac{\Delta f^{\mathrm{YM}}_{\boldsymbol{\xi}}}{\Delta \xi_k} \qquad s^f = -\frac{1+\boldsymbol{\xi}^2}{\xi_k} \frac{1}{T} \int_0^\infty dm_q \frac{\Delta \langle \bar{\psi}\psi \rangle_{\boldsymbol{\xi}}}{\Delta \xi_k}$$

- $\sim 5\%$ deviation from Stefan-Boltzmann limit

Conclusions & outlook

Results for $L/a = 96$, $L_0/a = 6$, $\beta = 8.8727$:

- Zs for non-singlet $T_{\mu\nu}$ with a precision of $\sim 3\%$ on Z_F and $\sim 5\%$ on Z_G
- Entropy density with a precision of $\sim 0.5\%$

Next steps:

- Finite volume effects: move to $L/a = 144$
- Discretization effects: extrapolate Zs $a/L_0 \to 0$ and $s(T)\ a \to 0$
- 8 temperatures in the range 1 GeV - 100 GeV

[Dalla Brida, Giusti, Harris, Laudicina, Pepe, JHEP 04 (2022), 034]

⋆ Target accuracy on extrapolated Zs: $\sim 2\%$

⋆ Target accuracy on the EoS in the continuum: $\sim 0.5\% - 1\%$