New gauge-independent transition dividing the confinement phase in the lattice gauge-adjoint scalar model.

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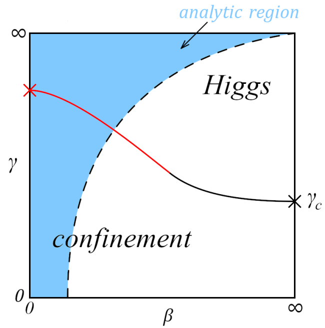
Introduction

- We investigate the gauge-scalar model to clarify the mechanism of confinement in the Yang-Mills theory in the presence of matter fields.
- We also investigate non-perturbative characterization of the Brout-Englert-Higgs (BEH) mechanism providing the gauge field with the mass, in the gauge-independent way (without gauge fixing).
- We reexamine the lattice $SU(2)$ gauge-scalar model with a radially-fixed scalar field (no Higgs mode) which transforms according to the adjoint representation of the gauge group $SU(2)$ without any gauge fixing.
- Note that it was impossible to realize the convenstional BEH mochanisim on the lattice unless the gauge fixing condition is imposed, since gauge non-invariant operators have vanishing vacuum expectation value on the lattice without gauge fixing due to the Elitzur theorem.
- This difficuty can be avoided by using the *gauge-independent description of the BEH mechanism* proposed recently by one of the authors, which *needs neither the spontaneous breakingof gauge symmetry*,
- Therefore, we can study the Higgs phase in the gauge-invariant way on the lattice without gauge fixing based on the lattice construction of gauge-independent description of the BEH mechanism.

Phase diagram of gauge-scalar model

In case of fundamental scalar field (talk by Ikeda)

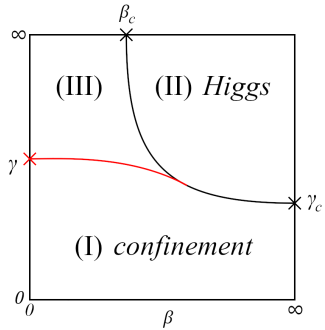
- Confinement and Higgs regions are subregions of analytically continued single phase.
  K. Ostewalder and E. Seiler, Annls. Phys. 110, 440 (1978)
  E. Fradkin and S.H. Shenker, PRD 19, 3682 (1979)
- We found a new transition line (red) which separates confinement and Higgs regions completely.



In case of adjoint scalar fields. (this talk)

- Confinement and Higgs regions are cpmpletely separated into the two differen phases by continuous transition line.

  R.C. Brower et. al. PRD 25, 3319 (1982)
- We found a new transition line (red) that divides completely the confinement phase into two parts.



Note that these results are obtained by investigatng the correlation functions between the gauge-invariant composite operators constructed from the scalar field and the the color-direction field obtained through the gauge-covariant decomposition

Lattice action and path integral measure

The $SU(2)$ gauge-scalar model with a radially-fixed scalar field in the adjoint representation of the gauge group:

$$S_{\mathrm{GS}} := S_g[U] + S_\phi[U, \boldsymbol{\phi}] \ ,$$

$$S_g[U] := \sum_x \sum_{\mu<\nu} \frac{\beta}{2} \mathrm{tr}\left(\mathbf{1} - U_{x,\mu} U_{x+\mu,\nu} U^\dagger_{x+\nu,\mu} U^\dagger_{x,\nu}\right) + c.c. \ ,$$

$$S_\phi[U, \phi] := \sum_{x,\mu} \frac{\gamma}{2} \mathrm{tr}\left((D^\epsilon_\mu[U]\boldsymbol{\phi}_x)^\dagger (D^\epsilon_\mu[U]\boldsymbol{\phi}_x) \right) \ ,$$

where $U_{x,\mu} = \exp(-ig\epsilon\mathscr{A}_{x,\mu}) \in SU(2)$ represents a gauge variable on a link $< x, \mu >$, $\boldsymbol{\phi}_x = \phi^A_x \sigma^A \in su(2)$ $(A = 1, 2, 3)$ represents a scalar field on a site x in the adjoint representation subject to the radially-fixed condition: $\boldsymbol{\phi}_x \cdot \boldsymbol{\phi}_x = \phi^A_x \phi^A_x = 1$, and $D^\epsilon_\mu[U]\boldsymbol{\phi}_x$ represents the covariant derivative in the adjoint representation defined as

$$D^\epsilon_\mu[U]\boldsymbol{\phi}_x = U_{x,\mu}\boldsymbol{\phi}_{x+\epsilon\hat{\mu}} - \boldsymbol{\phi}_x U_{x,\mu} \ .$$

