Coordinate-space calculation of isospin breaking corrections to the hadronic vacuum polarization contribution to $(g-2)_\mu$

En-Hung Chao, Harvey Meyer, Julian Parrino

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Hadronic contributions to $(g-2)_\mu$

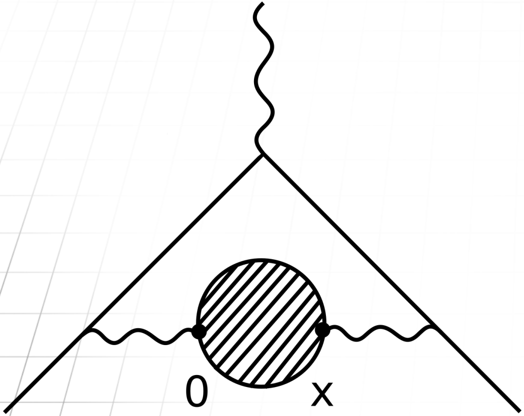


Fig. 1: Hadronic vacuum polarization (HVP) contribution

- $O(\alpha^2)$ contribution from hadronic vacuum polarization $\sim 700 \cdot 10^{-10}$, desirable accuracy $\sim 0.5\%$
- $O(\alpha^3)$ QED corrections become relevant
  - Understanding of tension between dispersive calculation of the HVP and lattice

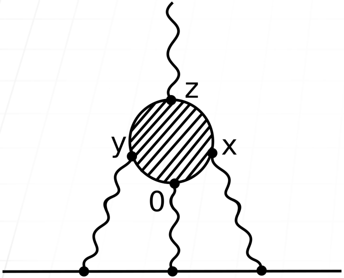


Fig. 2: Hadronic light-by-light scattering

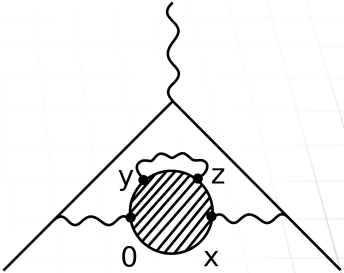


Fig. 3: HVP at NLO

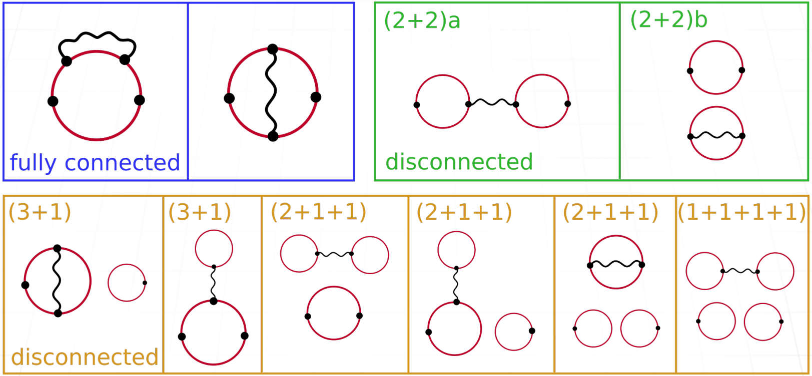
- HVP at NLO involes the same four-point function as Hlbl $\langle j_\mu(z) j_\nu(y) j_\rho(x) j_\sigma(0) \rangle$
- Propose calculation of HVP at NLO similar to Mainz calculation of Hlbl [2210.12263]
- QED$_\infty$: Kernel is calculated analytically in the continuum in infinite volume

QED corrections to the HVP

$$a_\mu^{HVP,NLO} = -\frac{e^2}{2}\int_{x,y,z} H_{\mu\sigma}(z)\delta_{\nu\rho}\Big[G_0(y-x)\Big]_\Lambda \langle j_\mu(z)j_\nu(y)j_\rho(x)j_\sigma(0)\rangle \quad (1)$$

- Covariant coordinate space (CCS) Kernel $H_{\mu\nu}(x) = -\delta_{\mu\nu}\mathcal{H}_1(|x|) + \frac{x_\mu x_\nu}{|x|^2}\mathcal{H}_2(|x|)$
  - Already successful calculation of LO HVP contribution to the window quantity a_μ^W in CCS formulation [2211.15581]
- With Pauli-Villars regularized photon propagator $\Big[G_0(y-x)\Big]_\Lambda = \frac{1}{4\pi^2|y-x|^2} - \frac{\Lambda K_1(\Lambda|y-x|)}{4\pi^2|y-x|^2}$
- Two options to restore QED:
  - take the limit $\Lambda \to \infty$
  - Evaluate missing piece on the lattice with massive photon in QED_m
- No power law finite-size effects, idea proposed in [2209.02149]

