How Gluon Pseudo-PDF Matrix Elements Depend on Gauge Smearing

2023 LATTICE

Presented by William Good[1,2]

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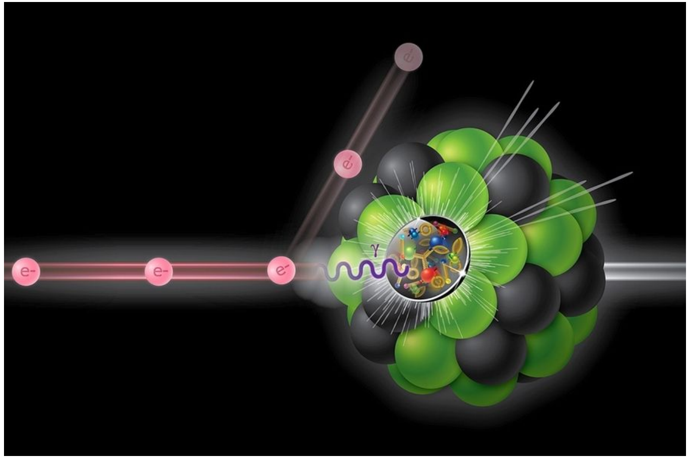
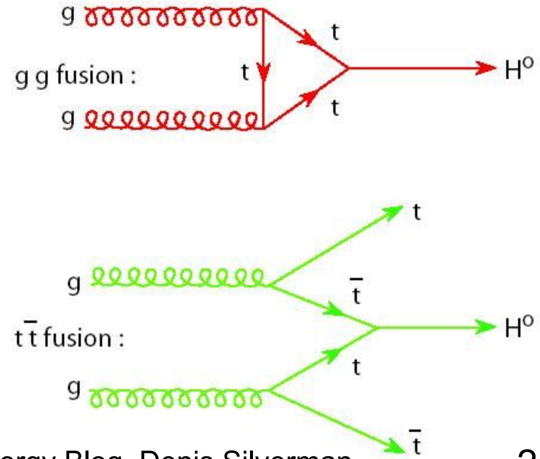
Introduction and Background

- The parton distribution function (PDF) of a hadron is an important quantity in calculating cross sections
- The nucleon gluon PDF is important in Higgs and J/ψ productions
- The pion gluon PDF is important because of the pion's role in nuclear binding forces
- Meson and nucleon gluon PDFs have only recently been calculated on the lattice due to their noisy matrix elements (MEs)

See section 3.1.1 of H.W. Lin, FBS 64:58, 2023.



Credit: Brookhaven National Lab

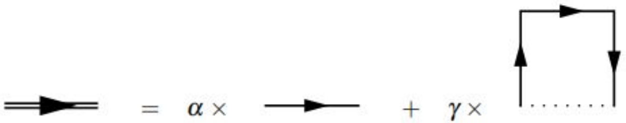


Credit: Energy Blog, Denis Silverman

Gauge Smearing

*We do not intend to call anyone out with our presentation as different ensembles will likely produce different results

- Gauge smearing is the process of taking a weighted average of a bare gauge link plus some “bypassing staples”
- This process suppresses UV fluctuations, used to improve signal
- Open questions brought up during [LaMET 2022 workshop]: How much is too much and can different smearing types be related?*
- We test smearing dependence on the unpolarized gluon PDF



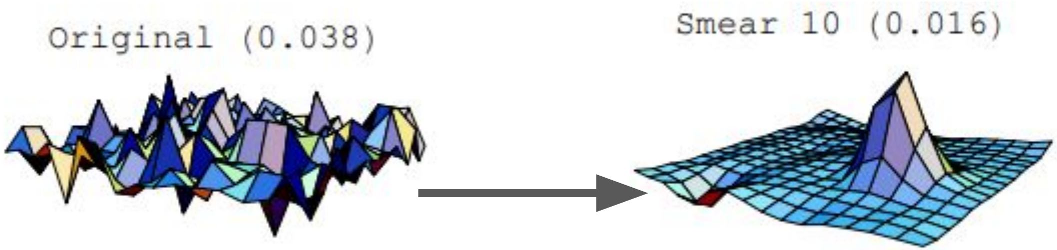


Image credit (for both): S. Solbrig, et al. arXiv:0710.0480 [hep-lat]

Smearing Types and Parameters

- All lattice configurations have 1 step of HYP smearing by default
- “HYP5” means 1 step + 4 additional steps of hypercubic smearing

$\alpha_1 = 0.75 \qquad \alpha_2 = 0.6 \qquad \alpha_3 = 0.3$

A. Hasenfratz, et al., PRD 64:034504, 2001.

- “STOUT10” means 1 step of HYP smearing + 10 steps of Stout smearing

$\rho = 0.125$

C. Morningstar, et al., PRD 69:054501, 2004.

- “WILSON3” means 1 step of HYP smearing + Wilson flow with $t = 3.0a^2$

$n = 100$

M. Lüscher, JHEP 08(2010)071.

  - Wilson is implemented as many small STOUT steps. Some interesting numerical work on this has been explored.

M. Nagatsuka, et al, arXiv:2303.09938 [hep-lat]

  - Relation between gradient flow and quasi-distributions has been explored in more detail as well

K Monahan, K. Orginos, JHEP 03(2017)116

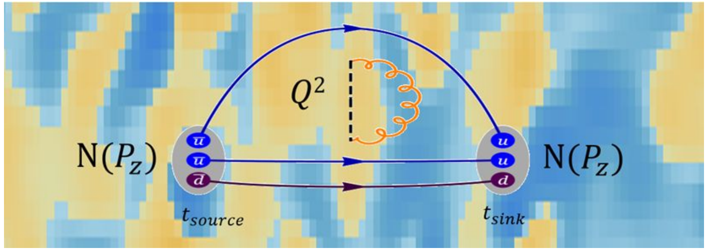
Lattice Details

Follana et al. PRD 75:054502, 2007.
A. Bazavov, et al. [MILC], PRD 82:074501, 2010.
A. Bazavov, et al. [MILC], PRD 87:054505, 2013.

- Calculation carried out with $N_f = 2+1+1$ highly improved staggered quarks (HISQ) generated by MILC collaboration
- Wilson-clover fermions used in valence sector
- Lattice spacing $a \approx 0.12$ fm
- Valence quarks tuned to reproduce light and strange pion masses $M_\pi \approx 310$ MeV and 690 MeV
- $O(10^5)$ 2pt correlator measurements over 1013 configurations
- Gaussian momentum smearing on quark fields
- Look at light and strange nucleon and pion:

  $N_l \quad N_s \quad \pi \quad \eta_s = s\bar{s}$

Ensemble	a12m310
a (fm)	0.1207(11)
$L^3 \times T$	$24^3 \times 64$
M_π^{val} (GeV)	0.309(1)
$M_{\eta_s}^{\mathrm{val}}$ (GeV)	0.6841(6)
P_z (GeV)	$[0, 2.14]$
N_{cfg}	1013
$N_{\mathrm{meas}}^{\mathrm{2pt}}$	324,160
t_{sep}	$[5, 9]$

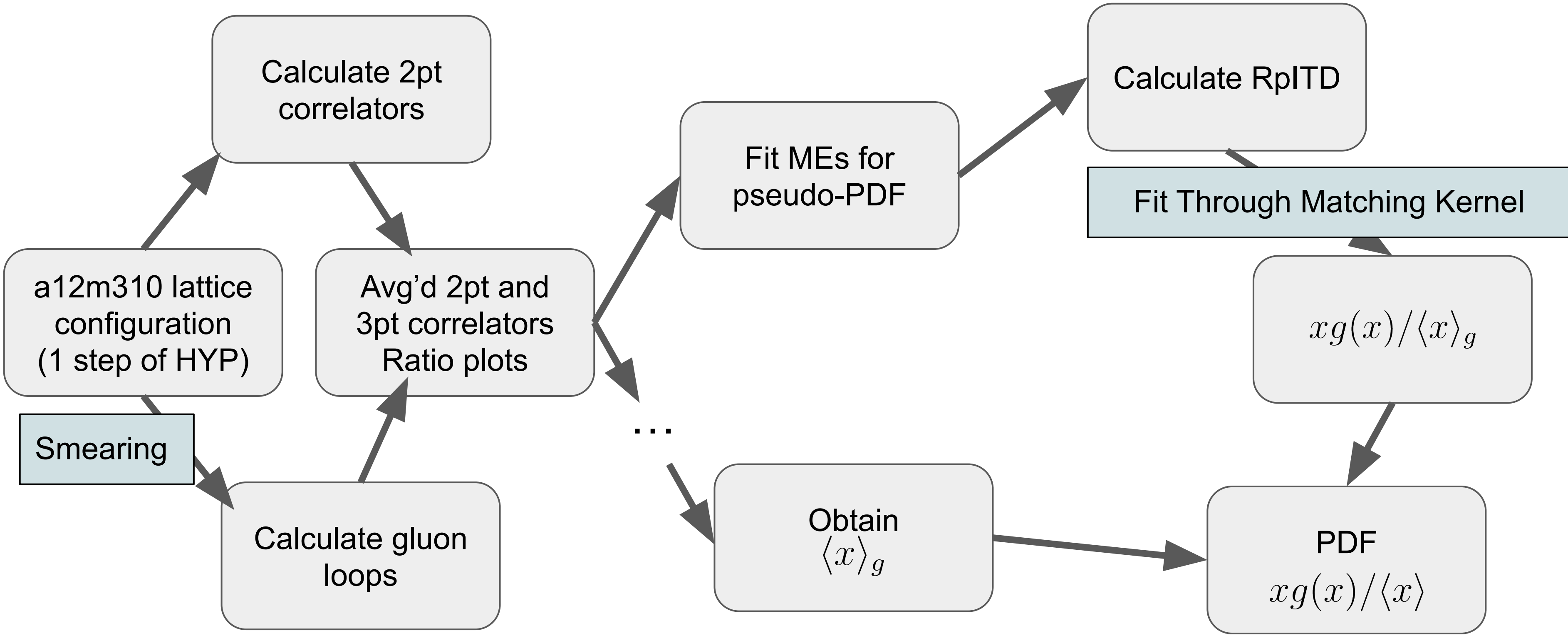


Pseudo-PDF Method

A. V. Radyushkin, PRD 96:034025, 2017.

$$\mathscr{M}(\nu,z^2)=\frac{\mathcal{M}(zP_z,z^2)/\mathcal{M}(0\cdot P_z,0)}{\mathcal{M}(z\cdot 0,z^2)/\mathcal{M}(0\cdot 0,0)}$$

(Reduced pseudo-Ioffe time distribution)
Removes divergences



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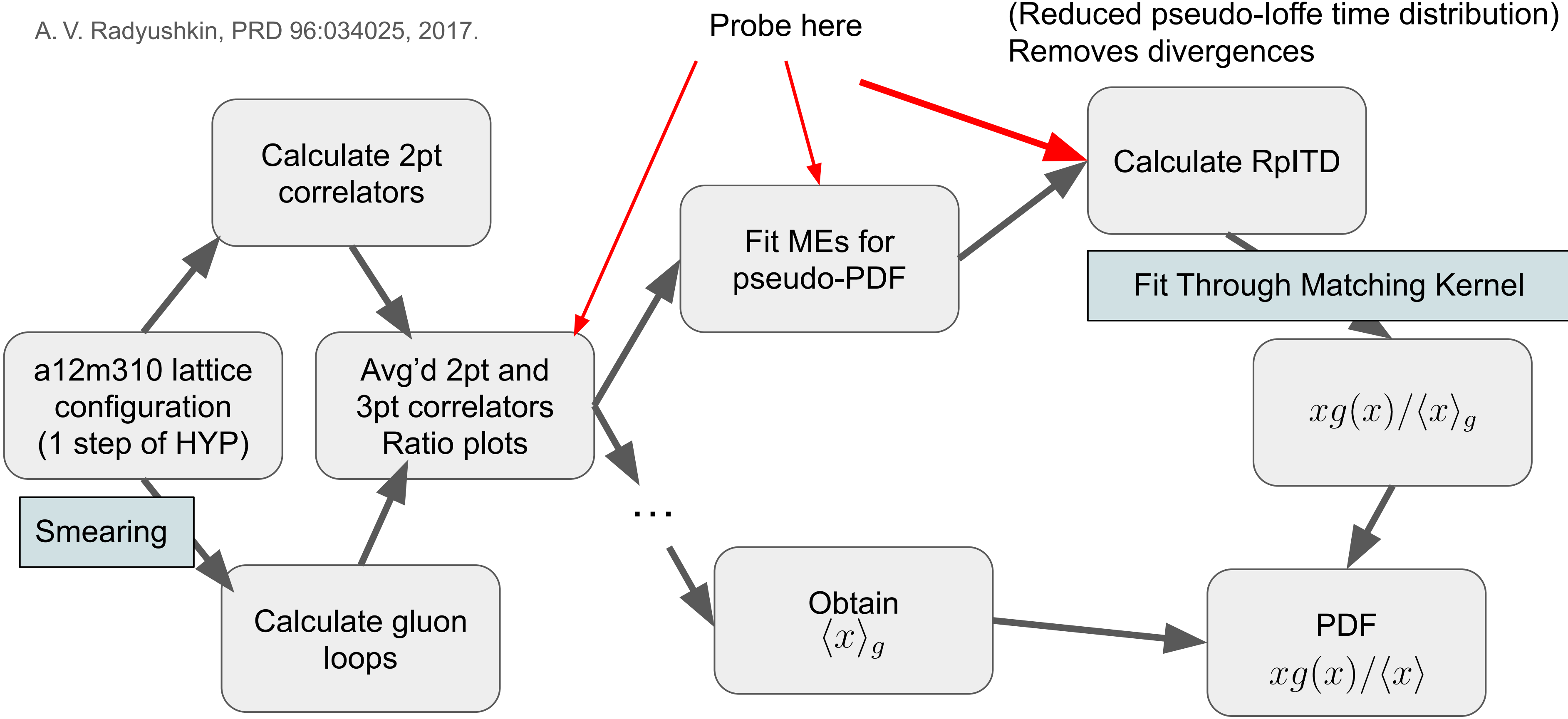

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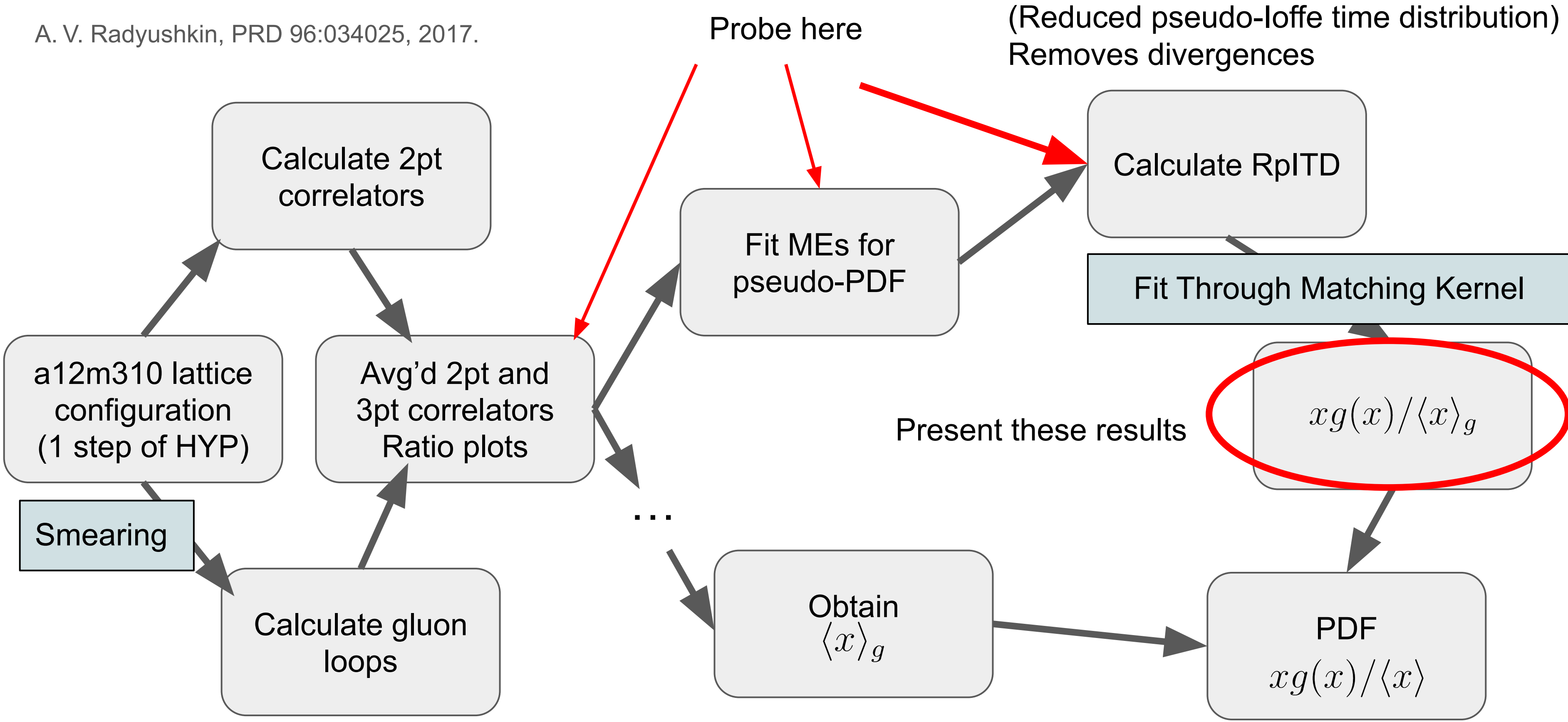


2pt and 3pt Correlator Form

- 2pt correlator expands as:

$$C_h^{\mathrm{2pt}}(P_z, t_{sep}) = |A_{h,0}|^2 e^{-E_{h,0} t_{sep}} + |A_{h,1}|^2 e^{-E_{h,1} t_{sep}} + \dots$$

- 3pt correlator expands as:

