Investigation of two-particle contributions to nucleon matrix elements

Constantia Alexandrou, Giannis Koutsou, **Yan Li**,
Marcus Petschlies, Ferenc Pittler

Background

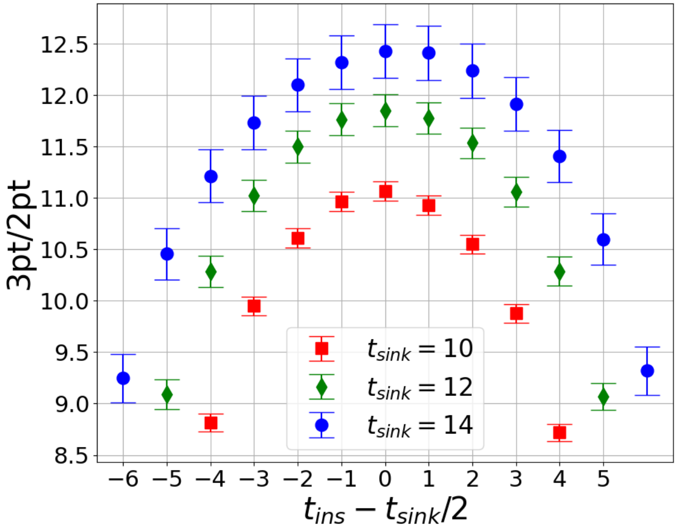
- Nucleon structure: nucleon matrix elements

$$\frac{\langle 0|O_N(t_{\rm sink})J(t_{\rm ins})\bar{O}_N(0)|0\rangle}{\langle 0|O_N(t_{\rm sink})\bar{O}_N(0)|0\rangle} \xrightarrow{\text{all } t \text{ well-separated}} \langle N|J|N\rangle$$

- Time-dependence indicates contamination from excited states

- Lowest excited state is a **Nucleon–Pion** state

$\mathcal{J} = S^{u+d} = \bar{u}u + \bar{d}d, \quad \vec{q} = (0, 0, 0)$



Simulation details

Ensembles	Flavors	$N_L^3 \times N_T$	m_π (MeV)	L (fm)	$m_\pi L$	$N_{\text{cfg}} \times N_{\text{src}}$
cA211.53.24	2+1+1	$24^3 \times 48$	346	2.27	3.99	300×120
cA2.09.48	2	$48^3 \times 96$	131	4.50	2.98	300×60

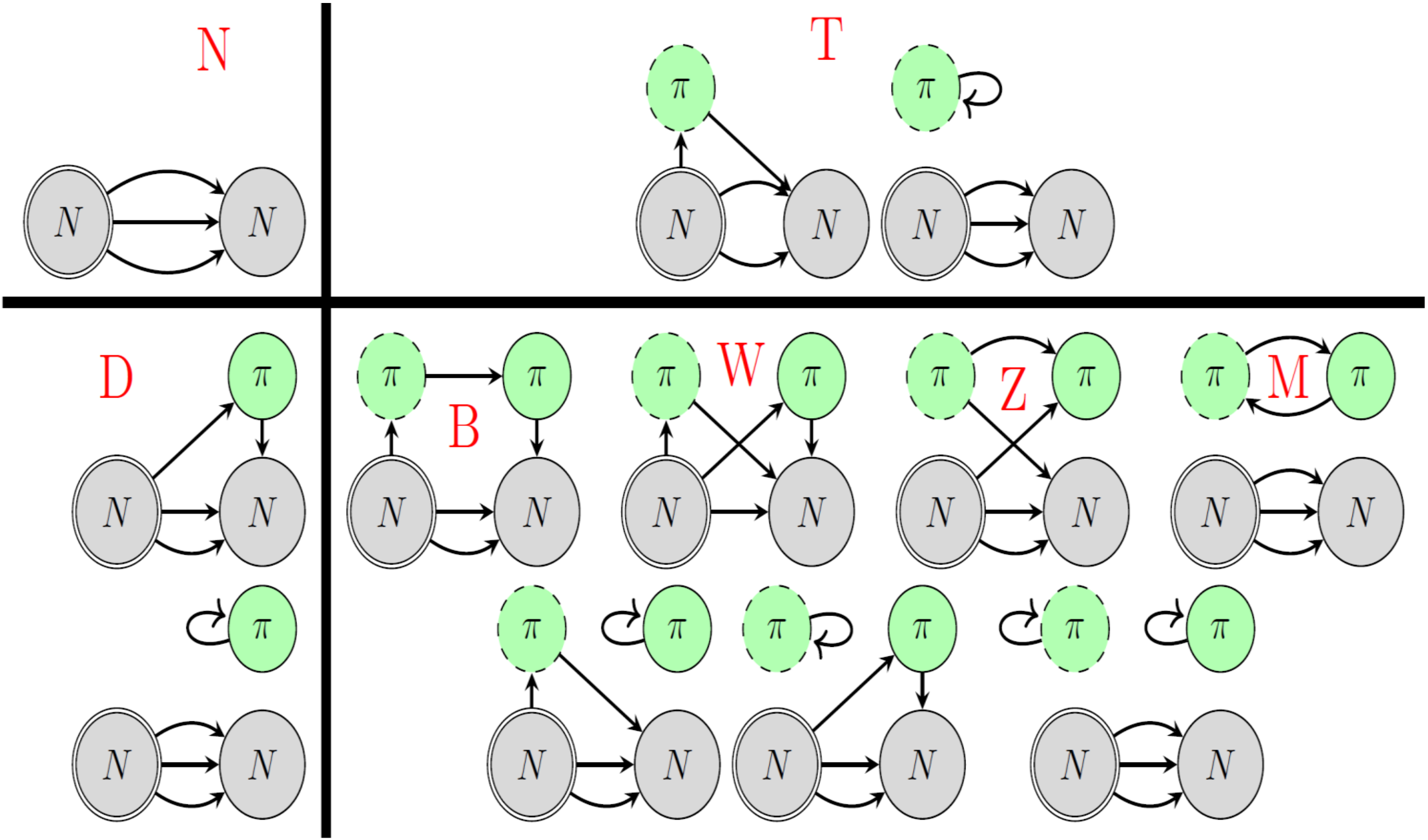
- Twisted−mass lattice
- Interpolating operators used:

