Non-equilibrium dynamics of topological defects in the 3D O(2) model

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Early Universe

The most widely accepted model of our Universe is the Standard Cosmological Model or Λ-CDM. The Universe began with a Big Bang. Cosmological constant Λ, Cold Dark Matter and Baryonic matter.

- Electroweak Epoch ($\lesssim 10^{-12}$ s or $\gtrsim 10^{2}$ GeV), interactions are dominated by electroweak interactions.
- Ends when the Higgs field acquires a non-zero vacuum expectation value (VEV) ≈ 246 GeV. Topological defects, such as cosmic strings, may form at this energy scale.

Kibble mechanism in the Early Universe

- Due to causality, the complex phases of Higgs field in different regions of the Universe are uncorrelated. Cosmic strings may form in the interfaces between these regions, where the phase change around a loop is 2π. (Kibble mechanism[1]).

[1] Kibble (1976).

Kibble mechanism in the Early Universe

- A random network of strings may arise.
- The characteristic scale is the correlation length, ξ.
- After cosmic string formation, the Universe is very dense, which dampens the string fluctuations. The damping force on strings increase the correlation length ξ.
- Cosmic strings lose energy through gravitational radiation and particle emission. Still very stable until today.

Kibble-Zurek mechanism (KZM) in condensed matter

- Zurek noticed that the Kibble mechanism can be observed in condensed matter systems. (Kibble-Zurek mechanism[2] or KZM).
- We consider a system with a second-order phase transition at T_c. The reduced temperature is defined as,

$$\epsilon = \frac{T_c - T}{T_c}.$$

- The correlation length ξ and the equilibrium relaxation time τ diverge at T_c,

$$\xi(\epsilon) = \frac{\xi_0}{|\epsilon|^\nu} \qquad \text{and} \qquad \tau(\epsilon) = \frac{\tau_0}{|\epsilon|^{z\nu}}.$$

- ν is a critical exponent, z is a dynamical critical exponent, and τ_0 and ξ_0 are proportionality constants.

[2] Zurek (1985).

Kibble-Zurek mechanism in condensed matter

- We consider a linear cooling process in time t, characterized by an inverse cooling rate τ_Q,

$$\epsilon(t) = \frac{t}{\tau_Q},$$

  where $t \in [-\tau_Q, \tau_Q]$.

- The phase transition occurs at $t = 0$,

$$\xi(t), \;\; \tau(t) \xrightarrow[t \to 0]{} \infty.$$

- Far from the critical point, $T \gg T_c$, the relaxation time is very small; Adiabatic cooling.
- In the vicinity of the critical point, $T \approx T_c$, the relaxation time becomes large. The dynamics is almost frozen. The system departs from equilibrium.

Kibble-Zurek mechanism in condensed matter

- The boundary between the adiabatic and frozen regions is estimated by comparing the time $\hat{t}$ elapsed after crossing the critical point with the relaxation time,

$$\tau(\hat{t}) = \hat{t},$$

yielding the time scale $\hat{t}$,

$$\hat{t} = \left(\tau_0\, \tau_Q^{z\nu}\right)^{\frac{1}{1+z\nu}}.$$

- At $\hat{t}$,

$$\hat{\epsilon} \equiv \epsilon(\hat{t}) = \left(\frac{\tau_0}{\tau_Q}\right)^{\frac{1}{1+z\nu}} \qquad \text{and} \qquad \hat{\xi} \equiv \xi(\hat{t}) = \xi_0 \left(\frac{\tau_Q}{\tau_0}\right)^{\frac{\nu}{1+z\nu}}.$$

Kibble-Zurek mechanism in condensed matter

- At $\hat{t}$, the density of persistent topological defects, ρ, is estimated by

$$\rho = \frac{\xi^d}{\xi^D} = \frac{1}{\xi_0^{D-d}} \left(\frac{\tau_0}{\tau_Q} \right)^{(D-d)\frac{\nu}{1+z\nu}} = \frac{1}{\xi_0^{D-d}} \left(\frac{\tau_0}{\tau_Q} \right)^{\zeta},$$

- Here, D is the spatial dimension of the system, d is the dimension of the topological defects, and τ_0 and ϵ_0 are constants.
- We introduce ζ to denote the exponent in this power law.

