Symmetric Mass Generation in Lattice Gauge Theory

Simon Catterall (Syracuse)
Goksu Can Toga (Syracuse)
Nouman Butt (UIUC)

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Symmetric Mass Generation SMG

Typically mass associated with symmetry breaking

- Higgs mechanism in Standard Model
- Chiral symmetry breaking in QCD
- Gross Neveu model ...

Typically broken symmetries are chiral ...

Is this necessarily always the case ?

Counter examples

- 4 flavors of reduced staggered fermion with four fermion (4f) interactions. [Catterall et al. 1810.06117]
- Boundary fermions in Fidkowski-Kitaev model in (0+1) D (4f interactions) [Kitaev et al. 1008.4138]
- 3-4-5-0 model of (1+1) chiral fermions (6f interations) [Wang et al. 2202.12355]

Four reduced staggered fermions in 4D

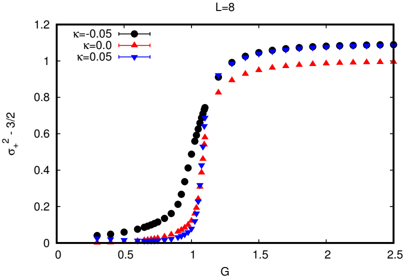
with D. Schaich [1810.06117]

$$S = \sum_{x,\mu} \eta_\mu(x)\mathrm{Tr}(\chi(x)\Delta_\mu\chi(x) + \sum_x G\mathrm{Tr}(\phi_+\chi\chi) + \mathrm{Tr}(\phi_+^2) + \kappa\mathrm{Tr}(\phi_+\Box\phi_+)$$

where χ^a in fundamental and ϕ_+^{ab} in **3** of $SO(4)$ ($\chi \equiv \overline{\chi}_+$ and χ_-)
Key global Z_4 symmetry:

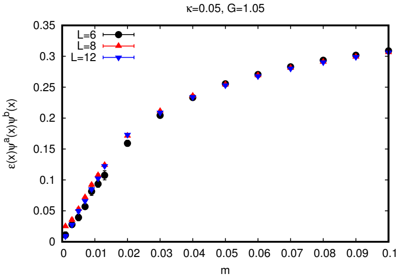
$$\chi(x) \to i\epsilon(x)\chi(x) \quad \phi_+(x) \to -\phi_+(x)$$

$$G^2 < \epsilon^{abcd}\chi^a\chi^b\chi^c\chi^d >$$

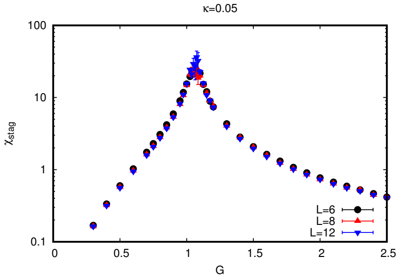


SMG

$B = \epsilon(x)\chi^a\chi^b$



Susceptibility χ_B



$< B > \neq 0 \rightarrow$ SSB $\rightarrow$ fermion mass but breaks Z_4

No volume dependence in $< B >$ or ϕ_+ as $m \rightarrow 0 \rightarrow$ no SSB

Peak χ_B shows transition near $\kappa = 0.05$ and $G = 1.05$ – observe correlation length ξ grows rapidly with volume ..

Conclusions

Likely continuous phase transition between massless and massive phases not associated with symmetry breaking/order parameter.
New fixed point for interacting 4d fermions ...!
Continuum interpretation ?

Topological defects ? - see Catterall et al. 1708.06715

Lots of previous lattice studies of Higgs-Yukawa models – all saw symmetry breaking. What new element distinguishes this model ?

ans: anomalies !

Anomaly constraints

't Hooft anomalies

Obstruction to gauging global symmetries

SMG $\equiv$ trivial gapped theory in I.R

Since anomalies are RG invariant and gapped theory can have no anomalies

$\rightarrow$ all 't Hooft anomalies must cancel in U.V for SMG

(Necessary but not sufficient condition)

What does this imply for staggered fermion model - Z_4 symmetry ?

Can add Z_4 link gauge fields to allow for local Z_4. But fermion measure $\int d\chi^1 \ldots d\chi^N$ locally invariant only if $N = 4k$

Thus to cancel 't Hooft anomaly for Z_4 requires multiples of four reduced fields or two regular staggered fields.