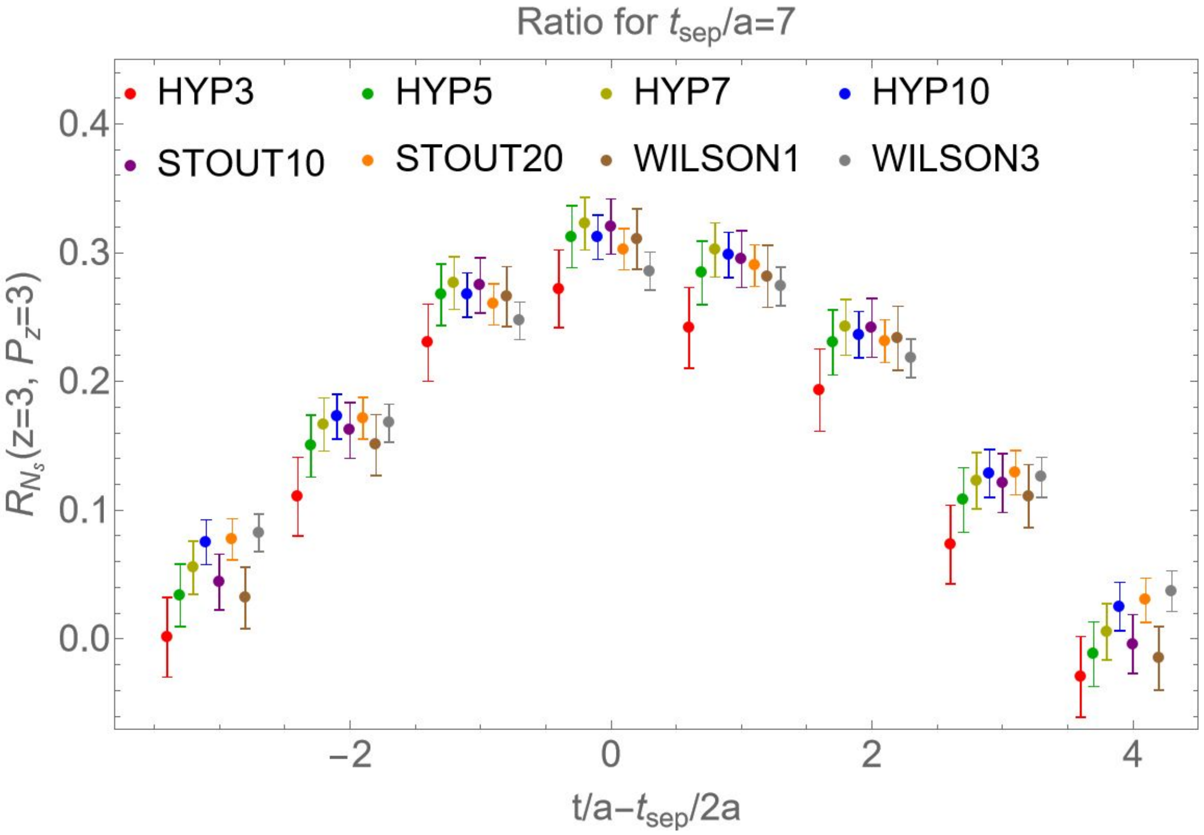
$$\begin{aligned} C_h^{\mathrm{3pt}}(z, P_z, t, t_{\mathrm{sep}}) = \\ & |A_{h,0}|^2 \langle 0|O_g|0\rangle e^{-E_{h,0} t_{\mathrm{sep}}} + |A_{h,0}||A_{h,1}| \langle 0|O_g|1\rangle e^{-E_{h,1}(t_{\mathrm{sep}}-t)} e^{-E_{h,0} t} \\ & + |A_{h,0}||A_{h,1}| \langle 1|O_g|0\rangle e^{-E_{h,0}(t_{\mathrm{sep}}-t)} e^{-E_{h,1} t} + |A_{h,1}|^2 \langle 1|O_g|1\rangle e^{-E_{h,1} t_{\mathrm{sep}}} + \dots \end{aligned}$$

- So we understand this:

$$R_h(z, P_z, t_{\mathrm{sep}}, t) = \frac{C_h^{\mathrm{3pt}}(z, P_z, t, t_{\mathrm{sep}})}{C_h^{\mathrm{2pt}}(P_z, t_{sep})} \xrightarrow{t_{sep} \to \infty} \langle 0|O_g|0\rangle$$

Gluon operator defined in Balitsky et al, PLB 808:135621, 2020

Ratio Data for N_s (t_{sep}=0.84 fm, z=0.36 fm, P_z=1.29 GeV)



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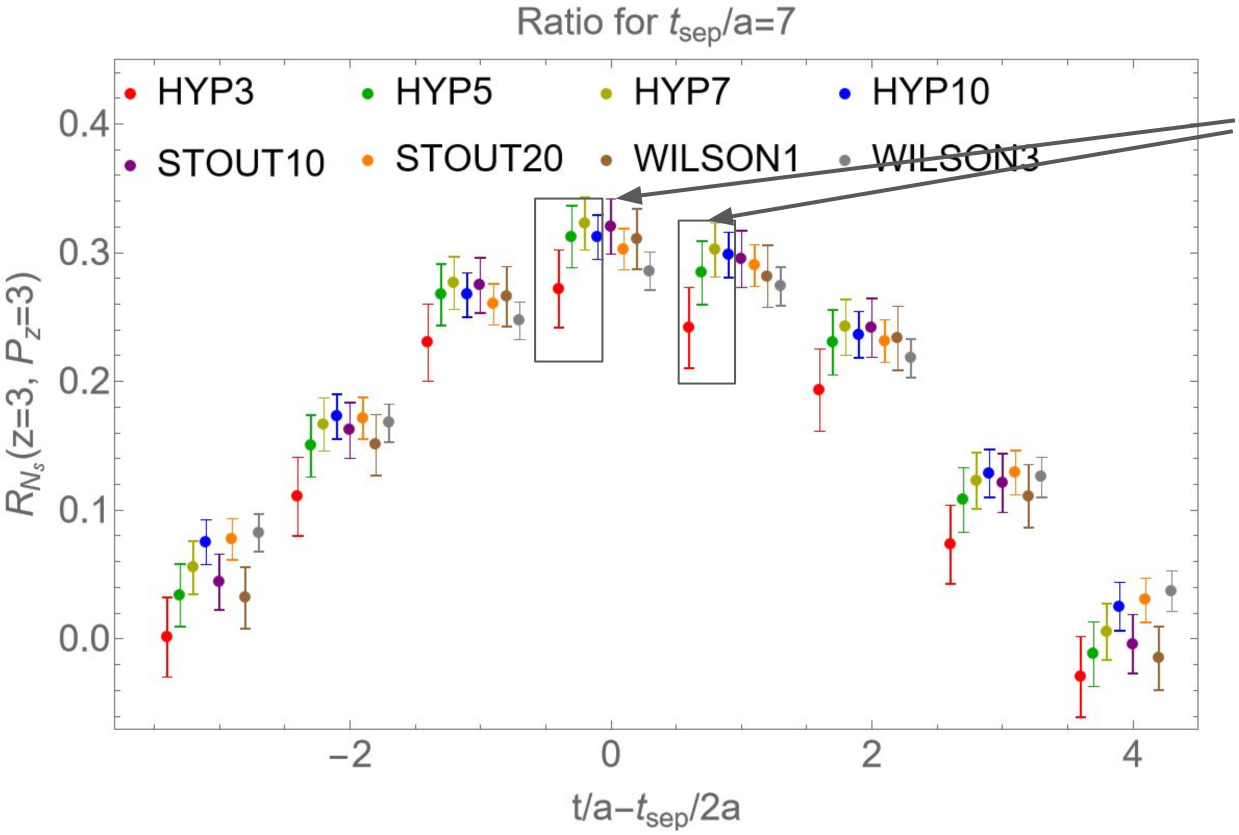
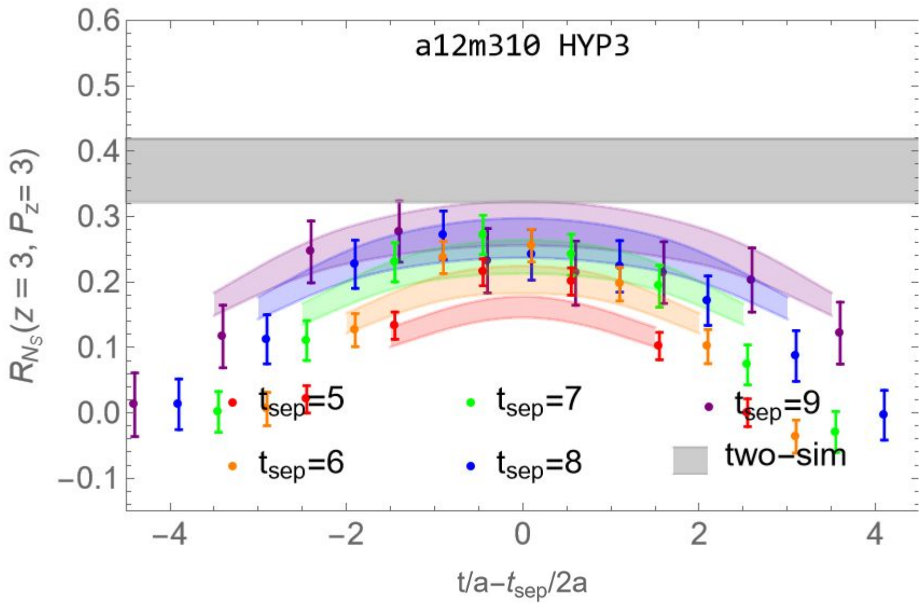


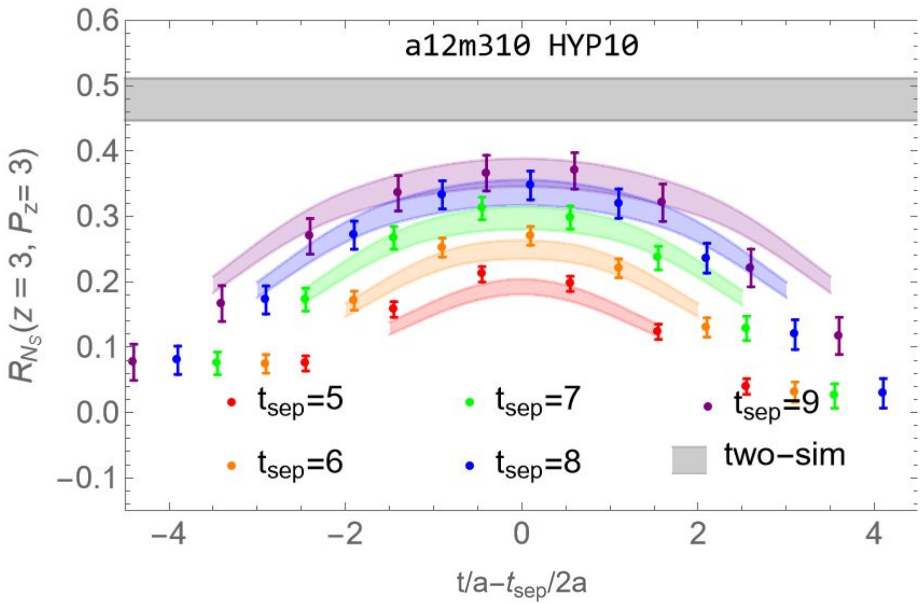
Better signal to noise ratio as expected

Reduced excited state contamination

Can’t say much about absolute value because of renormalization constants

Ratio Plots with ME Fits for N_s (z=0.36 fm, P_z=1.29 Gev)





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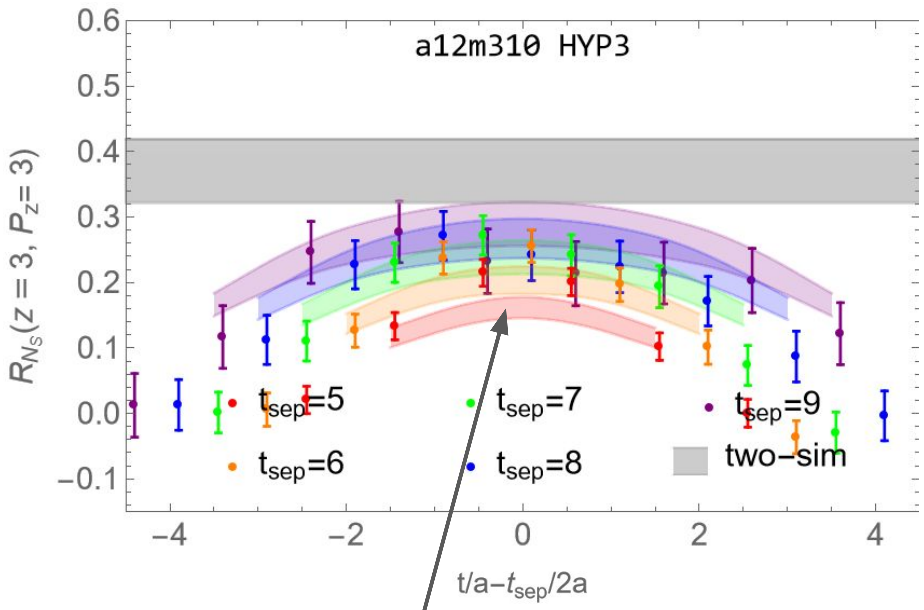

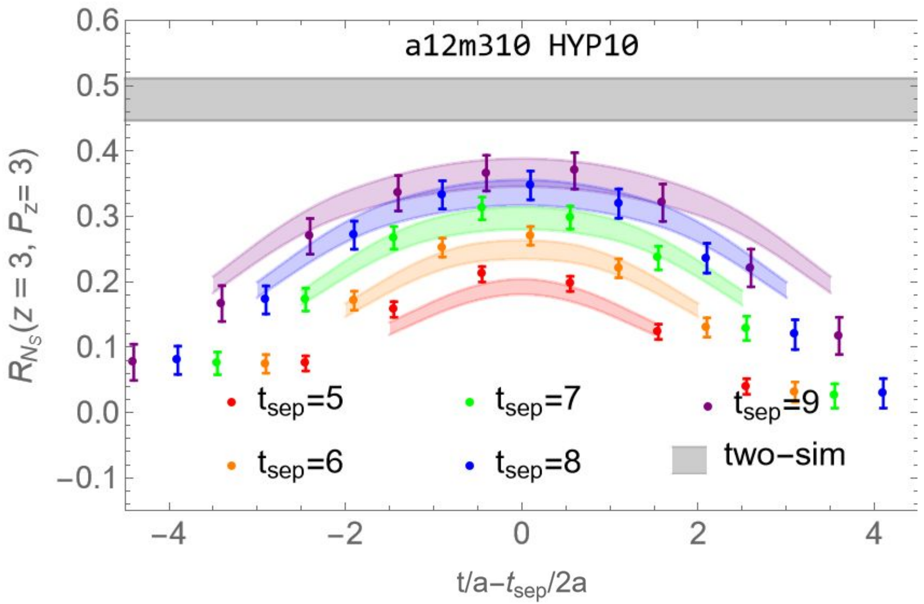

Overall, very noisy signal and loss of signal at larger t_{sep}

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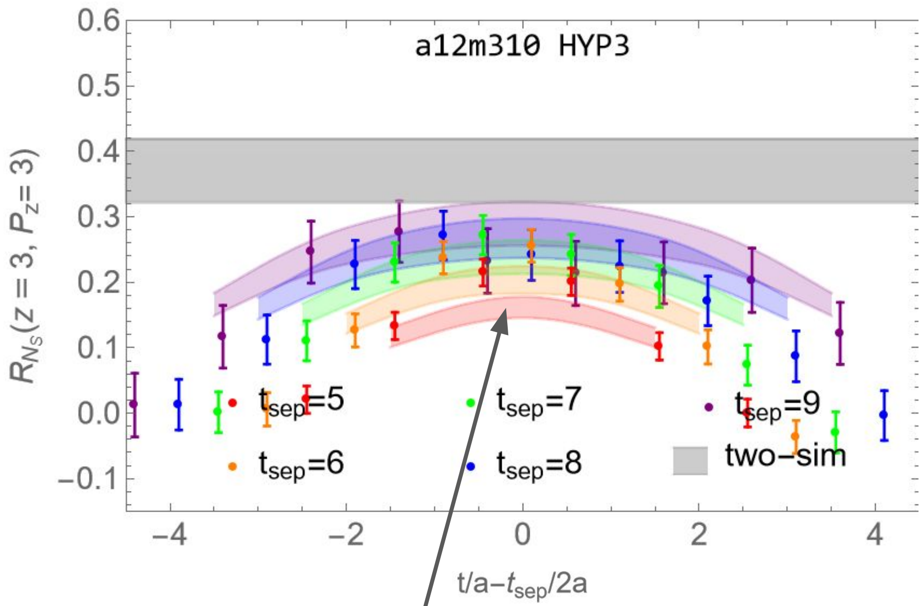
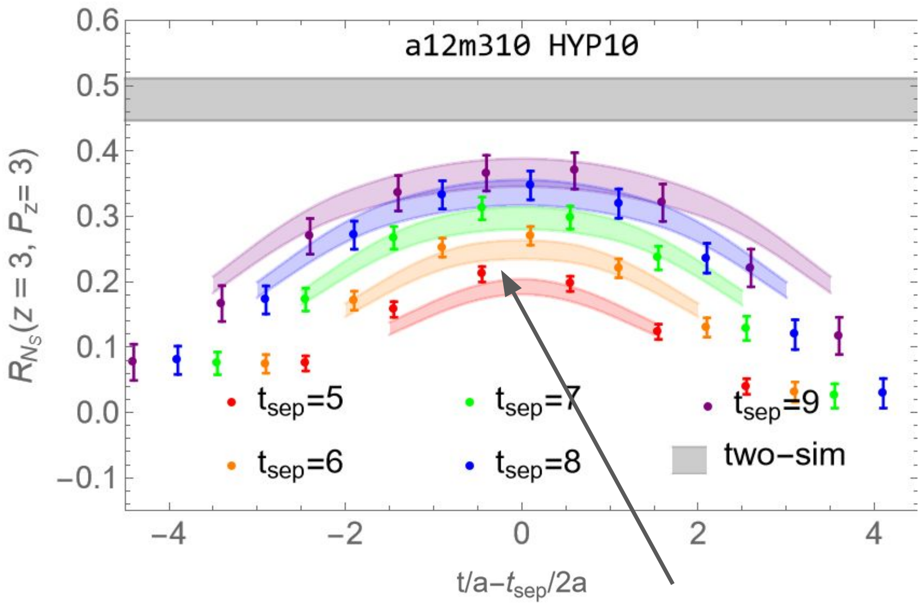
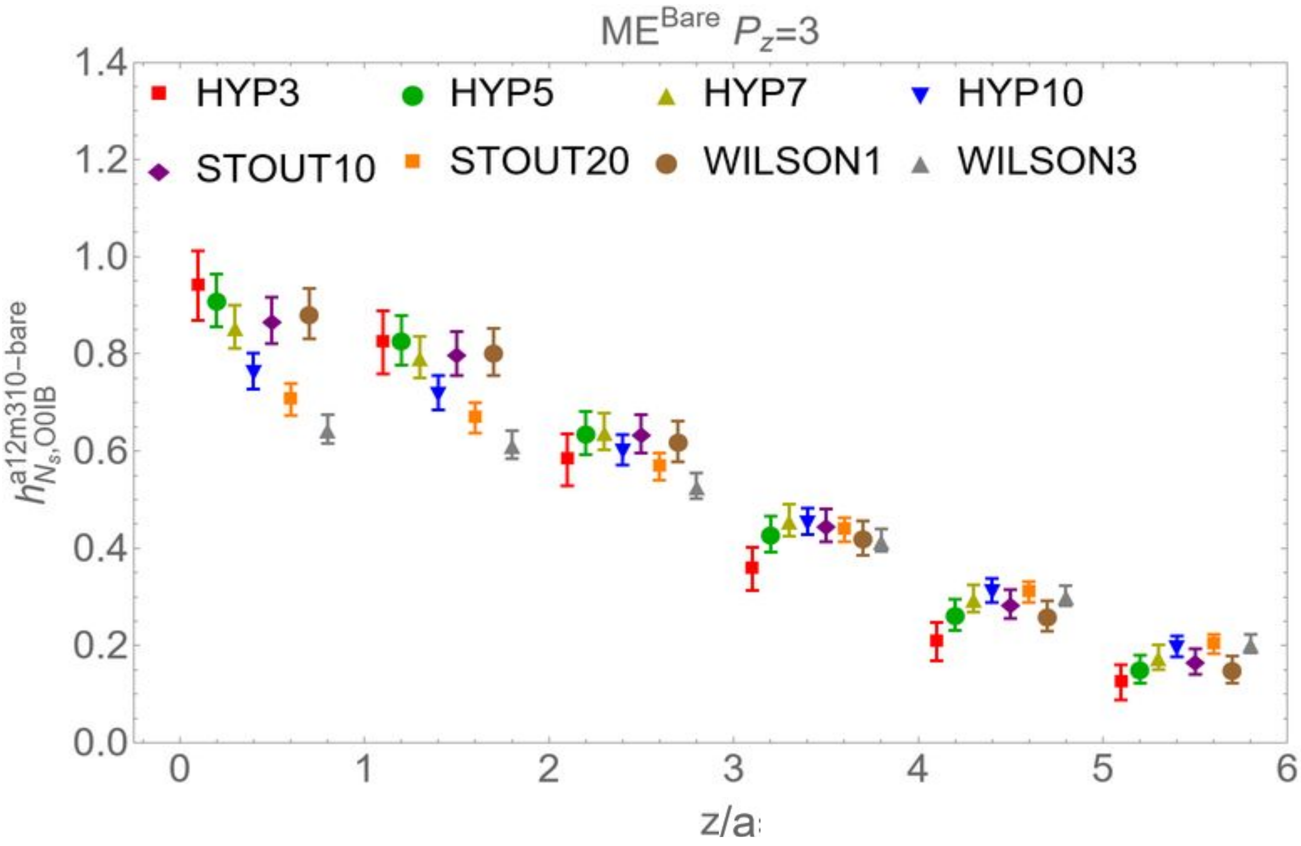




