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New gauge-independent transition separating confinement-Higgs phase in the lattice gauge-fundamental scalar model

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1. Introduction

■ We consider SU(2) LGST (with fundamental scalar)

Action of SU(2) LGST (with fund. scalar):

$$S[U,\hat{\Theta}] = \underbrace{\frac{\beta}{2}\sum_{x,\mu>\nu} \mathrm{Re}\,\mathrm{tr}\left(\mathbf{1} - U_{x,\mu}U_{x+\mu,\nu}U^{\dagger}_{x+\nu,\mu}U^{\dagger}_{x,\nu}\right)}_{S_G[U]\ :\ \text{gauge part}} + \underbrace{\frac{\gamma}{2}\sum_{x,\mu} \mathrm{Re}\,\mathrm{tr}(\mathbf{1} - \hat{\Theta}^{\dagger}_{x}U_{x,\mu}\hat{\Theta}_{x+\mu})}_{S_H[U,\hat{\Theta}]\ :\ \text{scalar part}}$$

- $U_{x,\mu} \in \mathrm{SU}(2)$: link variables, $\hat{\Theta}_x \in \mathrm{SU}(2)$: (normalized) scalar fields
- β : gauge coupling, γ : scalar coupling

$\hat{\Theta}_x$ transforms as the fundamental representation of the gauge group SU(2).

This model has the $\mathrm{SU}(2)_{\mathrm{local}} \times \mathrm{SU}(2)'_{\mathrm{global}}$ symmetry:

$$U_{x,\mu} \mapsto U'_{x,\mu} = \Omega_x U_{x,\mu}\Omega^{\dagger}_{x+\mu}\,, \quad \hat{\Theta}_x \mapsto \hat{\Theta}'_x = \Omega_x\hat{\Theta}_x\Omega'$$

where $\Omega_x \in \mathrm{SU}(2)_{\mathrm{local}}$, $\Omega' \in \mathrm{SU}(2)'_{\mathrm{global}}$.

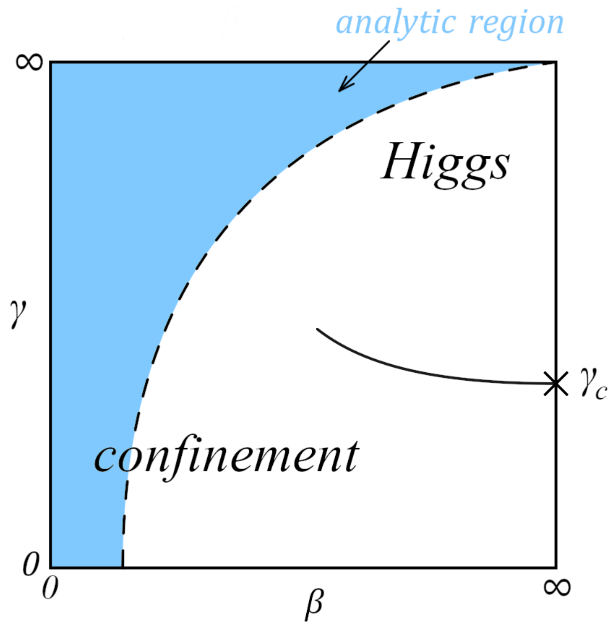
■ Motivation

□ <u>In case of fundamental scalar field</u> → This talk

Confinement ($\beta \geqslant 0, \gamma \ll 1$) and Higgs ($\beta \gg 1, \gamma_c \leqslant \gamma < \infty$) regions are subregions of an analytically continued single phase. The transition line starts from $(\beta, \gamma) = (\infty, \gamma_c)$ does not reach "analytic region" which connect these subregions.

[1] E. Fradkin and S. H. Shenker, Phys. Rev. **D19**, 3682 (1979),

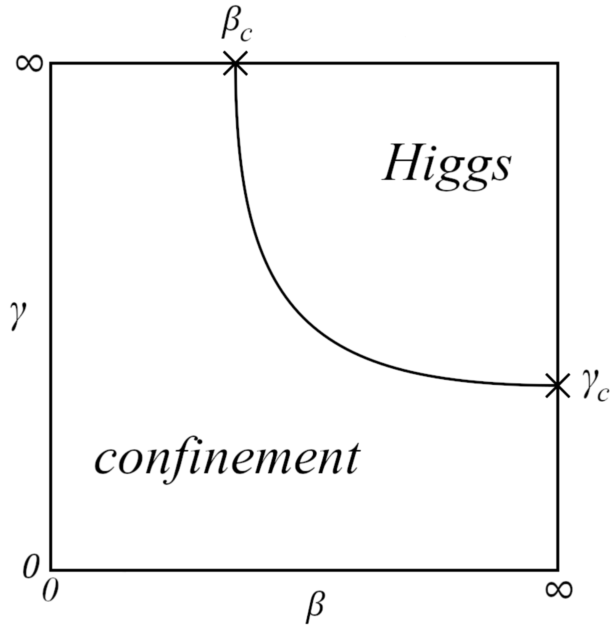
[2] K. Osterwalder and E. Seiler, Annals Phys. **110**, 440 (1978)



□ <u>In case of adjoint scalar field</u> → The next talk by Shibata

Confinement and Higgs regions are completely separated into the two different phases by the continuous transition line. The transition line has two endpoints, $(\beta, \gamma) = (\infty, \gamma_c)$ and $(\beta, \gamma) = (\beta_c, \infty)$.

[3] R. C. Brower et al., Phys. Rev. **D25**, 3319 (1982)



■ Motivation and Results

Re-examine the 4D SU(2) lattice gauge scalar theory (LGST) with fund. scalar field,

- We found the gauge-invariant composite operator of the original scalar field and the new "color-direction field", which enables to separate the confinement phase and the Higgs phase completely and gauge-independently.

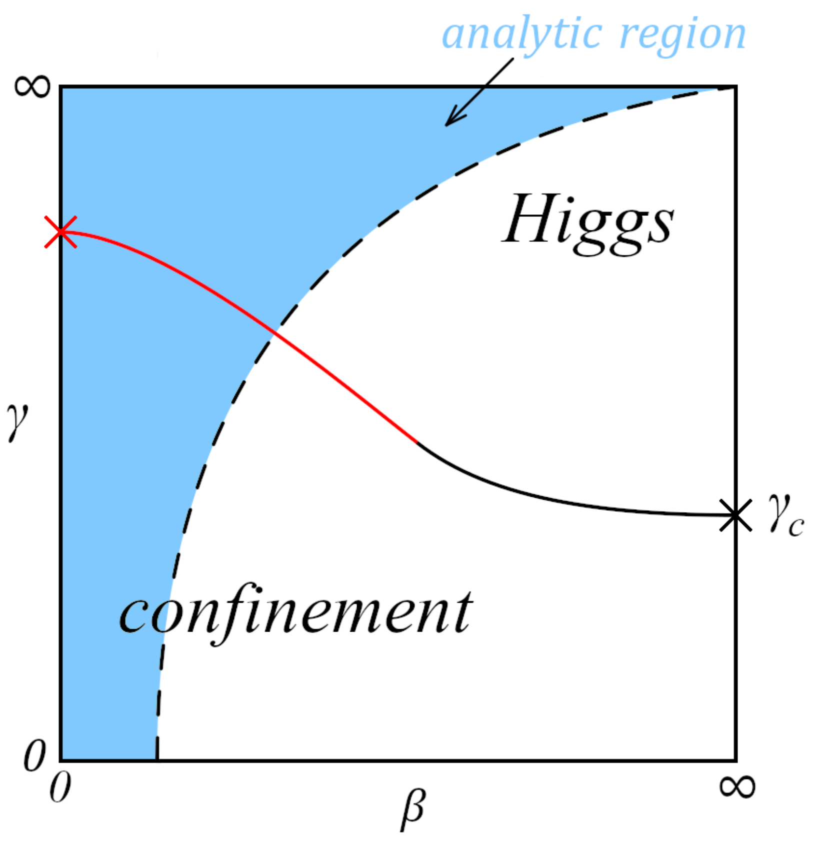
- We perform the numerical simulations for this model without any gauge fixing, and found a new transition line:
  - in the weak gauge coupling, it agrees with the conventional thermodynamical transition line.
  - in the strong gauge coupling, it divides the single confinement-Higgs phase into two separate phases, confinement and the Higgs.

