Anti-static-anti-static-light-light potentials from lattice QCD

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Motivation

- computation of a potential with two static anti-quarks and two dynamical light quarks $\bar{b}\bar{b}qq$ (e. g. T_{bb})
- could in principle also describe $\bar{b}\bar{c}qq$ or $\bar{c}\bar{c}qq$ (possibly with relativistic corrections)
  $\rightarrow$ relevant to recently found T_{cc} tetraquark [LHCb (2021) arXiv:2109.01038], [LHCb (2021) arXiv:2109.01056]
- $\bar{b}\bar{b}ud$ static potentials from the lattice useful for effective approaches like the Born-Oppenheimer approximation to study bound states and resonances [P. B., M. Cardoso, A. Peters, M. Pflaumer, M. W. (2017) arXiv:1704.02383], [J. Hoffmann, A. Zimermmane-Santos, M. W. (2022) arXiv:2211.15765]

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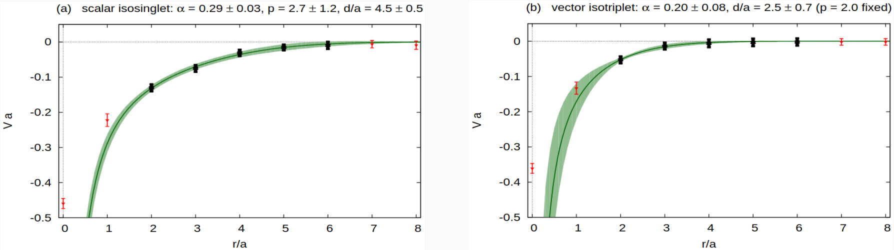
- ground state $\bar{b}\bar{b}ud$ static potentials firstly computed in [P. B., M. W. (2012) 1211.2165]



- more potentials and physical pion mass extrapolation in [P. B., K. Cichy, A. Peters, M. W. (2015) arXiv:1510.03441]

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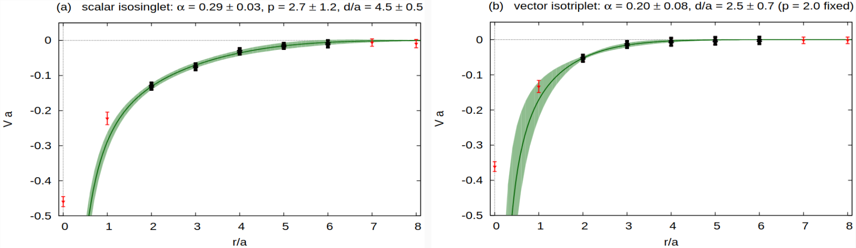
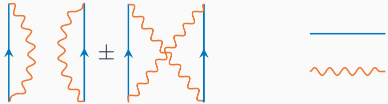


- more potentials and physical pion mass extrapolation in [P. B., K. Cichy, A. Peters, M. W. (2015) arXiv:1510.03441]
- the system was also studied in lattice QCD $\rightarrow$ discrepancies in bound state energies [A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman (2017), arXiv:1607.05214], [P. Junnarkar, N. Mathur, and M. Padmanath (2019) arXiv:1810.12285], [L. Leskovec, S. Meinel, M. Pflaumer, M. W. (2019) arXiv:1904.04197], [P. Mohanta and S. Basak (2020) arXiv:2008.11146],
- goal 1: improve on existing static $\bar{b}\bar{b}ud$ - potentials
- bound state for $\bar{b}\bar{b}us$ also predicted by lattice QCD recently [A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman (2017), arXiv:1607.05214], [P. Junnarkar, N. Mathur, and M. Padmanath (2019) arXiv:1810.12285], [S. Meinel, M. Pflaumer, M. W. (2022) arXiv:2205.13982]
- goal 2: compute static $\bar{b}\bar{b}us$ potential for the first time