Overall, very noisy signal and loss of signal at larger t_{sep}

Much “prettier” signal with a similar χ^2/dof
Much better signal to noise

Bare Matrix Elements for N_s at $P_z = 1.29$ GeV



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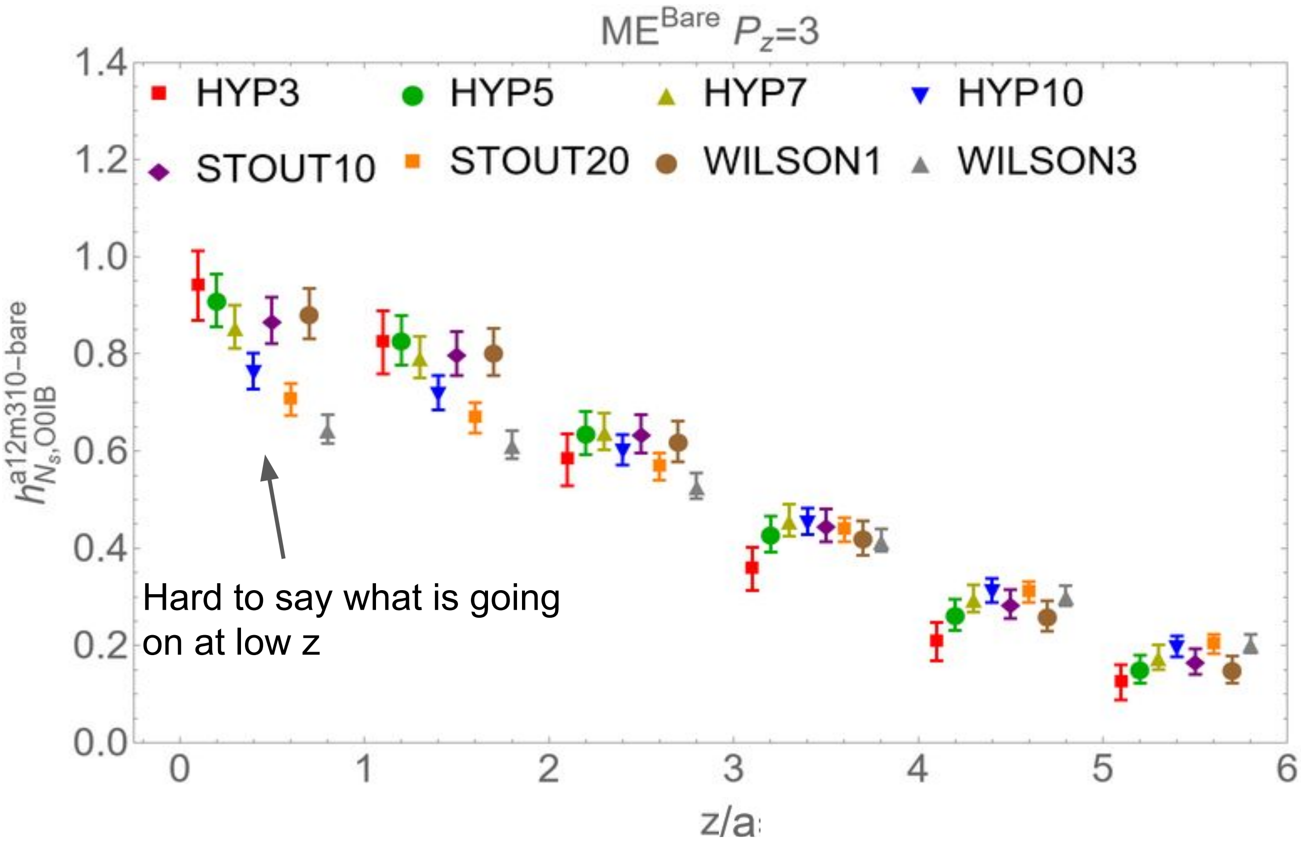

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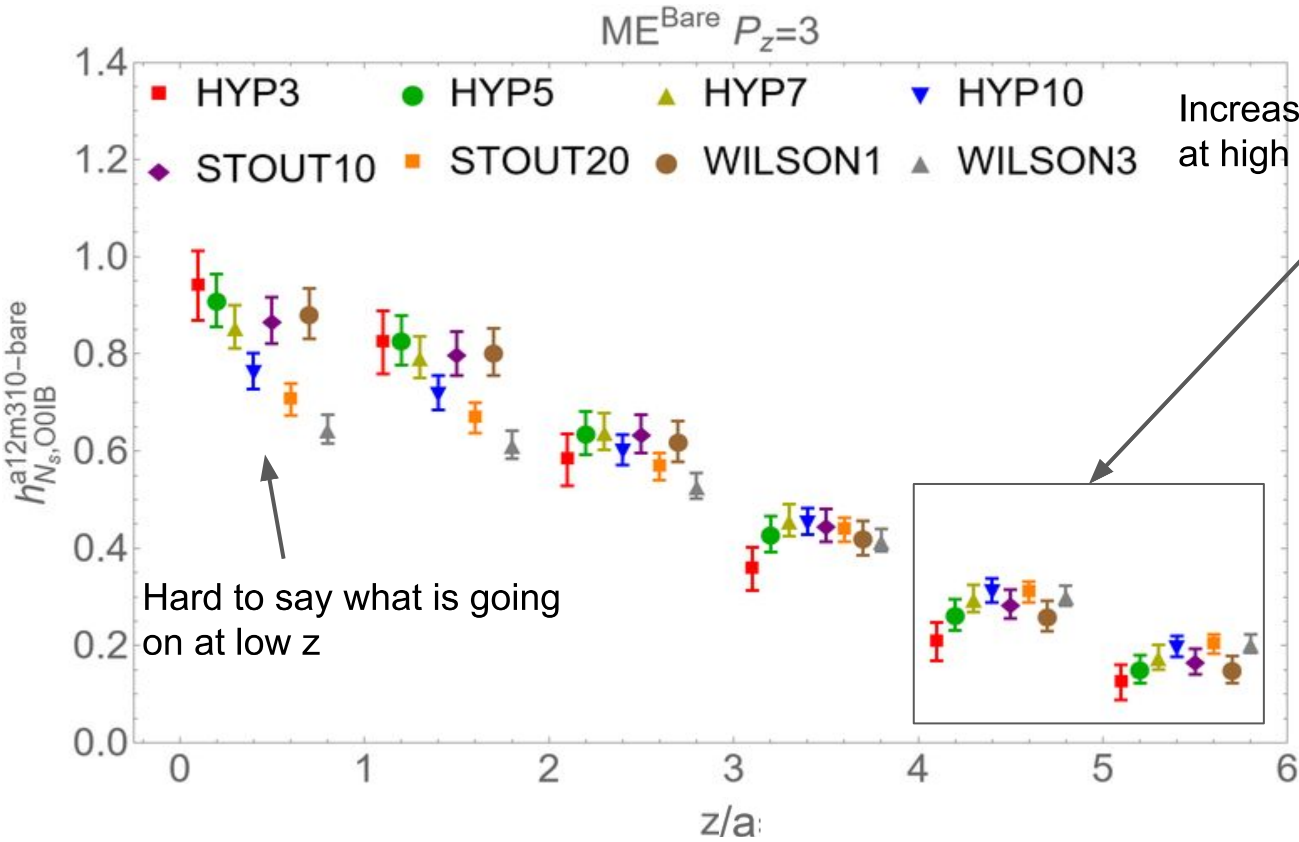
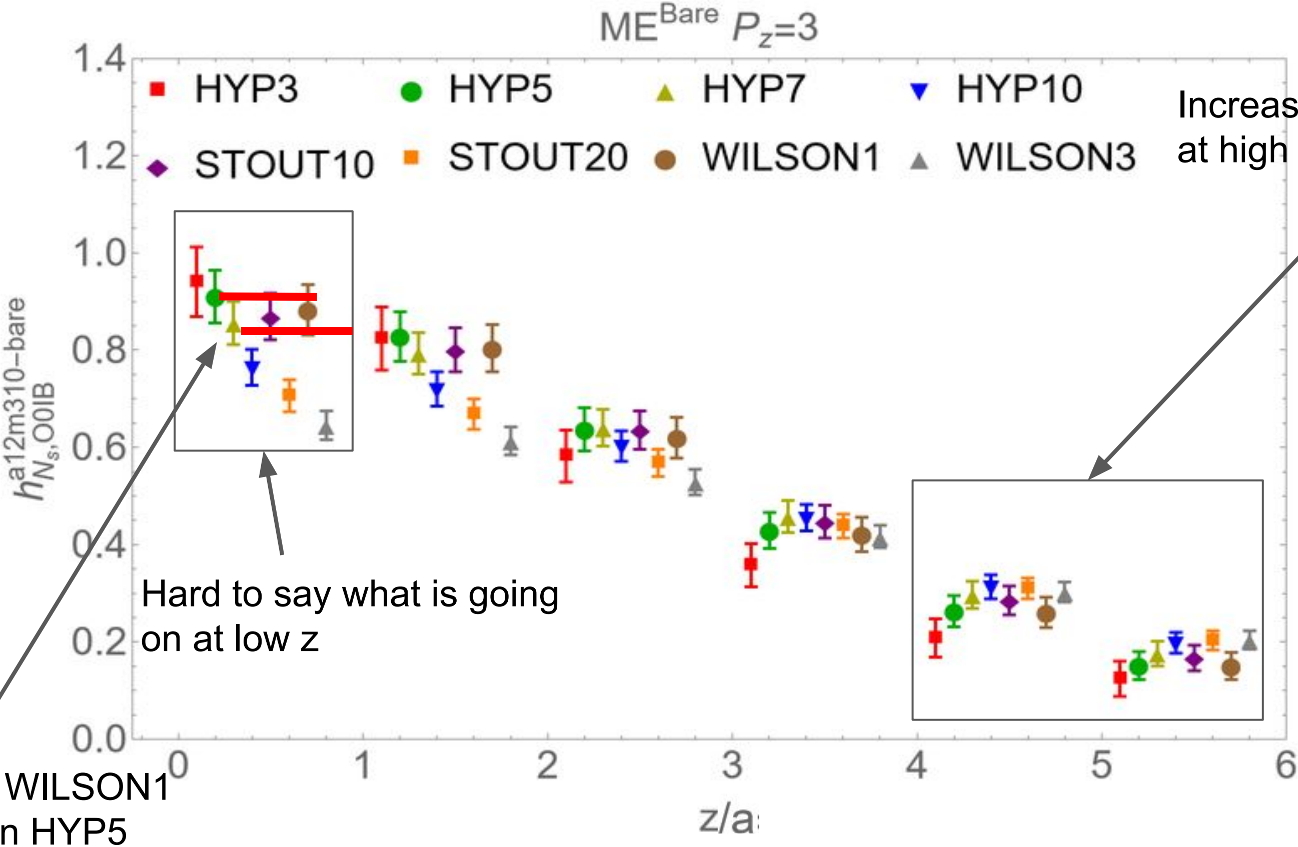


Increased signal to noise at high z as expected

Hard to say what is going on at low z

Bare Matrix Elements for N_s at P_z = 1.29 GeV



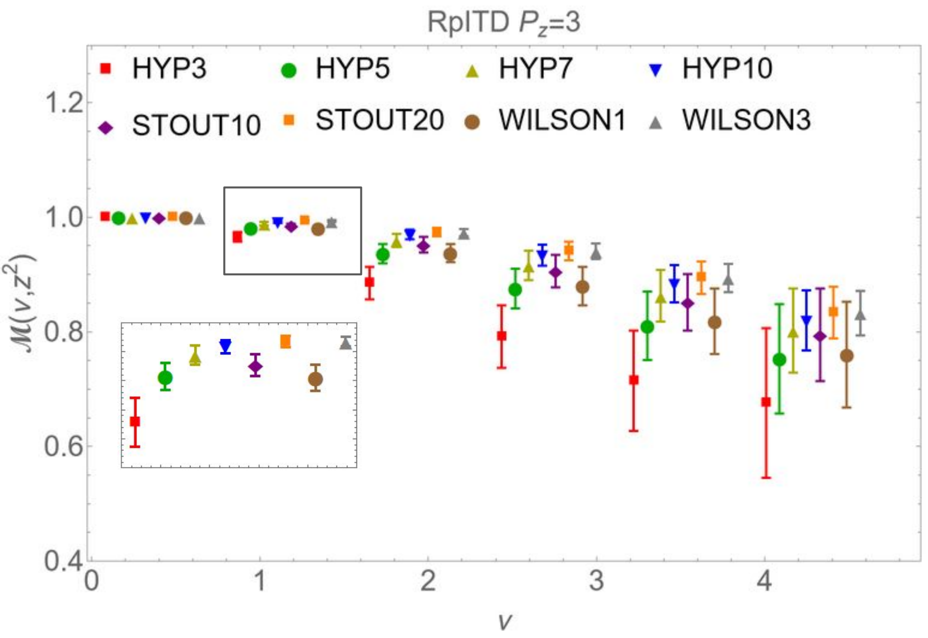
Increased signal to noise at high z as expected

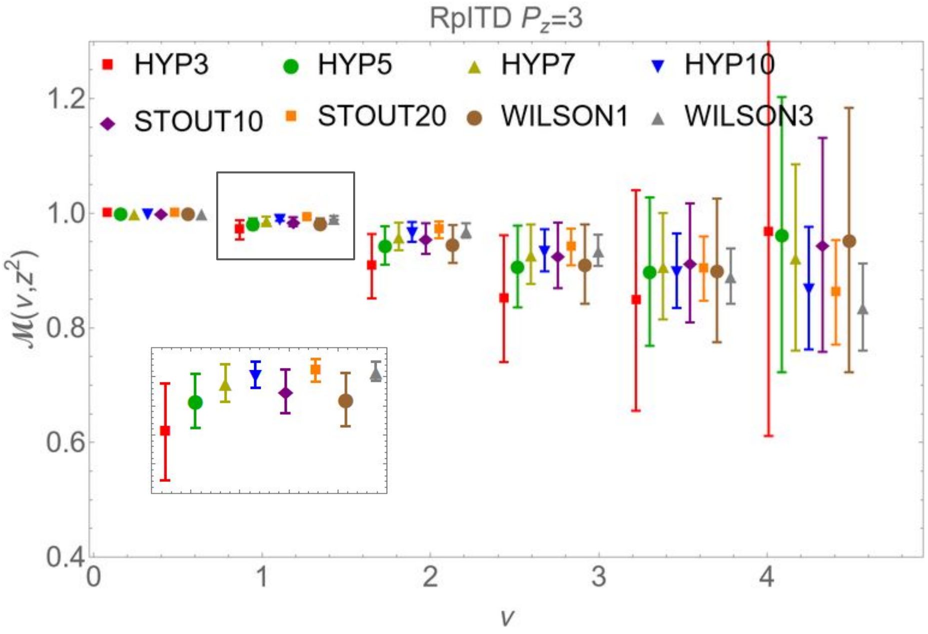
STOUT10 and WILSON1 always between HYP5 and HYP7. Coincidence?