In the naive continum limit this action reproduces $\epsilon \to 0$ the continum gauge-scalar theory with a radiallly-fixed scalar field $|\phi(x)| = v$ and a gauge coupling constant g where $\beta = 4/g^2$ and $\gamma = v^2/2$.

Lattice action and path integral measure (cont')

The numerical simulation can be performed by updating link varables and scalar fields alternatly.

For link variable $U_{x,\mu}$ we can apply the standard HMC algorithm.

For scalar field we reparametrized the variable $\boldsymbol{\phi}_x \in su(2)$ according to the adjoint-orbit representation:

$$\boldsymbol{\phi}_x := Y_x \sigma^3 Y_x^\dagger, \ \ Y_x \in SU(2),$$

which satisfies the normalization condition $\boldsymbol{\phi}_x \cdot \boldsymbol{\phi}_x = 1$ automatically.
The Haar measure is replaced by $D[\boldsymbol{\phi}] \prod_x \delta(\boldsymbol{\phi}_x \cdot \boldsymbol{\phi}_x - 1)$ to $D[Y]$, and we can apply the standard HMC algorithm for Y_x, to update configurations of the scalar fields $\boldsymbol{\phi}_x$.

Gauge-covariant decomposition (CDGSFN decomposition)

We introduce the site variable $\mathbf{n}_x := n_x^A \sigma_A \in SU(2)/U(1)$ which is called the color-direction (vector) field, in addition to the original link variable $U_{x,\mu} \in SU(2)$. The link variable $U_{x,\mu}$ and the site variable $\mathbf{n}_x$ transforms under the gauge transformation $\Omega_x \in SU(2)$ as

$$U_{x,\mu} \to \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger = U'_{x,\mu}, \quad \mathbf{n}_x \to \Omega_x \mathbf{n}_x \Omega_x^\dagger = \mathbf{n}'_x.$$

In the decomposition, a link variable $U_{x,\mu}$ is decomposed into two parts:

$$U_{x,\mu} := X_{x,\mu} V_{x,\mu}.$$

$$V_{x,\mu} \to \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger = V'_{x,\mu}. \quad X_{x,\mu} \to \Omega_x X_{x,\mu} \Omega_x^\dagger = X'_{x,\mu},$$

Such decomposition is obtained by solving the defining equations:

$$D_\mu[V]\mathbf{n}_x := V_{x,\mu}\mathbf{n}_{x+\mu} - \mathbf{n}_x V_{x,\mu} = 0, \quad \mathrm{tr}(\mathbf{n}_x X_{x,\mu}) = 0.$$

This defining equation has been solved exactly and the resulting link variable $V_{x,\mu}$ and site varible $X_{x,\mu}$ are of the form

$$V_{x,\mu} := \tilde{V}_{x,\mu} / \sqrt{\mathrm{tr}[\tilde{V}_{x,\mu}^\dagger \tilde{V}_{x,\mu}]/2}, \ \tilde{V}_{x,\mu} := U_{x,\mu} + \mathbf{n}_x U_{x,\mu} \mathbf{n}_{x+\mu},$$

$$X_{x,\mu} := U_{x,\mu} V_{x,\mu}^{-1} .$$

This decomposion is obtained uniquly for given set of link variable $U_{x,\mu}$ once the site variable $\mathbf{n}_x$ is given.