$$\mathcal{O}_{BB}^{I=0}(\mathbf{r}_1,\mathbf{r}_2,t) = \Gamma_{ab}\tilde{\Gamma}_{cd}\left(\bar{b}_c^A(\mathbf{r}_1,t)u_a^A(\mathbf{r}_1,t)\ \bar{b}_d^B(\mathbf{r}_2,t)d_b^B(\mathbf{r}_2,t) - (u \to d)\right)/\sqrt{2}$$

$$\mathcal{O}_{BB}^{I=1}(\mathbf{r}_1,\mathbf{r}_2,t) = \Gamma_{ab}\tilde{\Gamma}_{cd}\left(\bar{b}_c^A(\mathbf{r}_1,t)u_a^A(\mathbf{r}_1,t)\ \bar{b}_d^B(\mathbf{r}_2,t)d_b^B(\mathbf{r}_2,t) + (u \to d)\right)/\sqrt{2}$$

$$\begin{aligned}
&\mathcal{C}_{BB}^{I=0/I=1}(\mathbf{r}_1,t_1|\mathbf{r}_2,t_2) \\
&= \left\langle\Omega\right|\left(\mathcal{O}_{BB}^{I=0/I=1}(\mathbf{r}_1,\mathbf{r}_2,t_1)\right)^\dagger \mathcal{O}_{BB}^{I=0/I=1}(\mathbf{r}_1,\mathbf{r}_2,t_2)\left|\Omega\right\rangle \\
&\propto \Big\langle \left(\gamma_0\Gamma^\dagger\gamma_0\right)_{ba}\Gamma_{ef}\mathrm{Tr}_c\left[U(\mathbf{r}_1,t_1;\mathbf{r}_1,t_2)\left(M_q^{-1}\right)_{ea}(\mathbf{r}_1,t_2|\mathbf{r}_1,t_1)\right]\times\mathrm{Tr}_c\left[U(\mathbf{r}_2,t_1;\mathbf{r}_2,t_2)\left(M_q^{-1}\right)_{fb}(\mathbf{r}_2,t_2|\mathbf{r}_2,t_1)\right] \\
&+ \left(\gamma_0\Gamma^\dagger\gamma_0\right)_{ba}\Gamma_{ef}\mathrm{Tr}_c\left[U(\mathbf{r}_1,t_1;\mathbf{r}_1,t_2)\left(M_q^{-1}\right)_{ea}(\mathbf{r}_1,t_2|\mathbf{r}_2,t_1)\ U(\mathbf{r}_2,t_1;\mathbf{r}_2,t_2)\left(M_q^{-1}\right)_{fb}(\mathbf{r}_2,t_2|\mathbf{r}_1,t_1)\right]\Big\rangle .
\end{aligned}$$

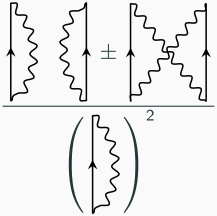
$\equiv$ 

Static quark propagator (Wilson line, gauge links)

light quark propagator (dynamical fermion field)

Instead, we compute

$$\frac{\mathcal{C}_{BB}^{I=0/I=1}(\mathbf{r}_1, 0|\mathbf{r}_2, t)}{\mathcal{C}_B(0|t)} =$$



$$\xrightarrow[t\to\infty]{} A \exp\Big(-(V_{BB,0}(|\mathbf{r}_2 - \mathbf{r}_1|) - 2m_B)t\Big)$$

$\rightarrow$ potential $V_{BB,0}(|\mathbf{r}_2 - \mathbf{r}_1|)$ automatically normalized to twice the B-meson mass $2m_B$

Lattice setup

ensemble	T/a	L/a	a[fm]	m_π[MeV]	N_{cfg}	α_{APE}	n_{APE}	κ_{G}	n_{G}
A5	64	32	0.0755	331	100	0.5	30	0.5	50
N6	96	48	0.0486	340	20	0.5	50	0.5	120

lattice ensemble details — smearing parameters

- gauge configurations generated as part of the CLS initiative
- O(a)-improved Wilson-quarks and Wilson plaquette action
- worked with "openQ*D" codebase [RC* Collaboration (2019) arXiv:1908.11673]
- computation of stochastic propagators
  - 12 per timeslice
  - 6-8 time slices per configuration
- HYP2 static action and APE smearing of gauge links
- Gaussian smearing of quark fields