RpITD for N_s (left) and N_l (right) (P_z= 1.29 Gev)

$$\mathscr{M}(\nu,z^2)=\frac{\mathcal{M}(zP_z,z^2)/\mathcal{M}(0\cdot P_z,0)}{\mathcal{M}(z\cdot 0,z^2)/\mathcal{M}(0\cdot 0,0)}$$

ν = zP_z





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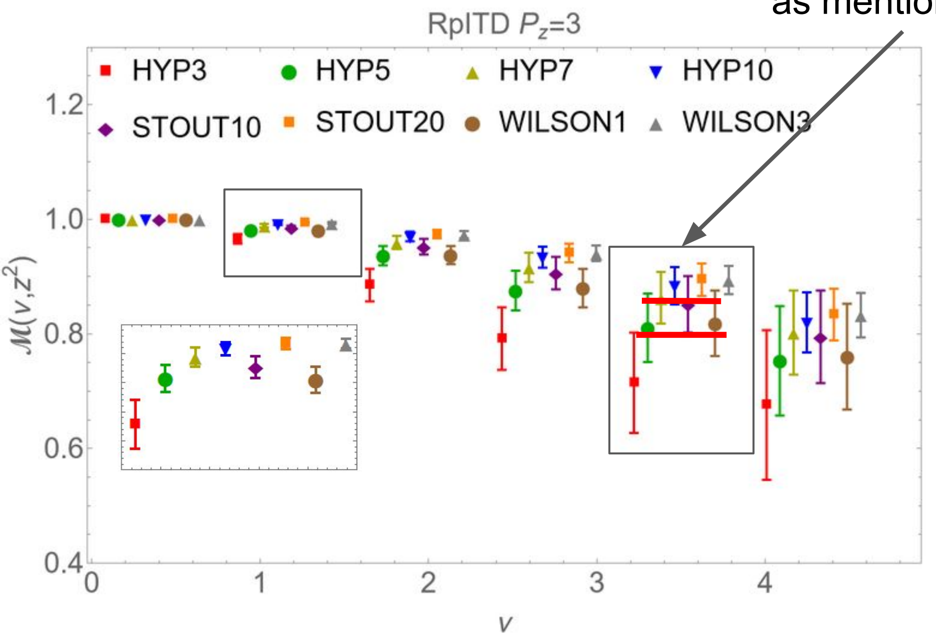
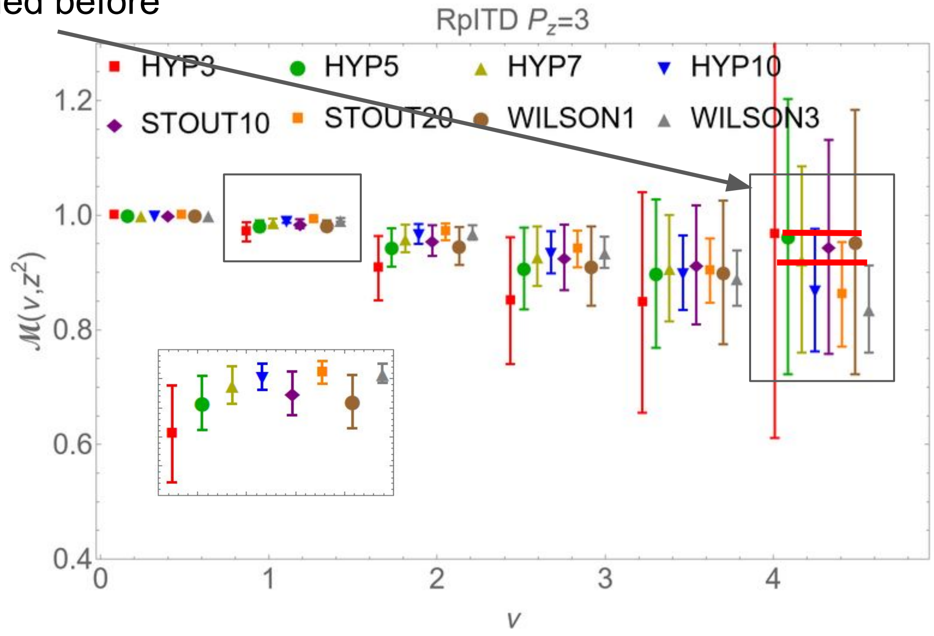
$\nu = zP_z$

Suspected correspondence as mentioned before





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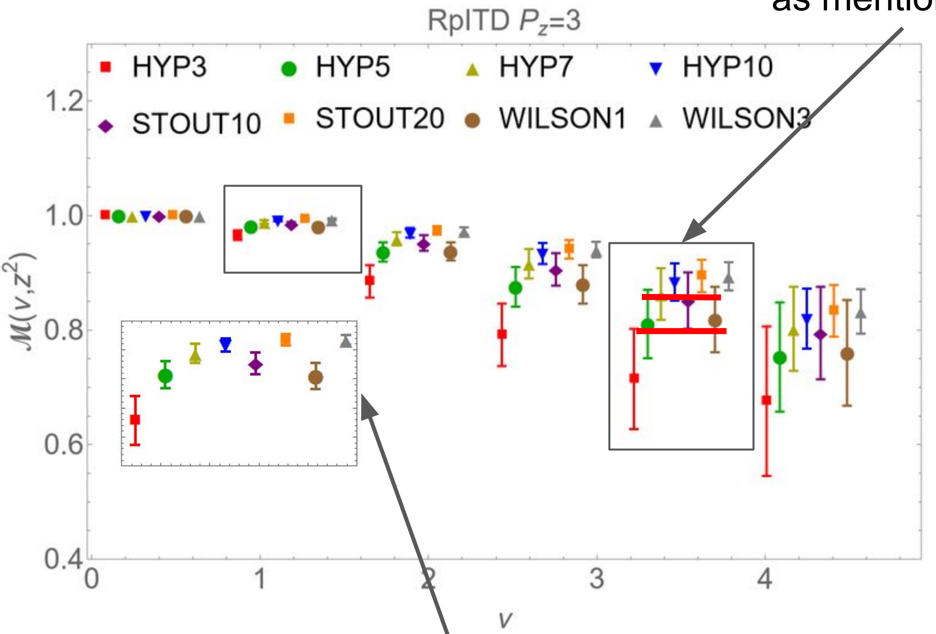

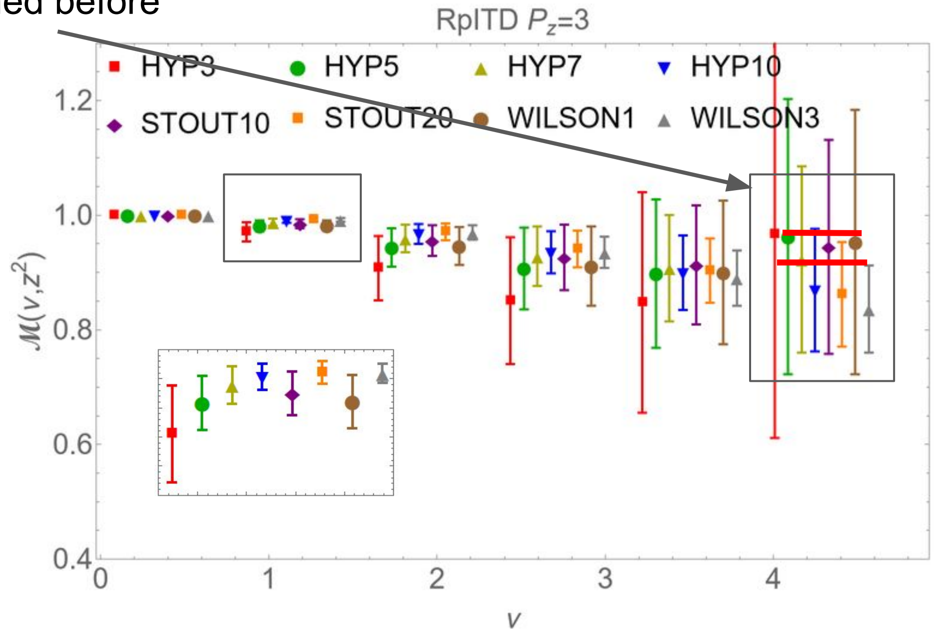

Slight tension between lowest and highest smearing for the heavy pion mass (somewhat unexpected)

RpITD for N_s (left) and N_l (right) (P_z= 1.29 Gev)

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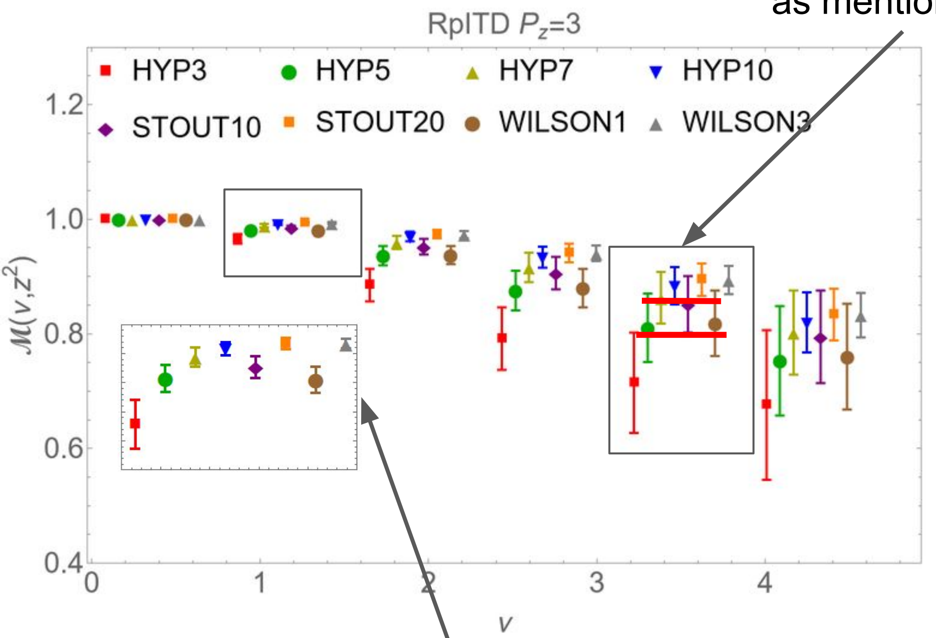

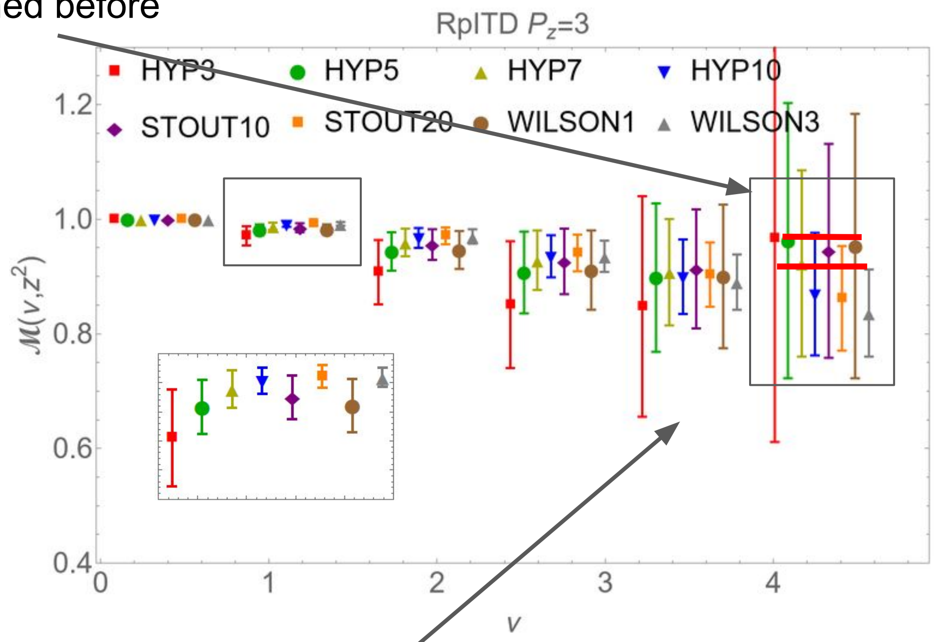

Slight tension between lowest and highest smearing for the heavy pion mass (somewhat unexpected)

Competing trends (at high z)?

Pion mass: ↓ RpITD: ↑

Smearing: ↑ RpITD: ↓

RpITD for η_s (left) and π (right) (P_z= 1.29 GeV)

$$\mathscr{M}(\nu,z^2)=\frac{\mathcal{M}(zP_z,z^2)/\mathcal{M}(0\cdot P_z,0)}{\mathcal{M}(z\cdot 0,z^2)/\mathcal{M}(0\cdot 0,0)}$$

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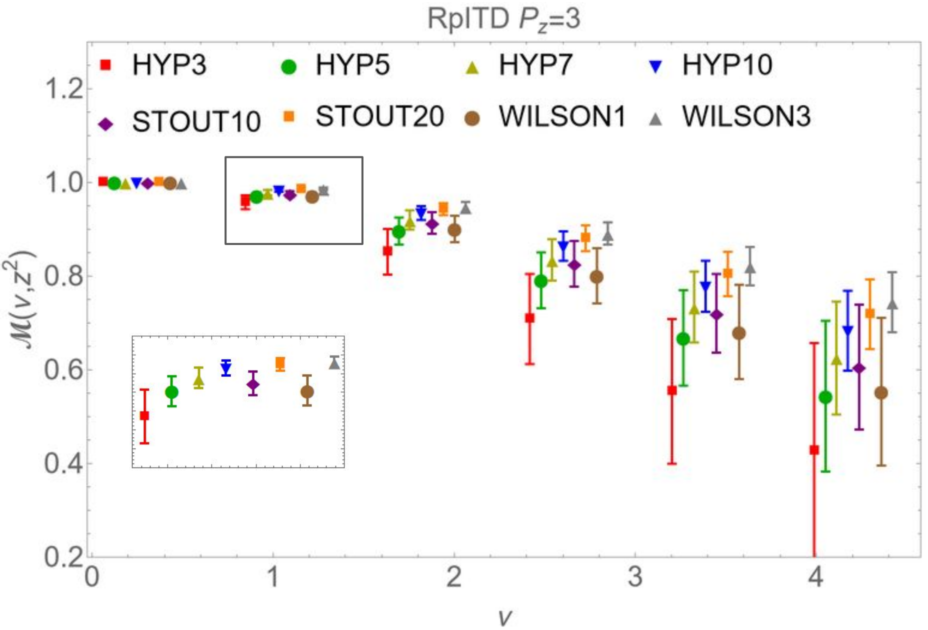

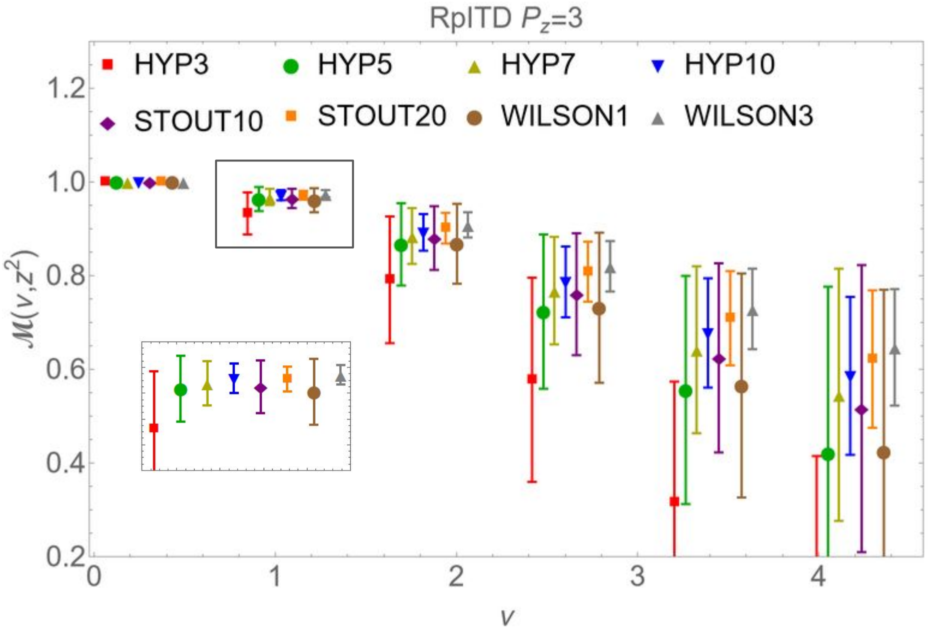

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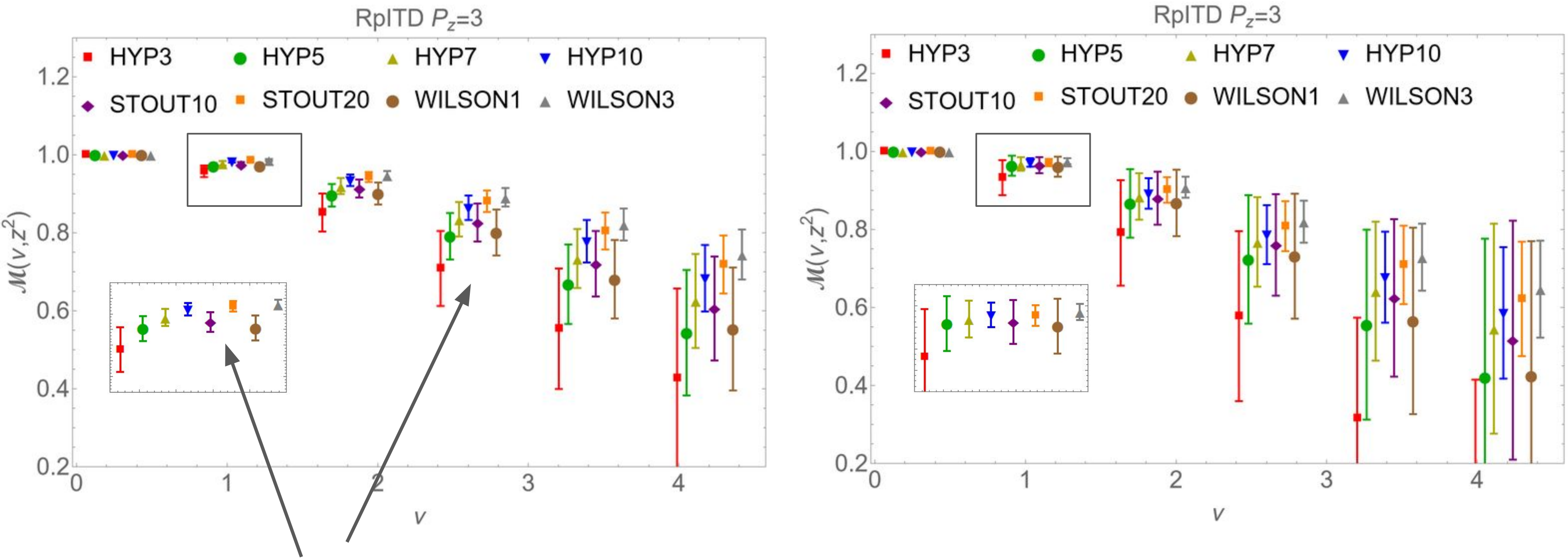

Similar tension just as in the nucleon case

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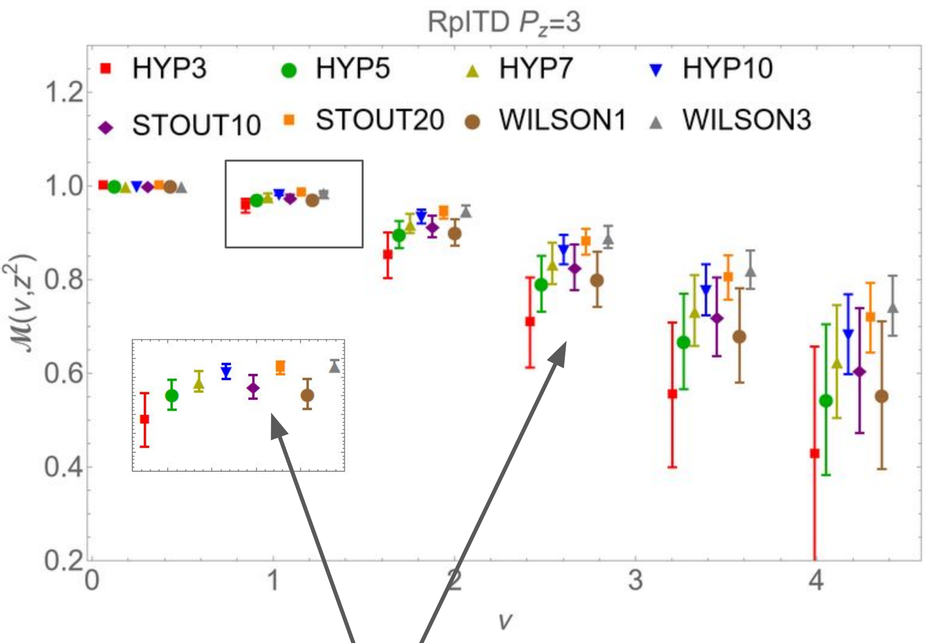

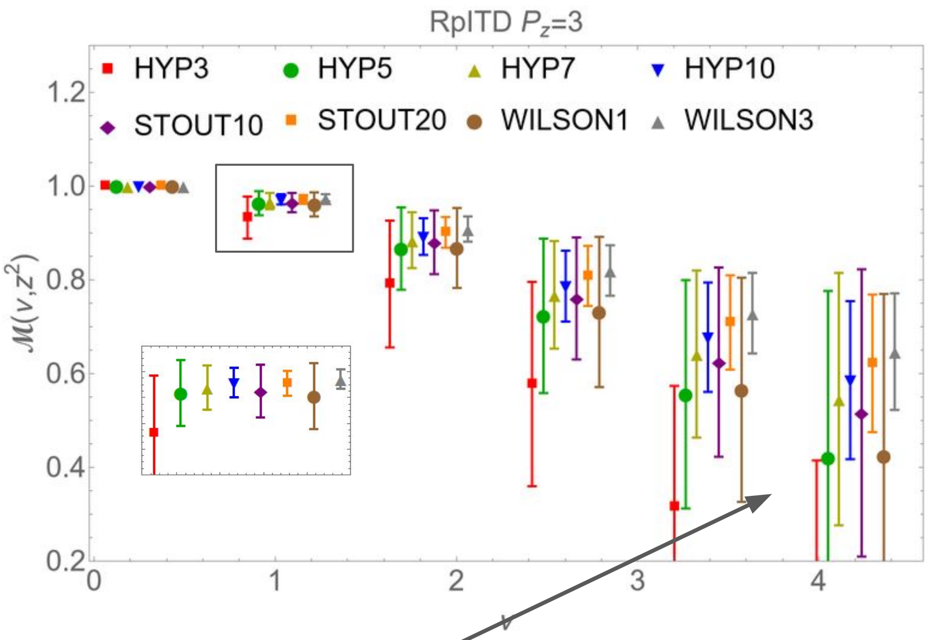

Similar tension just as in the nucleon case

Different trends than nucleon (at high z)?

