New gauge-independent transition separating confinement-Higgs phase in the lattice gauge-fundamental scalar model

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1. Introduction

We consider SU(2) LGST (with fundamental scalar)

Action of SU(2) LGST (with fund. scalar):

\[
S[U, \hat{\Theta}] = \frac{\beta}{2} \sum_{x, \mu > \nu} \text{Re} \text{tr} \left( 1 - U_{x,\mu} U_{x+\nu,\mu} U_{x,\nu} U_{x+\nu,\mu}^\dagger \right) + \frac{\gamma}{2} \sum_{x, \mu} \text{Re} \text{tr}(1 - \hat{\Theta}_{x,\mu} \hat{\Theta}_{x+\mu}^\dagger)
\]

- \( U_{x,\mu} \in \text{SU}(2) \): link variables, \( \hat{\Theta}_x \in \text{SU}(2) \): (normalized) scalar fields
- \( \beta \): gauge coupling, \( \gamma \): scalar coupling

\( \hat{\Theta}_x \) transforms as the fundamental representation of the gauge group SU(2).

This model has the \( \text{SU}(2)_{\text{local}} \times \text{SU}(2)'_{\text{global}} \) symmetry:

\[
U_{x,\mu} \mapsto U'_{x,\mu} = \Omega_{x} U_{x,\mu} \Omega_{x+\mu}^\dagger, \quad \hat{\Theta}_x \mapsto \hat{\Theta}'_x = \Omega_{x} \hat{\Theta}_x \Omega'
\]

where \( \Omega_x \in \text{SU}(2)_{\text{local}}, \Omega' \in \text{SU}(2)'_{\text{global}} \).
Motivation

- In case of fundamental scalar field \( \rightarrow \) This talk
  Confinement \( (\beta \geq 0, \gamma \ll 1) \) and Higgs \( (\beta \gg 1, \gamma_c \leq \gamma < \infty) \) regions are subregions of an analytically continued single phase. The transition line starts from \( (\beta, \gamma) = (\infty, \gamma_c) \) does not reach “analytic region” which connect these subregions.


- In case of adjoint scalar field \( \rightarrow \) The next talk by Shibata
  Confinement and Higgs regions are completely separated into the two different phases by the continuous transition line. The transition line has two endpoints, \( (\beta, \gamma) = (\infty, \gamma_c) \) and \( (\beta, \gamma) = (\beta_c, \infty) \).

Motivation and Results

Re-examine the 4D SU(2) lattice gauge scalar theory (LGST) with fund. scalar field,

- We found the gauge-invariant composite operator of the original scalar field and the new “color-direction field”, which enables to separate the confinement phase and the Higgs phase completely and gauge-independently.

- We perform the numerical simulations for this model without any gauge fixing, and found a new transition line:
  - in the weak gauge coupling, it agrees with the conventional thermodynamical transition line.
  - in the strong gauge coupling, it divides the single confinement-Higgs phase into two separate phases, confinement and the Higgs.
2. New transition line and Color-direction field

- Newly found transition line
  - As the result of the numerical simulations for the 4D SU(2) LGST with fund. scalar, we found a new transition line which separates confinement and Higgs regions completely, without any gauge fixing. (Red line in the right figure)
  - This transition line is obtained in the gauge-independent way by introducing the new composite operator of the original scalar field and the new “color-direction field”, based on the gauge-covariant decomposition of the gauge field due to Cho-Duan-Ge-Shabanov and Faddeev-Niemi.
gauge-covariant decomposition of the gauge field

$U_{x,\mu} \in \text{SU}(2)$ is gauge-covariantly decomposable:  

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu} \quad (X_{x,\mu}, V_{x,\mu} \in \text{SU}(2))$$

(We required the transformations,  

$$X_{x,\mu} \mapsto X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x^\dagger, \quad V_{x,\mu} \mapsto V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_x^\dagger$$

This decomposition is given uniquely by solving the defining equations for the “color-direction field” $n_x \in su(2) - u(1)$ with a unit :

$$D_\mu [V] n_x := V_{x,\mu} n_{x+\mu} - n_x V_{x,\mu} = 0, \quad \text{tr} (n_x X_{x,\mu}) = 0$$

(We required the transformation,  

$$n_x \mapsto n'_x = \Omega_x n_x \Omega_x^\dagger$$

For a given set of gauge fields $\{U_{x,\mu}\}$, a set of color-direction fields $\{n_x\}$ is determined as the configuration minimizing the reduction functional:

$$F_{\text{red}}[n; U] = \sum_{x,\mu} \frac{1}{2} \text{tr} \left\{ (D_\mu [U] n_x)^\dagger (D_\mu [U] n_x) \right\} = \sum_{x,\mu} \text{tr} \left( 1 - n_x U_{x,\mu} n_{x+\mu} U_{x,\mu}^\dagger \right)$$

$F_{\text{red}}[n; U]$ has the same form as the Higgs action of SU(2) LGST (with adj. scalar) (minimization of $F_{\text{red}}[n; U]$ extracts the DOF of $\{n_x\}$ from the gauge fields $\{U_{x,\mu}\}$)

construction of the scalar-color density \(<R>\)

- We required that \(\hat{\Theta}_x \in SU(2)\) and \(n_x \in su(2) - u(1)\) transform as

\[
\hat{\Theta}_x \mapsto \hat{\Theta}_x' = \Omega_x \hat{\Theta}_x \Omega_x', \quad n_x \mapsto n_x' = \Omega_x n_x \Omega_x^\dagger
\]

where \(\Omega_x \in SU(2)_{\text{local}}, \Omega' \in SU(2)_{\text{global}}\).