Quantum numbers and operators

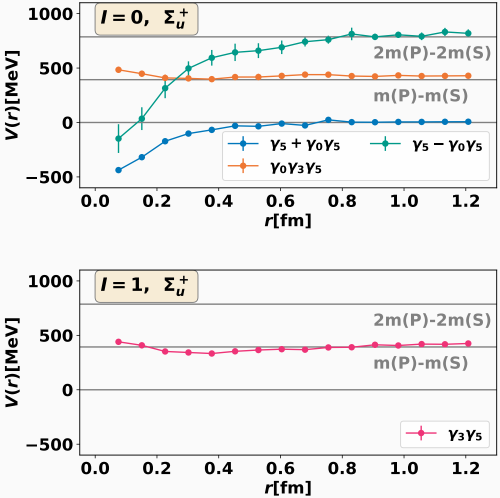
Isospin $I \in \{0, 1\}$
Rotational symmetry restricted to cylindrical symmetry around the separation axis.
$\Rightarrow$ use of quantum numbers Λ_η^ϵ:

- $\Lambda = \Sigma, \Pi$ angular momentum around the separation axis (corresponds to $|j_z| = 0, 1$)
- $\eta = +, - \equiv g, u$ parity
- $\epsilon = +, -$ reflection along an axis perpendicular to the separation axis

	$I = 0$		$I = 1$	
Γ	Λ_η^ϵ	shape	Λ_η^ϵ	shape
$\gamma_5 + \gamma_0\gamma_5$	Σ_u^+		Σ_g^+	
1	Σ_g^-		Σ_u^-	
γ_0	Σ_u^-		Σ_g^-	
$\gamma_5 - \gamma_0\gamma_5$	Σ_u^+		Σ_g^+	
$\gamma_3 + \gamma_0\gamma_3$	Σ_g^-		Σ_u^-	
$\gamma_3\gamma_5$	Σ_g^+		Σ_u^+	
$\gamma_0\gamma_3\gamma_5$	Σ_u^+		Σ_g^+	
$\gamma_3 - \gamma_0\gamma_3$	Σ_g^-		Σ_u^-	
$\gamma_{1/2} + \gamma_0\gamma_{1/2}$	Π_g		Π_u	
$\gamma_{1/2}\gamma_5$	Π_g		Π_u	
$\gamma_0\gamma_{1/2}\gamma_5$	Π_u		Π_g	
$\gamma_{1/2} - \gamma_0\gamma_{1/2}$	Π_g		Π_u	

Quantum numbers of BB trial states

BB results - Σ_u^+



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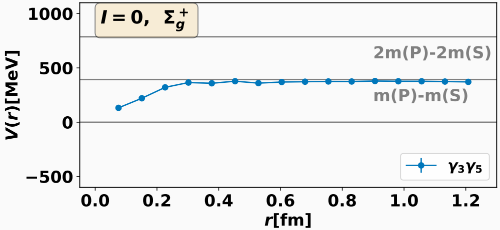
$\gamma_5 - \gamma_0\gamma_5$	Σ_u^+	A,PP	Σ_g^+	
$\gamma_3 + \gamma_0\gamma_3$	Σ_g^-		Σ_u^-	
$\gamma_3\gamma_5$	Σ_g^+		Σ_u^+	R,SP
$\gamma_0\gamma_3\gamma_5$	Σ_u^+	R,SP	Σ_g^+	
$\gamma_3 - \gamma_0\gamma_3$	Σ_g^-		Σ_u^-	
$\gamma_{1/2} + \gamma_0\gamma_{1/2}$	Π_g		Π_u	
$\gamma_{1/2}\gamma_5$	Π_g		Π_u	
$\gamma_0\gamma_{1/2}\gamma_5$	Π_u		Π_g	
$\gamma_{1/2} - \gamma_0\gamma_{1/2}$	Π_g		Π_u	