QED corrections to the HVP



- Successful crosscheck in QED between lattice calculation and infinite volume calculation
- Here we want to focus on (2+2) contributions in QCD with lattice data

(2+2)a contribution

- UV finite, derived from operator product expansion
- Interpret as QED correction to leading order disconnected contribution

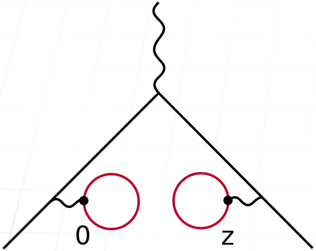


Fig. 4: leading order disconnected contribution contribution to $a_\mu^{HVP} \sim -(10 \sim 20) \cdot 10^{-10}$

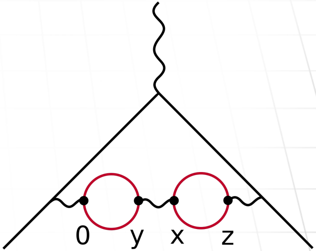
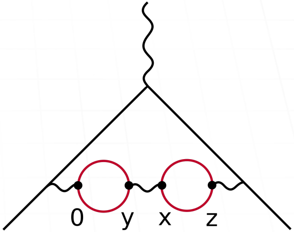


Fig. 5: QED correction to disconnected contribution

(2+2)a contribution

- Requieres only calculation of 2-point functions
- Result is UV finite → regulator can be dropped



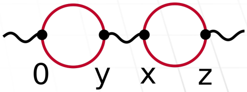
$$a_\mu^{HVP,NLO} = -\frac{e^2}{2} 2C \int_{x,y,z} H_{\mu\sigma}(z)\delta_{\nu\rho} G_0(y-x)\hat{\Pi}_{\mu\nu}(z,x)\hat{\Pi}_{\rho\sigma}(y,0) \quad (2)$$

with a chargefactor C ($C = 25/81$ for light contribution) and 2-pt function

$$\Pi_{\mu\nu}(x,y) = -Re\Big(Tr\Big[S(y,x)\gamma_\mu S(x,y)\gamma_\nu\Big]\Big), \quad \hat{\Pi}_{\mu\nu}(x,y) = \Pi_{\mu\nu}(x,y) - \langle \Pi_{\mu\nu}(x,y)\rangle_U \quad (3)$$

Strategy

- First step: Calculate quantity that is more short ranged
  - Subtracted vacuum polarization $\hat{\Pi}(Q_1^2, Q_2^2) = \Pi(Q_1^2) - \Pi(Q_2^2)$
- Only CCS Kernel needs to be changed
- Calculate integrand



$$f(|x|) = -\frac{e^2}{2} 2C2\pi^2 |x|^3 \int_{y,z} \tilde{H}_{\mu\sigma}(Q_1, Q_2, z) \delta_{\nu\rho} G_0(y-x) \hat{\Pi}_{\mu\nu}(z,x) \hat{\Pi}_{\rho\sigma}(y,0) \tag{4}$$

- Where $\hat{\Pi}(Q_1^2, Q_2^2)^{HVP,NLO} = \int_0^\infty f(|x|) d|x|$
- As consistency check, calculate with 2 version of the kernel $\tilde{H}_{\mu\sigma}^{TL}$ and $\tilde{H}_{\mu\sigma}^{XX}$

Strategy

$$f(|x|) = -\frac{e^2}{2} 2C2\pi^2 |x|^3 \int_{y,z} \tilde{H}_{\mu\sigma}(Q_1, Q_2, z)\delta_{\nu\rho} G_0(y-x) \hat{\Pi}_{\mu\nu}(z,x) \hat{\Pi}_{\rho\sigma}(y,0) \quad (5)$$

Id	β	$(\frac{L}{a})^3 \times (\frac{T}{a})$	a [fm]	m_π [MeV]	m_K [MeV]	$m_\pi L$	L [fm]	confs
N203	3.55	$48^3 \times 128$	0.06426	346(4)	442(5)	5.4	3.1	180

- With $N_f = 2+1$ dynamical flavors of non-perturbatively $O(a)$ improved Wilson quarks and tree-level $O(a^2)$ improved Lüscher-Weisz gauge action
- Compute and save $\Pi^{CL}_{\mu\nu}(x,y)$ for all lattice points x and 24 different source positions y