More on anomalies of staggered fermions

In absence of four fermi term the classical symmetry is $U(1)$ corresponding to $\chi \rightarrow e^{i\alpha\epsilon(x)}\chi$

However this is broken to Z_4 by quantum effects

Proof: map staggered fermions to Kähler-Dirac fermions $\rightarrow$ can be naturally formulated on general random triangulations of curved space. Can show that

$$Z \rightarrow e^{4\alpha i} Z$$

on a lattice with topology of sphere.

Exact lattice fermion anomaly
Only a Z_4 subgroup of staggered fermions remains non-anomalous after coupling to gravity

also: suggests four fermion interactions are not crucial for SMG ... (Catterall et al 1806.07845, 2101.01026, 2209.03828)

Why might SMG be important ?

Mirror models

One strategy to obtain chiral lattice gauge theories starts from a vector-like theory and uses eg strong Yukawa interactions to gap out say R-handed states.
This strategy failed in past because scalar fields pick up vevs associated with χ-symmetry breaking $\rightarrow$ yields Dirac mass terms

SMG offers prospect of gapping unwanted states without generating such mass terms
Indeed 3-4-5-0 model in Hamiltonian formalism gives an explicit example

Discrete 't Hooft anomaly structure of staggered fermions may also allow construction of mirror models for Z_4 symmetry.
Remarkably naive continuum limit $\rightarrow$ certain chiral theories

SMG without large Yukawa couplings

- G is perturbatively irrelevant. Previous SMG result depends on new non-perturbative physics/fixed pts.
- Can we use confinement to generate four fermion condensate for small G ?

Replace χ by 2x2 matrix ψ that transforms under $SU(2) \times SU(2)$

$$\psi \to G\psi H^\dagger$$

ψ satisfies a reality condition: $\psi^\dagger = \sigma_2 \psi \sigma_2 \to \mathrm{Tr}(\psi^\dagger \psi) = 0$
Gauge the $SU(2) \times SU(2)$ symmetry

Expect confinement. Does Z_4 remain unbroken with a four fermion condensate being formed instead of a bilinear ?

Simulations

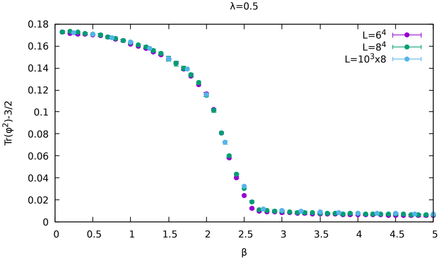
$\int D\chi e^{\chi^T D_U \chi} = \mathrm{Pf}(D_U)$ where D_U is complex, antisymmetric matrix.
For $SU(N), N > 2$ possesses a severe sign problem.

But for $SU(2)$ we have $D_U^* = \sigma_2 D_U \sigma_2$. Hence

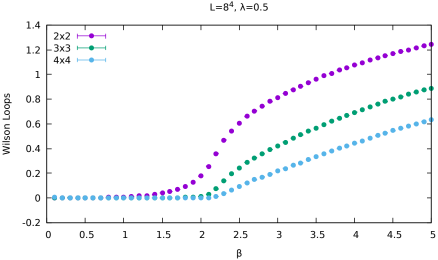
$$D_U \psi = \mu\psi \quad \text{then}$$
$$D_U(\sigma_2\psi) = \mu^*(\sigma_2\psi)$$

With antisymmetry $\rightarrow$ eigenvalues form quartets $(\mu, \mu^*, -\mu, -\mu^*)$ - guarantees that Pfaffian real positive
Can be simulated using RHMC. (Catterall et al. 2111.01001)

Gauge theory with small Yukawa



Four fermi condensate $\mathrm{Tr}[\psi^\dagger\psi\psi^\dagger\psi]$ vs β

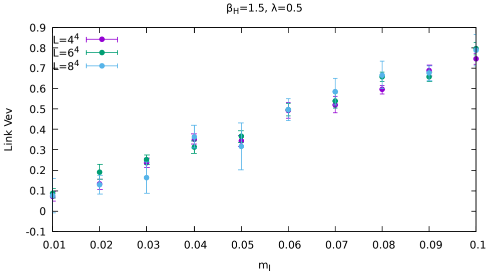


Wilson loops vs β

Other bilinears ..