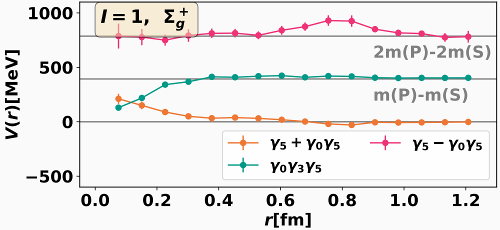
Quantum numbers of BB trial states

A = attractive, R = repulsive

SS = asymptotic value of $2m(S)$, SP = asymptotic value of $m(S) + m(P_-)$, PP = asymptotic value of $2m(P_-)$

BB results - Σ_g^+





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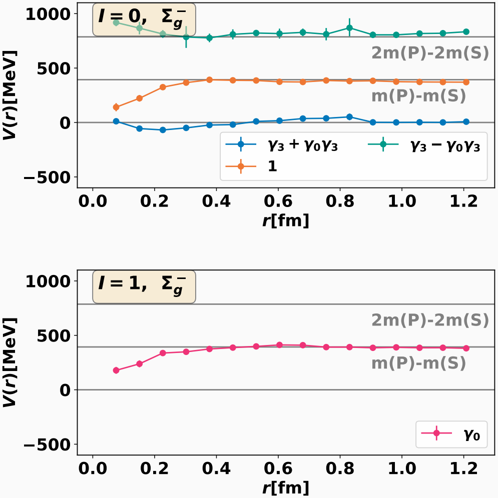

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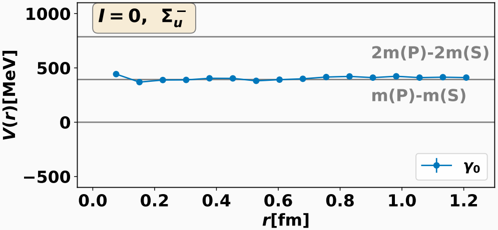

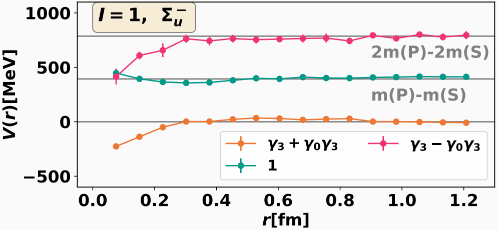

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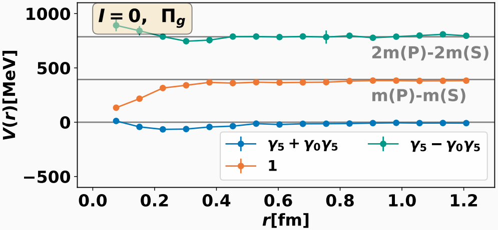

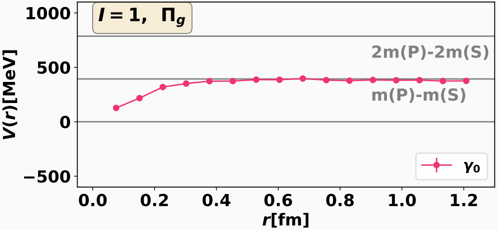

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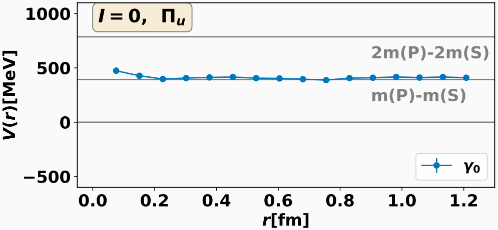
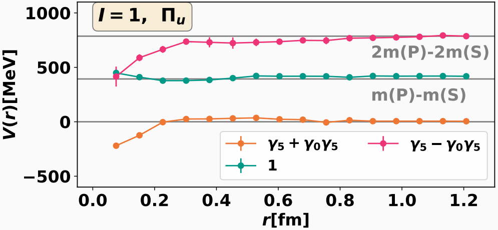
BB results - Π_u