Non-linear sigma model 3D O(2)

- In the lattice regularization, at each site there is a spin $\vec{\sigma}_x = (\cos\theta_x, \sin\theta_x)$, where $\theta_x \in (-\pi, \pi]$ and $x = (x_1, x_2, x_3)$.
- We use a cubic lattice with a volume of L^3 and periodic boundary conditions.

Hamiltonian

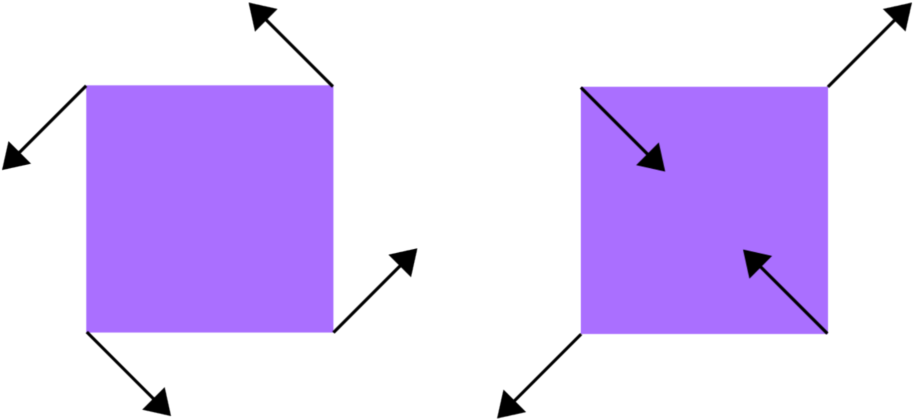
$$\mathcal{H}[\vec{\sigma}] = -\sum_{\langle xy\rangle} \vec{\sigma}_x \cdot \vec{\sigma}_y$$

- Second-order phase transition at $T_c = 2.201842$.[3]

[3] Campostrini et al. (2000)

Vortices

- The plaquettes may contain vortices or anti-vortices



Vorticity

- Each plaquette has a vorticity,

$$v_{\mu\nu}(x) = \frac{1}{2\pi}\left(\Delta\theta_{x,x+\hat{\mu}} + \Delta\theta_{x+\hat{\mu},x+\hat{\mu}+\hat{\nu}} + \Delta\theta_{x+\hat{\mu}+\hat{\nu},x+\hat{\nu}} + \Delta\theta_{x+\hat{\nu},x}\right),$$

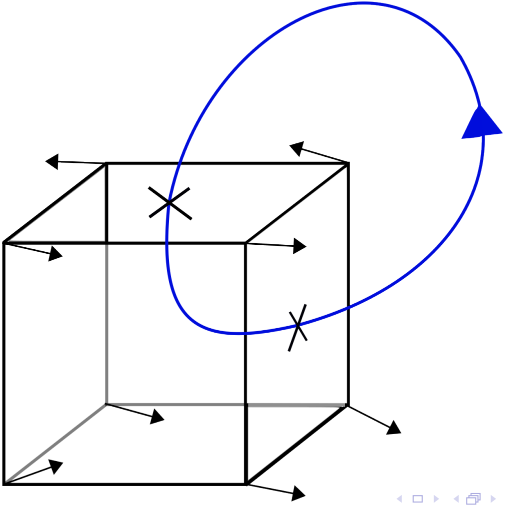
where $|\hat{\mu}|, |\hat{\nu}| = 1$ and $\Delta\theta_{x,x+\hat{\mu}} = \theta_{x+\hat{\mu}} - \theta_x \mod 2\pi \in [-\pi, \pi)$.