Continuum fields arise from 16 staggered fields in hypercube
single site bilinear vanishes
but maybe system condenses another bilinear eg
$\sum_\mu \epsilon(x)\xi_\mu(x)\mathrm{Tr}[\psi(x)U_\mu(x)\psi(x+\mu)V_\mu^\dagger(x)]$

Add such a term to action. Breaks shift symmetry
$\psi(x) \to \xi_\mu(x)\psi(x+\mu)$



No sign of SSB as $m_1 \to 0$

Connection to continuum

(Naive) continuum fermion Ψ gotten from

$$\Psi = \sum_b \chi(x+b)\gamma^b$$

where $b = (b_1, \ldots, b_4)$ with $b_i = 0, 1$ span hypercube (spin-taste basis) and $\gamma^b = \gamma_1^{b_1}\gamma_2^{b_2}\cdots\gamma_4^{b_4}$

For reduced field $\chi_- = \frac{1}{2}(1-\epsilon(x))\chi(x)$

Take Euclidean chiral basis for γ:

$$\Psi_- = \begin{pmatrix} 0 & \psi_R \\ \psi_L & 0 \end{pmatrix}$$

To cancel anomaly need 4 fields – $\Psi \to \Psi^a$. Take group to be $SU(4)$.

Replace $\psi_R \to i\sigma_2\psi_L^{*\prime}$

Global symmetries: $SU(4) \times SU(2) \times SU(2)$. Fermions live in $(4, 2, 1) \oplus (\bar{4}, 1, 2) \leftarrow$ Pati-Salam GUT !

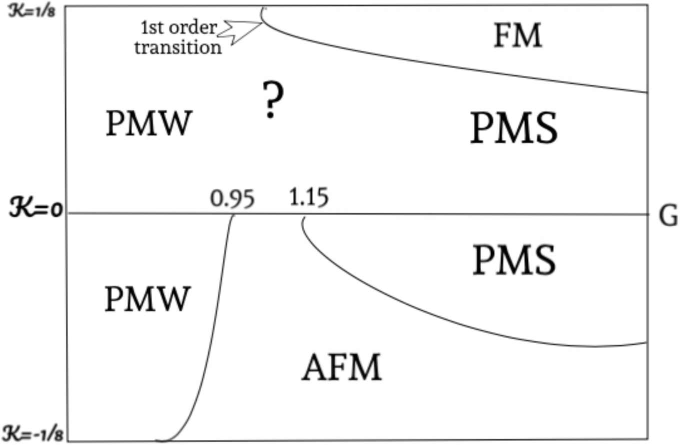
Summary

- Staggered fermions offer opportunities to realize SMG – gapping all states without breaking symmetries.
- Necessary (but not sufficient) condition – all (discrete) 't Hooft anomalies must cancel. Requires multiples of 4 reduced staggered fields. Exact lattice anomalies !
- Example of SMG in LGT that generates a four fermion condensate in its confining regime
- Extend symmetry $SO(4) \rightarrow SU(4)$ such a model can accommodate PS model.
- However $SU(4)$ has a sign problem ....

Thank you !

Backup

Phase diagram $SO(4)$ model:



$U(1)$ anomaly for KD fermions

Map staggered fields to antisymmetric tensors ϕ^p where p labels number of non-zero b's

$$\chi(x+b) \to \phi^p_{b_1 \ldots b_p}$$

Can place tensors on p-simplices in random triangulation.
Action:

$$\overline{\phi}^p \phi^{p-1}$$

invariant under $\phi^p_i \to e^{i\alpha(-1)^p} \phi^p_i$ recognize $[(-1)^p \equiv \epsilon(x)]$
Measure

$$D\overline{\Phi} D\Phi = \prod_{p=0}^{D} \prod_{i}^{N_p} d\phi^p_i d\overline{\phi}^p_i \to e^{2i\alpha \sum_{p=0}^{D} N_p (-1)^p} D\overline{\Phi} D\phi$$

But $\sum_{p=0}^{D} N_p \, (-1)^p = \chi$ - Euler character. Thus Z not invariant on sphere where $\chi = 2$.