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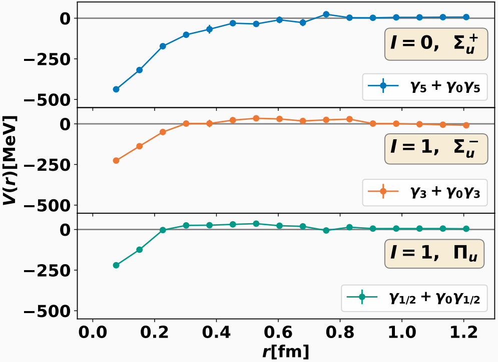
BB results - ground states



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$\gamma_{1/2}\gamma_5$	Π_g	A,SP	Π_u	R,SP
$\gamma_0\gamma_{1/2}\gamma_5$	Π_u	R,SP	Π_g	A,SP
$\gamma_{1/2} - \gamma_0\gamma_{1/2}$	Π_g	R,PP	Π_u	A,PP

Quantum numbers of *BB* trial states

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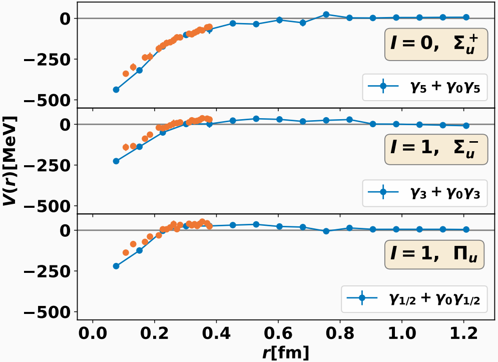
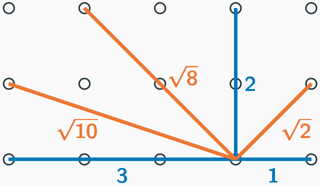
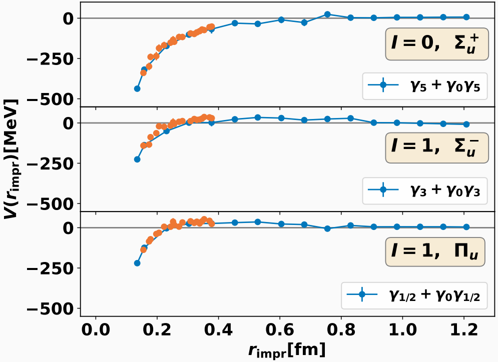
BB results - off-axis separations





- small separations most interesting
  → compute off-axis separations for more data points
- radius of $5a$ → 19 additional data points
- different discretization errors for off-axis separations
  → tree-level improvement

BB results - tree-level improvement



One-gluon exchange dominates at tree level perturbation theory

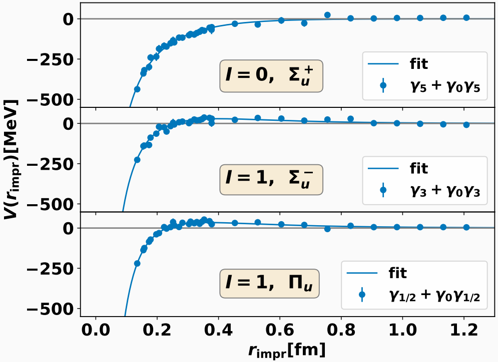
- $V_{\text{continuum}}(r) \propto \frac{1}{r}$
- $V_{\text{lattice}}(r) \propto G(\mathbf{r})$

where $G(\mathbf{r})$ is the tree-level lattice gluon propagator. Used to compute improved separations:

$$r \to r_{\text{impr}} = \frac{4\pi}{G(\mathbf{r})}$$

[R. Sommer (1994) arXiv:9310022 [hep-lat]]