- There are only three possible values,

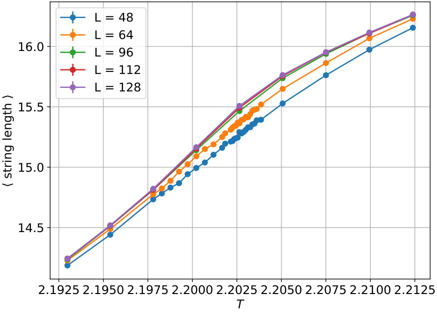
$$v_{\mu\nu}(x) \in \{-1 \ (\text{antivortex}), 0, 1 \ (\text{vortex})\}.$$

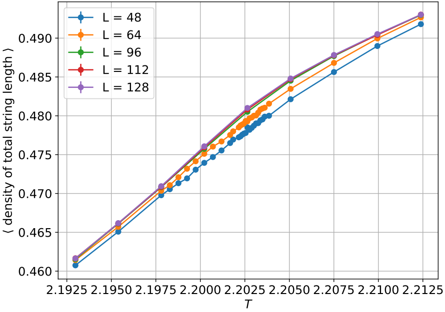
Vortex lines

- Vortices tend to form vortex lines. These objects share similarities with cosmic strings.
- We generate vortex lines connecting vortices stochastically.



Vortex lines in equilibrium





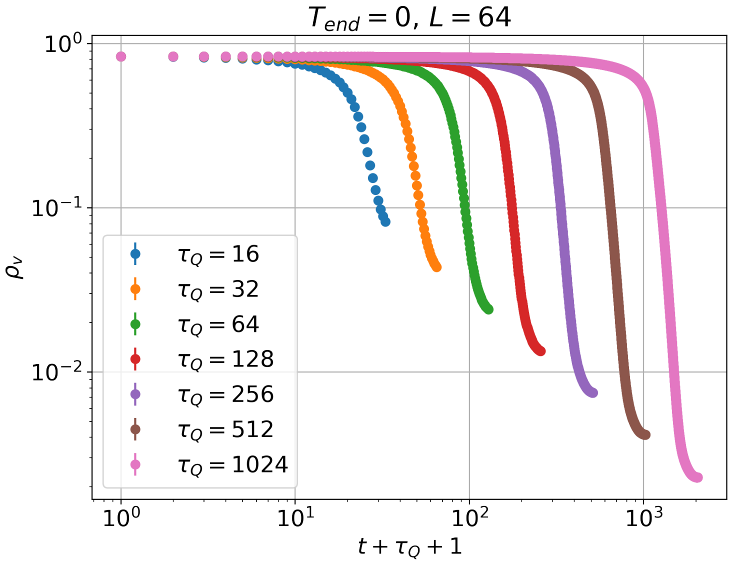
KZM in 3D O(2) model

- We use the Metropolis and Heat Bath algorithms, for which $z \approx 2$.
- In this model, $\nu = 0.67155(27)$.[4]
- Based on these values, $\zeta = 0.57321(26)$.
- Experiments conducted with hexagonal manganites ($R\mathrm{MnO_3}$) report a good agreement with this value for ζ at large τ_Q.[5]
- We use L^3 update steps (one sweep) as time unit.
- We thermalize the system at $T = 2T_c$, then, we reduce the temperature linearly in time until reaching a final temperature T_{end}.
- We measure ζ for several T_{end} values.

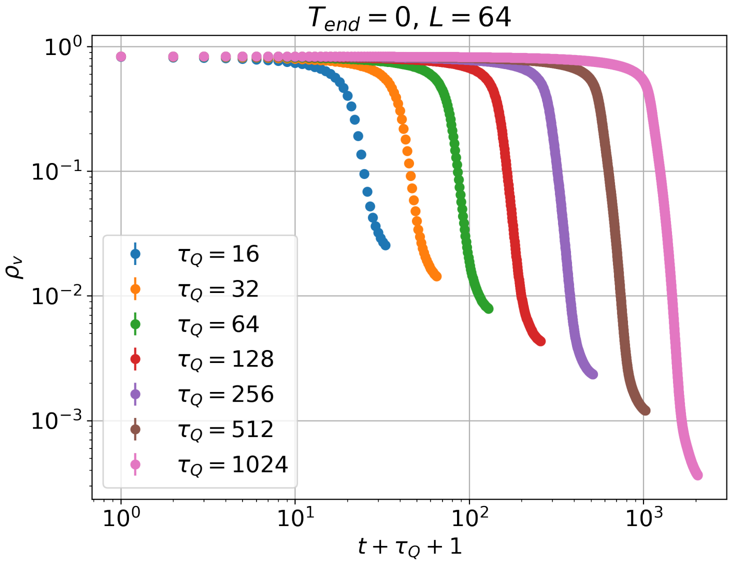
[4] Campostrini et al. (2000)