Pion mass: ↓ RpITD: ↓

Smearing: ↑ RpITD: ↑

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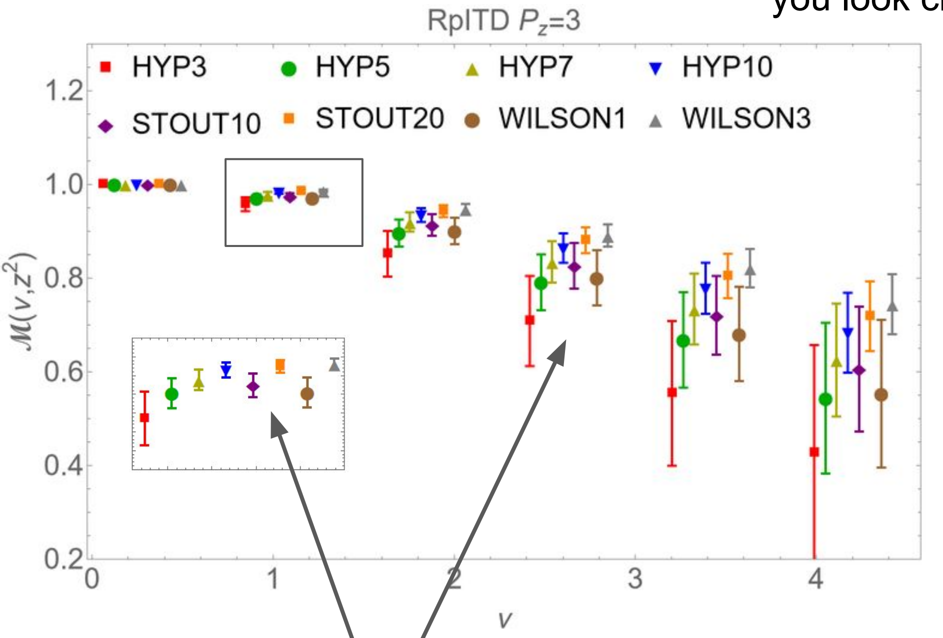
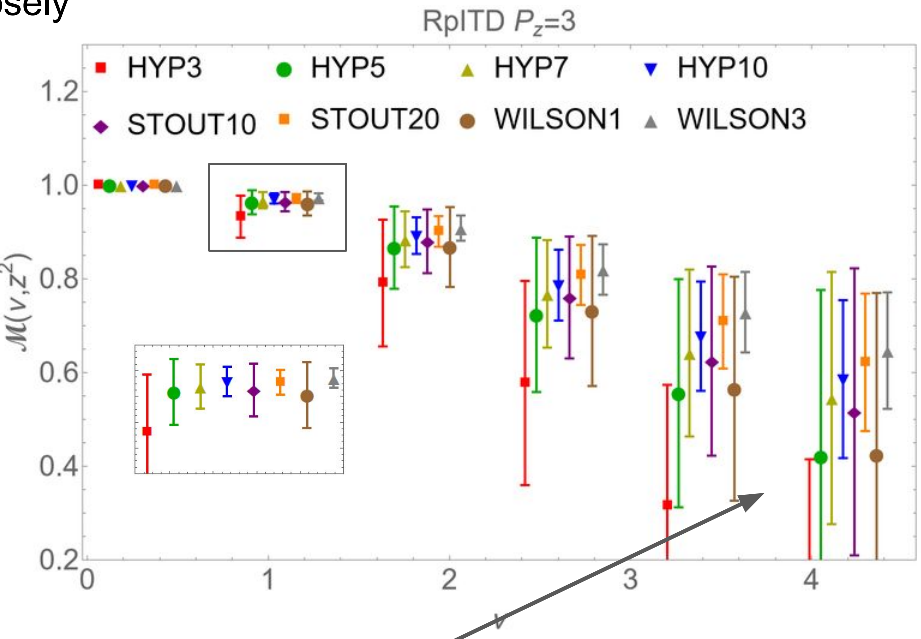
Correspondence still here if you look closely





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PDF (Divided by Moment) Fit

Gluon matching kernel R_{gg} connects the RpITD to the PDF as shown

$$\mathscr{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x,\mu^2)}{\langle x \rangle_g} R_{gg}(x\nu, z^2\mu^2)$$

Use a typical global analysis fit form

Balitsky et al, PLB 808:135621, 2020.

$B(A+1, C+1)$ is beta function (integral of numerator)

(μ=2 GeV is the renormalization scale in the MS-bar scheme)

$$f_g(x,\mu) = \frac{xg(x,\mu)}{\langle x \rangle_g(\mu)} = \frac{x^A(1-x)^C}{B(A+1,C+1)}$$

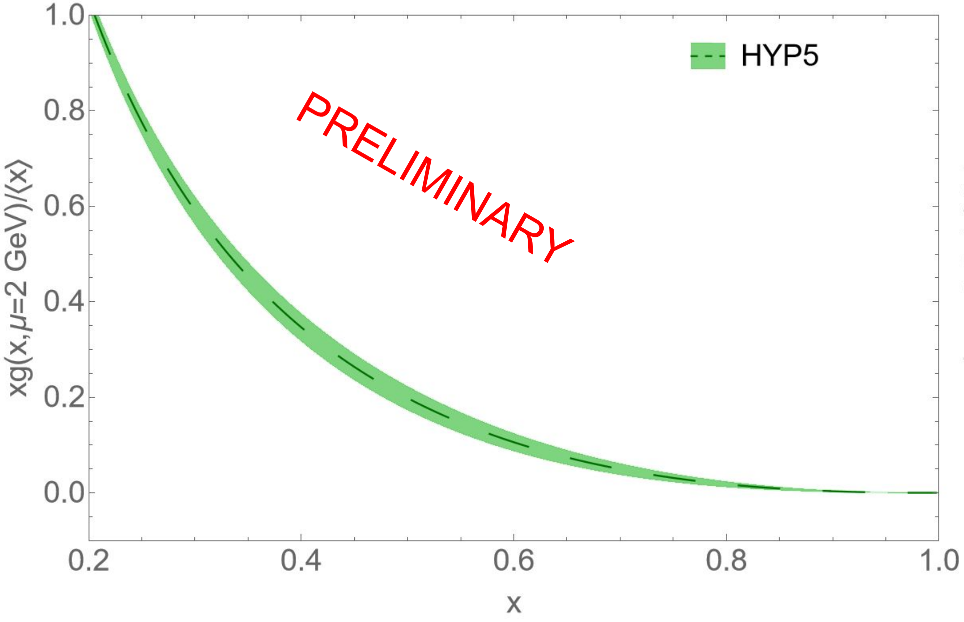
Minimize

$$\chi^2(\mu, a, M_\pi) = \sum_{\nu,z} \frac{(\mathscr{M}^{\rm fit}(\nu,\mu,z^2,a,M_\pi) - \mathscr{M}^{\rm lat}(\nu,z^2,a,M_\pi))^2}{\sigma^2_{\mathscr{M}}(\nu,z^2,a,M_\pi)}$$

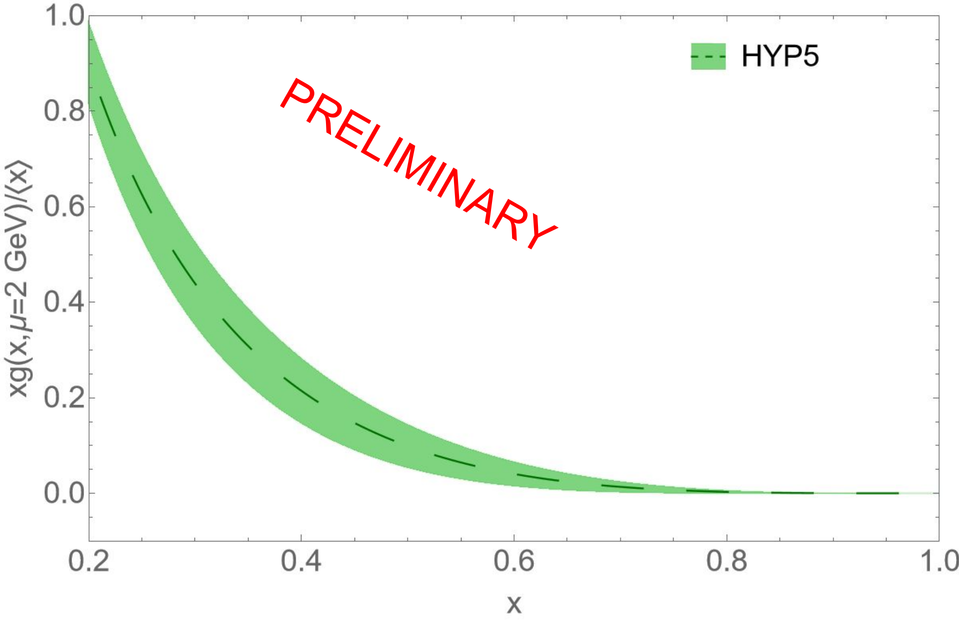
Unpolarized Nucleon PDFs

Same ensemble calculation plotted with other lattice and global analysis results can be found here:
Z. Fan, WG, H. W. Lin, PRD 108:014508 (2023)

a=0.12 fm M_π~690 MeV



a=0.12 fm M_π~310 MeV



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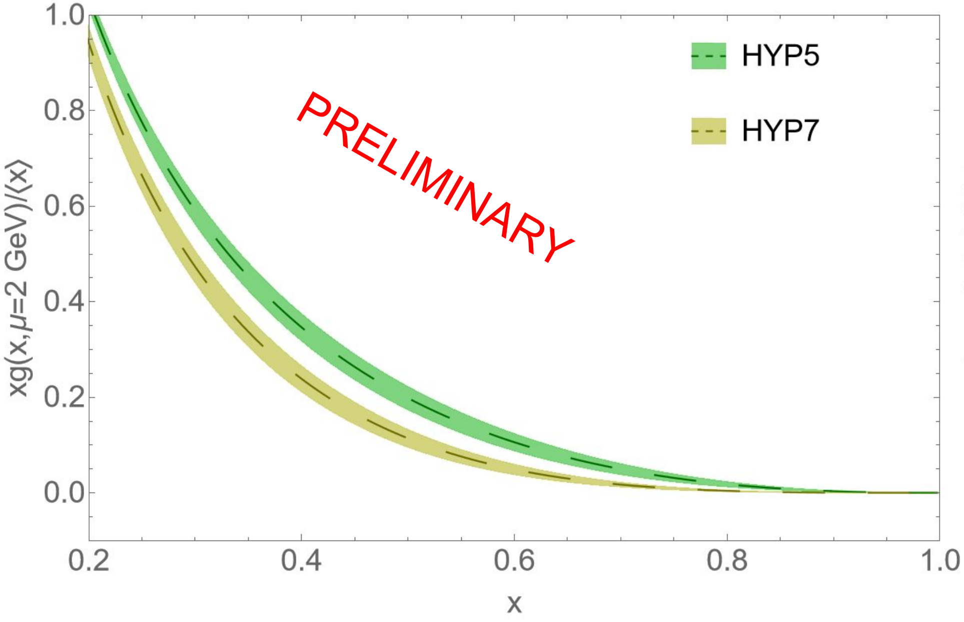

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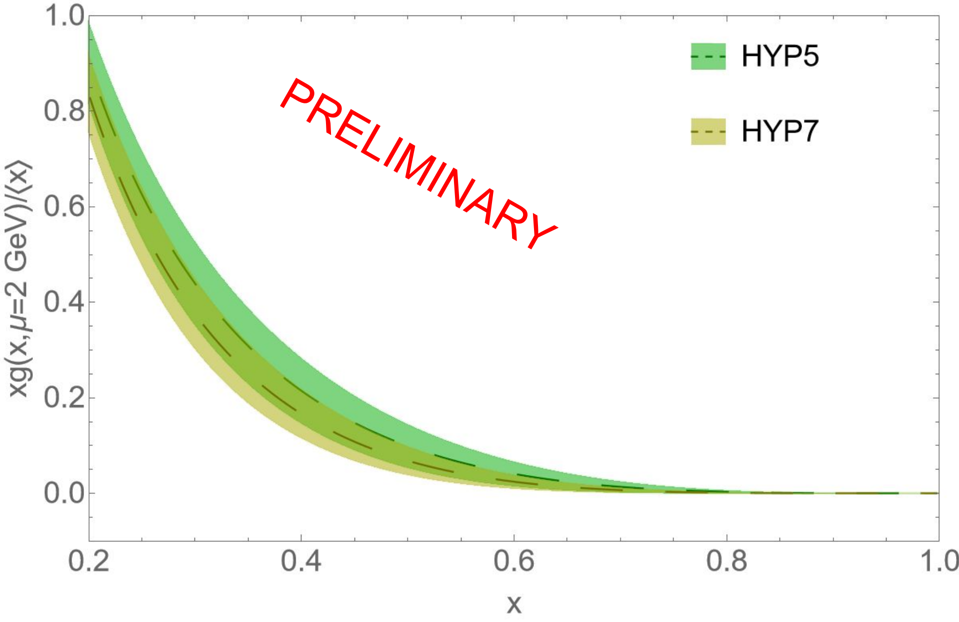

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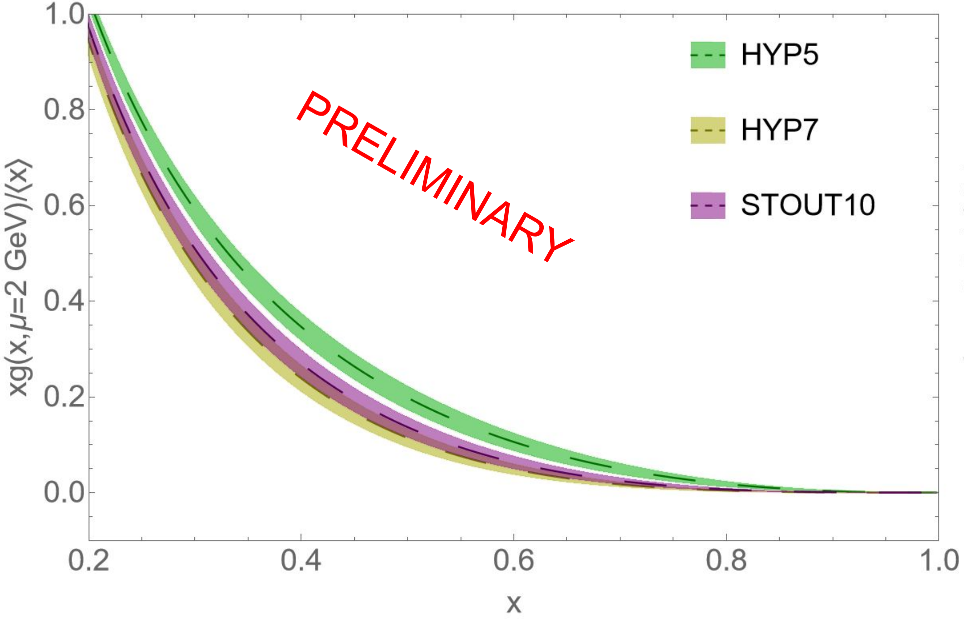

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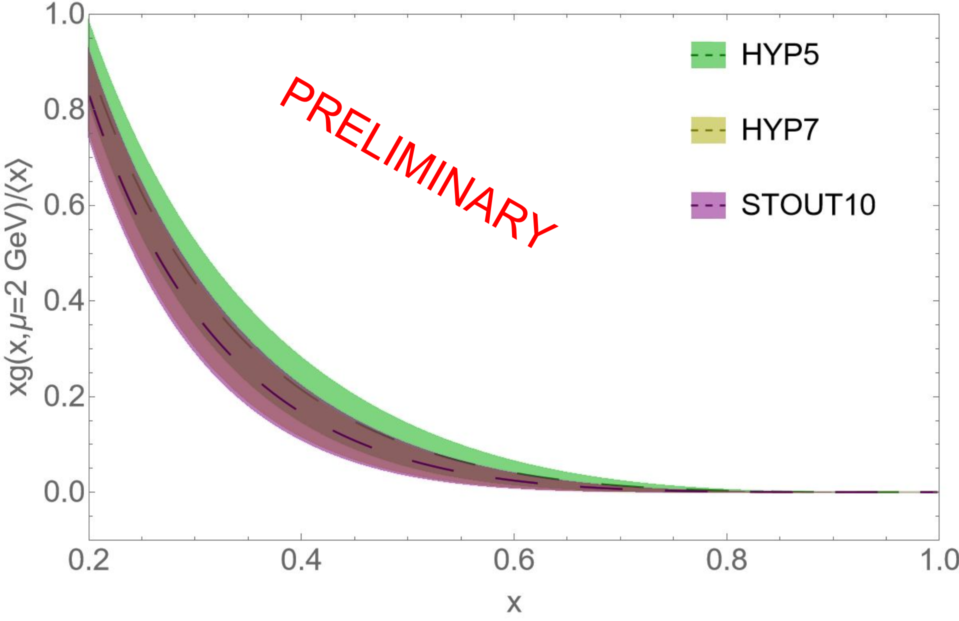