Fitting the potential



$I = 0$: Screened Coulomb-like potential

$$V_1(r) = -\frac{\alpha_1}{r} \exp\left(-\left(\frac{r}{d}\right)^p\right)$$

$I = 1$: Screened Coulomb-like potential plus Yukawa term

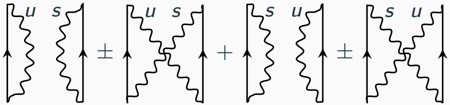
$$V_2(r) = V_1(r) + \frac{\alpha_2}{r} \exp(-\mu r)$$

A5	Γ	α	d/a	p	α_2	$a \cdot \mu$
$I = 0$	$\gamma_5 + \gamma_0\gamma_5$	0.35	4.4	1.8		
$I = 1$	$\gamma_3 + \gamma_0\gamma_3$	2.7	2.2	1.2	2.5	0.46
$I = 1$	$\gamma_{1/2} + \gamma_0\gamma_{1/2}$	0.80	1.9	1.2	0.23	0.17

BB_s correlation function

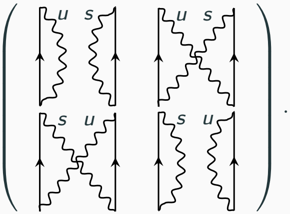
In the $\bar{b}\bar{b}us$-system the two light quark propagators are distinguishable.

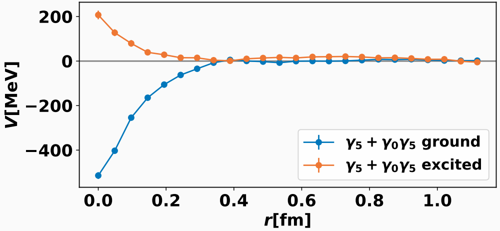
$$\mathcal{C}_{BB_s}(\mathbf{r}_1, t_1|\mathbf{r}_2, t_2) \equiv$$

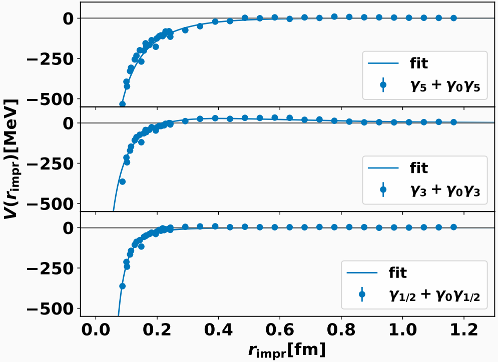


Without isospin symmetry many correlators are now trial states for the same sector. They can be disentangled by constructing a correlation matrix and solving the GEVP, e. g.

$$\tilde{\mathcal{C}}_{BB_s}(\mathbf{r}_1, t_1|\mathbf{r}_2, t_2) =$$







We again obtain three ground state (SS) potentials

Γ	α_1	d/a	p	α_2	$a \cdot \mu$
$\gamma_5 + \gamma_0\gamma_5$	0.29	5.2	1.3		
$\gamma_3 + \gamma_0\gamma_3$	2.8	5.7	1.0	2.6	0.16
$\gamma_{1/2} + \gamma_0\gamma_{1/2}$	1256	0.0004	0.2		

Summary and outlook

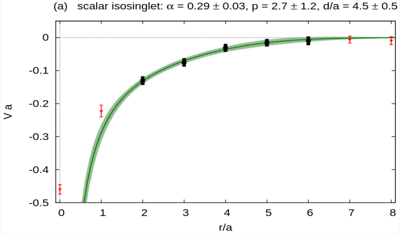
We

- improved on previous results significantly by
  - including off-axis separations
  - applying tree level improvement
- computed for the first time the static $\bar{b}\bar{b}us$ potential

Our next plans are to

- compute more statistics on our current ensembles
- investigate the pion mass dependence on an ensemble with smaller pion mass
- use the static potentials as input in the Born-Oppenheimer approximation to compute bound states and resonances

[P. B., M. W. (2012) 1211.2165]:



this work: