Nonperturbative renormalization of HQET operators in Position Space

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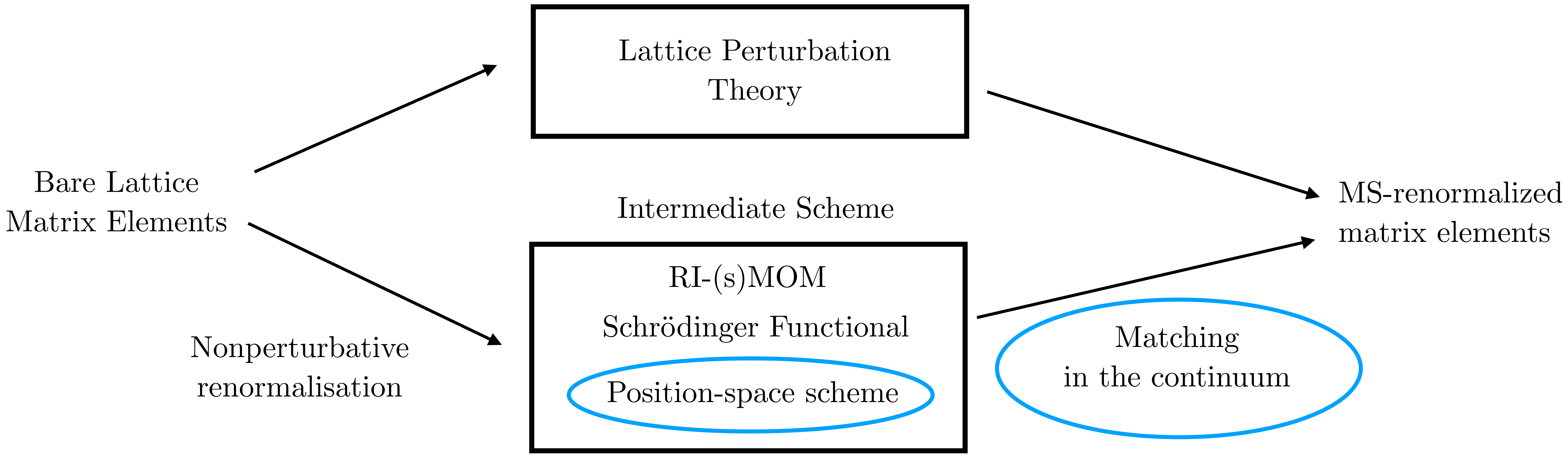
Lattice Renormalizations

✱ To connect bare lattice quantities to the continuum, we need to renormalize our operators. There are many different approaches:



✱ Goal: Matching of HQET operators in position-space

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Position Space Schemes (X-space)

- Renormalization conditions in momentum space (RI-(s)MOM)

$$Z_\Gamma \langle p \,|\, \mathcal{O}_\Gamma \,|\, p \rangle \Big|_{p_1^2=p_2^2=q^2=-\mu^2} = \langle p \,|\, \mathcal{O} \,|\, p \rangle \Big|_{\text{tree}}$$

- Renormalization conditions in position space

$$Z_\Gamma^2 \langle \mathcal{O}_\Gamma(x)\mathcal{O}_\Gamma(0) \rangle \Big|_{x^2=-\mu^2} = \langle \mathcal{O}_\Gamma(x)\mathcal{O}_\Gamma(0) \rangle \Big|_{\text{tree}}$$

The good — The bad

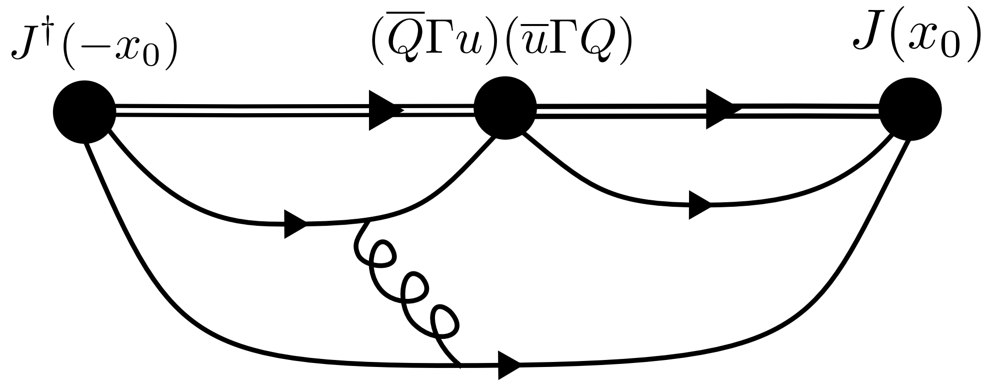
- Easy to implement!
- Gauge invariant
  - no gauge fixing required
  - no Gribov copies
  - no gauge non-invariant operators