$$O_p; \qquad O_{N\pi}^{1/2} = \sqrt{2/3}\, O_{n\pi^+} - \sqrt{1/3}\, O_{p\pi^0}$$

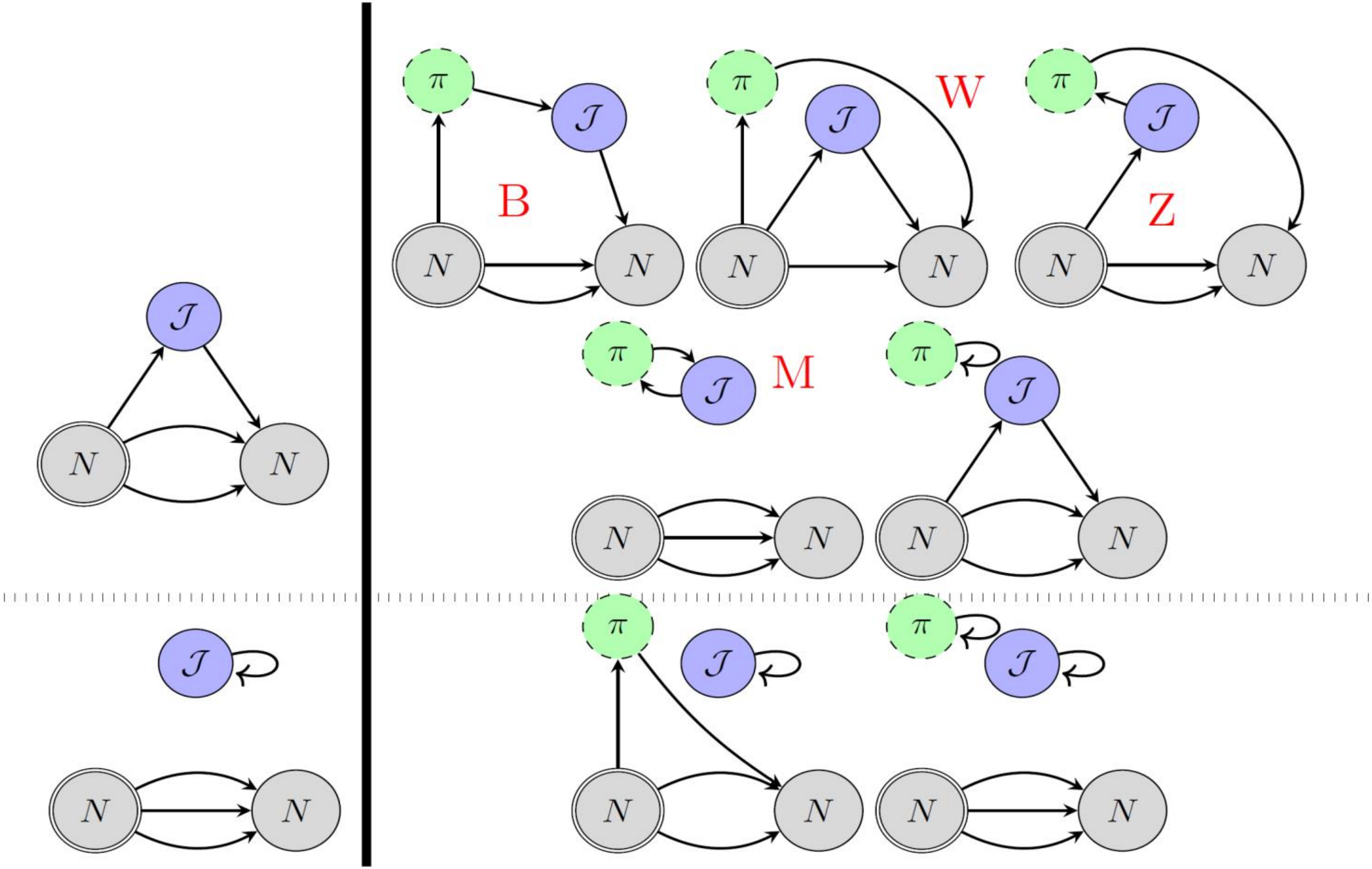
- Generalized eigenvalue problem (GEVP)
  - Do GEVP on 2pt functions
  - Use the results to improve 3pt functions

- Based on: https://github.com/cylqcd/PLEGMA
  https://github.com/lattice/quda/

Diagrams: 2pt functions



Diagrams: 3pt functions



Generalized eigenvalue problem (GEVP)

- GEVP starts with 2pt functions:

$$C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger(0)|0\rangle$$

- GEVP returns eigenvalues and eigenvectors:

$$C_{ij}(t)v_j^n = \lambda^n(t,t_0)C_{ij}(t_0)v_j^n$$

$$\lambda^n(t,t_0) = e^{-E_n(t-t_0)}\,, \quad v_j^n O_j^\dagger(0)\,|0\rangle = |n\rangle$$