Gauge-covariant decomposition (cont')

The configurations of the color-direction field $\{\mathbf{n}_x\}$ are obtained by minimizing the functional:

$$F_{\text{red}}[\{\mathbf{n}_x\}|\{U_{x,\mu}\}] := \sum_{x,\mu} \text{tr}\left\{ \left(D_{x,\mu}[U]\mathbf{n}_x\right)^\dagger \left(D_{x,\mu}[U]\mathbf{n}_x\right) \right\},$$

which we call the *reduction condition*.

Note that this functional has the same form as the action of the scalar field:

$$S_\phi = \frac{\gamma}{2} F_{\text{red}}[\{\boldsymbol{\phi}_x\}|\{U_{x,\mu}\}].$$

Lattice result and gauge-independent analyses

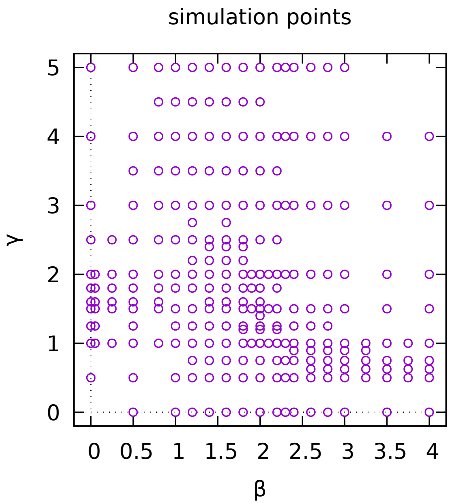


Figure: Simulation points in β-γ plane

- Simulation
  16^4 lattice with PBC in the gauge-independent way (without gauge fixing).
  After thermalization 5000 sweeps and store 800 configurations every 25 sweeps.

- The search for the phase boundary
  by measuring the expectation value $\langle \mathcal{O} \rangle$ of a chosen operator $\mathcal{O}$ by changing γ (or β) along the $\beta =$const. (or $\gamma =$const.) lines.
- identify the boundary,
  Used the bent, step, and gap observed in the graph of the plots for $\langle \mathcal{O} \rangle$.

Numerical Result I

plaquette-action density

$$P = \frac{1}{6N_{\text{site}}} \sum_x \sum_{\mu<\nu} \frac{1}{2} \text{tr}(U_{x,\mu\nu}), \qquad U_{x,\mu\nu} = U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu},$$

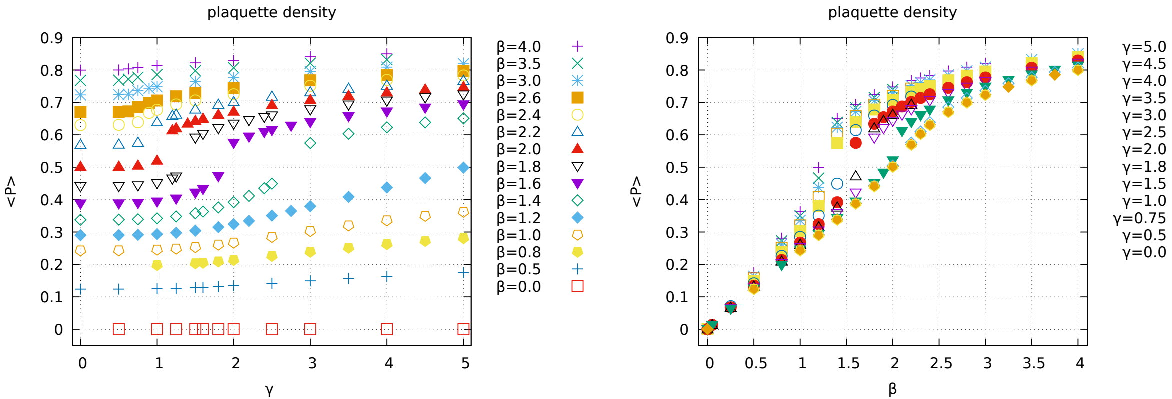


Figure: Average of the plaquette-action density $\langle P \rangle$: (Left) $\langle P \rangle$ versus γ on various $\beta =$ const. lines, (Right) $\langle P \rangle$ versus β on various $\gamma =$const. lines

scalar-action density

,

$$M = \frac{1}{4N_{\text{site}}} \sum_x \sum_\mu \frac{1}{2} \mathrm{tr} \left(\left(D_\mu[U_{x,\mu}]\boldsymbol{\phi}_x\right)^\dagger \left(D_\mu[U_{x,\mu}]\boldsymbol{\phi}_x\right) \right),$$