- Similar window problem to other schemes:

$$a \ll x_0 \ll \Lambda_{\text{QCD}}^{-1}$$

- Mixing often requires three (or more) point-function calculations to have enough constraints

- Perturbative calculations contain more loops. e.g. $O(\alpha_S)$ matching for four-quark operators requires 3-loop calculations



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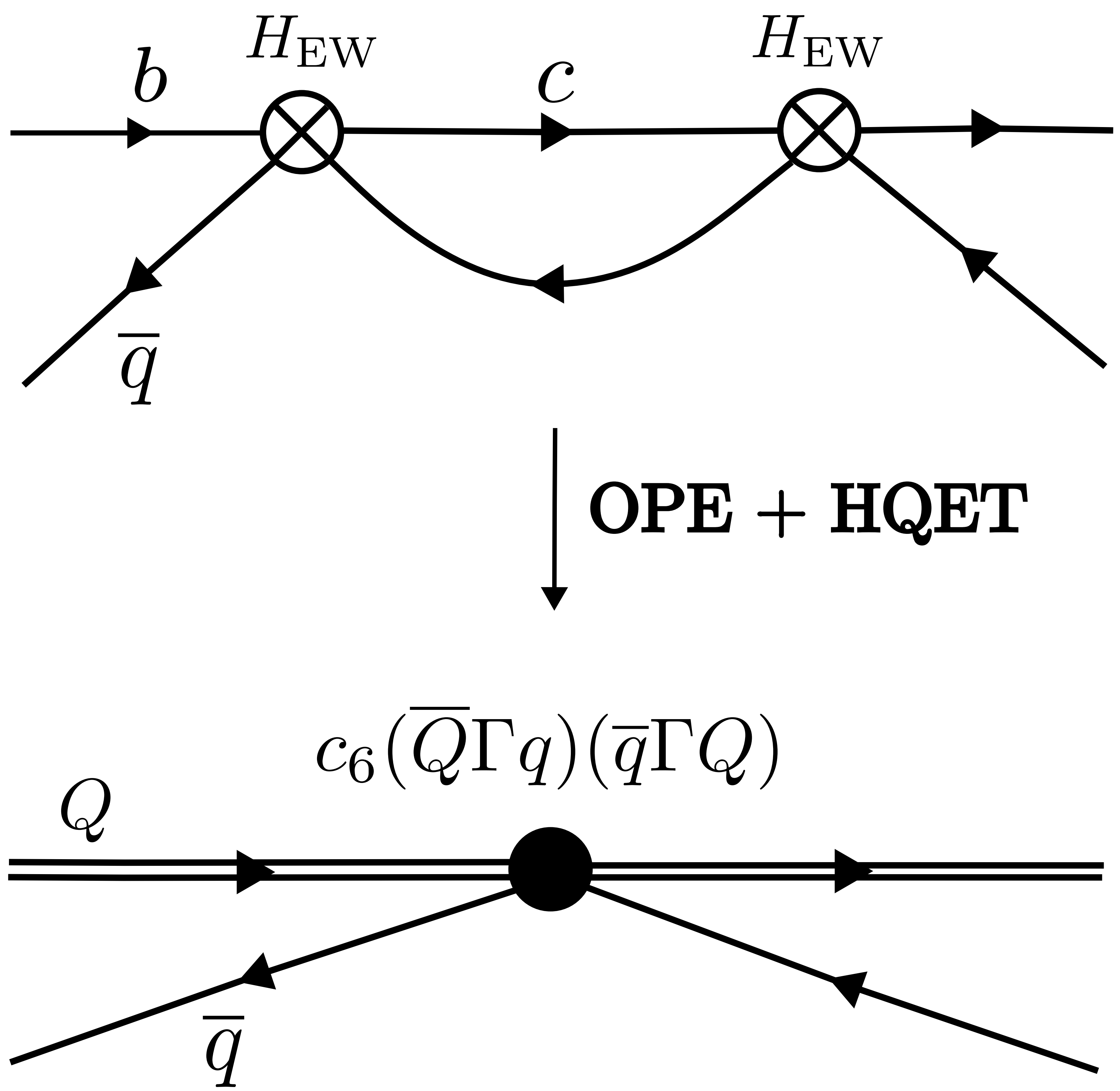
HQET four-quark operators of interest



- When studying inclusive decay rates of B-hadrons, an OPE leads to four-quark operators:

  [Review: A. Lenz, hep-ph/1405.3601]

$$O_q^1 = \left(\overline{Q}\gamma_\mu P_L q\right)\left(\bar{q}\gamma_\mu P_L Q\right) \qquad O_q^3 = \left(\overline{Q}T_a\gamma_\mu P_L q\right)\left(\bar{q}T_a\gamma^\mu P_L Q\right)$$

$$O_q^2 = \left(\overline{Q}P_L q\right)\left(\bar{q}P_R Q\right) \qquad O_q^4 = \left(\overline{Q}T_a P_L q\right)\left(\bar{q}T_a P_R Q\right)$$

- We'll focus for now on isospin non-singlet operators, $O_u - O_d$. These are protected from power-divergent mixings arising from eye contractions of the light quark. (these are the enhanced "Spectator Effects")

- The four-quark operators showing up in B-meson mixing are similar, ΔB=2 rather than ΔB=0. The matching of these operators in the continuum will not be a lot of additional work.

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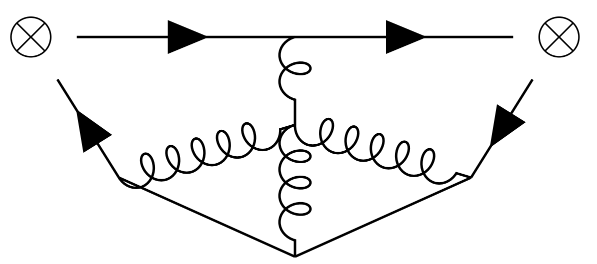
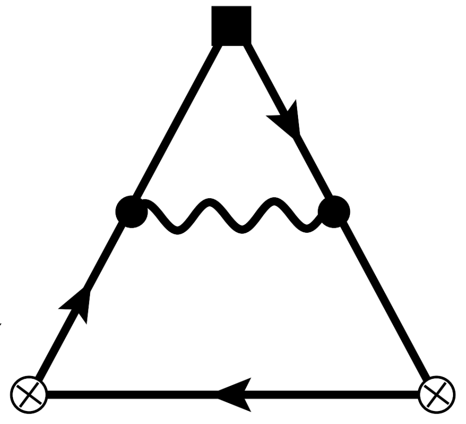
X-space dimension-3 light bilinears

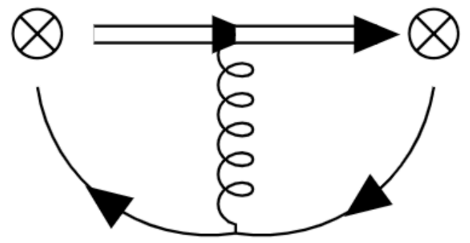
- $O(\alpha_S^4)$ matching for scalar, vector bilinears
  K. G. Chetyrkin, A. Maier [hep-ph/1010.1145]
- Wilson fermions
  V. Gimenez et al [hep-lat/0406019]
  S. Calì et al [hep-lat/2003.05781]
- $N_f = 2$, twisted-mass fermions
  ETMC [hep-lat/1207.0628]
- $N_f = 2 + 1$, Domain-wall fermions
  JLQCD [hep-lat/1604.08702]