2. New transition line and Color-direction field

■ Newly found transition line

- As the result of the numerical simulations for the 4D SU(2) LGST with fund. scalar, we found a new transition line which separates confinement and Higgs regions completely, without any gauge fixing. (Red line in the right figure)

- This transition line is obtained in the gauge-independent way by introducing the new composite operator of the original scalar field and the new "color-direction field", based on the gauge-covariant decomposition of the gauge field due to Cho-Duan-Ge-Shabanov and Faddeev-Niemi.



■ gauge-covariant decomposition of the gauge field

$U_{x,\mu} \in \mathrm{SU}(2)$ is gauge-covariantly decomposable: $U_{x,\mu} = X_{x,\mu} V_{x,\mu} \quad (X_{x,\mu}, V_{x,\mu} \in \mathrm{SU}(2))$

(We required the transformations, $X_{x,\mu} \mapsto X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x^\dagger \,, \quad V_{x,\mu} \mapsto V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger$)

- This decomposition is given uniquely by solving the defining equations for the "color-direction field" $\boldsymbol{n}_x \in su(2) - u(1)$ with a unit :

$$D_\mu[V]\boldsymbol{n}_x := V_{x,\mu}\boldsymbol{n}_{x+\mu} - \boldsymbol{n}_x V_{x,\mu} = 0 \,, \quad \mathrm{tr}\,(\boldsymbol{n}_x X_{x,\mu}) = 0$$

  (We required the transformation, $\boldsymbol{n}_x \mapsto \boldsymbol{n}'_x = \Omega_x \boldsymbol{n}_x \Omega_x^\dagger$)

- For a given set of gauge fields $\{U_{x,\mu}\}$, a set of color-direction fields $\{\boldsymbol{n}_x\}$ is determined as the configuration minimizing the reduction functional:

$$F_{\mathrm{red}}[\boldsymbol{n};U] = \sum_{x,\mu} \frac{1}{2}\,\mathrm{tr}\left\{ (D_\mu[U]\boldsymbol{n}_x)^\dagger (D_\mu[U]\boldsymbol{n}_x) \right\} = \sum_{x,\mu} \mathrm{tr}\left(\mathbf{1} - \boldsymbol{n}_x U_{x,\mu} \boldsymbol{n}_{x+\mu} U_{x,\mu}^\dagger \right)$$

- $F_{\mathrm{red}}[\boldsymbol{n};U]$ has the same form as the Higgs action of SU(2) LGST (with adj. scalar) (minimization of $F_{\mathrm{red}}[\boldsymbol{n};U]$ extracts the DOF of $\{\boldsymbol{n}_x\}$ from the gauge fields $\{U_{x,\mu}\}$)

[4] Kondo et al., Phys. Rep. **579**, 1-226 (2015)

■ construction of the scalar-color density <R>

- We required that $\hat{\Theta}_x \in \mathrm{SU}(2)$ and $\boldsymbol{n}_x \in su(2) - u(1)$ transform as

$$\hat{\Theta}_x \mapsto \hat{\Theta}'_x = \Omega_x \hat{\Theta}_x \Omega' \,, \quad \boldsymbol{n}_x \mapsto \boldsymbol{n}'_x = \Omega_x \boldsymbol{n}_x \Omega_x^\dagger$$

where $\Omega_x \in \mathrm{SU}(2)_{\mathrm{local}}$, $\Omega' \in \mathrm{SU}(2)'_{\mathrm{global}}$.

- Then it is possible to find a local quantity R_x which has the global covariance:

$$R_x := \hat{\Theta}_x^\dagger \boldsymbol{n}_x \hat{\Theta}_x \,, \quad R_x \mapsto R'_x = \Omega'^\dagger R_x \Omega'$$

- The spacetime average of R_x also has the global covariance:

$$\bar{R} := \frac{1}{V} \sum_x R_x \,, \quad \bar{R} \mapsto \bar{R}' = \frac{1}{V} \sum_x \Omega'^\dagger R_x \Omega' = \Omega'^\dagger \left(\frac{1}{V} \sum_x R_x \right) \Omega' = \Omega'^\dagger \bar{R} \Omega'$$

- Then there exists a non-trivial gauge invariant defined as

$$R := |\bar{R}| = \sqrt{\frac{1}{2} \mathrm{tr}(\bar{R}^\dagger \bar{R})} \,, \quad R \mapsto R' = R$$

R also can be defined as the absolute value of the two eigenvalues of $\bar{R}$.

3. Numerical simulation

■ Setting for lattice simulation

- 8^4 -lattice, pseudo heat bath method, cold start ($U_{x,\mu} = \mathbf{1}$, $\hat{\Theta}_x = \mathbf{1}$)

For a point of the couplings (β, γ),

- 5000 sweeps were discarded for thermalization, before sampling.
- Per 100 sweeps, a set of gauge and scalar configuration was sampled.
  Total 100 samples were taken per point.