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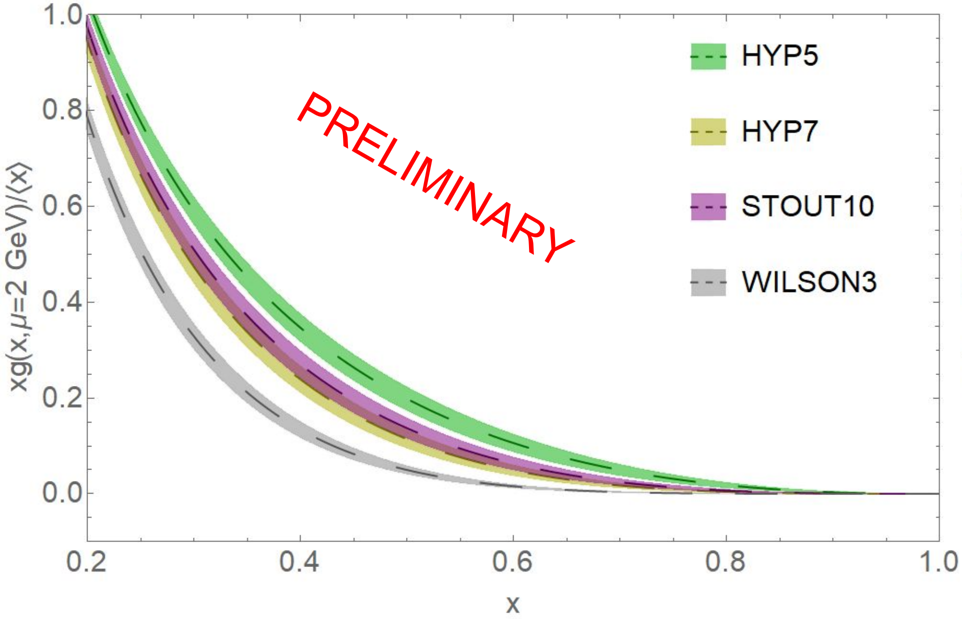
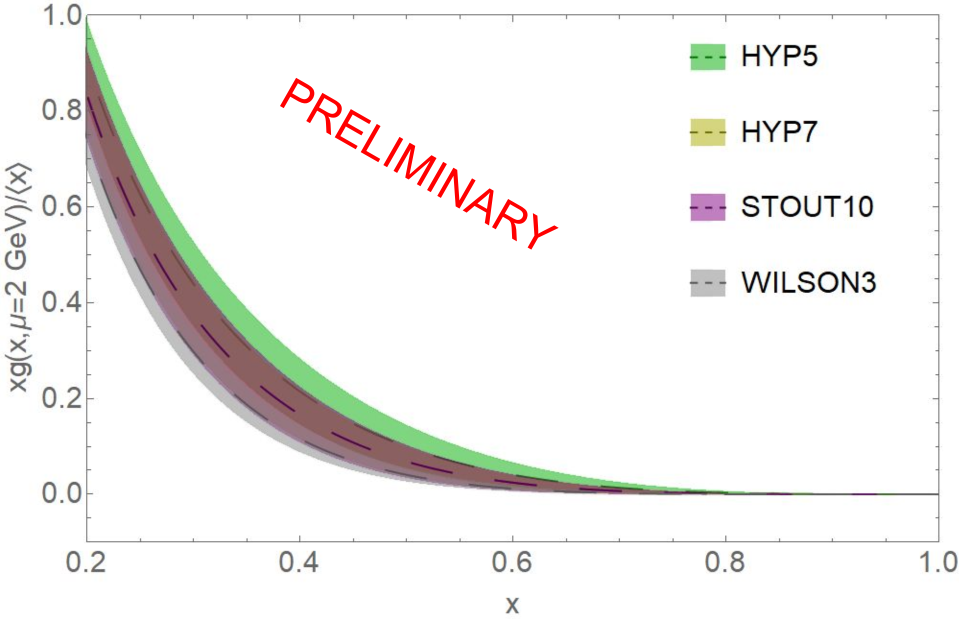


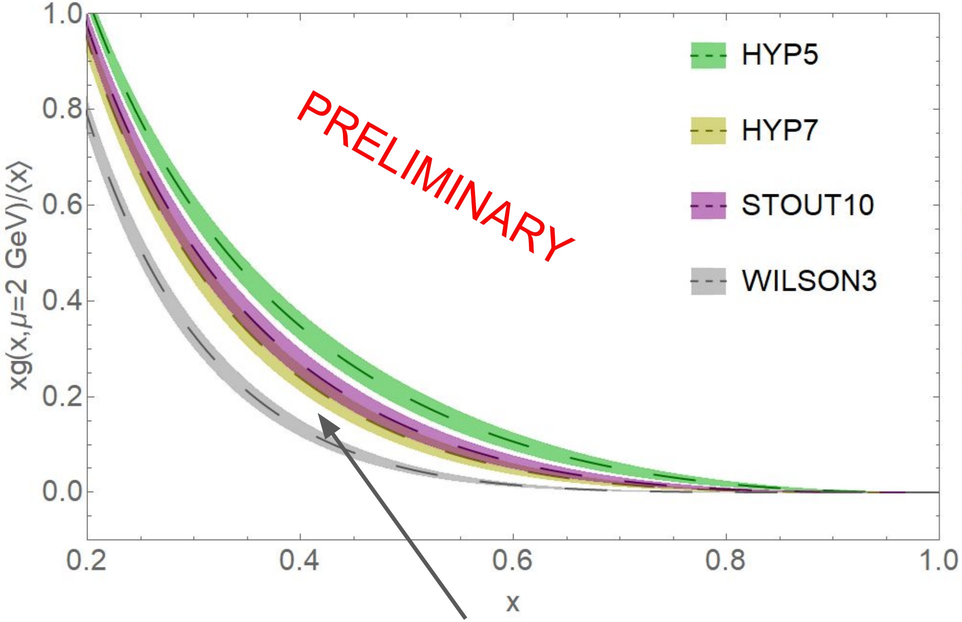
a=0.12 fm M_π~310 MeV



Unpolarized Nucleon PDFs

Same ensemble calculation plotted with other lattice and global analysis results can be found here:
Z. Fan, WG, H. W. Lin, PRD 108:014508 (2023)

a=0.12 fm M_π~690 MeV



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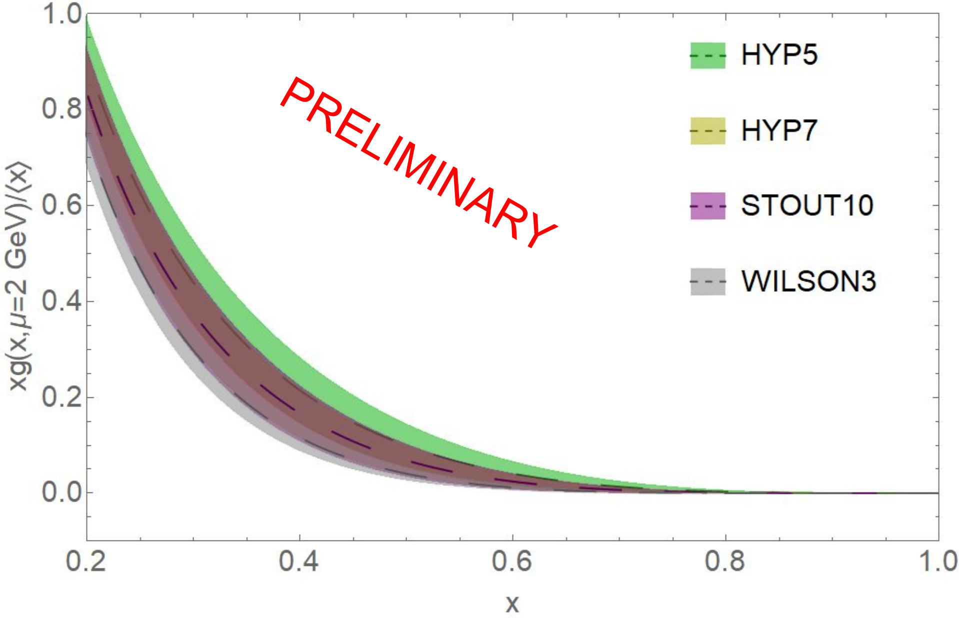

CLEAR tension between WILSON3 and the rest
Some tension between HYP5 and HYP7/STOUT10

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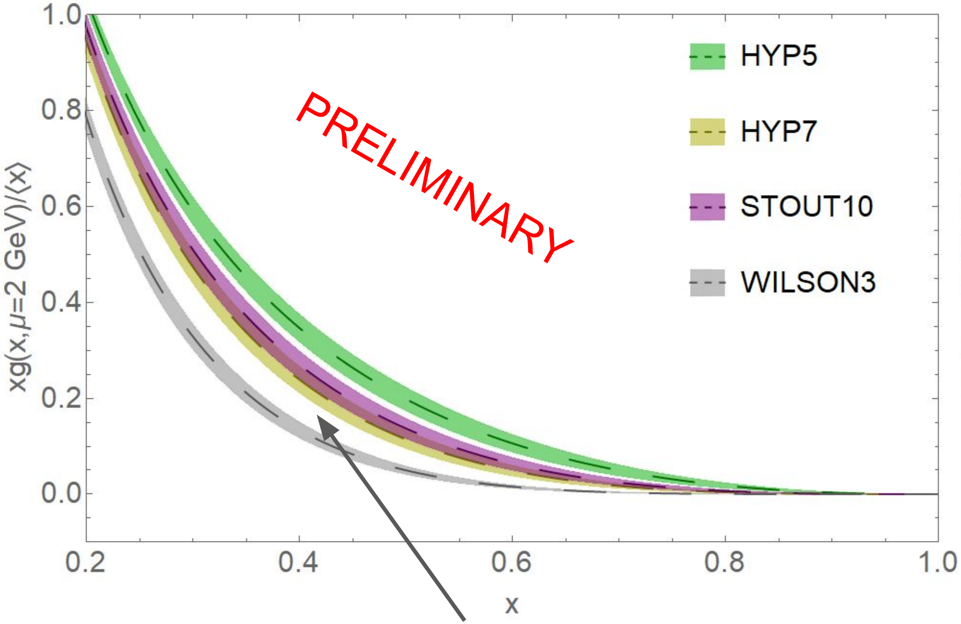

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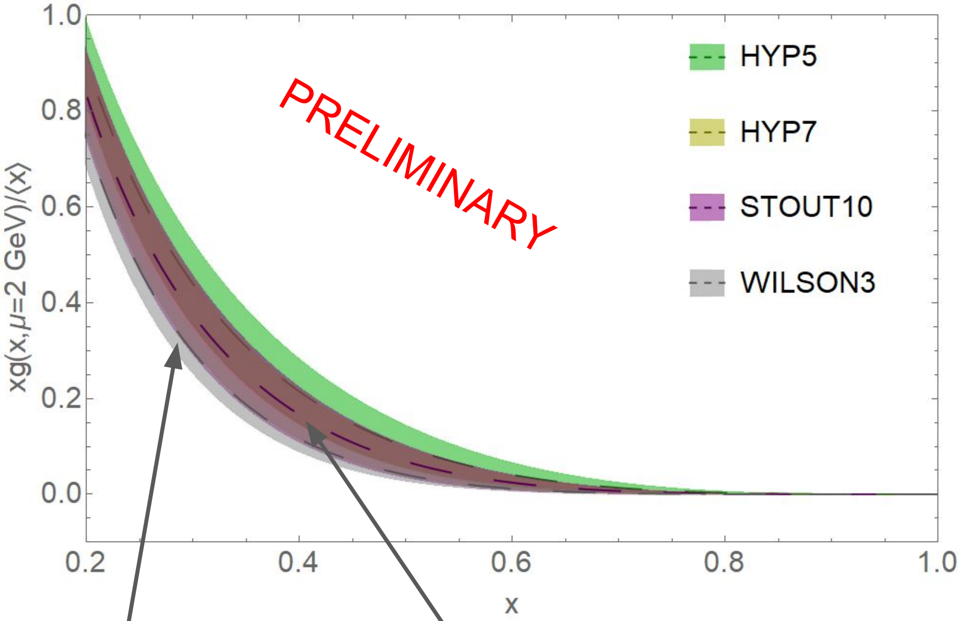

CLEAR tension between WILSON3 and the rest
Some tension between HYP5 and HYP7/STOUT10

Very happy agreement
HYP7 and STOUT10 are right on top of each other
Tension between light and strange nucleon

Unpolarized Pion PDFs

Same ensemble calculation plotted with other lattice and global analysis results can be found here:
Z. Fan, H. W. Lin, PLB 823:136778 (2021)

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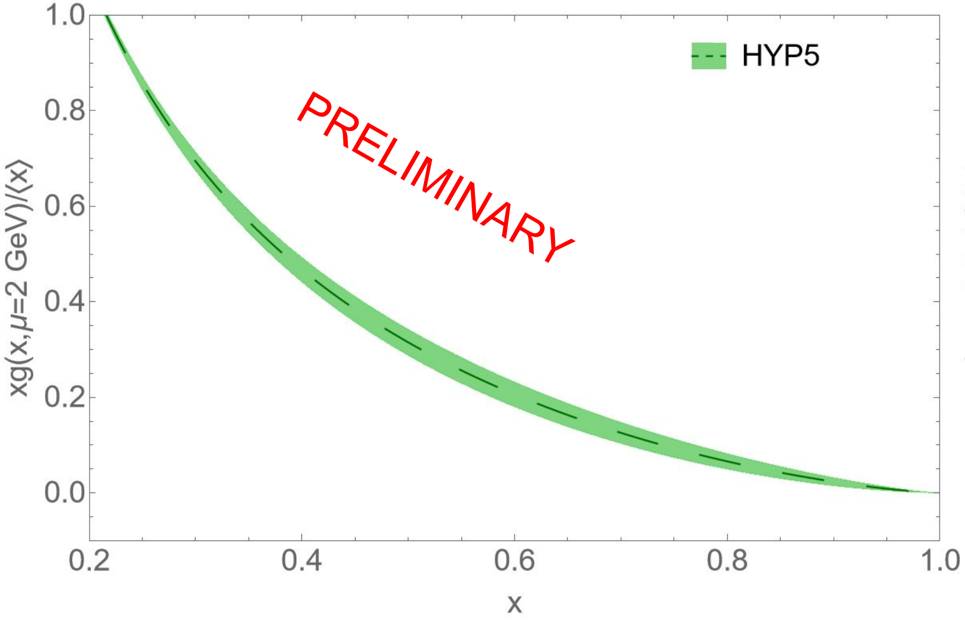

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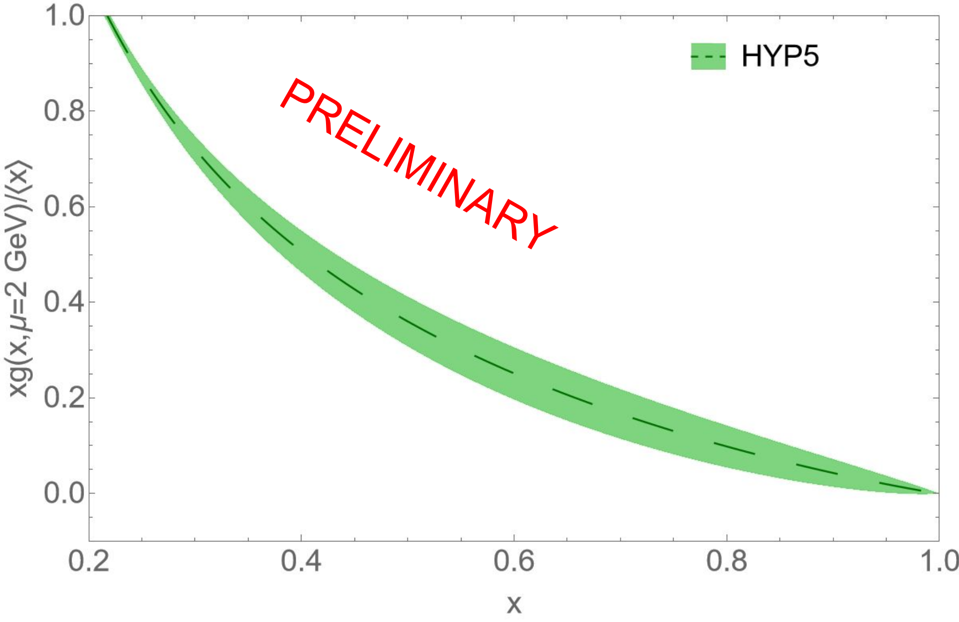

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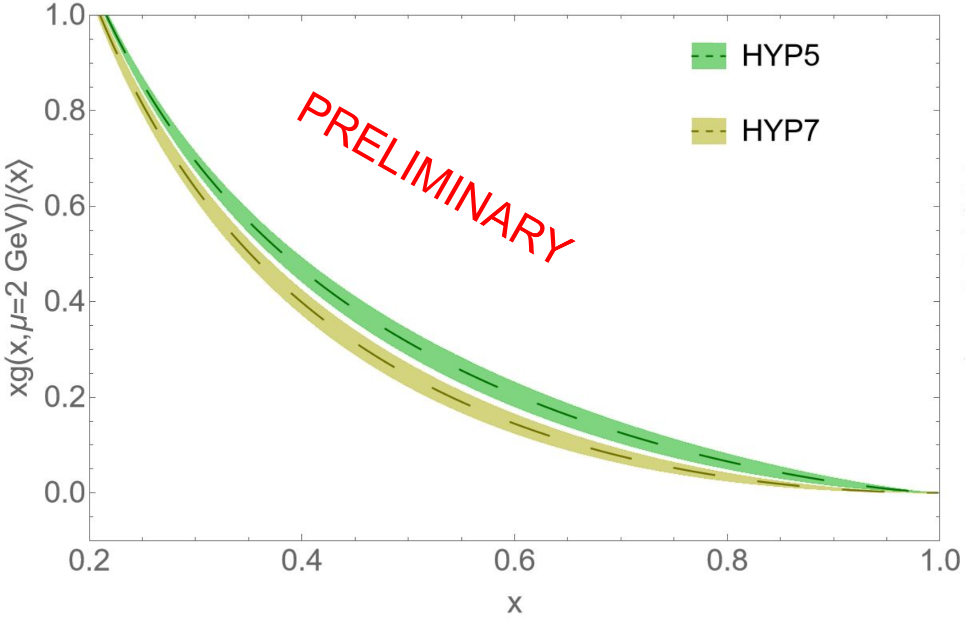

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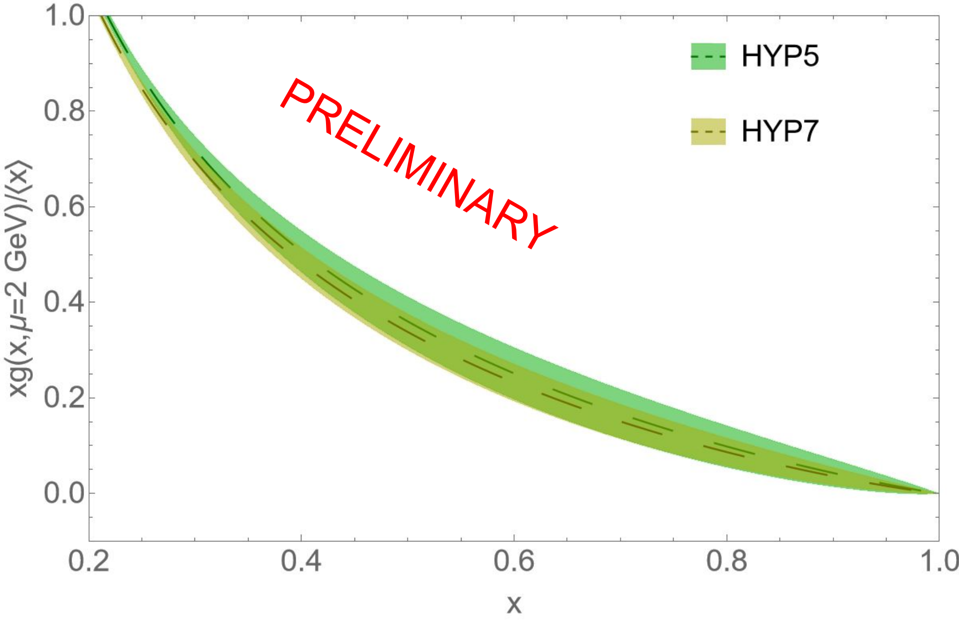

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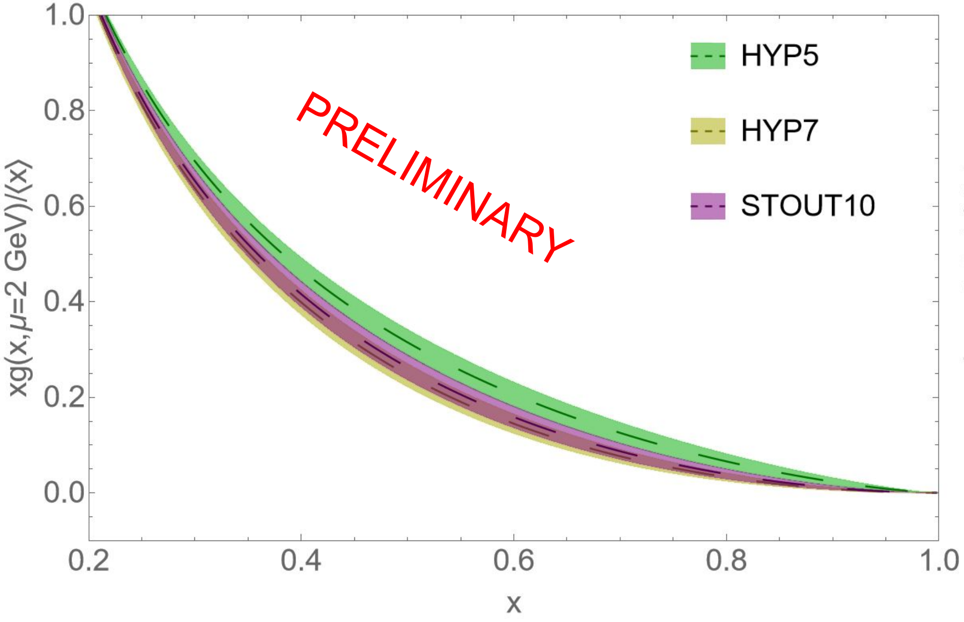

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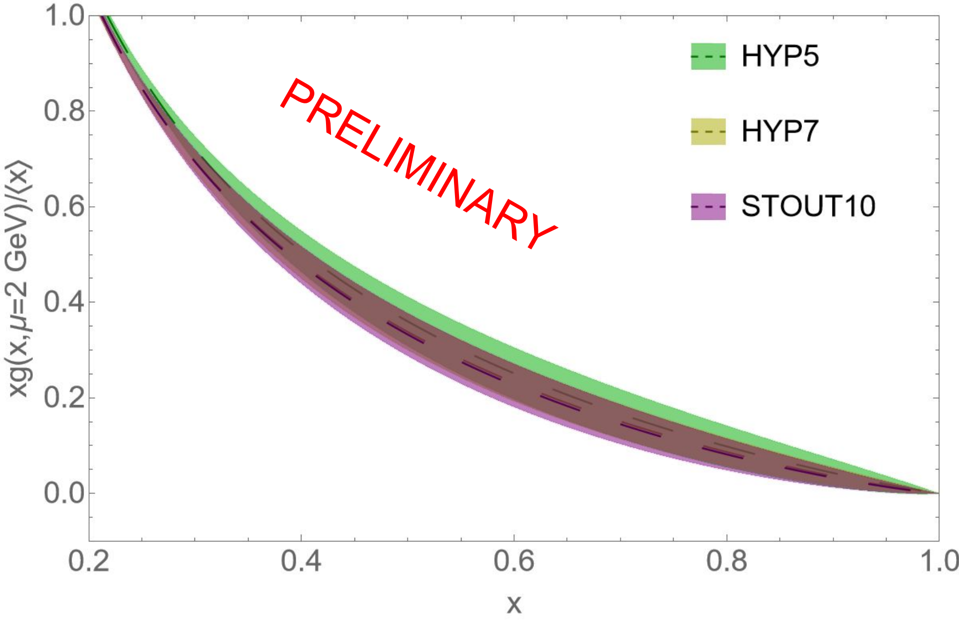

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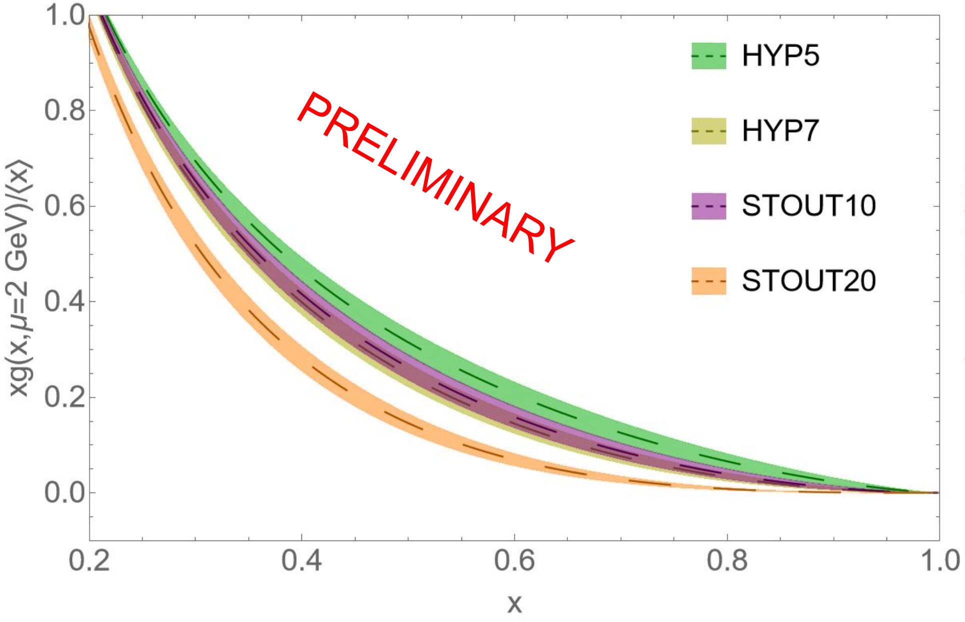

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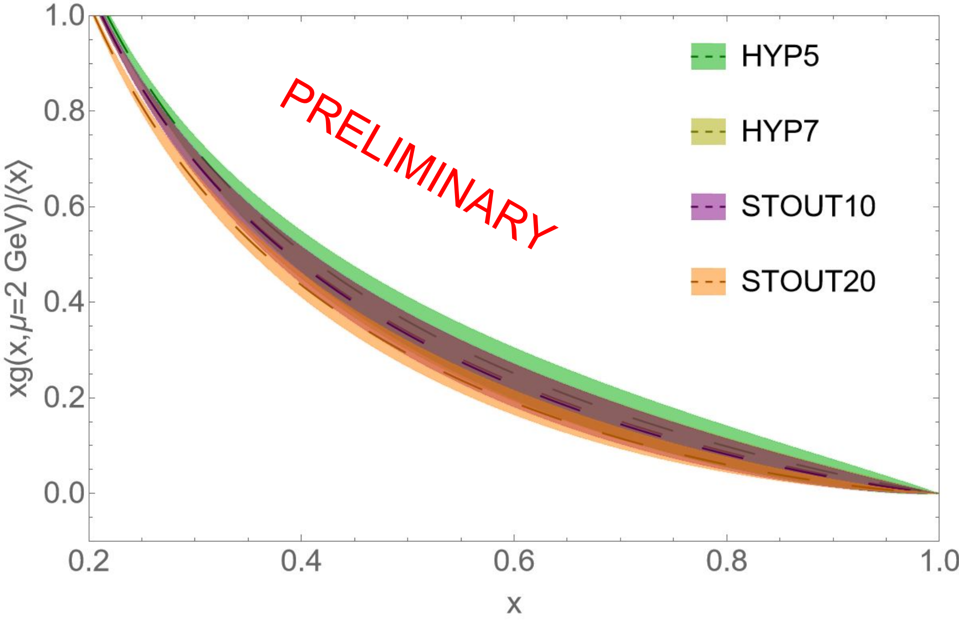

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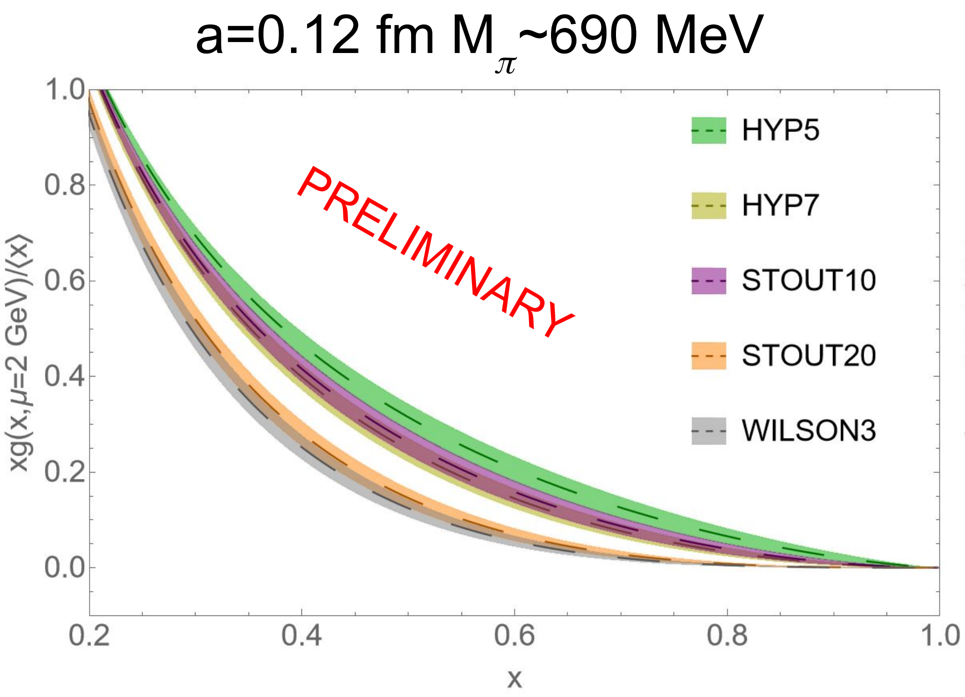

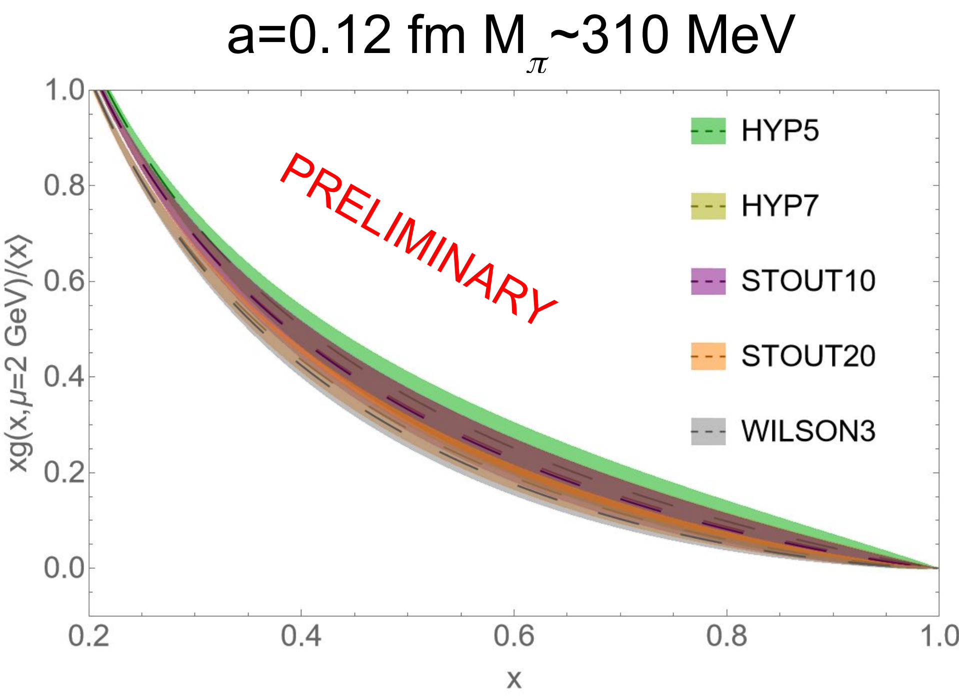

Unpolarized Pion PDFs

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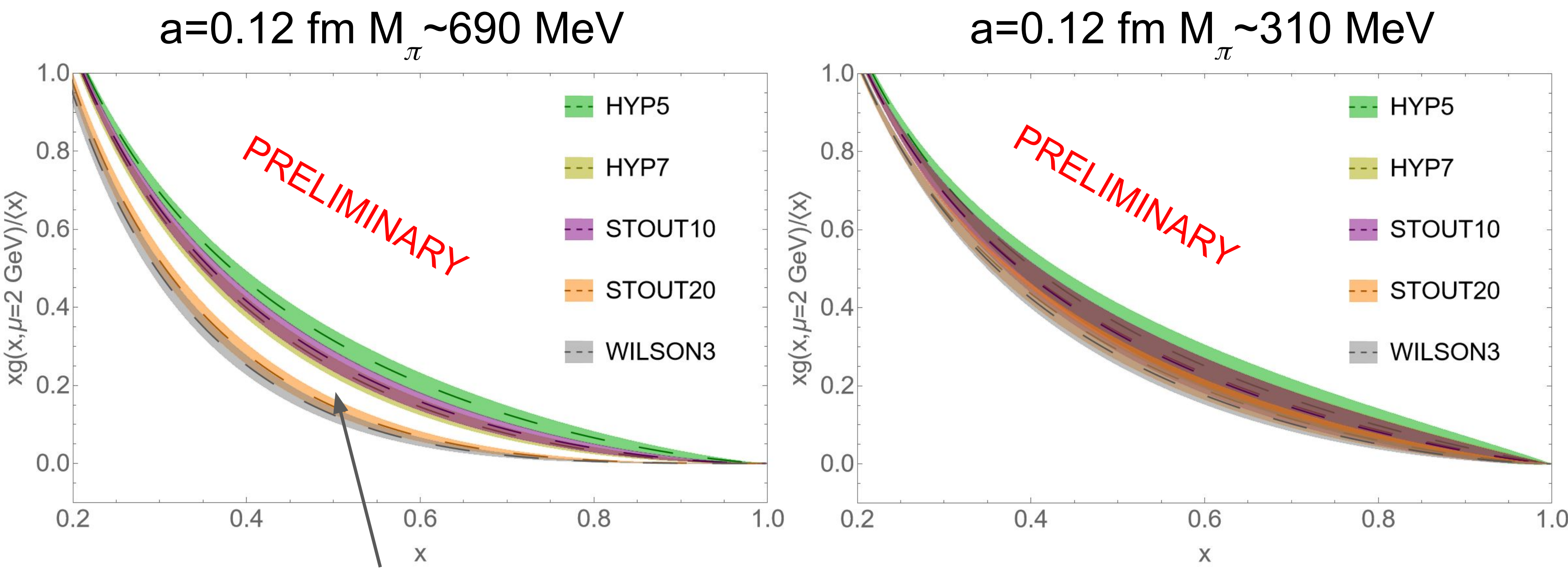

CLEAR tension between WILSON3/STOUT20 and the rest
Slight tension between HYP5 and HYP7

Unpolarized Pion PDFs

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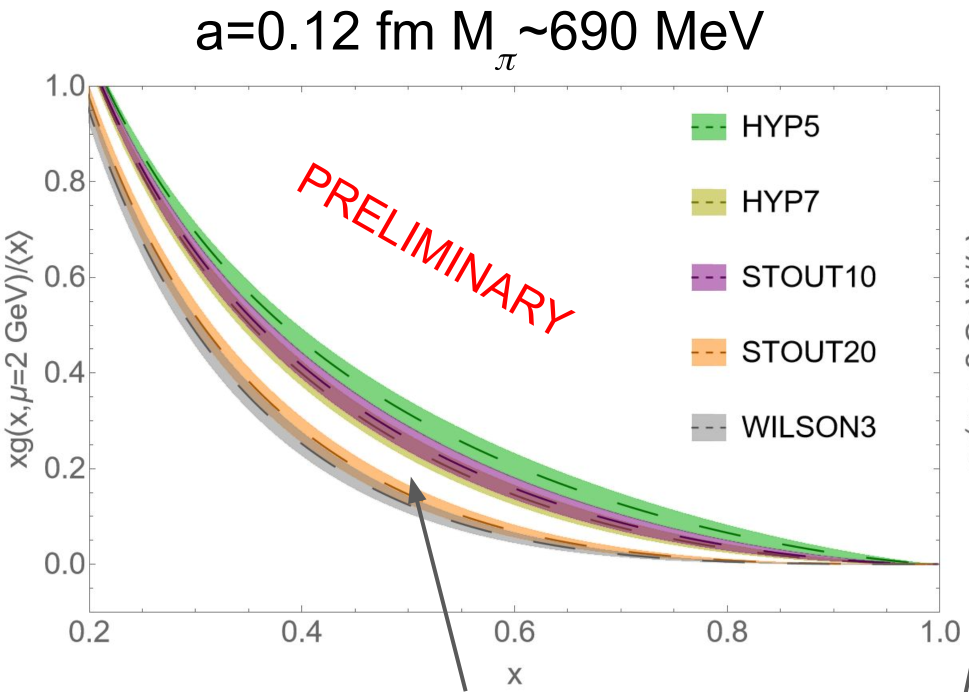
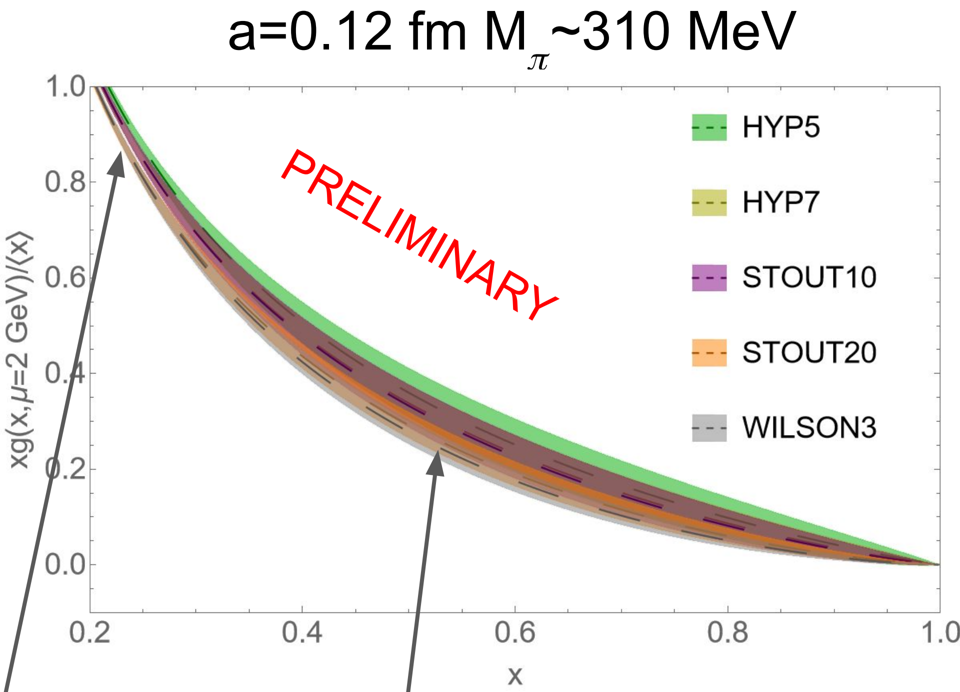
Z. Fan, H. W. Lin, PLB 823:136778 (2021)





CLEAR tension between WILSON3/STOUT20 and the rest
Slight tension between HYP5 and HYP7

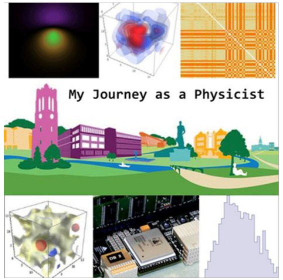
Very happy agreement
VERY SLIGHT tension at small-x (pseudo-PDF isn't supposed to be accurate there anyway)

Conclusion *From Our Lattice*

- Can draw some conclusions from raw data and bare MEs
- The double ratio of the RpITD removes most smearing dependencies except in the heavier pion masses and some relationships can be seen here
  - STOUT10 ~ HYP6? More samples to make some empirical relation??
  - WILSON3 appears to be too much smearing for heavy pion masses, would like to explore extrapolation to zero flow time as seen in some of HadStruct's work
- This affects the PDFs as we would expect
  - Smaller error bars on higher smearing, similar correspondences to the RpITD results, tension for heavier pion mass
- A Couple Caveats:
  - How does this change on different lattices? Can one work harder to get better ME fist for different smearing types? Zero flow/smear extrap.? Fill in gaps between smearing amounts? Other smearing parameters? Different PDF fit form?