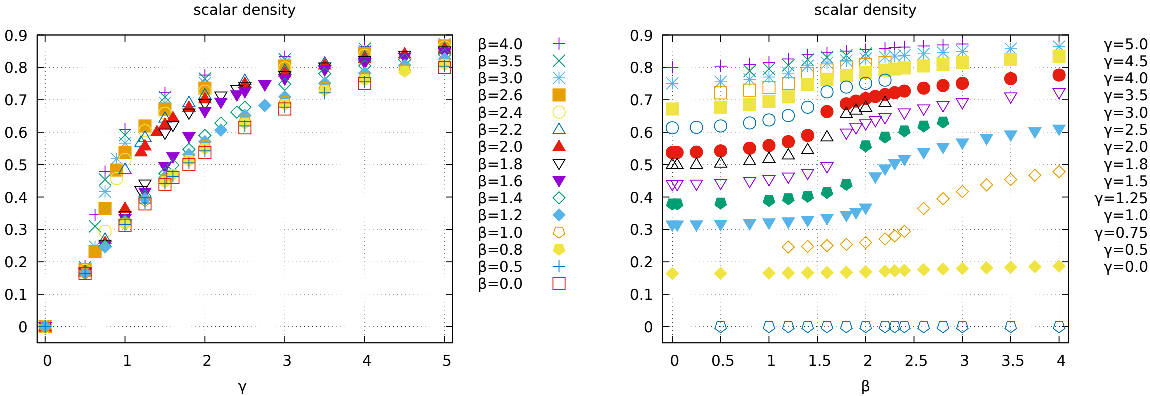


Figure: Average of the scalar-action density $\langle M \rangle$: (Left) $\langle M \rangle$ versus γ on various $\beta =$ const. lines, (Right) $\langle M \rangle$ versus β on various $\gamma =$const. lines

Phase boundary from action density

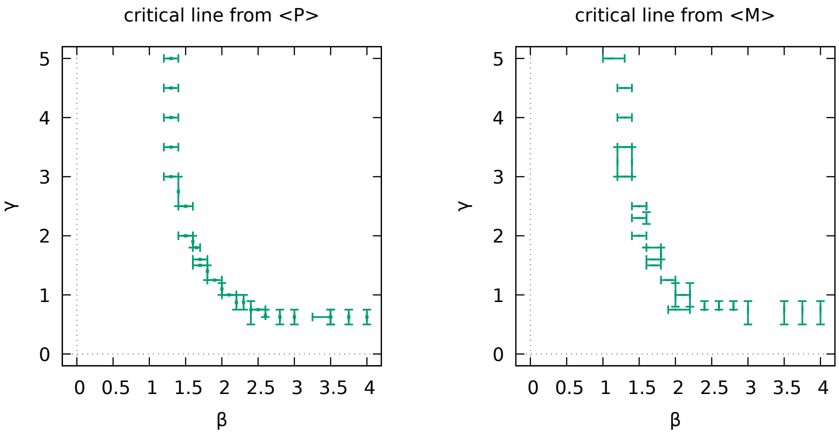


Figure: The phase boundary determined by the action densities: (Left) $\langle P \rangle$, (Right) $\langle M \rangle$.

We find that the gauge-independent numerical simualtions reproduce the critical line obtained by Brower et al.

Numerical Result II

We investigate the correlations between the scalar field and the color-direction field through the gauge covariant decomposition:

$$Q = \frac{1}{N_{\text{site}}} \sum_x \frac{1}{2} \mathrm{tr}(\mathbf{n}_x \boldsymbol{\phi}_x),$$

We need to solve the reduction condition to obtain the color-direction field $\mathbf{n}_x$, which however has two kinds of ambiguity.
One comes form so-called the Gribov copies that arethe local minimal solutions of the reduction condition.
Another comes from the choice of a global sign factor, which originates from the fact that whenever a configuration $\{\mathbf{n}_x\}$ is a solution, the flipped one $\{-\mathbf{n}_x\}$ is also a solution, since the reduction functional is quadratic in the color fields.
To avoid these issues, we propose to use $\langle |Q| \rangle$ and $\langle Q^2 \rangle$.The phase boundary is searched for based on two ways:

(i) the location at which $\langle |Q| \rangle$ changes from $\langle |Q| \rangle \simeq 0$ to $\langle |Q| \rangle > 0$. This is also the case for $\langle Q^2 \rangle$.

(ii) the location at which $\langle |Q| \rangle$ changes abruptly, as was done for $\langle P \rangle$ and $\langle M \rangle$. This is also the case for $\langle Q^2 \rangle$.

scalar-color correlation

$$Q = \frac{1}{N_{\text{site}}} \sum_x \frac{1}{2} \text{tr}(\mathbf{n}_x \boldsymbol{\phi}_x),$$

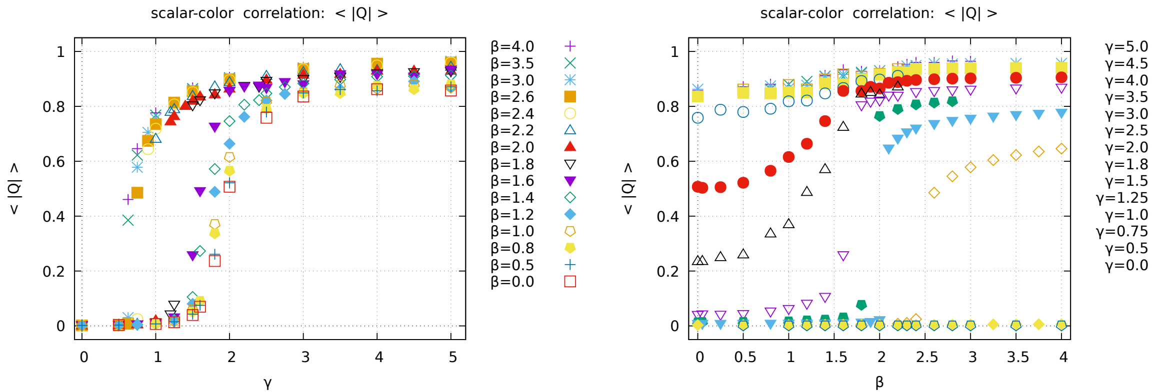


Figure: Average of the scalar-color composite field $\langle |Q| \rangle$: (Left) $\langle |Q| \rangle$ versus γ on various $\beta =$const. lines, (Right) $\langle |Q| \rangle$ versus β on various $\gamma =$const. lines.

Phase boundary (critical line) from scalar-color correlation

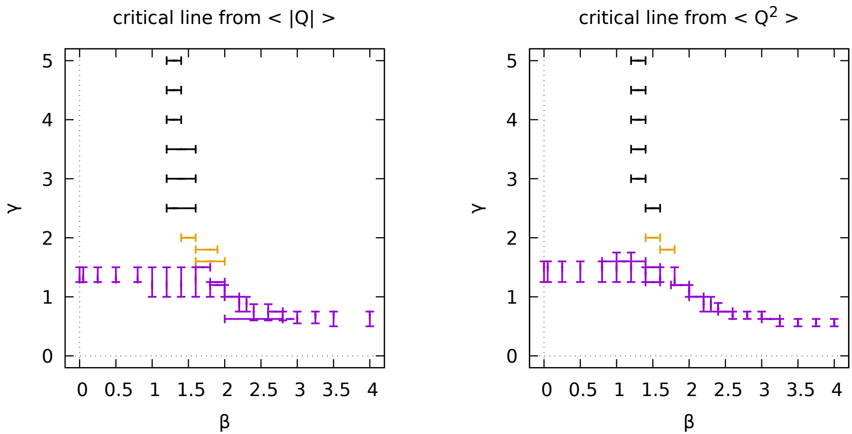
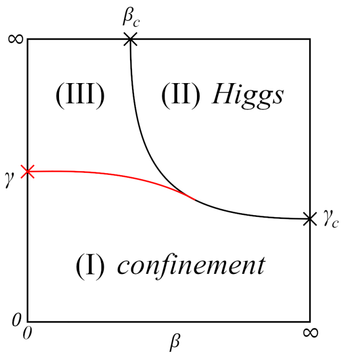


Figure: Critical lines determined (Left) from $\langle |Q| \rangle$, (Right) from $\langle Q^2 \rangle$.

Understanding the new phase structure



(I) confinement phase

$\gamma < \gamma_c(\beta)$: the effect of the scalar field would be relatively smal and confinement would occur in the way similar to the pure $SU(2)$ gauge theory.

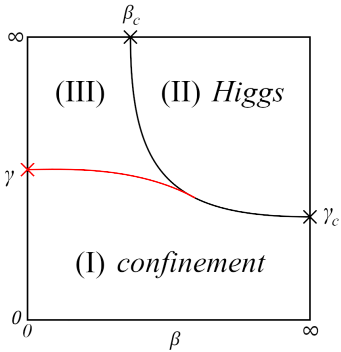
disordered phase:in the sense that the color direction field $\boldsymbol{n}_x$ takes various directions with no specific direction in color space, i.e., very small or vanishing value of the average $\langle Q \rangle = 0$.

Connement isexpected to occur due to vacuum condensations of non-Abelian magnetic monopoles.

(II) Higgs phase

The gauge fields become massive due to different physical origins. that is, the off-diagonal gauge fields for the modes $SU(2)/U(1)$ become massive due to the BEH mechanism, which is a consequence of the (partial) spontaneous symmetry breaking $SU(2) \to U(1)$ according to the conventional understanding of the BEH mechanism,

The diagonal gauge field for the mode $U(1)$ always remains massless everywhere



(III) confinement phase
the gauge fields become massive due to self-interactions among the gauge fields, as in the phase (I).

difference between (II) and (III)
orderd phase:In these phase, the color-direction field $\boldsymbol{n}_x$ correlates strongly with the given scalar field $\boldsymbol{\phi}_x$ which tends to align to a specific direction.
Consequently, the $SU(2)$ gauge-scalar model reduces to the pure compact $U(1)$ gauge model.
The pure compact $U(1)$ gauge model in four space-time dimensions has two phases:confinement phase with massive $U(1)$ gauge field in the strong gauge coupling region $\beta < \beta_*$ and the Coulomb phase with massless $U(1)$ gauge field in the weak gauge coupling region $\beta > \beta_*$, which has been proved rigorously. (A.H. Guth 1980, Fröhlich and T. Spencer 1982)

Conclusion and discussion

- We have investigated the lattice $SU(2)$ gauge-scalar model with the scalar field in the adjoint representation of the gauge group in a gauge-independent way.
- We have re-examined this phase structure in the gauge-independent way based on the numerical simulations performed without any gauge fixing, which should be compared with the preceding studies by Brower et.al..
- This is motivated to confirm the recently proposed gauge-independent Brout-Englert-Higgs mechanics for giving the mass of the gauge field without relying on any spontaneous symmetry breaking
- For this purpose we have investigated correlation functions between gauge-invariant operators obtained by combining the original adjoint scalar field and the new field called the color-direction field which is constructed from the gauge field based on the gauge-covariant decomposition of the gauge field due to Cho-Duan-Ge-Shabanov and Faddeev-Niemi.
- we have reproduced gauge-independently the transition line separating confinement and Higgs phase obtained in the preseeding study, and show surprisingly the existence of a new transition line that divides completely the confinement phase into two parts.
- The result obtained in this paper should be compared with the lattice $SU(2)$ gauge-scalar model with the scalar field in the fundamental representation of the gauge group in a gauge-independent way.