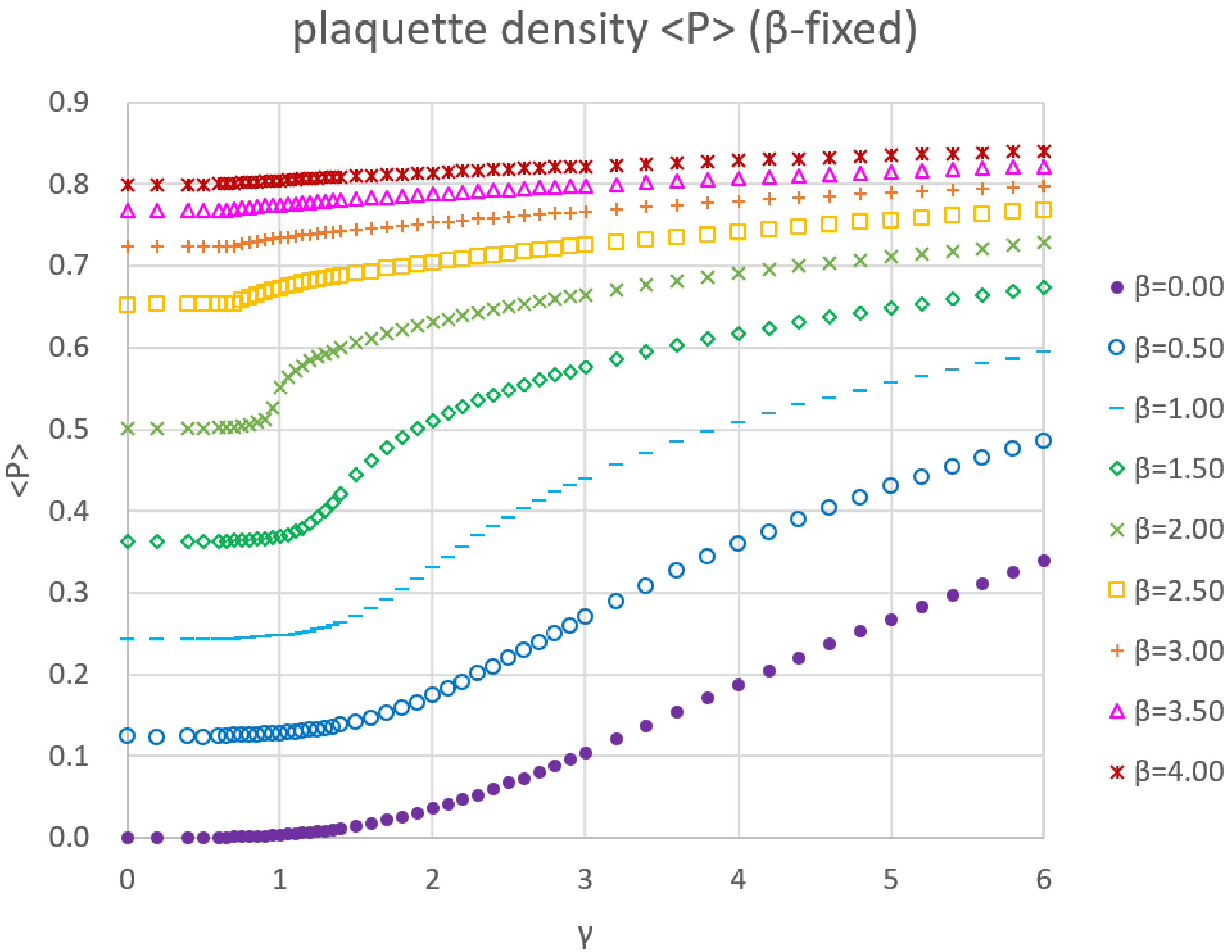
To determine a set of color-direction fields $\{\boldsymbol{n}_x\}$ from a given set of $\{U_{x,\mu}\}$ numerically,

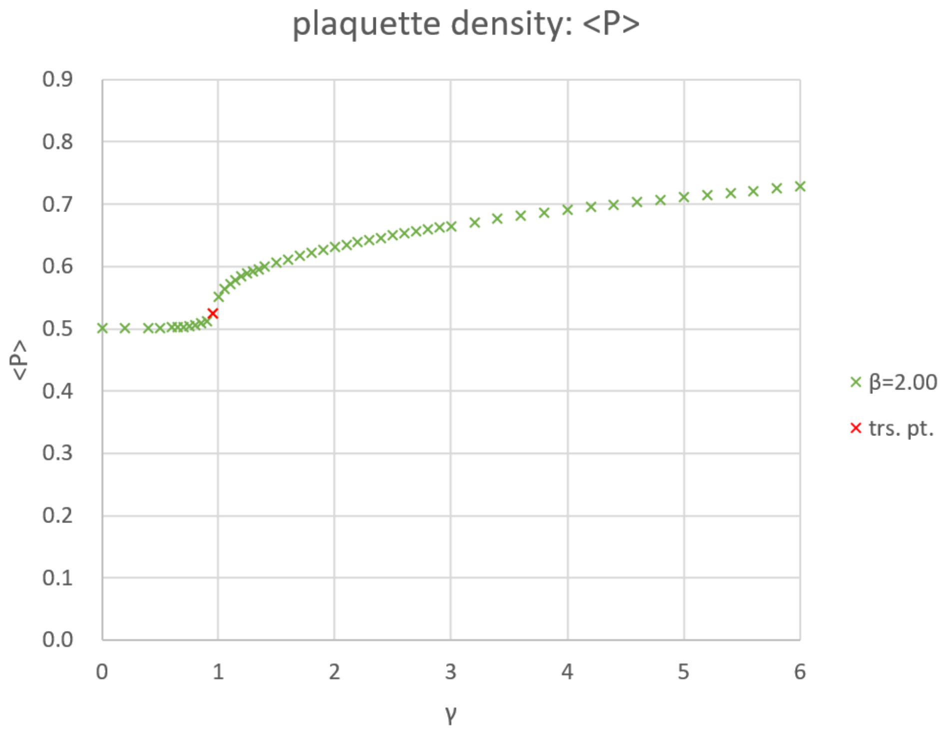
- $F_{\rm red}[\boldsymbol{n}; U]$ is minimized by using the iterative method with over-relaxation.
- For searching the global optimal configuration, 10 trials are taken per point.

The above simulations were performed for 17 × 52=884 sets of parameter points (β, γ).

■ plaquette density <P>

$$P = \frac{1}{6V}\sum_{x,\mu<\nu} \mathrm{tr}\left(U_{x,\mu}U_{x+\mu,\nu}U^\dagger_{x+\nu,\mu}U^\dagger_{x,\nu}\right)$$





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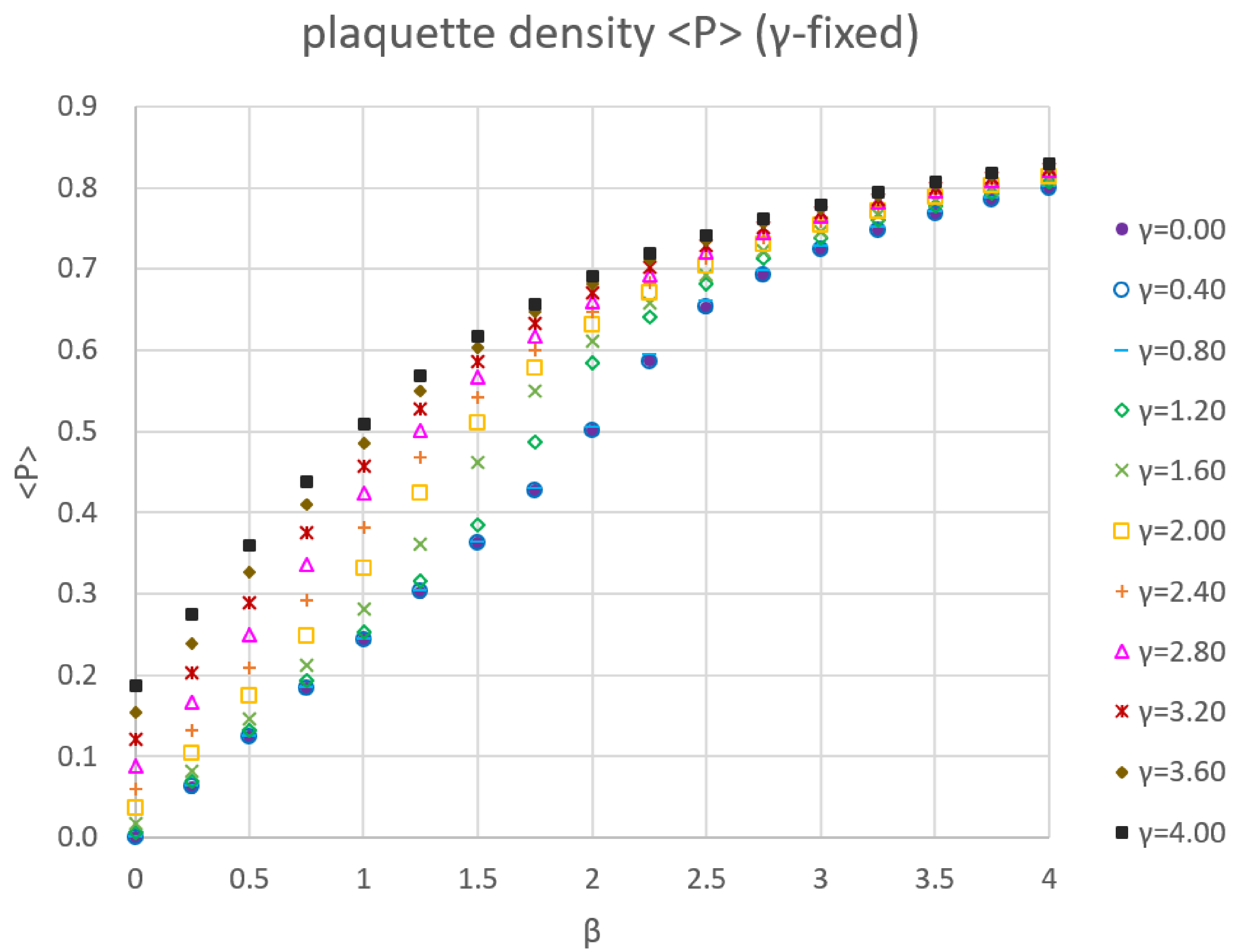
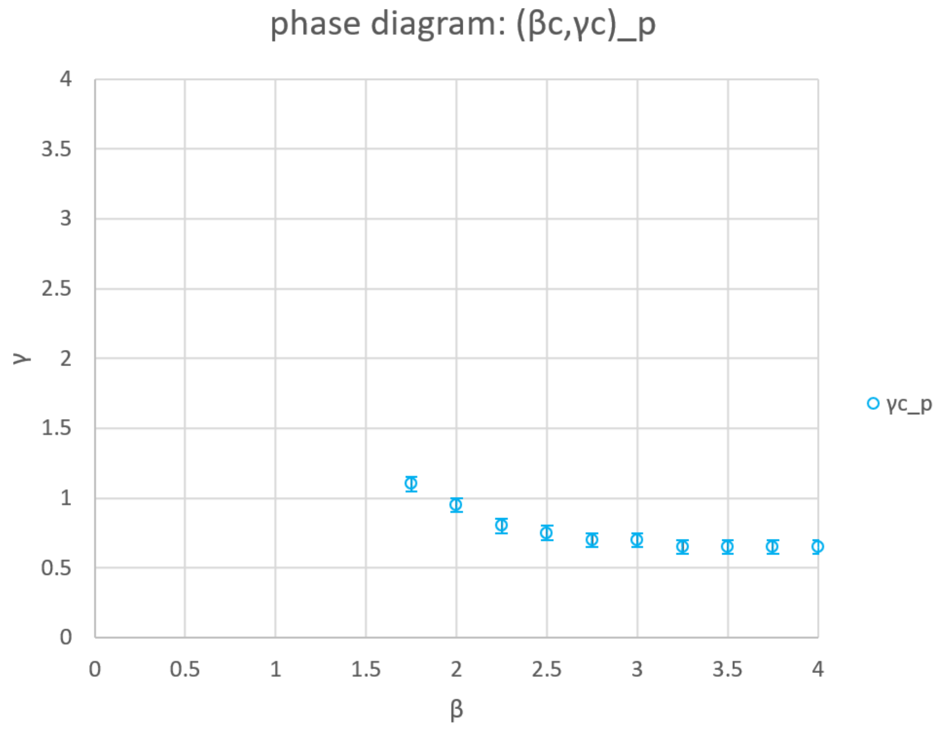
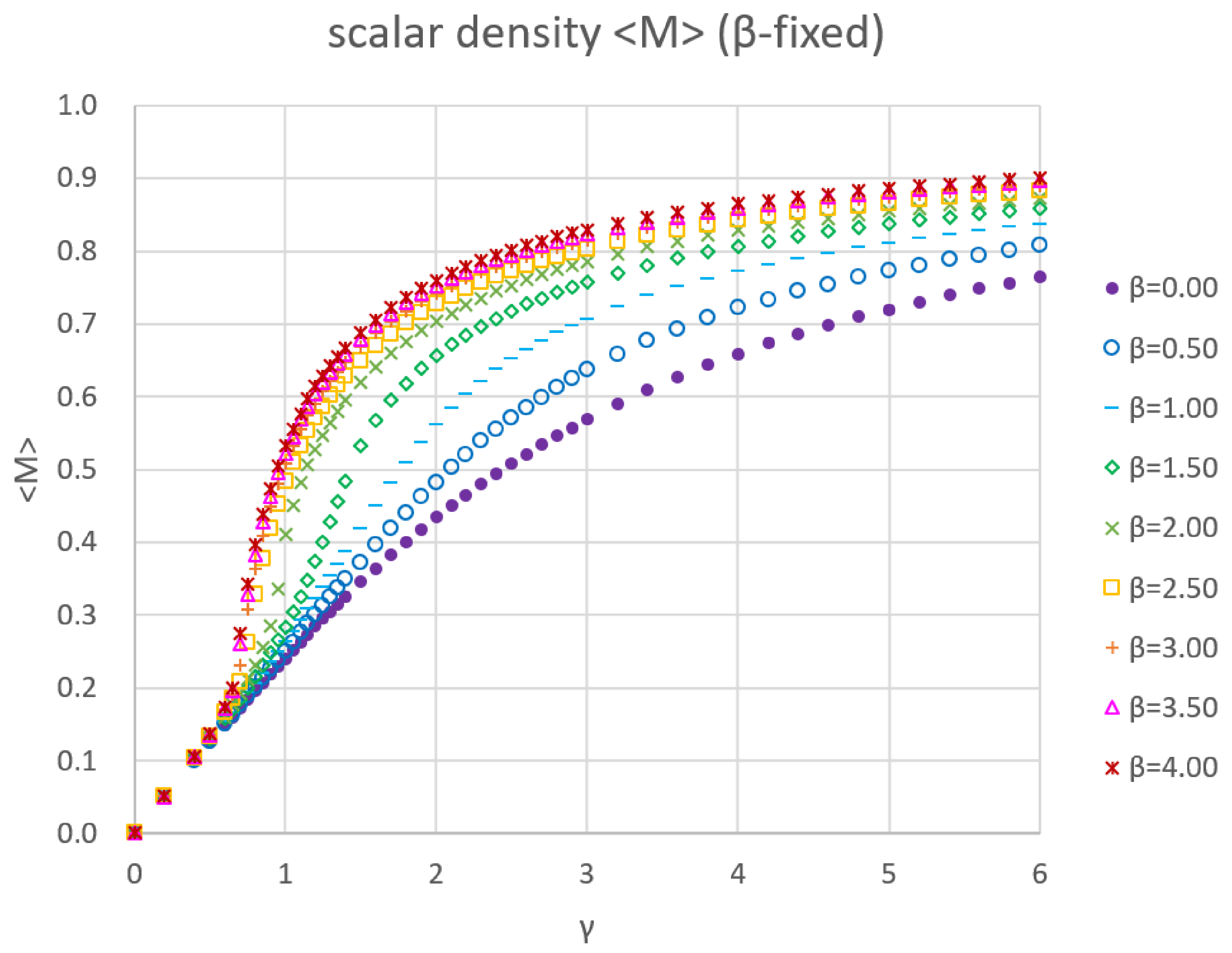
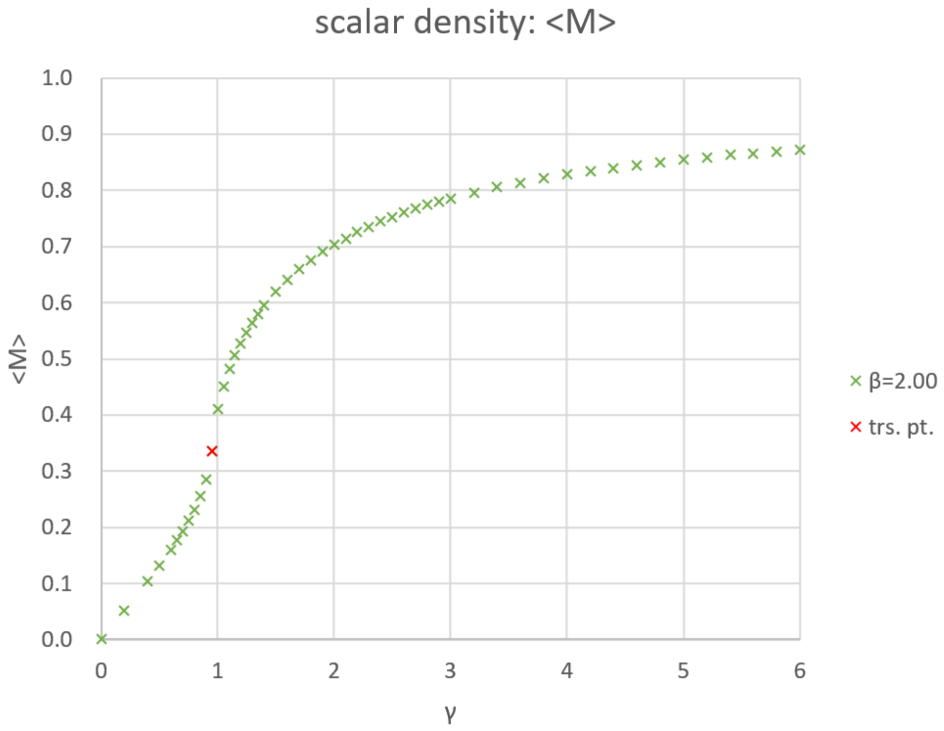




■ scalar density <M>

$$M = \frac{1}{4V}\sum_{x,\mu}\mathrm{tr}\left(\hat{\Theta}_x^\dagger U_{x,\mu}\hat{\Theta}_{x+\mu}\right)$$





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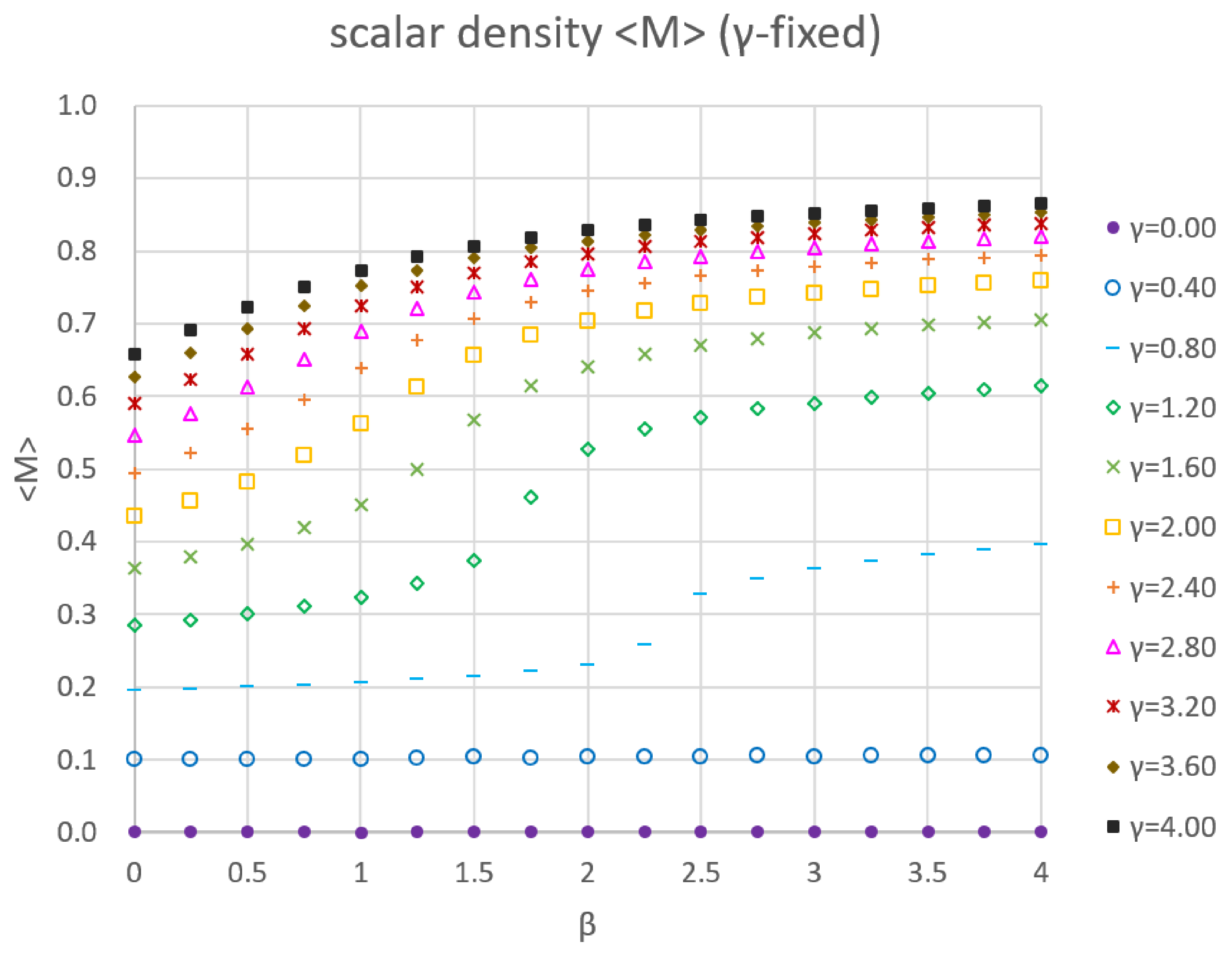
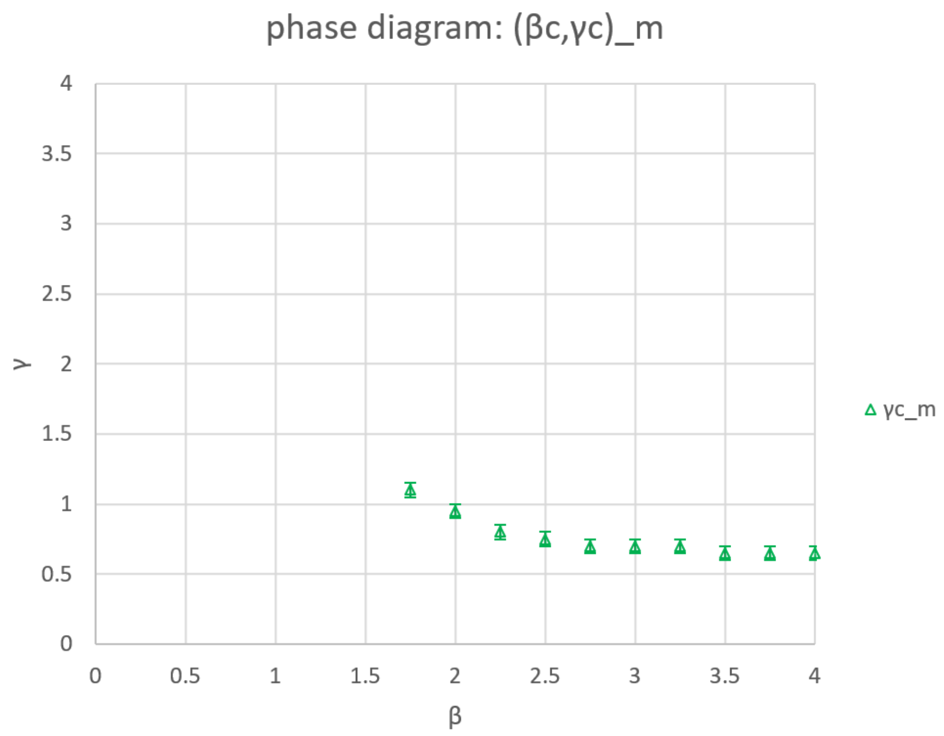
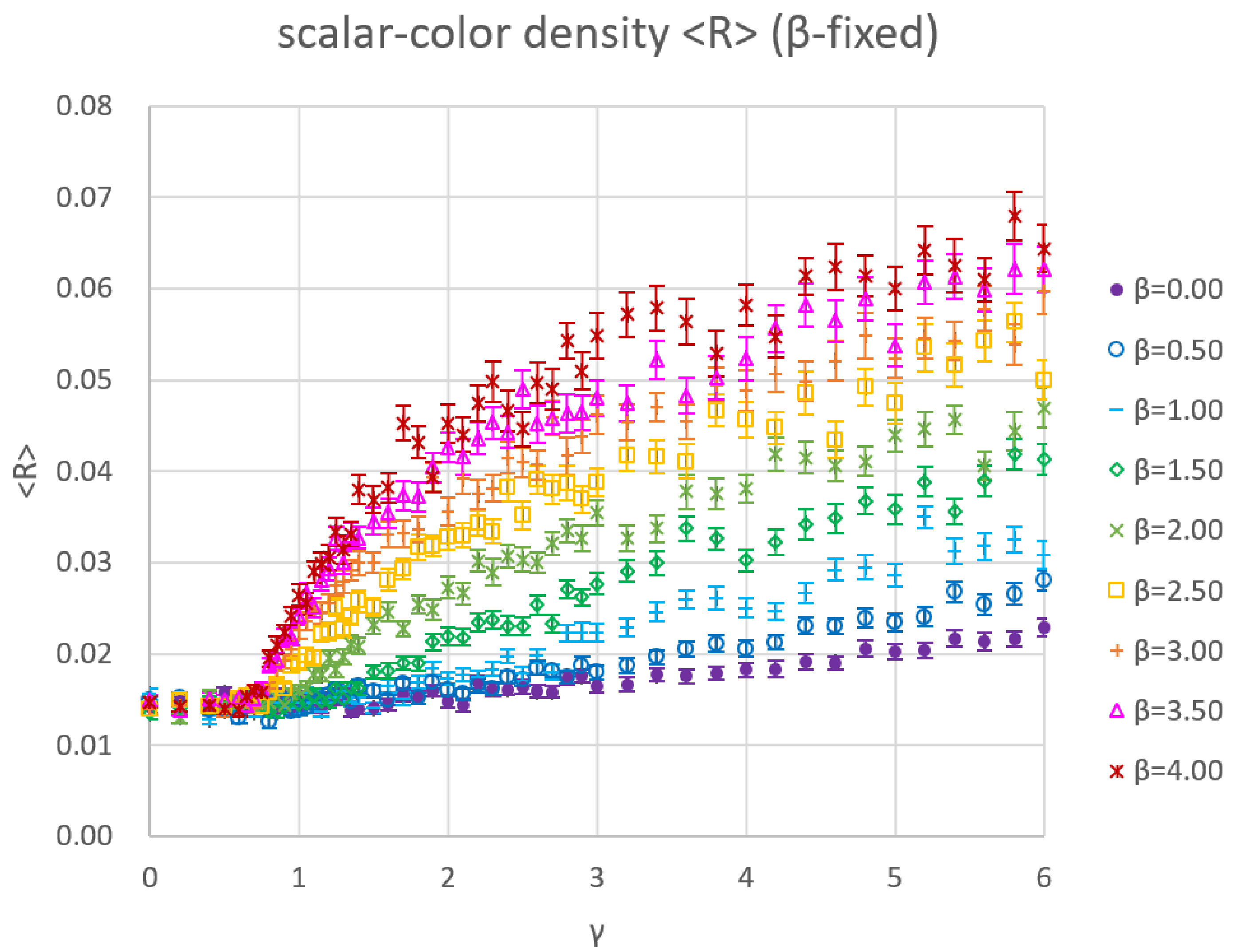




■ scalar-color density <R>

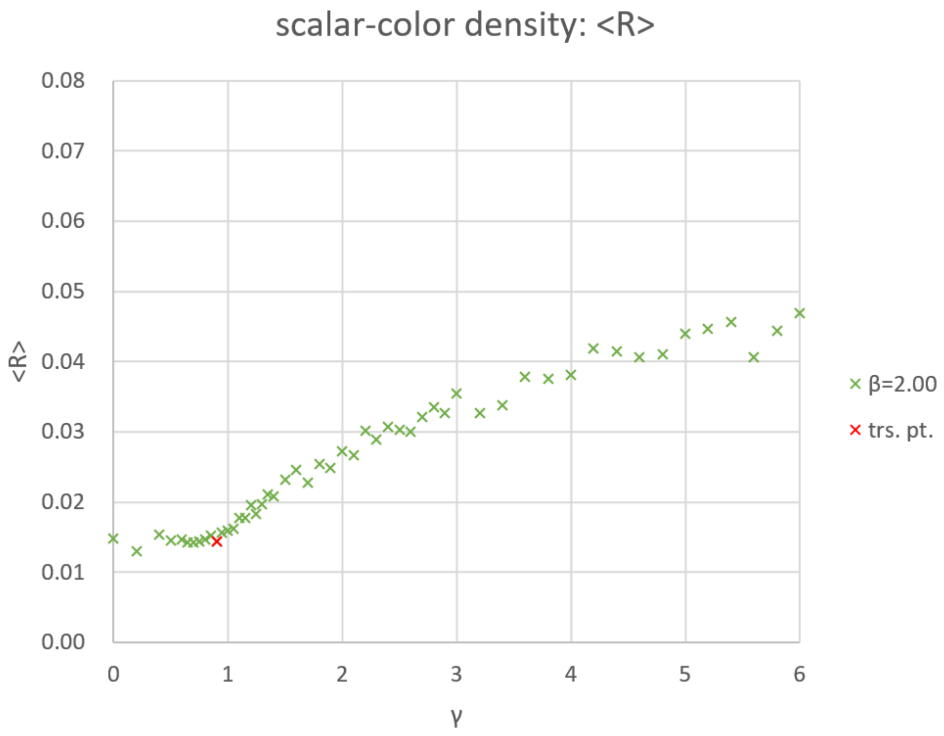
$$R = \left| \frac{1}{V} \sum_x \hat{\Theta}_x^\dagger \boldsymbol{n}_x \hat{\Theta}_x \right| := \sqrt{\frac{1}{2} \mathrm{tr} \left[\left(\frac{1}{V} \sum_x \hat{\Theta}_x^\dagger \boldsymbol{n}_x \hat{\Theta}_x \right)^\dagger \left(\frac{1}{V} \sum_x \hat{\Theta}_x^\dagger \boldsymbol{n}_x \hat{\Theta}_x \right) \right]}$$



$$R_x := \hat{\Theta}_x^\dagger \boldsymbol{n}_x \hat{\Theta}_x \qquad (R_x \mapsto R'_x = \Omega'^\dagger R_x \Omega')$$

$$\bar{R} := \frac{1}{V} \sum_x R_x \qquad (\bar{R} \mapsto \bar{R}' = \Omega'^\dagger \bar{R} \Omega')$$

$$R := |\bar{R}| = \sqrt{\mathrm{tr}(\bar{R}^\dagger \bar{R})/2} \qquad (R \mapsto R' = R)$$



■ scalar-color density <R>

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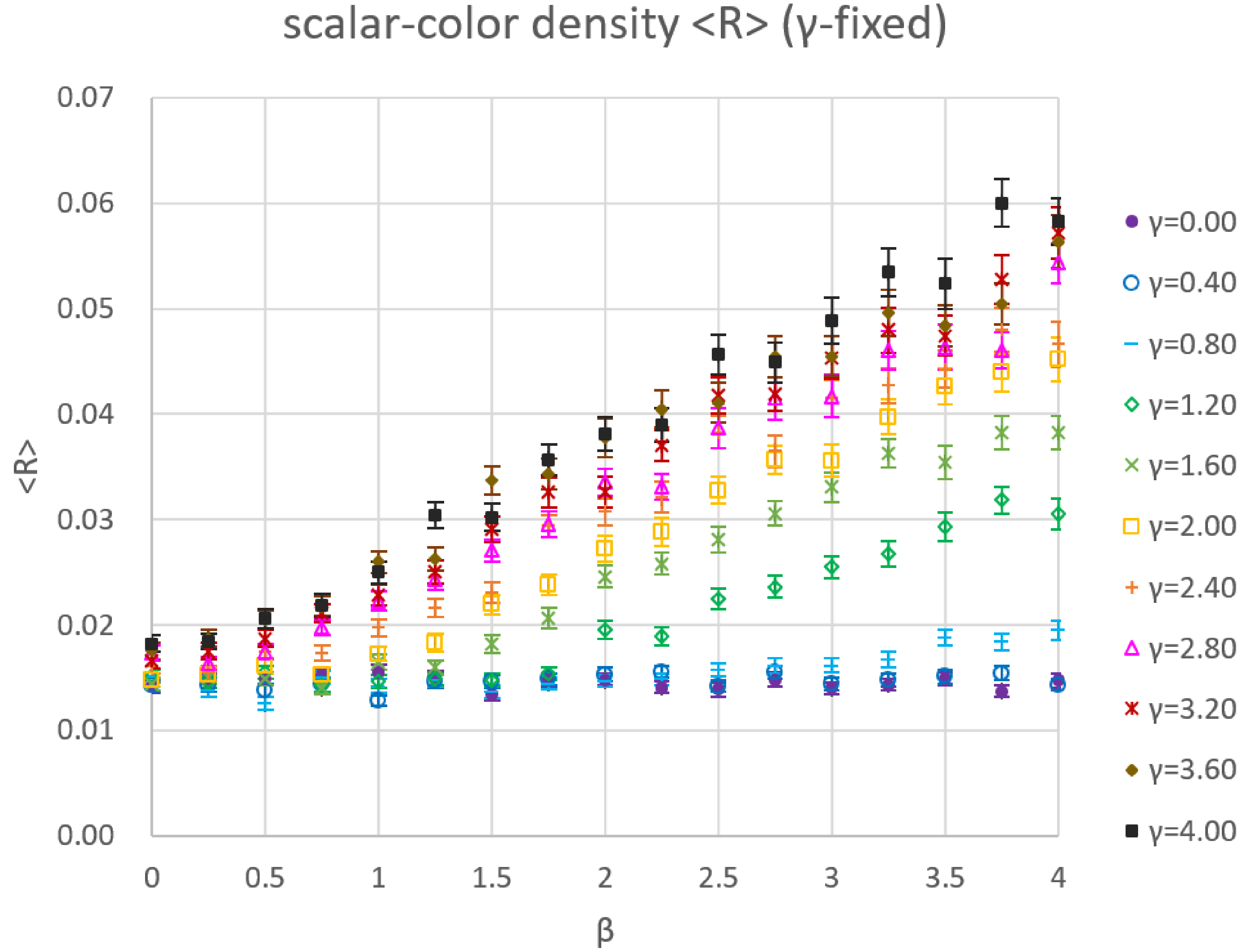
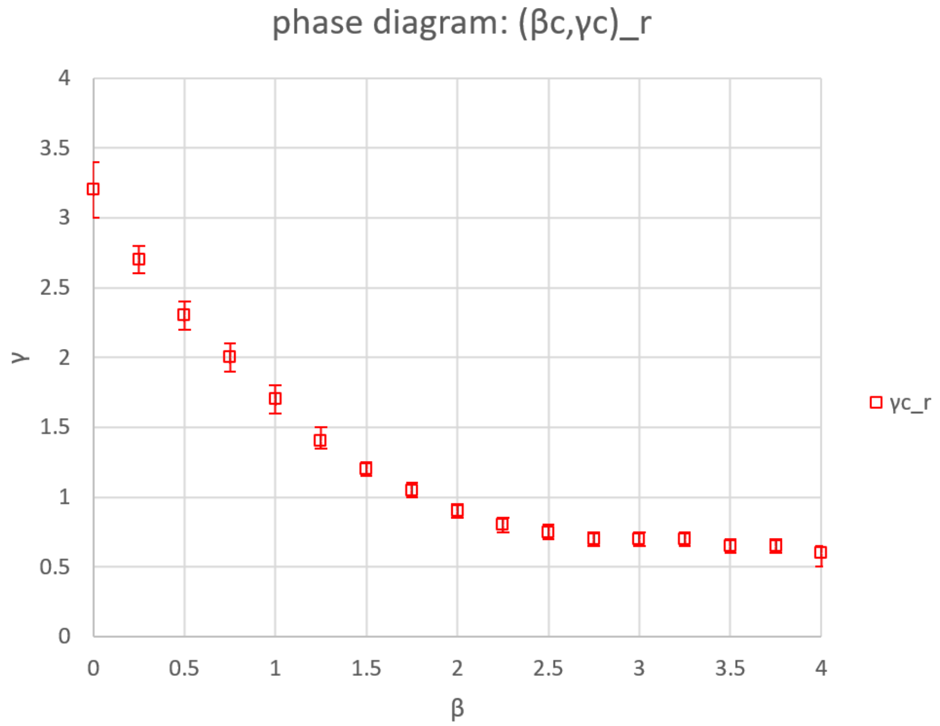
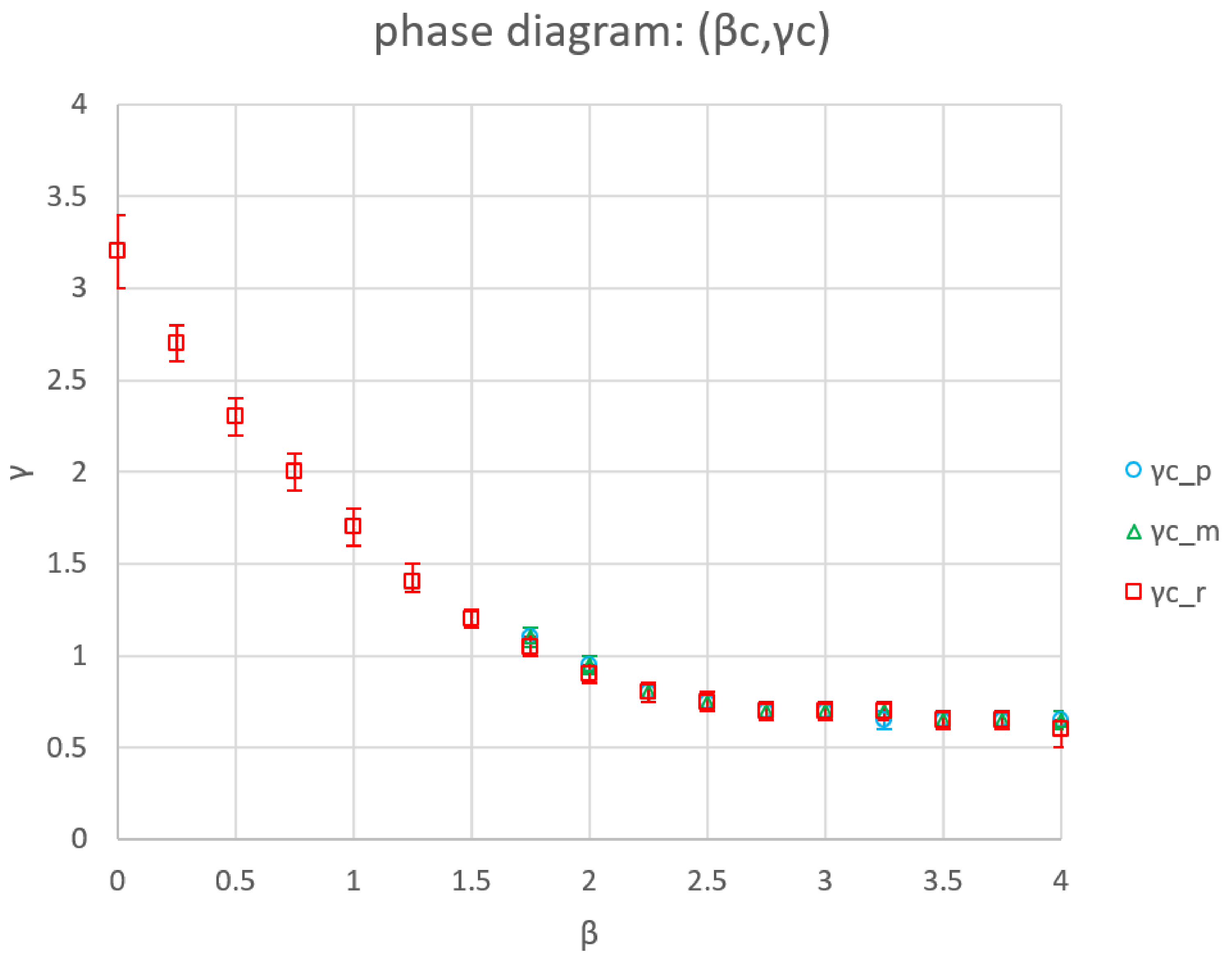




■ Phase diagram



4. Conclusion and outlook

We studied the new type of operator in the SU(2) LGST with fundamental scalar field.

- Combining the scalar field $\hat{\Theta}_x$ and newly introduced "color-direction field" $\boldsymbol{n}_x$ (representing the gauge DOF) as the composite operator, we found the complete and gauge-independent separation between the confinement phase and the Higgs phase.
- We performed the gauge-fixing-free numerical simulations and checked that there is a new transition line which overlaps with the known thermodynamical transition line in the weak gauge coupling, and divides a single confinement-Higgs phase into the two phases in the weak gauge coupling.

Outlook:

- Physical meaning and implications of the new transition
  (e.g. quark-hadron continuity, Schäfer and Wilczek, 1999, $\cdots$)
- Relationship with the spontaneous breaking of the custodial $\mathrm{SU(2)}'_{\mathrm{global}}$ symmetry, Nambu-Goldstone theorem (vs. Greensite and Matsuyama, 2020 ?)
- Improvement of the accuracy for the numerical simulation
- Extension to the case of LGST with SU(N) gauge group (N≥3)