- Then it is possible to find a local quantity \(R_x\) which has the global covariance:

\[
R_x := \hat{\Theta}_x^\dagger n_x \hat{\Theta}_x, \quad R_x \mapsto R_x' = \Omega'^\dagger R_x \Omega'
\]

- The spacetime average of \(R_x\) also has the global covariance:

\[
\bar{R} := \frac{1}{V} \sum_x R_x, \quad \bar{R} \mapsto \bar{R}' = \frac{1}{V} \sum_x \Omega'^\dagger R_x \Omega' = \Omega'^\dagger \left( \frac{1}{V} \sum_x R_x \right) \Omega' = \Omega'^\dagger \bar{R} \Omega'
\]

- Then there exists a non-trivial gauge invariant defined as

\[
R := |\bar{R}| = \sqrt{\frac{1}{2} \text{tr}(\bar{R}^\dagger \bar{R})}, \quad R \mapsto R' = R
\]

\(R\) also can be defined as the absolute value of the two eigenvalues of \(\bar{R}\).
3. Numerical simulation

- Setting for lattice simulation
  - $8^4$-lattice, pseudo heat bath method, cold start ($U_{x,\mu} = 1$, $\hat{\Theta}_x = 1$)

  For a point of the couplings $(\beta, \gamma)$,
  - 5000 sweeps were discarded for thermalization, before sampling.
  - Per 100 sweeps, a set of gauge and scalar configuration was sampled.
    Total 100 samples were taken per point.

To determine a set of color-direction fields $\{n_x\}$ from a given set of $\{U_{x,\mu}\}$ numerically,
- $F_{\text{red}}[n;U]$ is minimized by using the iterative method with over-relaxation.
- For searching the global optimal configuration, 10 trials are taken per point.

The above simulations were performed for $17 \times 52 = 884$ sets of parameter points $(\beta, \gamma)$. 
plaquette density $<P>$

\[ P = \frac{1}{6V} \sum_{x, \mu < \nu} \text{tr} \left( U_{x, \mu} U_{x+\mu, \nu} U_{x+\nu, \mu}^\dagger U_{x, \nu}^\dagger \right) \]
plaquette density $\langle P \rangle$

\[
P = \frac{1}{6V} \sum_{x, \mu < \nu} \text{tr} \left( U_{x, \mu} U_{x+\mu, \nu} U_{x+\nu, \mu} U_{x, \nu}^\dagger \right)
\]
\[ M = \frac{1}{4V} \sum_{x, \mu} \text{tr} \left( \hat{\Theta}^\dagger_x U_{x, \mu} \hat{\Theta}_{x+\mu} \right) \]
scalar density $\langle M \rangle$

$$M = \frac{1}{4V} \sum_{x, \mu} \text{tr} \left( \hat{\Theta}^\dagger_x U_{x, \mu} \hat{\Theta}_{x+\mu} \right)$$
\[ R = \left| \frac{1}{V} \sum_x \hat{\Theta}_x n_x \hat{\Theta}_x \right| := \sqrt{\frac{1}{2} \text{tr} \left[ \left( \frac{1}{V} \sum_x \hat{\Theta}_x n_x \hat{\Theta}_x \right) \left( \frac{1}{V} \sum_x \hat{\Theta}_x n_x \hat{\Theta}_x \right)^\dagger \right]} \]

\[ R_x := \hat{\Theta}_x n_x \hat{\Theta}_x \quad (R_x \mapsto R'_x = \Omega_{\beta}^t R_x \Omega_{\beta}'^t) \]

\[ \bar{R} := \frac{1}{V} \sum_x R_x \quad (\bar{R} \mapsto \bar{R}' = \Omega_{\beta}^t \bar{R} \Omega_{\beta}'^t) \]

\[ R := |\bar{R}| = \sqrt{\text{tr}(\bar{R}^t \bar{R})/2} \quad (R \mapsto R' = R) \]
scalar-color density $\langle R \rangle$

$$R = \left| \frac{1}{V} \sum_x \hat{\Theta}_x^\dagger n_x \hat{\Theta}_x \right| := \sqrt{\frac{1}{2} \text{tr} \left[ \left( \frac{1}{V} \sum_x \hat{\Theta}_x^\dagger n_x \hat{\Theta}_x \right) \left( \frac{1}{V} \sum_x \hat{\Theta}_x^\dagger n_x \hat{\Theta}_x \right)^\dagger \right]}$$

scalar-color density $\langle R \rangle$ (γ-fixed)
Phase diagram

phase diagram: $(\beta_c, \gamma_c)$

- $\gamma_{c_p}$
- $\gamma_{c_m}$
- $\gamma_{c_r}$
4. Conclusion and outlook

We studied the new type of operator in the SU(2) LGST with fundamental scalar field.

- Combining the scalar field $\hat{\Theta}_x$ and newly introduced “color-direction field” $n_x$ (representing the gauge DOF) as the composite operator, we found the complete and gauge-independent separation between the confinement phase and the Higgs phase.

- We performed the gauge-fixing-free numerical simulations and checked that there is a new transition line which overlaps with the known thermodynamical transition line in the weak gauge coupling, and divides a single confinement-Higgs phase into the two phases in the weak gauge coupling.

Outlook:

- Physical meaning and implications of the new transition (e.g. quark-hadron continuity, Schäfer and Wilczek, 1999, ⋯)

- Relationship with the spontaneous breaking of the custodial $SU(2)_{\text{global}}$ symmetry, Nambu-Goldstone theorem (vs. Greensite and Matsuyama, 2020)

- Improvement of the accuracy for the numerical simulation

- Extension to the case of LGST with $SU(N)$ gauge group ($N \geq 3$)