X-space other bilinears

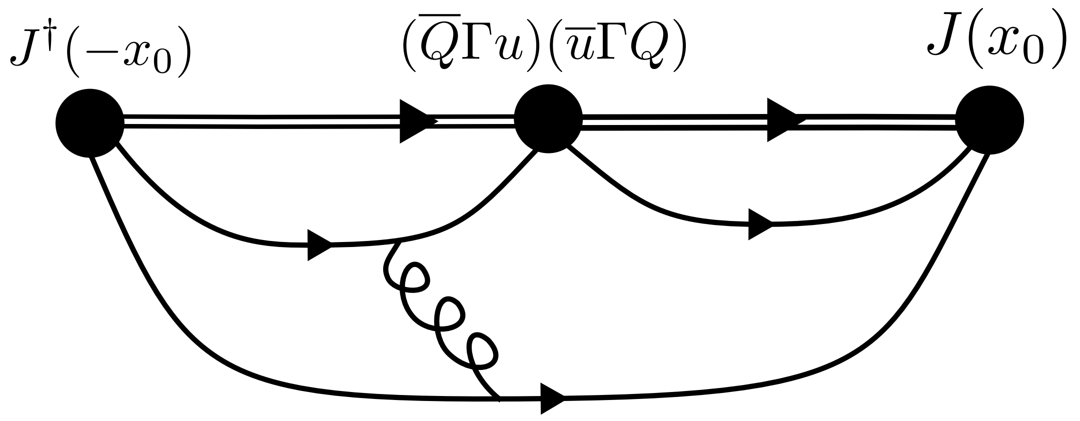
- Energy Momentum Tensor (dimension-4)
  Cyprus group [hep-lat/2102.00858, hep-lat/2212.07730]
- Staple-shaped operators
  C. Alexandrou et al [hep-lat/2305.11824]
- Heavy-light Bilinears
  P. Korcyl et al [hep-lat/1512.00069]



X-space Four-quark operators

- $\Delta s = 1$, $K \to \pi\pi$ operators
  matching between 3 and 4-flavor theories.
  M. Tomii et al [hep-lat/1811.11238,1901.04107]

Matching for X-space ↔ MS has not yet been calculated for four-quark operators



The scheme

- Requires three-point renormalization condition:

$$\frac{\langle J_\alpha(x_0)\mathcal{O}_j^{(X)}(\mu = x_0^{-1},0)J_\alpha^\dagger(-x_0)\rangle}{\langle J_\alpha(x_0)J_\alpha^\dagger(-x_0)\rangle} = \frac{\langle J_\alpha(x_0)\mathcal{O}_j^{(0)}(0)J_\alpha^\dagger(-x_0)\rangle}{\langle J_\alpha(x_0)J_\alpha^\dagger(-x_0)\rangle}\Bigg|_{\text{Tree Value}} =: T_{j,\alpha}$$

Measured on the lattice → $M_{i,\alpha}$

$$M_{i,\alpha} = \frac{\langle J_\alpha(x_0)\mathcal{O}_i^{(0)}(0)J_\alpha^\dagger(-x_0)\rangle}{\langle J_\alpha(x_0)J_\alpha^\dagger(-x_0)\rangle} = Z_{ij}(\mu = x_0^{-1})\frac{\langle J_\alpha(x_0)\mathcal{O}_j^{(X)}(\mu = x_0^{-1},0)J_\alpha^\dagger(-x_0)\rangle}{\langle J_\alpha(x_0)J_\alpha^\dagger(-x_0)\rangle} = Z_{ij}(x_0)T_{j,\alpha}$$

$$Z_{ij} = M_{i,\alpha}T^{-1}_{\alpha,j}$$

- Ratios are nice because Z-factors of source cancels, and linear divergence of Wilson line cancels.