[5] Chae et al. (2012), Griffin et al. (2012)

Vortex number density (Metropolis)

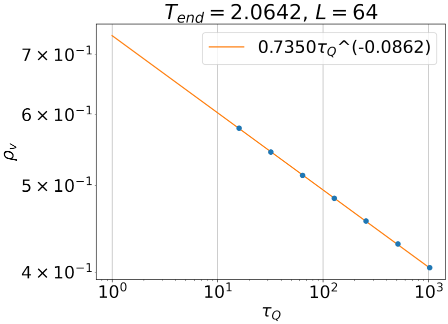


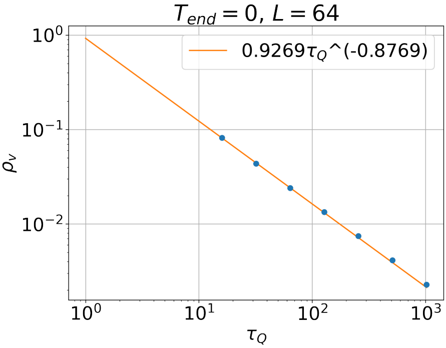
Vortex number density (Heat Bath)



Power law (Metropolis)

- $T_{\mathrm{end}} < T_c = 2.201842$.





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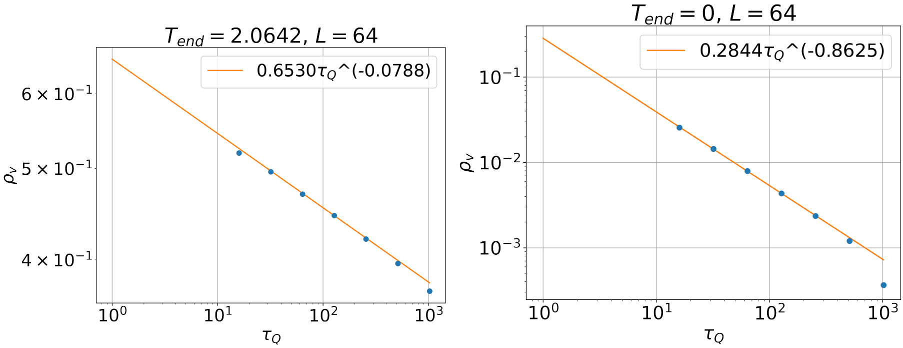
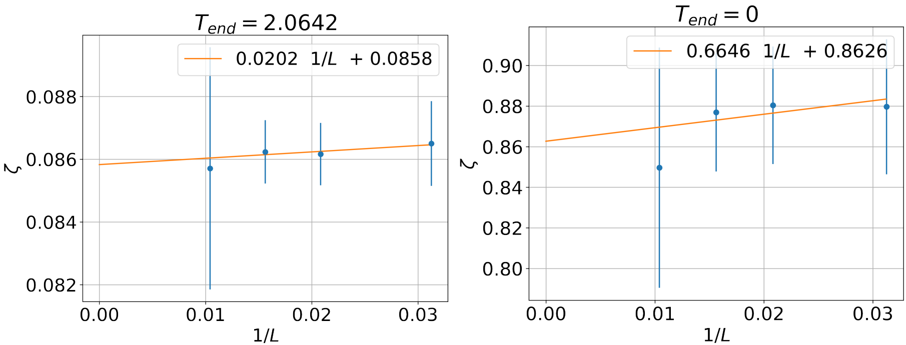
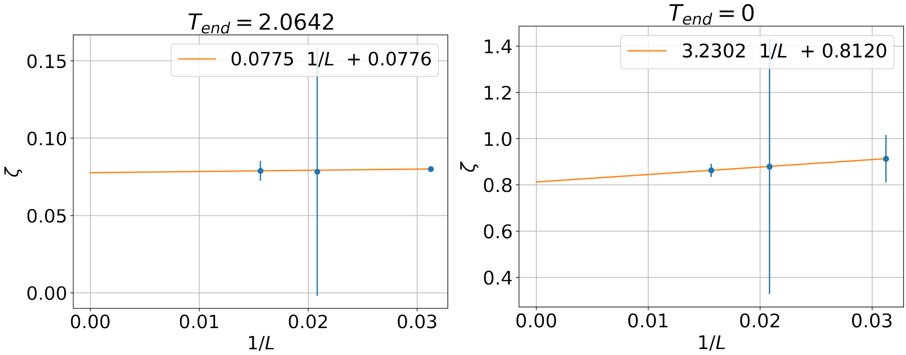


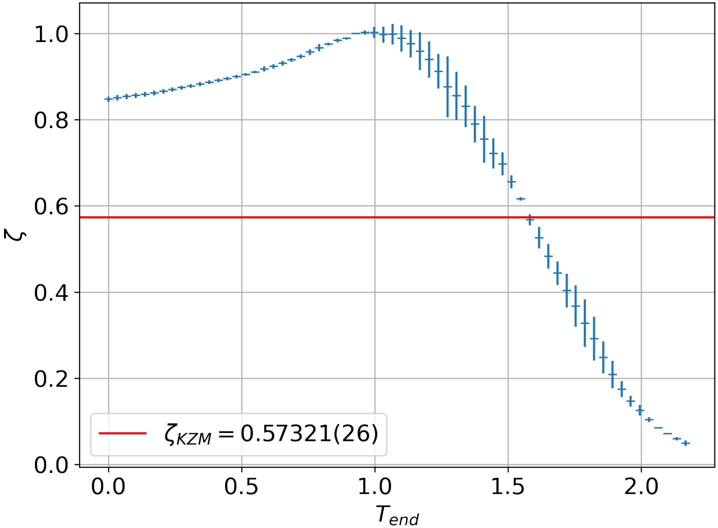
ζ exponent (Metropolis)



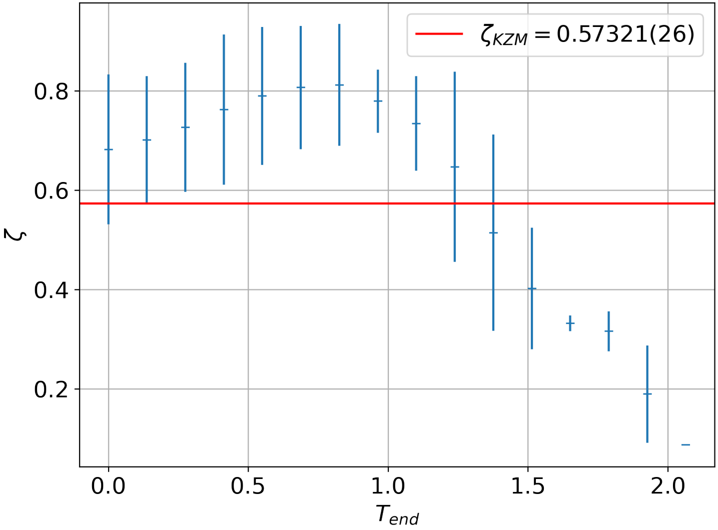
ζ exponent (Heat Bath)



ζ vs T_{end} (Metropolis)



ζ vs T_{end} (Heat Bath)



Conclusions

- We simulated the 3D O(2) non-linear σ model as an effective theory for cosmic strings.
- Vortex lines share similarities with cosmic strings.
- The vortex density follows a power law in the cooling rate after crossing the critical point.
- The exponent ζ depends on T_{end} in a non-monotonic manner.
- Values for ζ depend on the algorithm.

Thank you!