Thank You!

Podcast Plug:



Season 3 of H.W. Lin and B. Stanley’s My Journey as a Physicists is hosted by Bill Good and features physicists working on the Long-Range Plan for Nuclear Science

Season 1 is all about people working in the field of Lattice QCD and is hosted by Bryan Stanley

Backup

Appendix: Correlators and Operator

$$C_N^{\mathrm{2pt}}(P_z;t) = \langle 0|\Gamma \int d^3y\, e^{-iyP_z} \chi(\vec{y},t)\chi(\vec{0},0)|0\rangle$$

$$\Gamma = \tfrac{1}{2}(1+\gamma_4) \qquad \chi(\vec{y},t) = \epsilon^{lmn}[u(y)^{l^T} i\gamma_4\gamma_2\gamma_5 d^m(y)]u^n(y)$$

$$C_N^{\mathrm{3pt}}(z,P_z;t_{\mathrm{sep}},t) =$$

$$= \langle 0|\Gamma \int d^3y\, e^{-iyP_z} \chi(\vec{y},t_{\mathrm{sep}})\mathcal{O}_g(z,t)\chi(\vec{0},0)|0\rangle$$

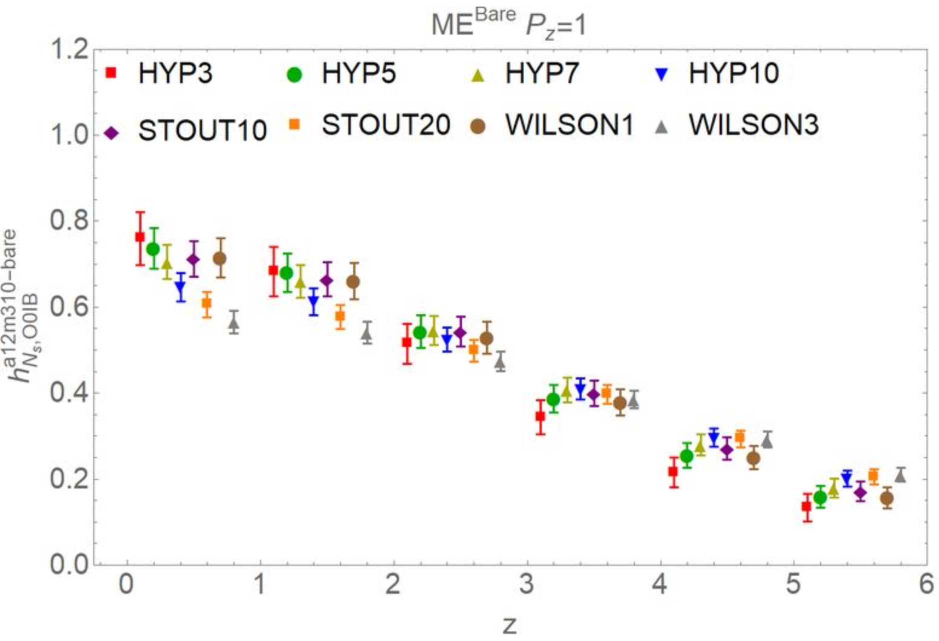
(Meson correlators and interpolation operator defined similarly)

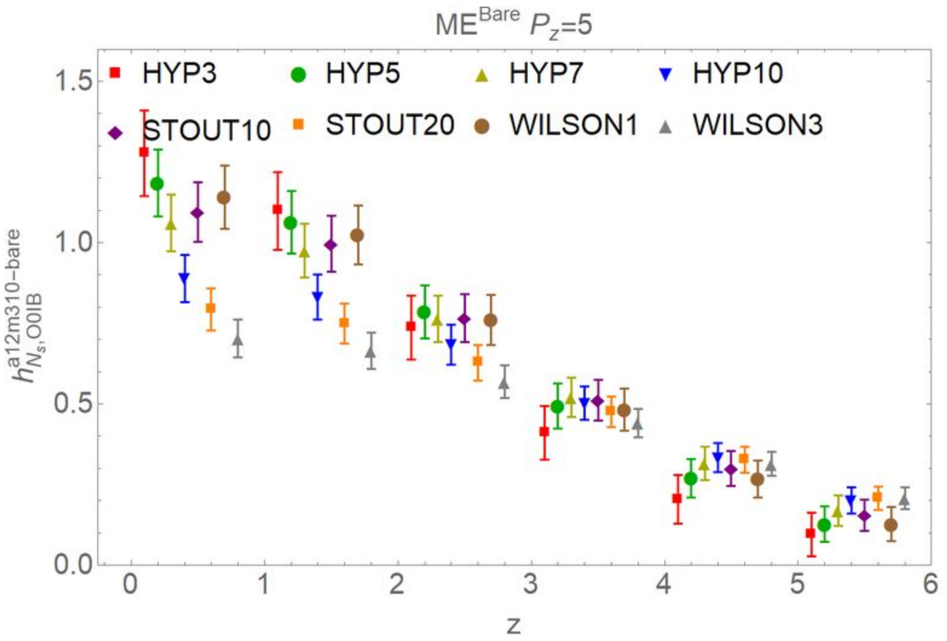
$$\mathcal{O}(z) \equiv \sum_{i\neq z,t} \mathcal{O}(F^{ti},F^{ti};z) - \frac{1}{4}\sum_{i,j\neq z,t} \mathcal{O}(F^{ij},F^{ij};z)$$

Balitsky et al, PLB 808:135621, 2020.

$$\mathcal{O}(F^{\mu\nu},F^{\alpha\beta};z) = F^{\mu}_{\nu}(z)U(z,0)F^{\alpha}_{\beta}(0)$$

Bare Matrix Elements for N_s

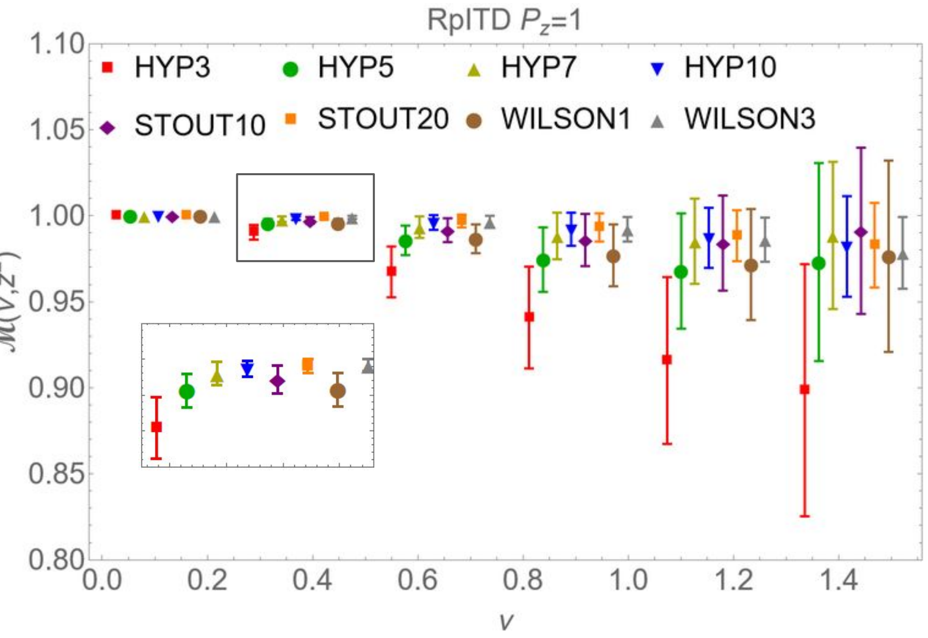


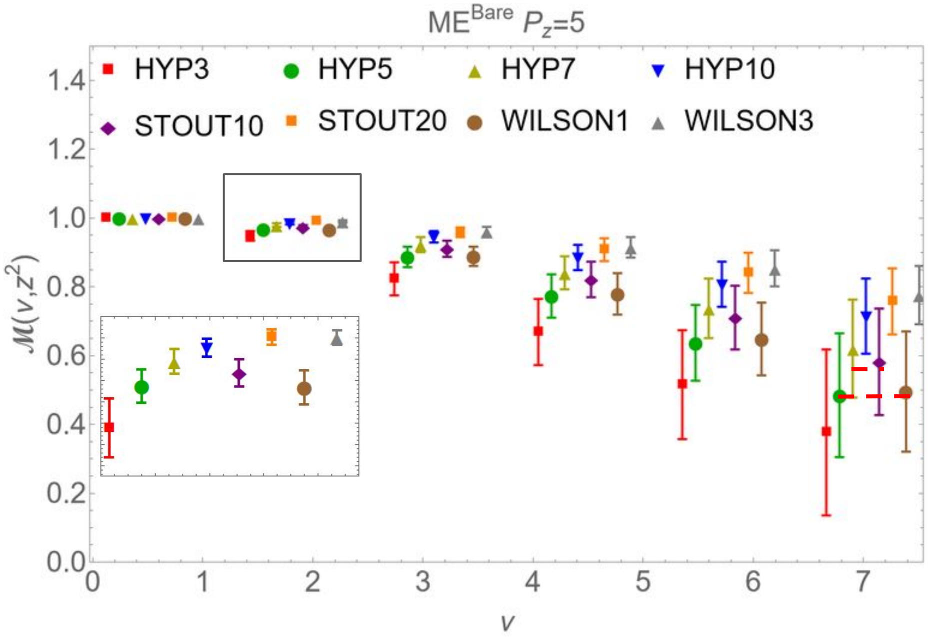


RpITD for N_s

$\nu = zP_z$

$$\mathcal{M}(\nu,z^2)=\frac{\mathcal{M}(zP_z,z^2)/\mathcal{M}(0\cdot P_z,0)}{\mathcal{M}(z\cdot 0,z^2)/\mathcal{M}(0\cdot 0,0)}$$

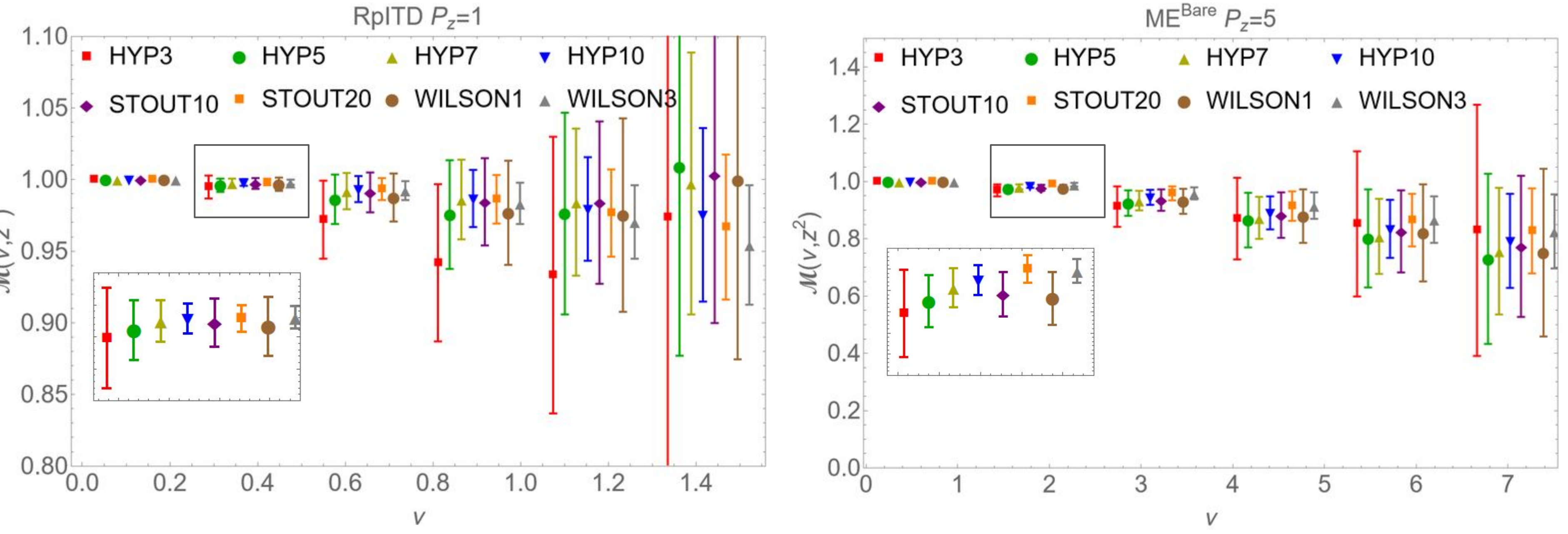




RpITD for N_l

$\nu = zP_z$

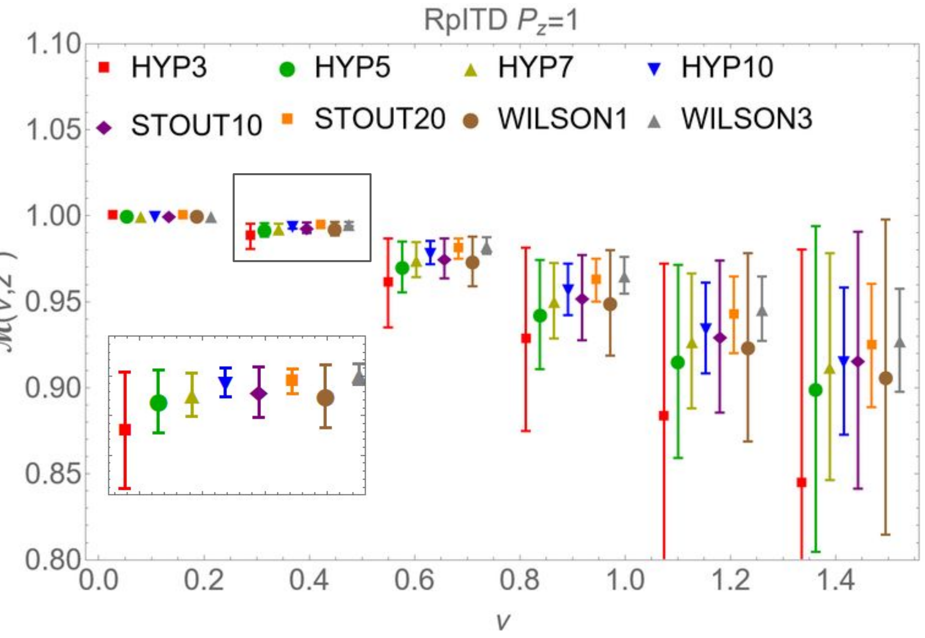
$$\mathscr{M}(\nu, z^2) = \frac{\mathcal{M}(zP_z, z^2)/\mathcal{M}(0\cdot P_z, 0)}{\mathcal{M}(z\cdot 0, z^2)/\mathcal{M}(0\cdot 0, 0)}$$

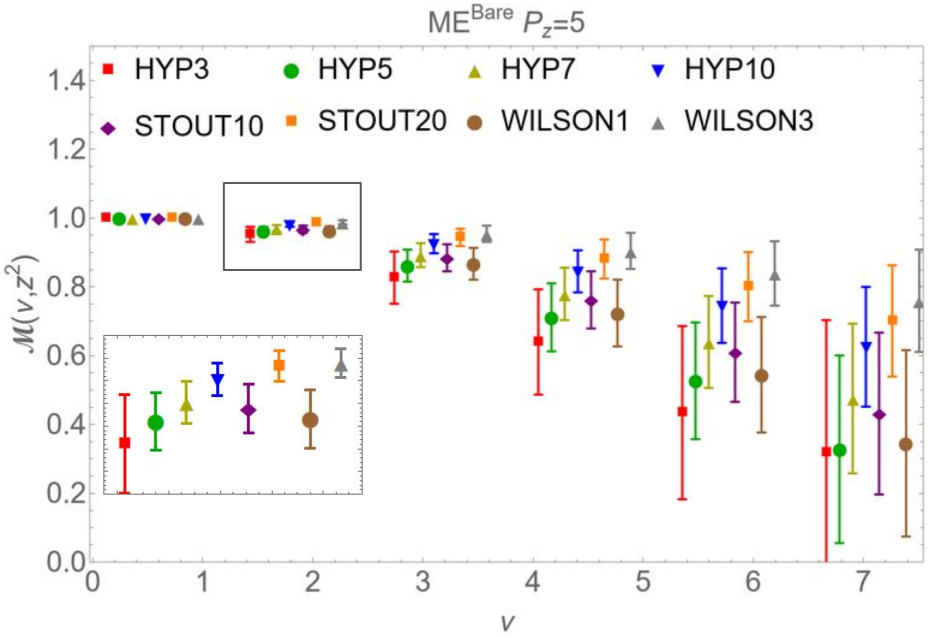


RpITD for η_s

$\nu = zP_z$

$$\mathscr{M}(\nu, z^2) = \frac{\mathcal{M}(zP_z, z^2)/\mathcal{M}(0 \cdot P_z, 0)}{\mathcal{M}(z \cdot 0, z^2)/\mathcal{M}(0 \cdot 0, 0)}$$





RpITD for π

ν = zP_z

$$\mathscr{M}(\nu,z^2)=\frac{\mathcal{M}(zP_z,z^2)/\mathcal{M}(0\cdot P_z,0)}{\mathcal{M}(z\cdot 0,z^2)/\mathcal{M}(0\cdot 0,0)}$$