(2+2)a contribution to $\hat{\Pi}(Q_1^2, Q_2^2)$

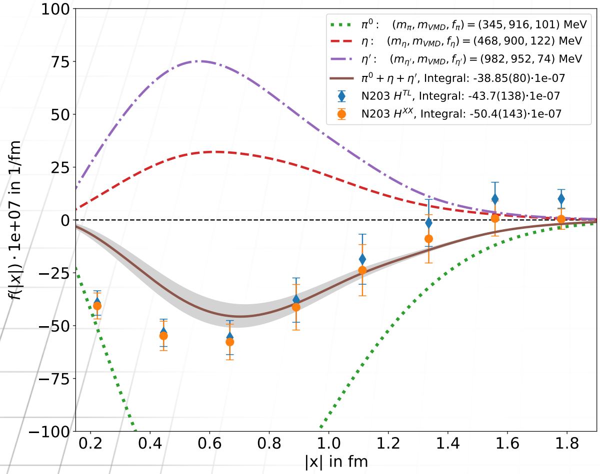
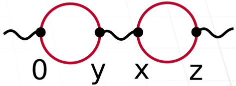


Fig. 6: (2+2)a contribution to $\hat{\Pi}^{HVP,NLO}(1\,GeV^2, 0.25\,GeV^2)$



- Good agreement between both kernels
- Compare with model of pseudoscalar meson exchange with VMD form factor
  - Use the corresponding parameters for π^0 and η on the given gauge ensemble
  - physical parameters for η'

(2+2)a contribution to $a_\mu^{HVP,NLO}$

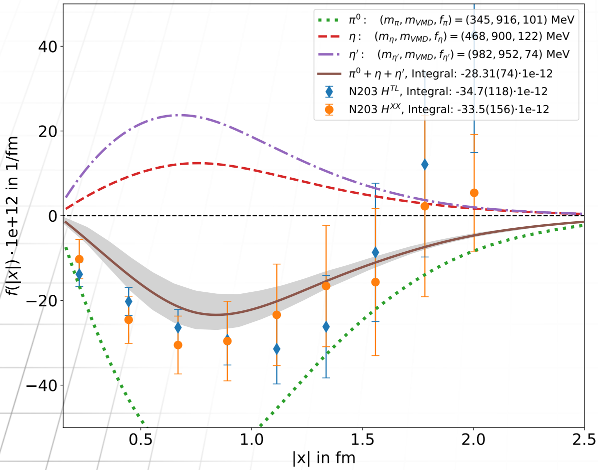
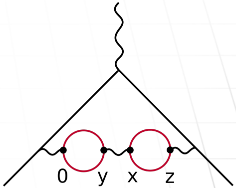


Fig. 7: (2+2)a contribution to $a_\mu^{HVP,NLO}$



- Choice of suitable kernel is important
- Total size of order 0.1% of leading order HVP, but: 5% - 10% of disconnected contribution to HVP

(2+2)a contribution to $a_\mu^{HVP,NLO}$

	physical	$m_\pi = 345\text{MeV}$ $m_K = 441\text{MeV}$
model π^0 model η model η'	-1.01 0.22 0.25	-0.68 0.15 0.25(?)
model combined	-0.54(2)	-0.28(1)
H^{TL} $(48^3 \times 128,\ a = 0.06426\text{fm})$ H^{XX} $(48^3 \times 128,\ a = 0.06426\text{fm})$		-0.35(12) -0.34(16)
RBC/UKQCD [1801.07224]	-6.9(2.1)(2.0)	
BMW (3+1 diagram included) [2002.12347]	-0.55(15)(11)	

Table: Results for (2+2)a contribution to $a_\mu^{HVP,NLO}$ in units of 10^{-10}

- The model describe the lattice data well at $m_\pi = 345$ MeV
- Only rough estimate on model uncertainty from fit to data
- (3+1) is UV divergent, can not be compared on its own

(2+2)b contribution

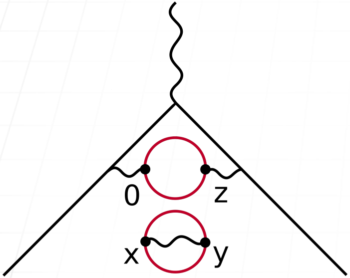


Fig. 8: (2+2)b contribution to $a_\mu^{HVP,NLO}$

- Need to keep regulator and use $\Lambda = 3m_\mu$
- Can be computed using the same 2-pt functions
- Interpret as sea-quark effect on the leading order HVP
- Quantity is very long-ranged
  - We focus first on the intermediate window quantity a_μ^W, where long distance effects are suppressed