- Sources need to be chosen so that the tree level matrix is invertible, otherwise don't have enough constraints on the Z-matrix. Need baryonic sources, otherwise the colour-mixed operators will have vanishing matrix elements.

$$O_q^3 = \left(\bar{Q}T_a\gamma_\mu P_L q\right)\left(\bar{q}T_a\gamma^\mu P_L Q\right) \quad O_q^4 = \left(\bar{Q}T_a P_L q\right)\left(\bar{q}T_a P_R Q\right)$$

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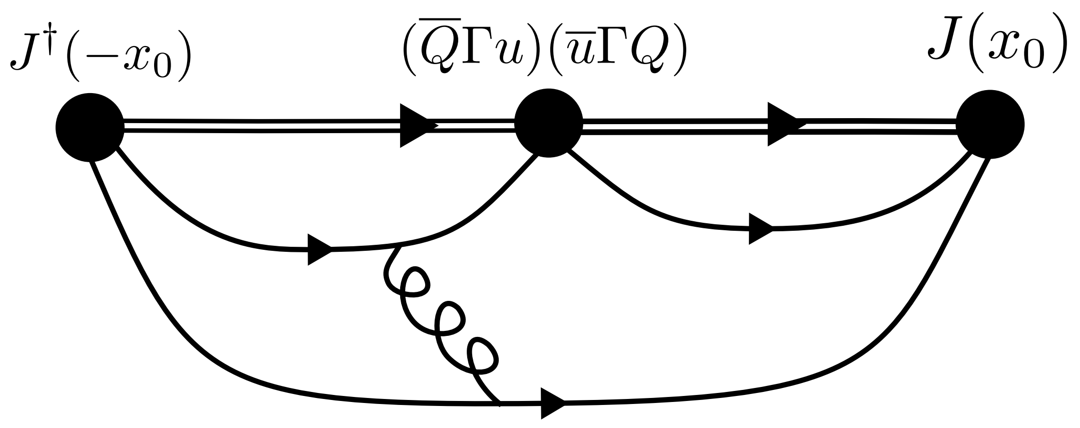
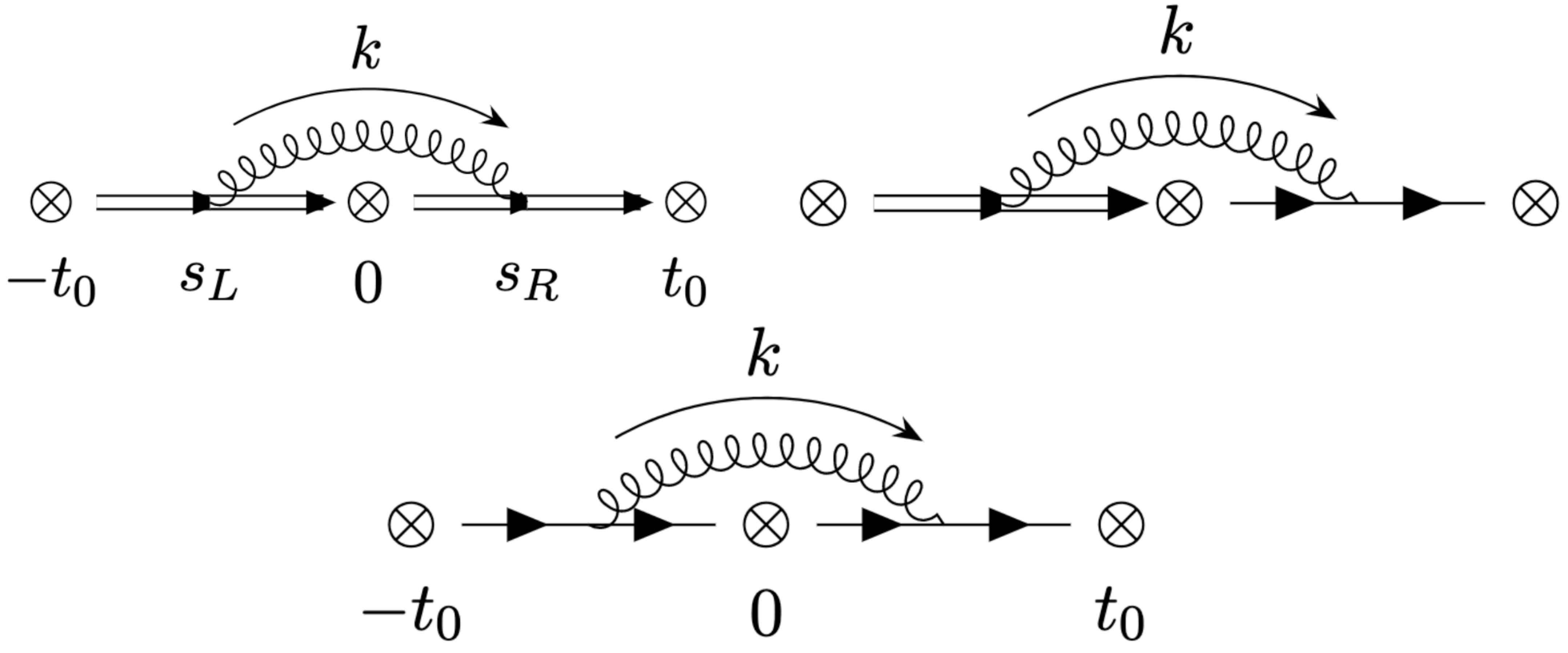
- ✱ $O(\alpha_S)$ position-space correlation functions break up into building blocks:





$$\int \frac{d^d p_L}{(2\pi)^d}\frac{d^d p_R}{(2\pi)^d}\frac{d^d k}{(2\pi)^d}\frac{e^{ip_L x_L - ip_R x_R}\, p_R^\alpha (p_R-k)^\beta (p_L-k)^\rho p_L^\delta}{(-p_L^2)(-(p_L-k)^2)(-(p_R-k)^2)(-p_R^2)(-k^2)}$$

$$\int \frac{d^d p_L}{(2\pi)^d}\frac{d^d p_R}{(2\pi)^d}\frac{d^d k}{(2\pi)^d}\frac{e^{ix_L p_L} e^{-ix_R p_R}\, p_R^\alpha (p_R-k)^\beta}{(-p_R^2)(-(p_R-k)^2)(-k^2)(v\cdot(p_L-k))(v\cdot p_L)}$$

→  Integration by parts

$$\int d^d k \left(\frac{\partial}{\partial k_\mu}\cdot p_\mu\right)\circ f(k) = 0$$

→ Two-loop master integrals

- ✱ In practice, in-house codes written in Mathematica to handle this
- ✱ HypExp: [T.Huber, D. Maitre, hep-ph/0507094]
- ✱ TRACER; M. Jamin, M. Lautenbacher

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Master integrals and blocks

* Master Integrals look like:

$$T_{LL}(x_L, x_R; n_1, n_2, n_3) = \int \frac{d^d p_L d^d p_R}{(2\pi)^{2d}} \frac{e^{ip_L x_L} e^{-ip_R x_R}}{(-p_L^2)^{n_1}(-p_R^2)^{n_2}(-(p_L - p_R)^2)^{n_3}}$$

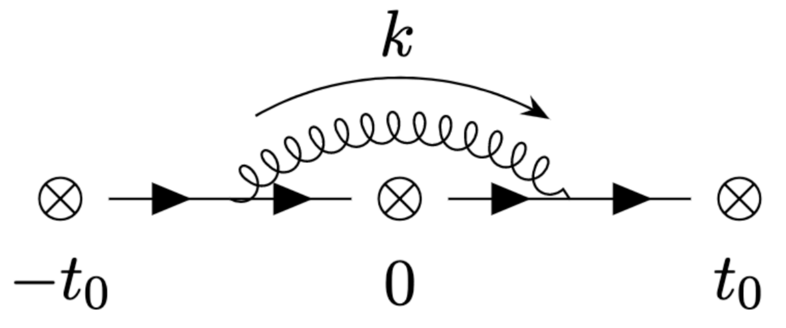
(agrees with hep-lat/2102.00858)

$$= \frac{-\Gamma(\frac{d}{2} - n_1)\Gamma(d - n_1 - n_2 - n_3)}{\Gamma(n_2)\Gamma(n_3)\Gamma(\frac{d}{2})4^{n_1+n_2+n_3}\pi^d}(-x_R^2)^{-d+n_1+n_2+n_3}\int_0^1 \mathrm{d}x(1 - x_1)^{-\frac{d}{2}+n_1+n_2-1}x_1^{-\frac{d}{2}+n_1+n_3-1}{}_2F_1\left(\frac{d}{2} - n_1, d - n_1 - n_2 - n_3, \frac{d}{2}, \frac{-(x_L - x_1 x_R)^2}{x_1(1 - x_1)x_R^2}\right)$$

$$T_{HH}(x_L, x_R; n_1, n_2, n_3) = \int \frac{d^d p_L d^d p_R}{(2\pi)^{2d}} \frac{e^{ip_L x_L} e^{-ip_R x_R}}{(v \cdot p_L)^{n_1}(v \cdot p_R)^{n_2}(-(p_L - p_R)^2)^{n_3}}$$

$$= \frac{1}{4^{n_3}\pi^{\frac{d}{2}}} \frac{\Gamma(\frac{d}{2} - n_3)}{\Gamma(n_3)\Gamma(n_1 + n_2)} {}_2F_1\left(n_1, d - 2n_3, n_1 + n_2, \frac{v \cdot (x_L - x_R)}{v \cdot x_L}\right)(-iv \cdot (x_R - x_L))^{-1+n_1+n_2}(-(v \cdot x_L)^2)^{-\frac{d}{2}+n_3}\delta_\perp(v \cdot (x_L - x_R))\theta(v \cdot (xR - xL) > 0)$$

* Building blocks look like:



$$= g^2(\mu^2)^{\frac{4-d}{2}}(-t_0^2)^{8-\frac{7d}{4}}\left[\left(\frac{-1}{32\pi^6\epsilon} + \frac{10\log 2 - 3\log\pi - 3\gamma_E}{64\pi^6}\right)(\not{x})(\not{x}) + \left(\frac{-1}{32\pi^6\epsilon} + \frac{3 + 4\log 2 - 6\log\pi - 6\gamma_E}{128\pi^6}\right)(\gamma_\mu)(\gamma_\mu) + \left(\frac{1}{128\pi^6\epsilon} + \frac{-1 - 2\log 2 + 3\log\pi + 3\gamma_E}{256\pi^6\epsilon}\right)(\gamma_\alpha\gamma_\beta\not{x})(\gamma_\alpha\gamma_\beta\not{x})\right]$$

Matching

- In the MS scheme at $O(\alpha_S)$, the operators mix in 2x2 sub-blocks. So as a check of our machinery, we can work in a scheme that forces this mixing structure, meaning we only need two sources in our X-space scheme:

$$J_\alpha = \bar{b}\gamma^5 q,\ \epsilon^{abc} b_a (u_b^T C \gamma_5 d_c)$$

$$Z_{2\times 2}^{(X)} = 1 + g^2(-\mu^2 x_0^2)^{\frac{4-d}{2}} \begin{pmatrix} \frac{1}{2\pi\epsilon} + \frac{24 + 9\gamma_E + 4\pi^2 - 18\log 2 + 9\log\pi}{36\pi^2} & \frac{-3}{8\pi^2\epsilon} + \frac{-4 - 3\gamma_E + 4\pi^2 + 6\log 2 - 3\log\pi}{16\pi^2} \\ \frac{-1}{12\pi^2\epsilon} - \frac{51 + 18\gamma_E - 8\pi^2 - 36\log 2 + 18\log\pi}{432\pi^2} & \frac{1}{16\pi^2\epsilon} + \frac{27 + 9\gamma_E + 28\pi^2 + 9\log\pi - 18\log 2}{288\pi^2} \end{pmatrix}$$

- Important check: divergent parts match!
  [M. Neubert, C.T. Sachrajda hep-ph/9603202]

- This scheme exploits the divergence structure at $O(\alpha_S)$, a full treatment requires four sources (in progress).

Future Prospects

Completed

- ☑ Master integrals and building blocks computed
  - ☑ Cross-checks that three-point renormalizations of bilinears has correct divergent pieces

In progress

- ☐ Writing down matching coefficients for explicit choices of sources for $\Delta B = 0$ four-quark operators
- ☐ Investigating the window problem on RBC/UKQCD 2+1 Domain Wall fermion ensembles

Wishlist

- ☐ Writing down matching coefficients for all light four-quark operators

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Thanks!