- We determine the optimal interpolating field:

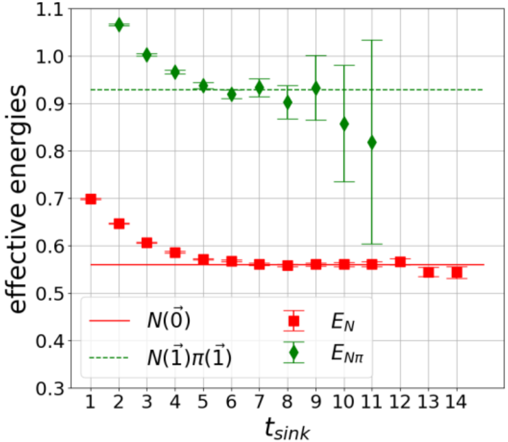
$$O'_N\,|0\rangle = (O_N + v^N_{N\pi}O_{N\pi})\,|0\rangle \propto |N\rangle$$

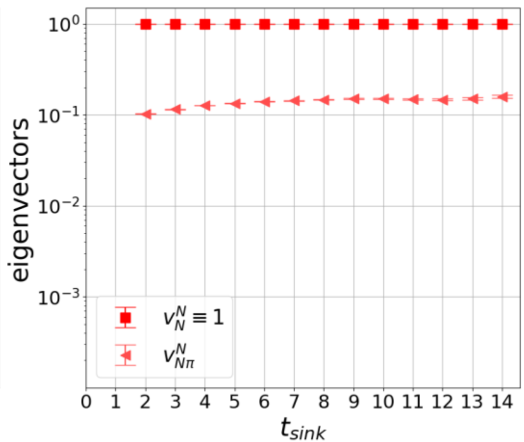
- We can use it to improve matrix elements:

$$\frac{\langle 0|O_N J\bar{O}_N|0\rangle}{\langle 0|O_N\bar{O}_N|0\rangle} \xrightarrow{\text{GEVP improving}} \frac{\langle 0|O'_N J\bar{O}'_N|0\rangle}{\langle 0|O'_N\bar{O}'_N|0\rangle}$$

GEVP for 2pt functions:

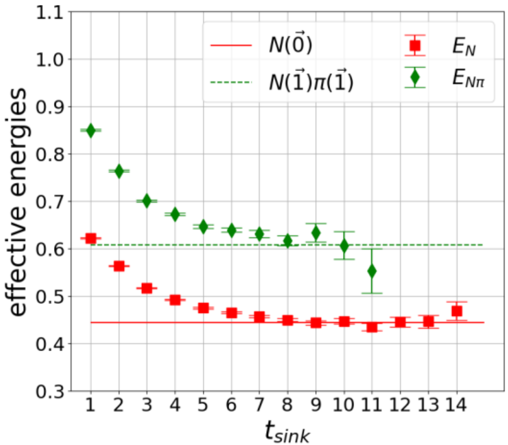
$\vec{p}_{tot} = (0,0,0); \mathbf{N}\left(\vec{0}\right), \mathbf{N}\left(\vec{1}\right)\pi(-\vec{1})$

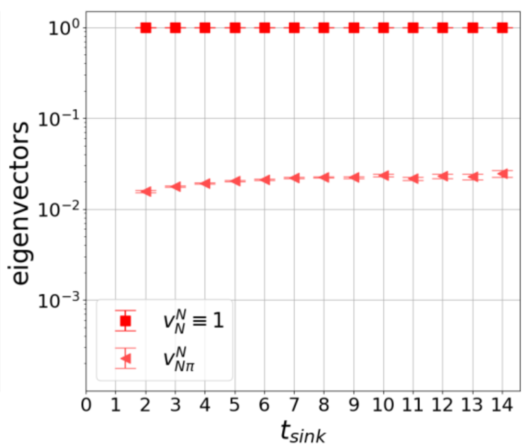




$m_\pi = 346$ MeV

$$O'_N = O_N + v_{N\pi}^N O_{N\pi}$$

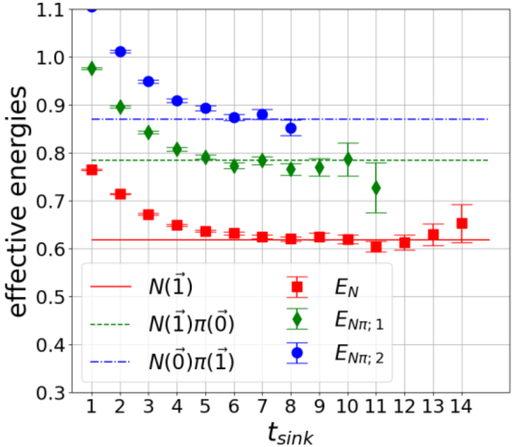


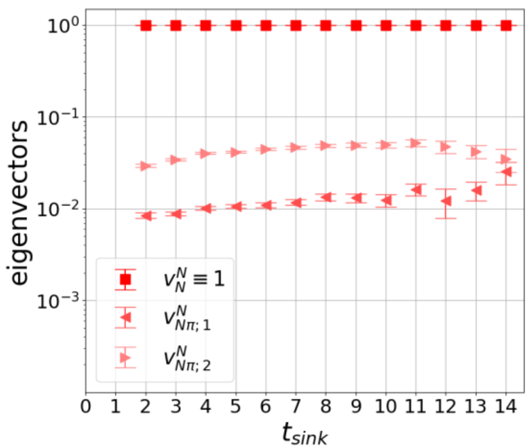


$m_\pi = 131$ MeV

GEVP for 2pt functions:

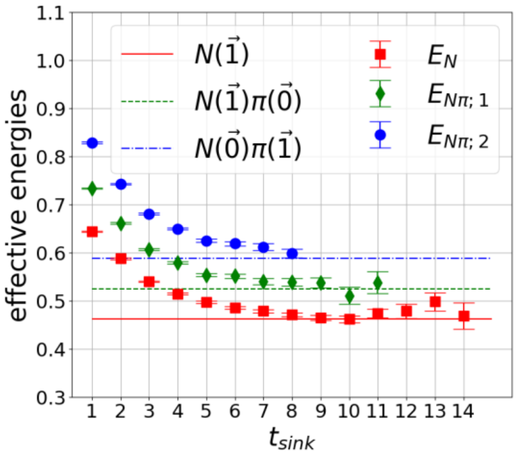
$$\vec{p}_{tot} = (0,0,1); N(\vec{1}), N(\vec{1})\pi(\vec{0}), N(\vec{0})\pi(\vec{1})$$

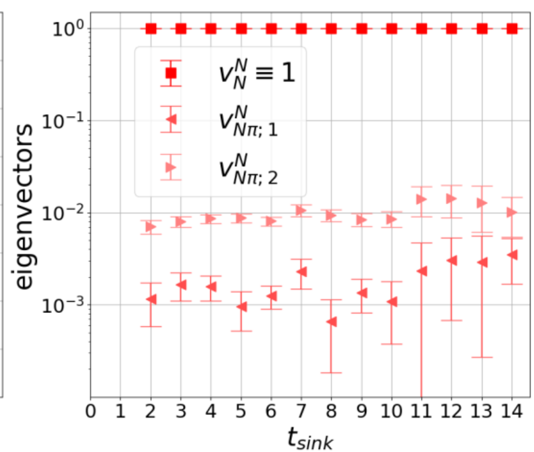




$m_\pi = 346$ MeV

$$O'_N = O_N + v^N_{N\pi;1}\, O_{N\pi;1} + v^N_{N\pi;2}\, O_{N\pi;2}$$





$m_\pi = 131$ MeV

GEVP improvement on 3pt functions

$$\frac{\langle 0|O_N J \bar{O}_N|0\rangle}{\langle 0|O_N \bar{O}_N|0\rangle} \xrightarrow{\text{GEVP improving}} \frac{\langle 0|O'_N J \bar{O}'_N|0\rangle}{\langle 0|O'_N \bar{O}'_N|0\rangle}$$

$$\langle 0|O'_N J \bar{O}'_N|0\rangle = \langle 0|(O_N - x\,O_{N\pi}) J (\bar{O}_N - x\,\bar{O}_{N\pi})|0\rangle$$

$$= \langle 0|O_N J \bar{O}_N|0\rangle \mathbf{-x\,\langle 0|O_N J \bar{O}_{N\pi}|0\rangle} - x\,\langle 0|O_{N\pi} J \bar{O}_N|0\rangle + x^2\,\langle 0|O_{N\pi} J \bar{O}_{N\pi}|0\rangle$$

$$\langle 0|O_N J \bar{O}_{N\pi}|0\rangle = [\langle N| + x\,\langle N\pi|]\; J\; [y\,|N\rangle + |N\pi\rangle]$$

$$= \mathbf{y}\,\langle N|J|N\rangle + \mathbf{1}\,\langle N|J|N\pi\rangle + \mathbf{xy}\,\langle N\pi|J|N\rangle + \mathbf{x}\,\langle N\pi|J|N\pi\rangle$$

Up to time-dependent terms

	$\langle N\|J\|N\rangle$	$\langle N\|J\|N\pi\rangle$	$\langle N\pi\|J\|N\rangle$	$\langle N\pi\|J\|N\pi\rangle$
$\langle 0\|O_N J \bar{O}_N\|0\rangle$	1	x	x	x^2
$\mathbf{-x\,\langle 0\|O_N J \bar{O}_{N\pi}\|0\rangle}$	$\mathbf{-x * y}$	$\mathbf{-x * 1}$	$\mathbf{-x * xy}$	$\mathbf{-x * x}$
$-x\,\langle 0\|O_{N\pi} J \bar{O}_N\|0\rangle$	$-xy$	$-x^2 y$	$-x$	$-x^2$
$x^2\,\langle 0\|O_{N\pi} J \bar{O}_{N\pi}\|0\rangle$	$x^2 y^2$	$x^2 y$	$x^2 y$	x^2
$\langle 0\|O'_N J \bar{O}'_N\|0\rangle$	$(1 - xy)^2$	0	0	0

GEVP improvement on 3pt functions

- We have: $\langle 0|O_N J\bar{O}_N|0\rangle \qquad \langle 0|O_N J\bar{O}_{N\pi}|0\rangle$
- When $\vec{q} = (0,0,0)$, by symmetry:

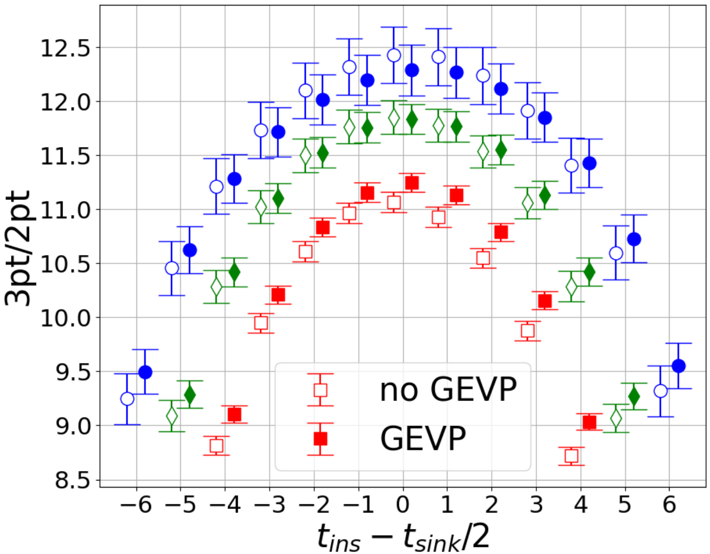
$$\langle 0|O_N J\bar{O}_{N\pi}|0\rangle \xrightarrow{\text{when } \vec{q}=(0,0,0)} \langle 0|O_{N\pi} J\bar{O}_N|0\rangle$$

- So we can include three rows

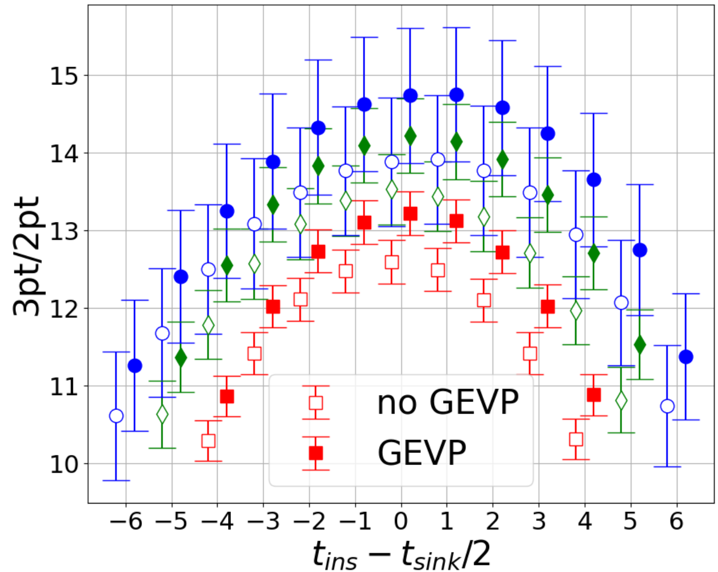
	$\langle N\|J\|N\rangle$	$\langle N\|J\|N\pi\rangle$	$\langle N\pi\|J\|N\rangle$	$\langle N\pi\|J\|N\pi\rangle$
$\langle 0\|O_N J\bar{O}_N\|0\rangle$	1	x	x	x^2
$-x\,\langle 0\|O_N J\bar{O}_{N\pi}\|0\rangle$	$-xy$	$-x$	$-x^2y$	$-x^2$
$-x\,\langle 0\|O_{N\pi} J\bar{O}_N\|0\rangle$	$-xy$	$-x^2y$	$-x$	$-x^2$
$x^2\,\langle 0\|O_{N\pi} J\bar{O}_{N\pi}\|0\rangle$	x^2y^2	x^2y	x^2y	x^2
$\langle 0\|O'_N J\bar{O}'_N\|0\rangle$	$1-2xy$	$-x^2y$	$-x^2y$	$-x^2$

GEVP improvement on 3pt functions:

J= $S^{u+d} = \bar{u}u + \bar{d}d$; $\vec{q} = (0,0,0)$



$m_\pi = 346$ MeV

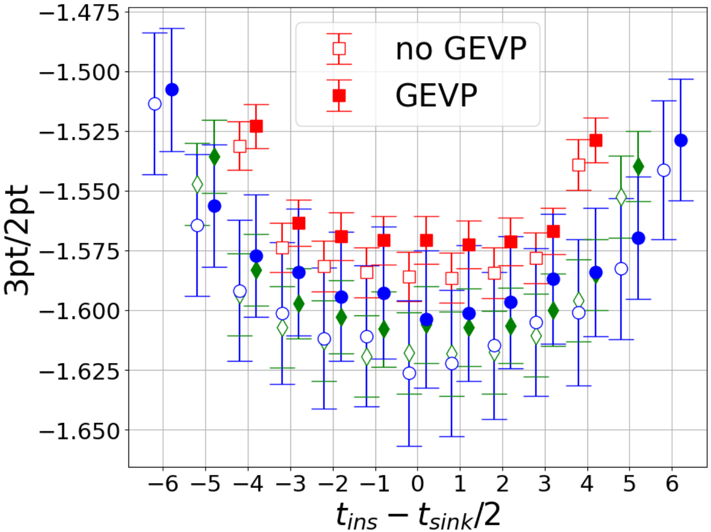


$m_\pi = 131$ MeV

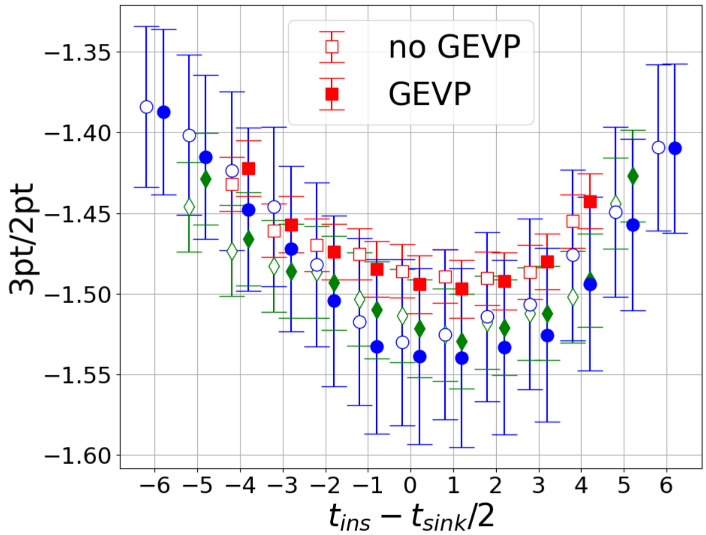
- Different color & shape denote different t_{sink}
- No significant improvement

GEVP improvement on 3pt functions:

J= $A_3^{u-d} = \bar{u}\gamma_5\gamma_3 u - \bar{d}\gamma_5\gamma_3 d$; $\vec{q} = (0,0,0)$



$m_\pi = 346$ MeV



$m_\pi = 131$ MeV

- Similar behavior with the previous case

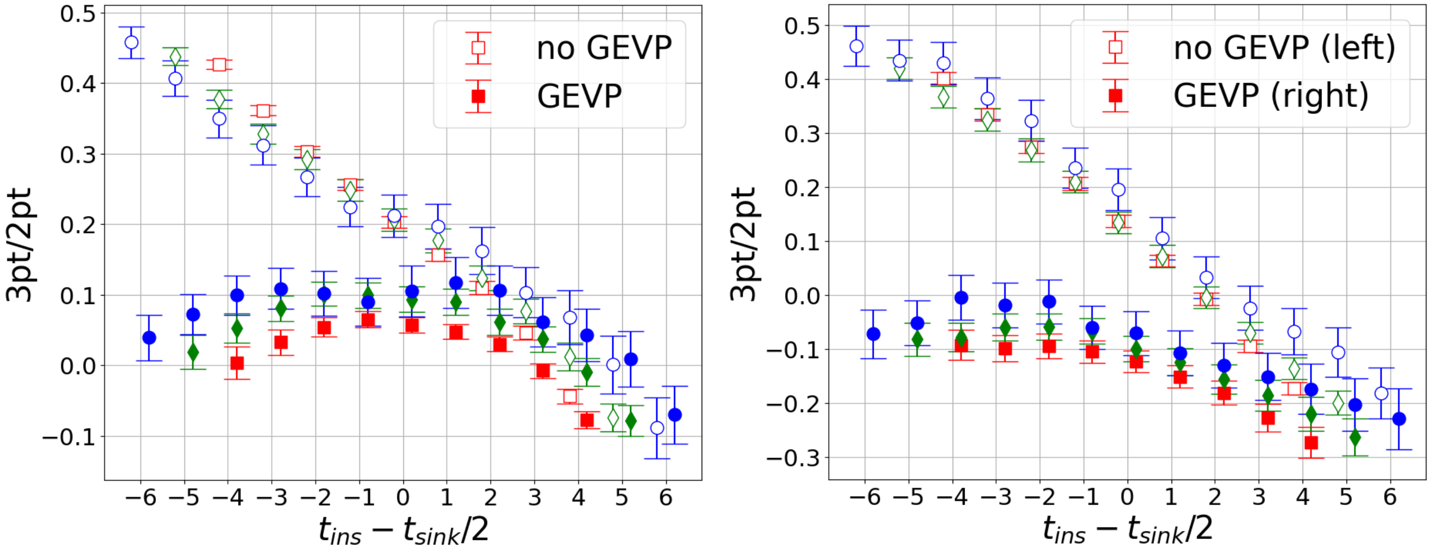
GEVP improvement on 3pt functions

- We have: $\langle 0|O_N J\bar{O}_N|0\rangle$ $\langle 0|O_N J\bar{O}_{N\pi}|0\rangle$
- $\vec{q} \neq (0,0,0)$
- We can include two rows

	$\langle N\|J\|N\rangle$	$\langle N\|J\|N\pi\rangle$	$\langle N\pi\|J\|N\rangle$	$\langle N\pi\|J\|N\pi\rangle$
$\mathbf{\langle 0\|O_N J\bar{O}_N\|0\rangle}$	$\mathbf{1}$	$\mathbf{x}$	$\mathbf{x}$	$\mathbf{x^2}$
$\mathbf{-x\,\langle 0\|O_N J\bar{O}_{N\pi}\|0\rangle}$	$\mathbf{-xy}$	$\mathbf{-x}$	$\mathbf{-x^2y}$	$\mathbf{-x^2}$
$-x\,\langle 0\|O_{N\pi} J\bar{O}_N\|0\rangle$	$-xy$	$-x^2y$	$-x$	$-x^2$
$x^2\,\langle 0\|O_{N\pi} J\bar{O}_{N\pi}\|0\rangle$	x^2y^2	x^2y	x^2y	x^2
$\mathbf{\langle 0\|O'_N J\bar{O}'_N\|0\rangle}$	$\mathbf{1-xy}$	$\mathbf{0}$	$\mathbf{x-x^2y}$	$\mathbf{0}$

GEVP improvement on 3pt functions:

$J = A_4^{u-d} = \bar{u}\gamma_5\gamma_4 u - \bar{d}\gamma_5\gamma_4 d;\ \vec{q} = (0,0,1)$



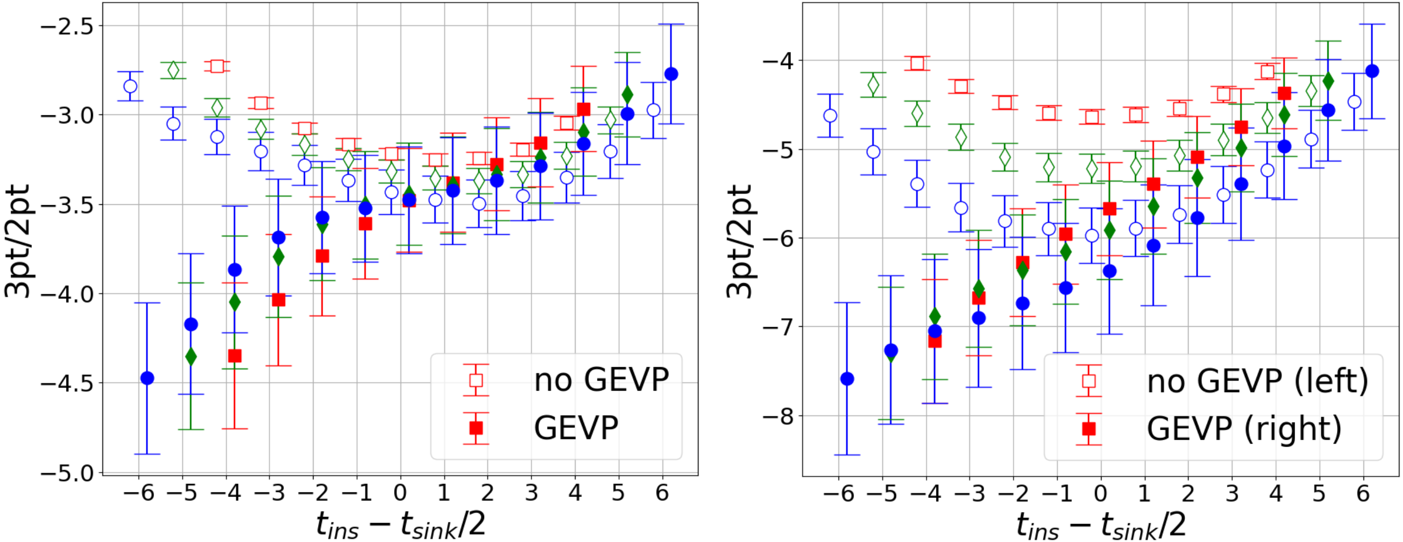
$m_\pi = 346$ MeV

$m_\pi = 131$ MeV

➢ Significant improvement compared with the two previous cases

GEVP improvement on 3pt functions:

J= $P^{u-d} = \bar{u}\gamma_5 u - \bar{d}\gamma_5 d$; $\vec{q} = (0,0,1)$



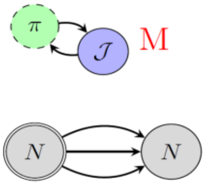
$m_\pi = 346$ MeV

$m_\pi = 131$ MeV

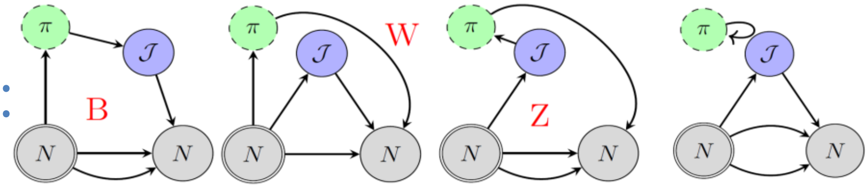
- $N\pi$ is one source of contamination

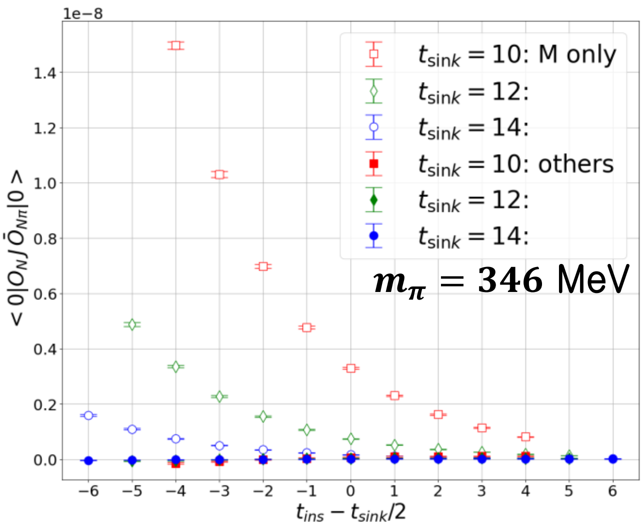
GEVP on 3pt: dominant contribution

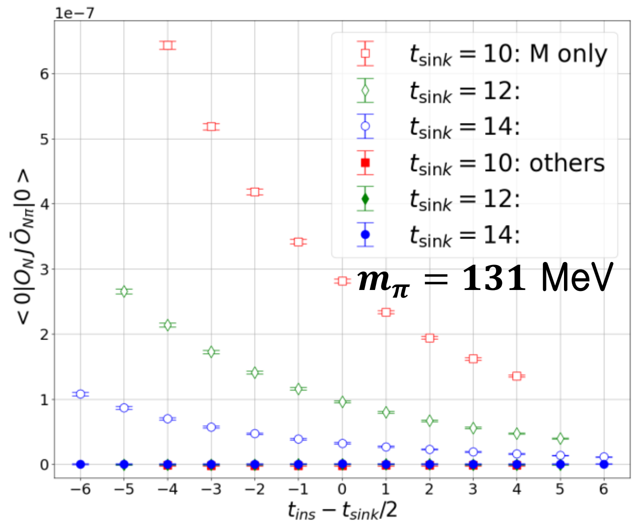
M:



others:







- M is the dominant diagram in both ensembles
- Agree with Regensburg group

Barca, Bali, Collins PRD 107, L051505 (2023)

Summary

- Studied two ensembles:
  - $N_L^3 = 24^3$ and $m_\pi = 346$ MeV
  - $N_L^3 = 48^3$ and $m_\pi = 131$ MeV
- Performed a GEVP analysis for the 2pt functions
- Projected 3pt functions using the optimized eigenvectors

Outlook

- Improve statistics
- Do the remaining 3pt functions: $\langle 0|O_{N\pi} J \bar{O}_N|0\rangle$ $\langle 0|O_{N\pi} J \bar{O}_{N\pi}|0\rangle$
- Include more states in the GEVP
  - Other momenta ($\vec{p}_{tot}$ and $\vec{q}$)
  - Other two-particle states?

THANKS

Με τη συγχρηματοδότηση
της Ευρωπαϊκής Ένωσης

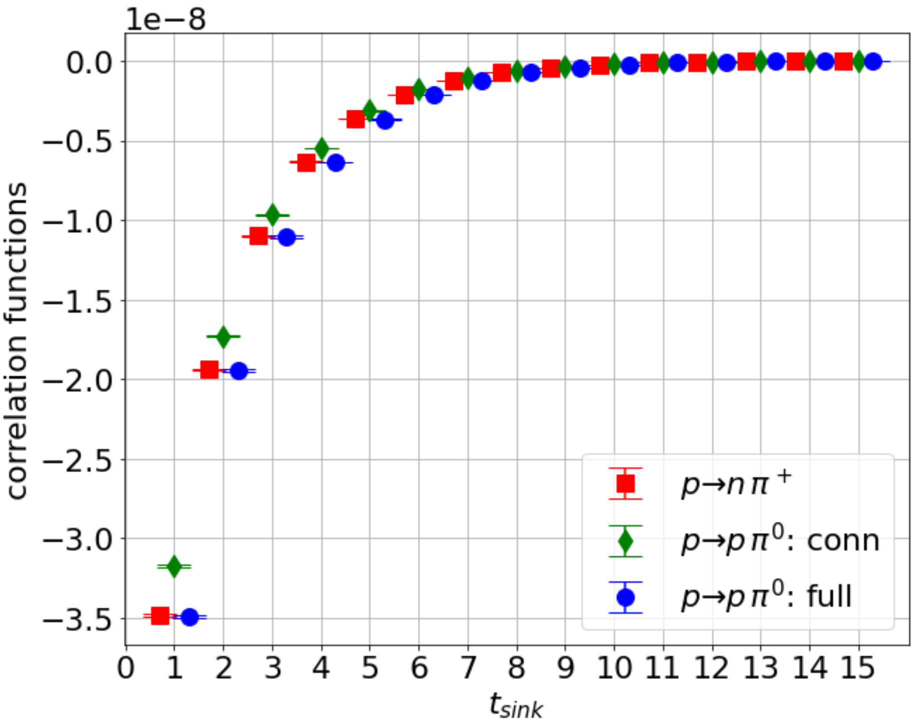
Κυπριακή Δημοκρατία

ΙΔΡΥΜΑ
ΕΡΕΥΝΑΣ ΚΑΙ
ΚΑΙΝΟΤΟΜΙΑΣ

EXCELLENCE/0421/0043

Isospin symmetry

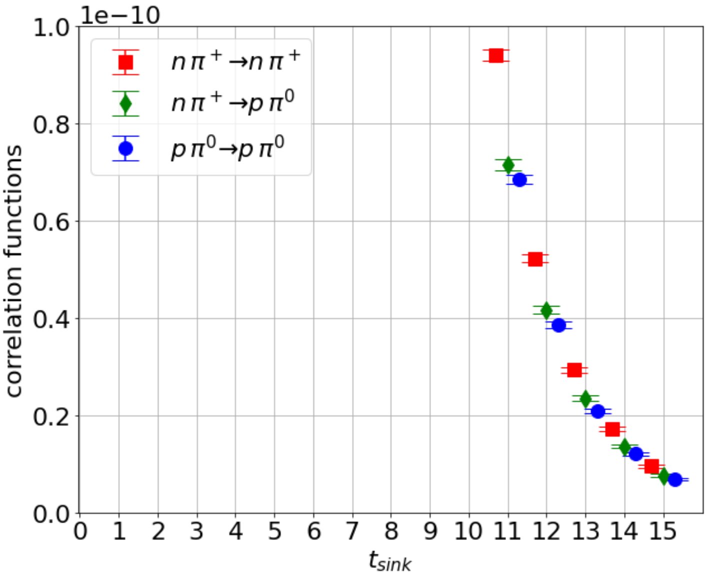
$$\langle 0|O_p \begin{bmatrix} \bar{O}_{n\pi^+} = \sqrt{\tfrac{2}{3}}O_{\frac{1}{2}} + \sqrt{\tfrac{1}{3}}O_{\frac{3}{2}} \\ \bar{O}_{p\pi^0} = -\sqrt{\tfrac{1}{3}}O_{\frac{1}{2}} + \sqrt{\tfrac{2}{3}}O_{\frac{3}{2}} \end{bmatrix} |0\rangle \qquad \langle 0|O_p\bar{O}_{n\pi^+}|0\rangle = \langle 0|O_p\bar{O}_{p\pi^0}|0\rangle \times (-\sqrt{2})$$



Isospin symmetry

$$\langle 0|O_{n\pi^+}\bar{O}_{n\pi^+}|0\rangle \xrightarrow{t\to\infty} \begin{bmatrix} \langle 0|O_{n\pi^+}\bar{O}_{p\pi^0}|0\rangle \times (-\sqrt{2}) \\ \langle 0|O_{p\pi^0}\bar{O}_{p\pi^0}|0\rangle \times 2 \end{bmatrix}$$

No loops



With loops