- (2+2)b contribution for window quantity

$$f(|x|) = -\frac{e^2}{2} 2C2\pi^2|x|^3 \int_{y,z} H^W_{\mu\sigma}(z)\delta_{\nu\rho}\Big[G_0(y-x)\Big]_\Lambda \hat{\Pi}_{\mu\sigma}(z,0)\hat{\Pi}_{\rho\nu}(y,x) \tag{6}$$

Comparison between both (2+2) diagrams for window quantity $a_\mu^{W,HVP,NLO}$

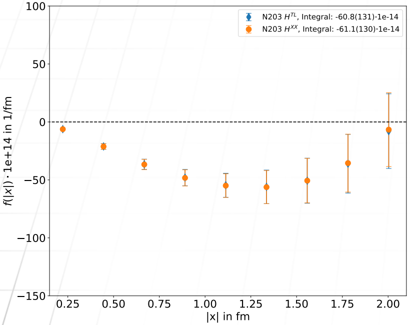


Fig. 9: (2+2)b contribution to $a_\mu^{W,HVP,NLO}$

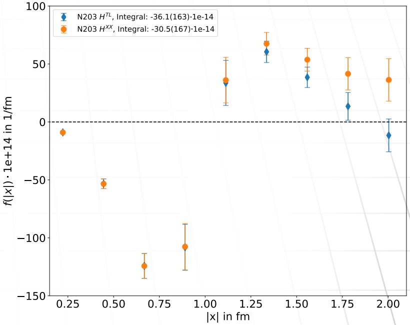
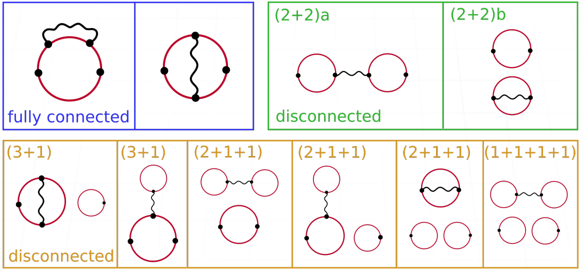


Fig. 10: (2+2)a contribution to $a_\mu^{W,HVP,NLO}$

- (2+2)b contribution to $a_\mu^{W,HVP,NLO}$ can be calculated with regulator Λ and is not negligible

Conclusion and Outlook



- Coordinate-space approach successful in calculating (2+2) disconnected contributions
- (2+2)a diagram can be compared in infinite volume and continuum limit
- Still much work to do for result of full $O(\alpha^3)$ hadronic corrections and also for calculation of strong isospin breaking effects

Backup Slides

Pseudoscalar exchange model

$$f(|x|) = \int_{z,y} H_{\sigma\lambda}(z)\Big[G_0(x-y)\Big]_\Lambda \int_{q,k,p} e^{i(p\cdot z+q\cdot y+k\cdot x)}\Pi_{\sigma\mu\mu\lambda}(p,q,k) \tag{7}$$

- VMD form factor

$$\mathcal{F}(-p^2,-k^2) = \frac{c_\pi}{(p^2+m_V^2)(k^2+m_V^2)} \tag{8}$$

- Chargefactors $(-25/9, 1, 1)$ for (π^0, η, η') respectively
- Use parameters for m_π and f_π on ensemble
- $m_\eta^2 \sim 4/3m_K^2 - 1/3m_\pi^2$ and f_η from linear interpolation between $SU(3)$ symmetric and physical point
- Physical parameters for η'

(2+2) Contribution

- Total (2+2) Contribution can be written as

$$a_\mu^{HVP,NLO} = -\frac{e^2}{2} C 2\pi^2 \int d|x||x|^3 \left[I^1(x) I^4(x) + 2 I^2_{\rho\sigma}(x) I^3_{\sigma\rho}(x) \right] \tag{9}$$

$$I^1(x) = \int_z G_{\mathrm{PV}}(\mu; x-z) \hat{\Pi}_{\nu\nu}(x,z) \tag{10}$$

$$I^2_{\rho\sigma}(x) = \int_y G_{\mathrm{PV}}(\mu; x-y) \hat{\Pi}_{\rho\sigma}(y,0) \tag{11}$$

$$I^3_{\sigma\rho}(x) = \int_z H_{\nu\sigma}(z) \hat{\Pi}_{\nu\rho}(z,x) \tag{12}$$

$$I^4(x) = \int_y H_{\mu\sigma}(y) \hat{\Pi}_{\mu\sigma}(y,0) \tag{13}$$